Bandpass Sampling (2B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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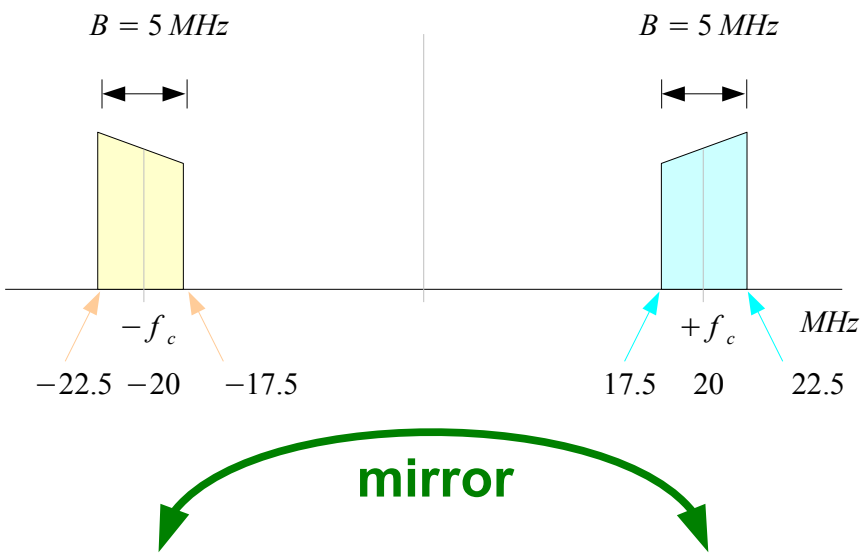
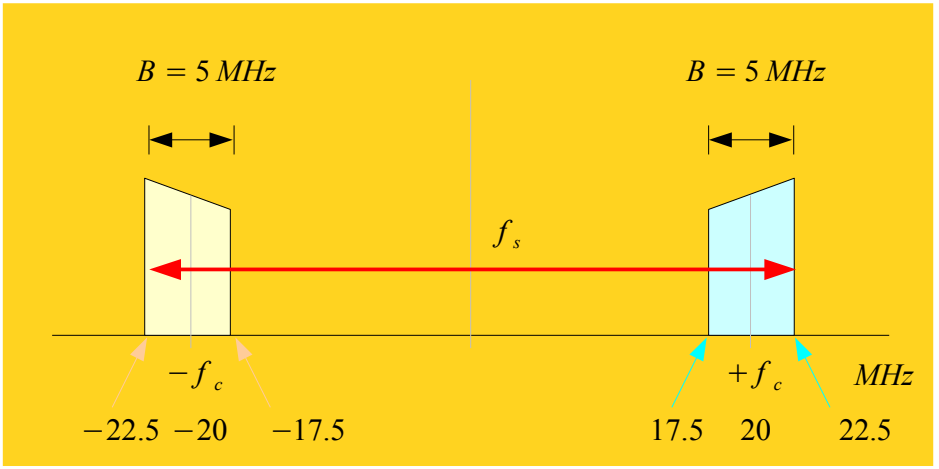
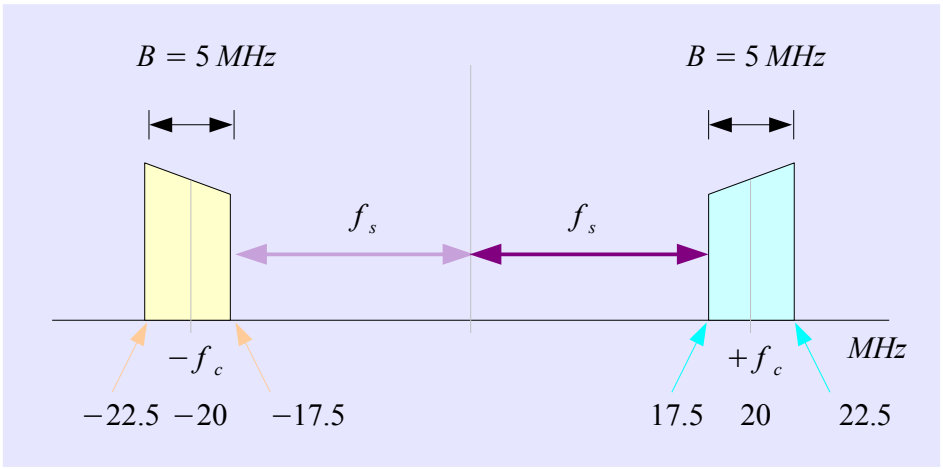
Band-limited Signal



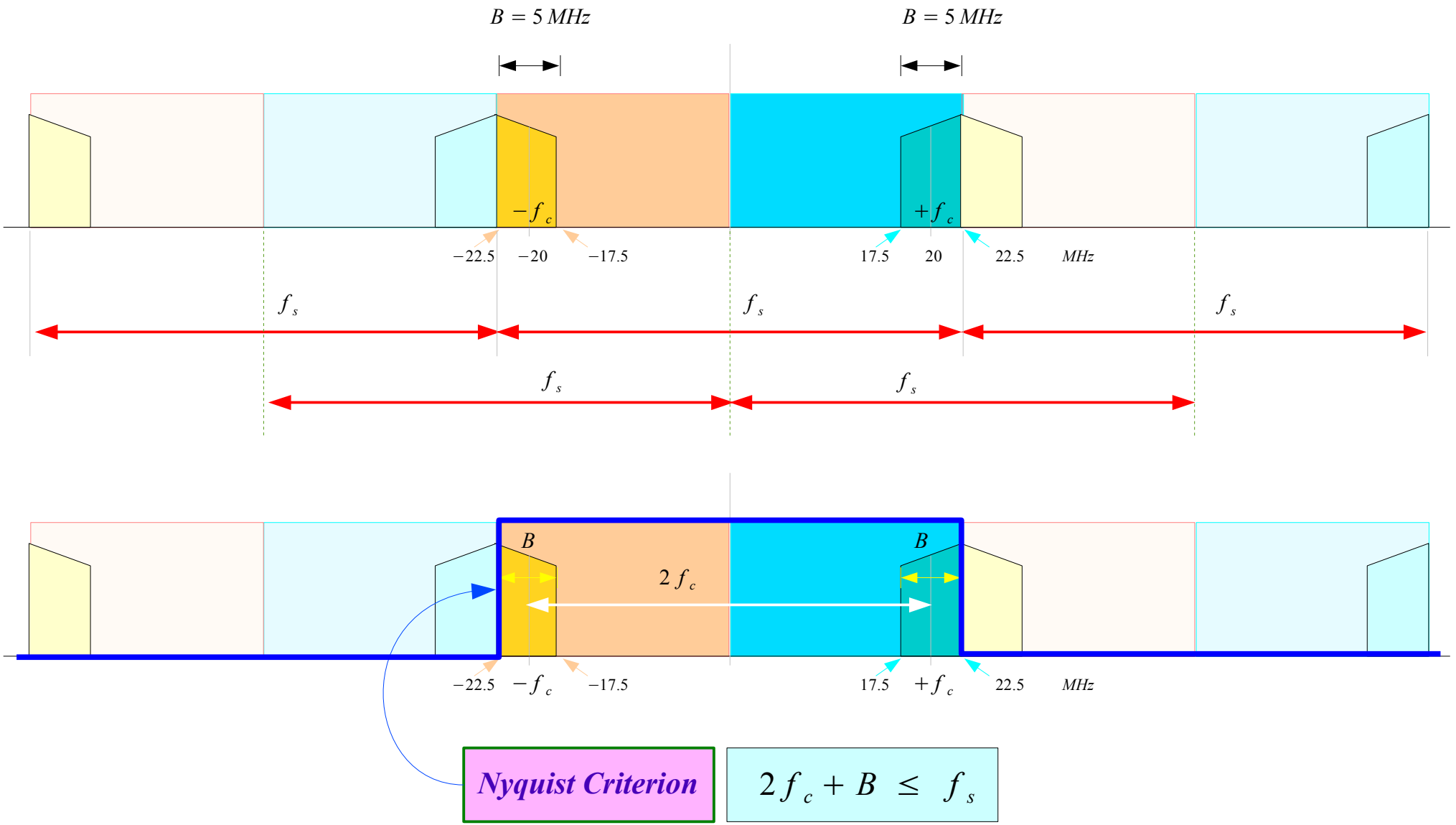


- **Lowpass Sampling**

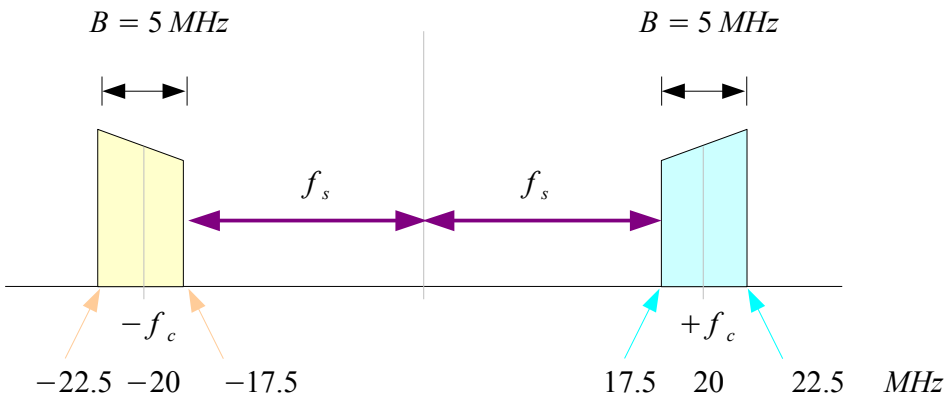
- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

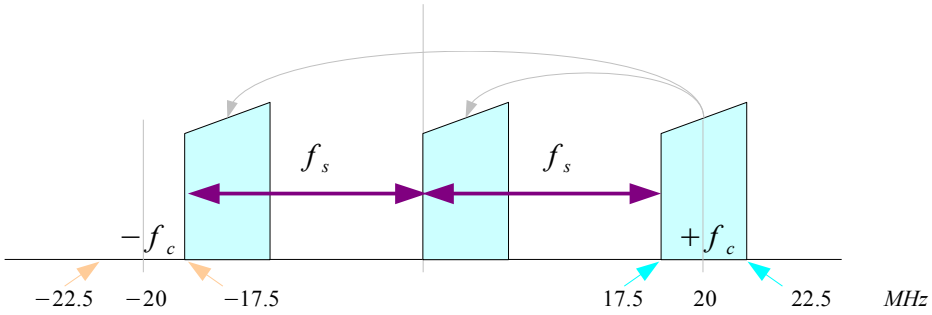


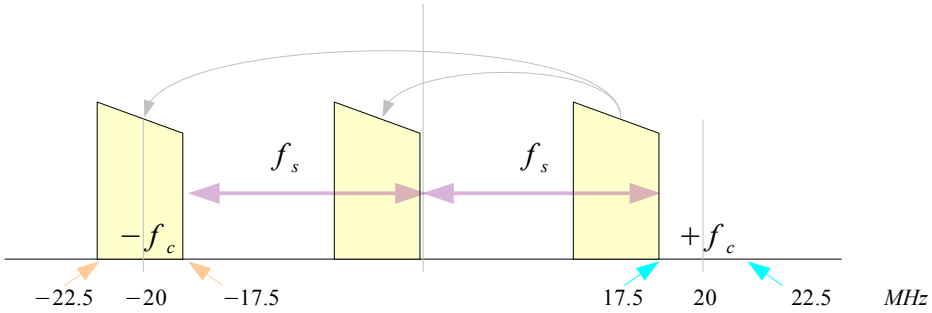
Low-pass Signal Sampling



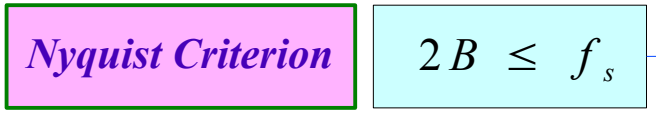
Band-pass Signal Sampling

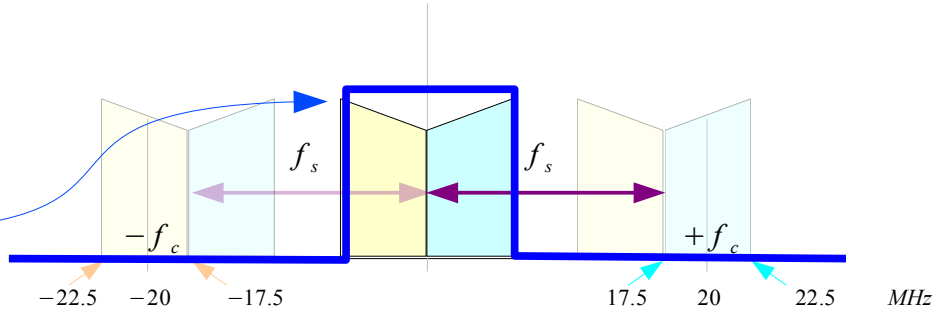






- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**





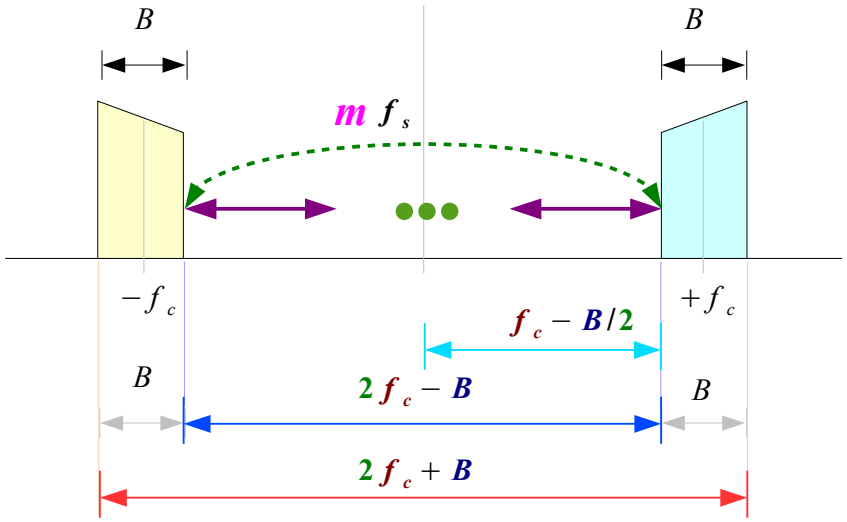
Sampling Frequency f_s (1)

Assume there are m multiples of f_s — *Given an integer m*

$$2f_c - B = m \cdot f_s$$ ⟸ *Max* f_s *condition*

f_s *can be decreased according to the following condition without introducing aliasing problems*

$$2f_c + B = (m+1) \cdot f_s$$ ⟸ *Min* f_s *condition*



Given Band-pass Signal is characterized by

- ***Bandwidth B***
- ***Carrier Frequency*** f_c

$$\frac{2f_c + B}{m+1} \leq f_s \leq \frac{2f_c - B}{m}$$

Sampling Frequency f_s (2)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{(m+1)B} \le \frac{f_s}{B} \le \frac{2f_c - B}{mB}$$

$$\frac{2f_H}{(m+1)B} \le \frac{f_s}{B} \le \frac{2f_L}{mB}$$

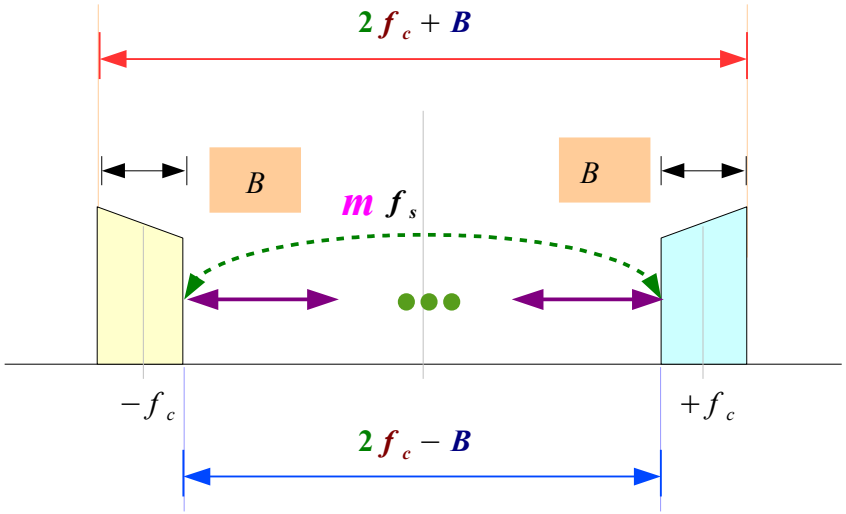
Given Band-pass Signal is characterized by

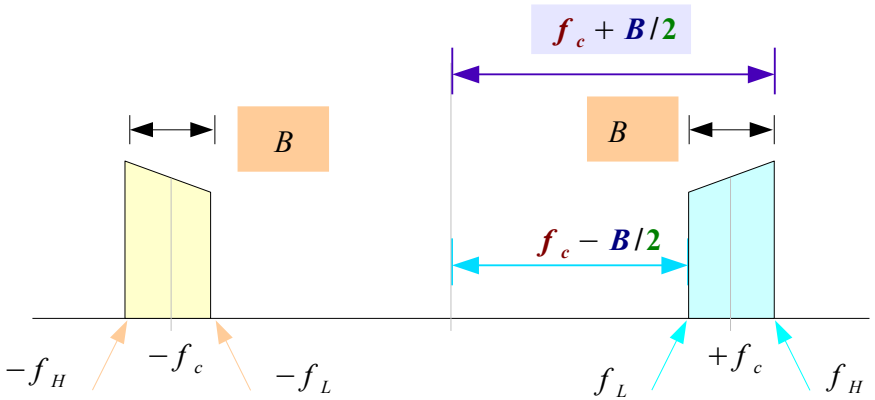
- ***Bandwidth B***
- ***Carrier Frequency f_c***

➡ ***Normalization by B***

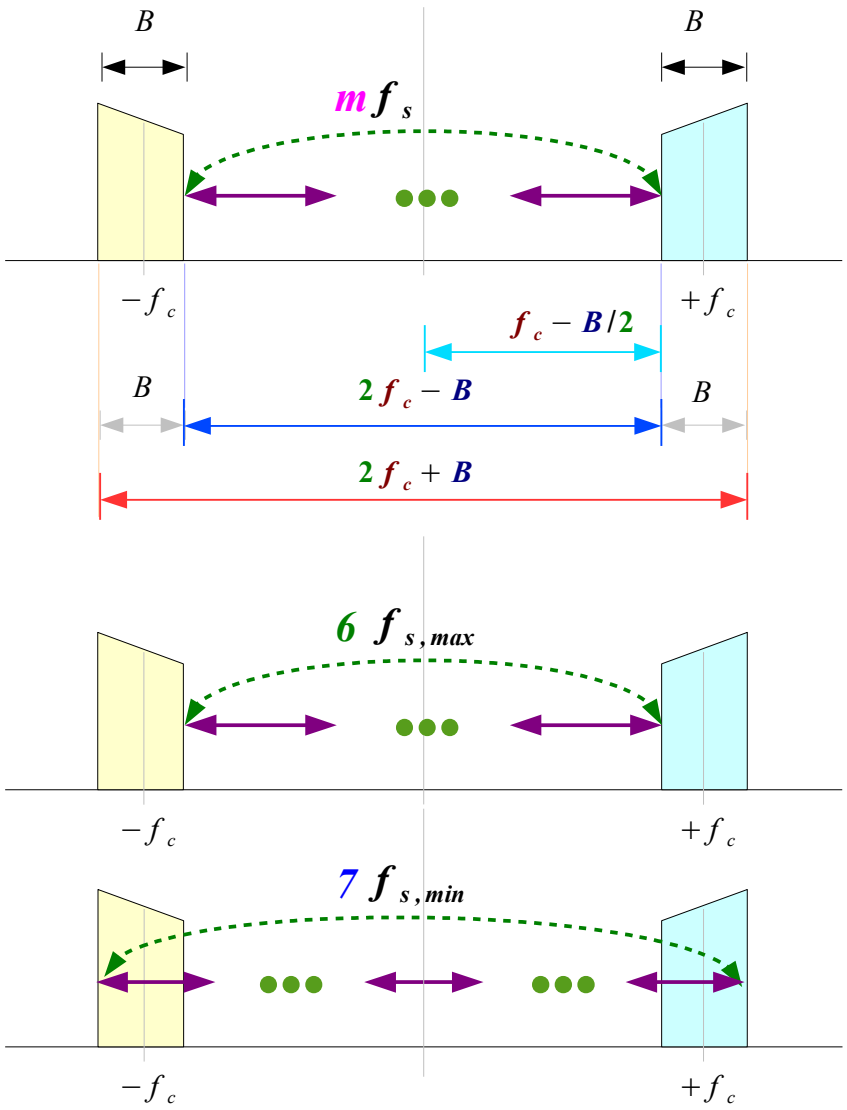
$$f_H = f_c + B/2 \quad \textit{Highest frequency}$$

$$f_L = f_c - B/2 \quad \textit{Lowest frequency}$$





Example m=6 (1)



$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

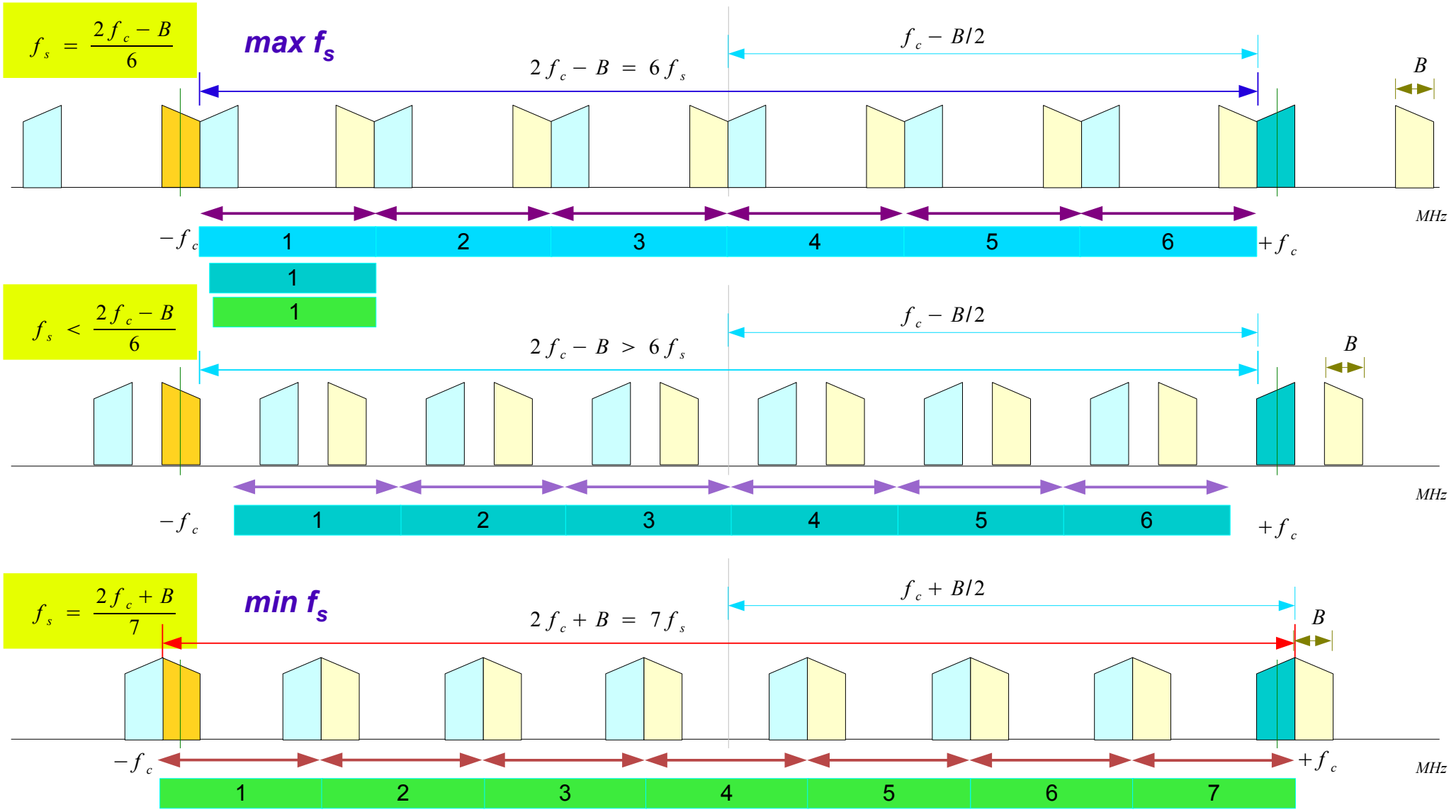
When $m = 6$

$$\min f_s \quad \frac{2f_c + B}{7} \le f_s \le \frac{2f_c - B}{6} \quad \max f_s$$

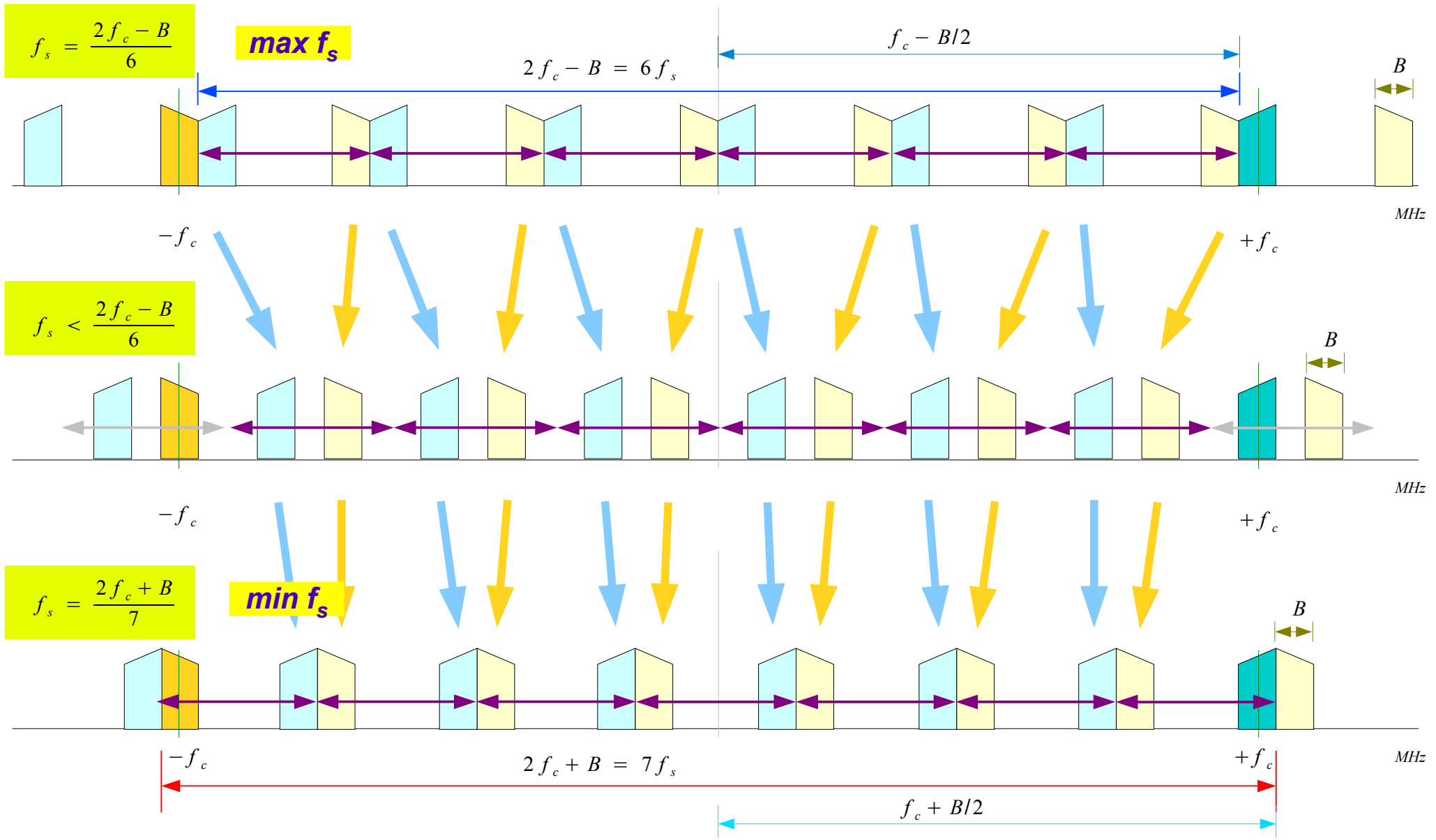
$$\max f_s = \frac{2f_c - B}{6}$$

$$\min f_s = \frac{2f_c + B}{7}$$

Example m=6 (2)


$f_s = \frac{2f_c - B}{6}$
max f_s
$2f_c - B = 6f_s$
$f_c - B/2$
B
MHz
$-f_c$
1
2
3
4
5
6
$+f_c$
$f_s < \frac{2f_c - B}{6}$
$2f_c - B > 6f_s$
$f_s = \frac{2f_c + B}{7}$
min f_s
$2f_c + B = 7f_s$
$f_c + B/2$
7

Example m=6 (3)


$f_s = \frac{2f_c - B}{6}$
max f_s
$2f_c - B = 6f_s$
$f_c - B/2$
B
MHz
$-f_c$
$+f_c$
$f_s < \frac{2f_c - B}{6}$
B
MHz
$-f_c$
$+f_c$
$f_s = \frac{2f_c + B}{7}$
min f_s
B
$-f_c$
$2f_c + B = 7f_s$
$+f_c$
MHz
$f_c + B/2$

Minimum f_s Plot (1)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

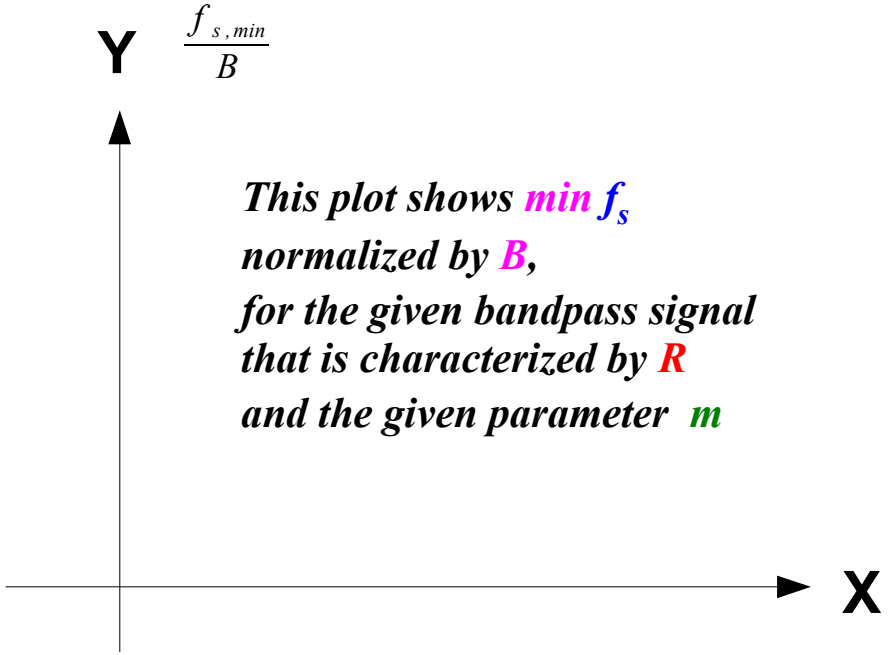
$$\frac{f_c + B/2}{B} = R \rightarrow X$$

$$R = \frac{\text{highest signal frequency}}{\text{bandwidth } B}$$

$$\frac{2f_c + B}{(m+1)B} = \frac{f_{s,min}}{B} \rightarrow Y$$

$$\frac{f_{s,min}}{B} = \frac{\text{minimum sampling rate}}{\text{bandwidth } B}$$

X-Y Plot



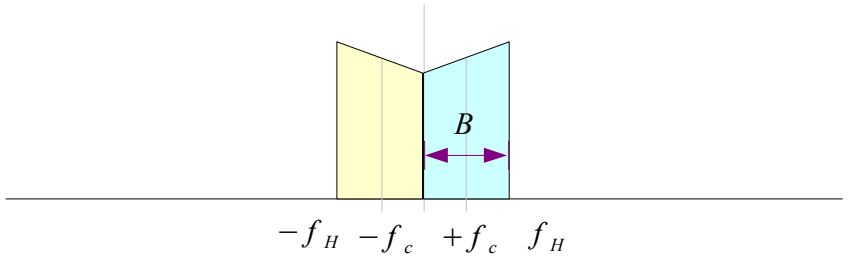
This plot shows min f_s normalized by B, for the given bandpass signal that is characterized by R and the given parameter m

Characterized by
- *Bandwidth B*
- *Carrier Frequency f_c*

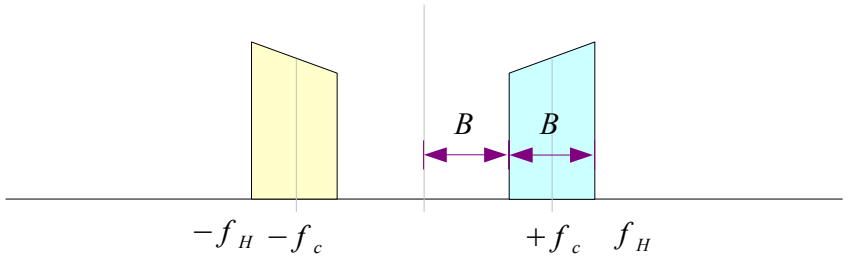
$$R = \frac{f_H}{B} = \frac{f_c + B/2}{B}$$

Minimum f_s Plot (2)

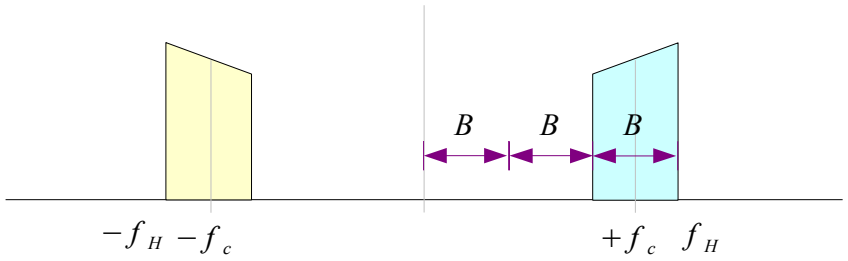
$f_H = f_c + B/2 = 1B$ $\quad R = f_H / B = 1$



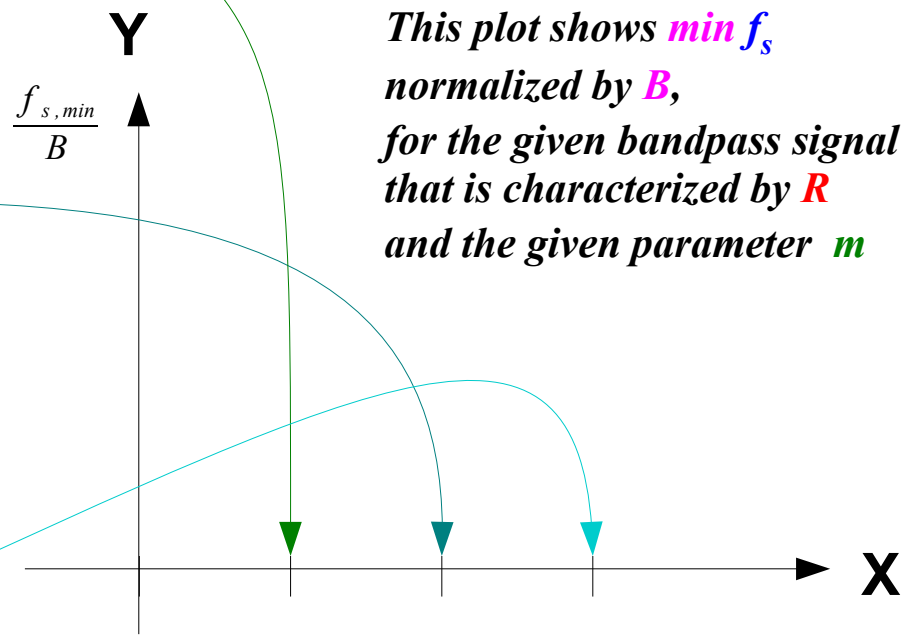
$f_H = f_c + B/2 = 2B$ $\quad R = f_H / B = 2$



$f_H = f_c + B/2 = 3B$ $\quad R = f_H / B = 3$



X-Y Plot



This plot shows min f_s normalized by B, for the given bandpass signal that is characterized by R and the given parameter m

Characterized by

- ***Bandwidth B***
- ***Carrier Frequency f_c***

$$R = \frac{f_H}{B} = \frac{f_c + B/2}{B}$$

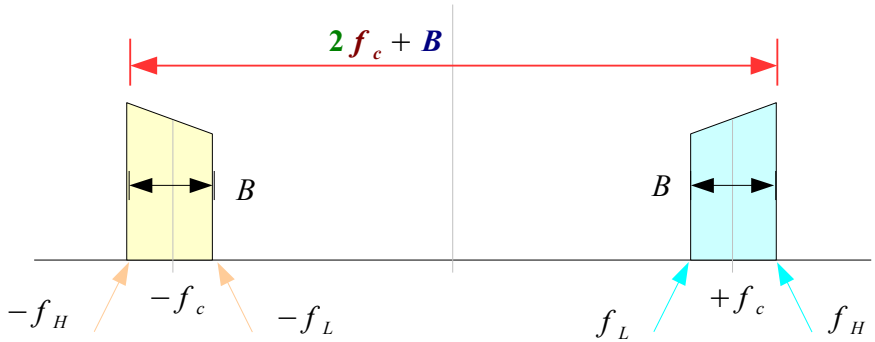
Minimum f_s Plot (3)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$g(m,R) = \frac{2}{(m+1)}\frac{f_H}{B} = \frac{2}{(m+1)}R$$

$$\frac{f_H}{B} = X \Rightarrow \frac{f_c + B/2}{B} = R$$

$$\frac{f_{s,min}}{B} = Y \Rightarrow \frac{2f_c + B}{(m+1)B} = \frac{2f_H}{(m+1)B} \Rightarrow g(m,R)$$



$$m=0 \quad g(0,R) = 2R \quad slope = 2$$
$$m=1 \quad g(1,R) = R \quad slope = 1$$
$$m=2 \quad g(2,R) = \tfrac{2}{3}R \quad slope = 2/3$$
$$m=3 \quad g(3,R) = \tfrac{1}{2}R \quad slope = 1/2$$
$$m=4 \quad g(4,R) = \tfrac{2}{5}R \quad slope = 2/5$$
$$m=5 \quad g(5,R) = \tfrac{1}{3}R \quad slope = 1/3$$
$$m=6 \quad g(6,R) = \tfrac{2}{7}R \quad slope = 2/7$$
$$m=7 \quad g(7,R) = \tfrac{1}{4}R \quad slope = 1/4$$
$$m=8 \quad g(8,R) = \tfrac{2}{9}R \quad slope = 2/9$$

Minimum f_s Plot (4)

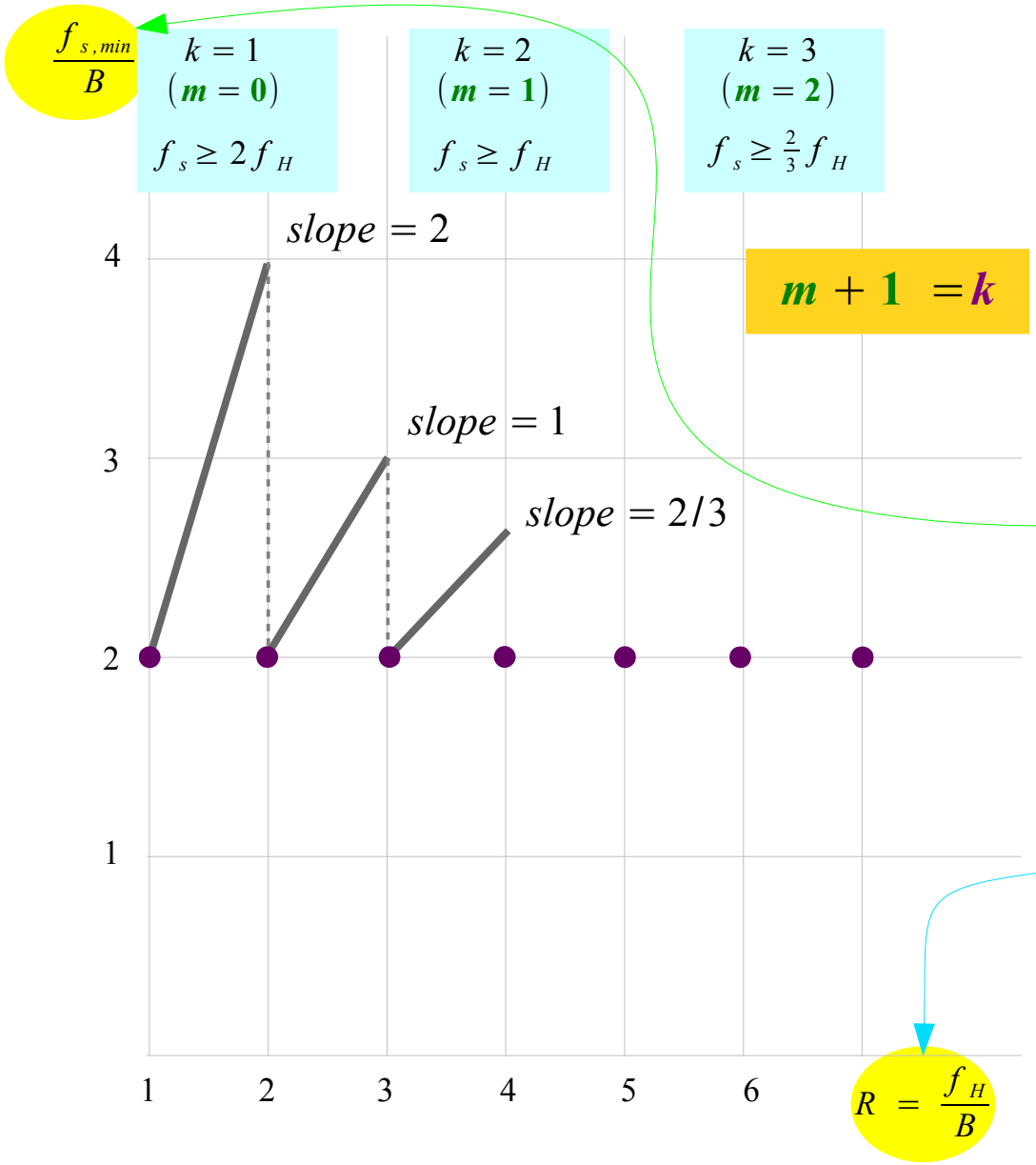
$$g(m,R) = \frac{2}{(m+1)}\frac{f_H}{B} = \frac{2}{(m+1)}R$$

$$R = m+1 \quad \Rightarrow \quad g(m,m+1) = 2$$

$m=0$	$g(0,R) = 2R$	$slope = 2$
$m=1$	$g(1,R) = R$	$slope = 1$
$m=2$	$g(2,R) = \frac{2}{3}R$	$slope = 2/3$
$m=3$	$g(3,R) = \frac{1}{2}R$	$slope = 1/2$
$m=4$	$g(4,R) = \frac{2}{5}R$	$slope = 2/5$
$m=5$	$g(5,R) = \frac{1}{3}R$	$slope = 1/3$
$m=6$	$g(6,R) = \frac{2}{7}R$	$slope = 2/7$
$m=7$	$g(7,R) = \frac{1}{4}R$	$slope = 1/4$
$m=8$	$g(8,R) = \frac{2}{9}R$	$slope = 2/9$

$m=0$	$R=1$	$\Rightarrow$	$g(0,1) = 2$
$m=1$	$R=2$	$\Rightarrow$	$g(1,2) = 2$
$m=2$	$R=3$	$\Rightarrow$	$g(2,3) = 2$
$m=3$	$R=4$	$\Rightarrow$	$g(3,4) = 2$
$m=4$	$R=5$	$\Rightarrow$	$g(4,5) = 2$
$m=5$	$R=6$	$\Rightarrow$	$g(5,6) = 2$
$m=6$	$R=7$	$\Rightarrow$	$g(6,7) = 2$
$m=7$	$R=8$	$\Rightarrow$	$g(7,8) = 2$
$m=8$	$R=9$	$\Rightarrow$	$g(8,9) = 2$

Minimum f_s Plot (5)



$$m + 1 = k$$

$$\frac{2 f_c + B}{m + 1} \leq f_s \leq \frac{2 f_c - B}{m}$$

$$\frac{2 f_c + B}{k} \leq f_s \leq \frac{2 f_c - B}{k - 1}$$

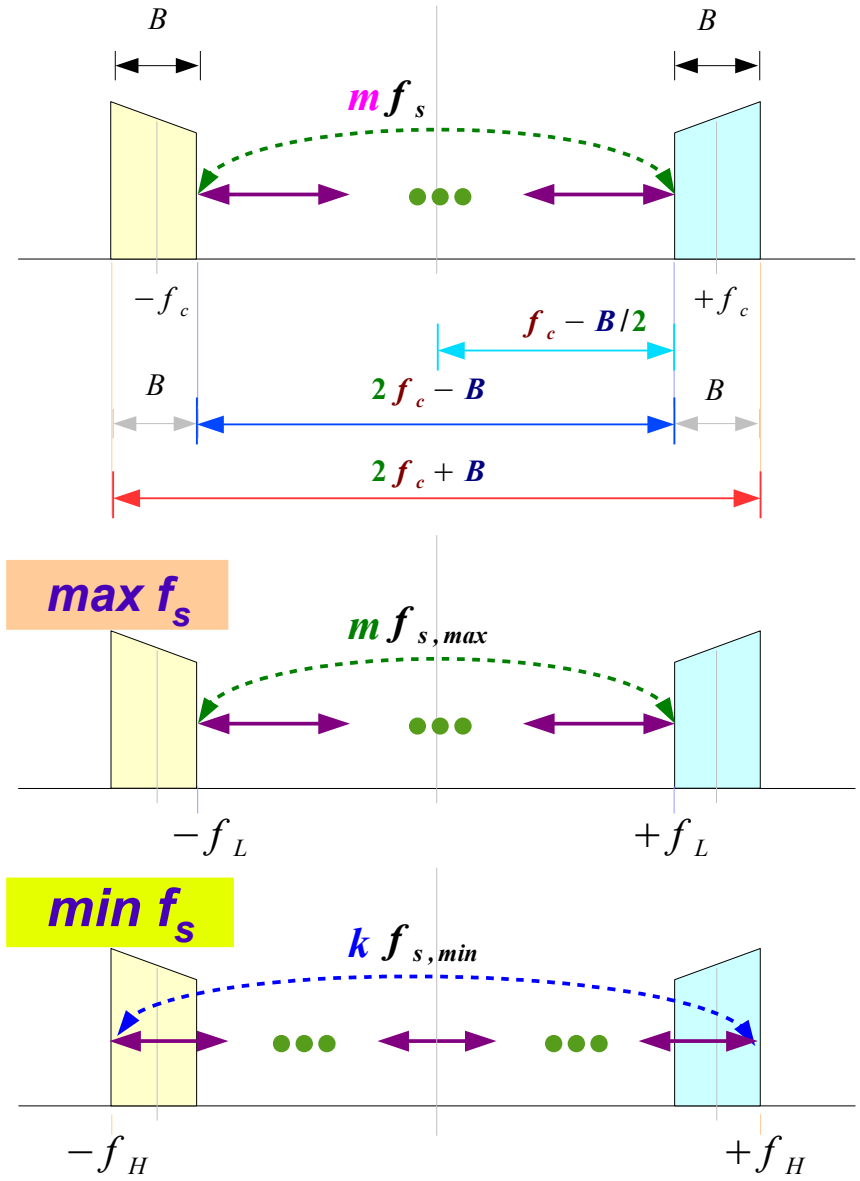
$$\frac{2 f_c + B}{(m + 1) B} = \frac{f_{s,min}}{B} = g(m, R)$$

$$\frac{\text{minimum sampling rate}}{\text{bandwidth } B}$$

$$\frac{f_c + B/2}{B} = R$$

$$\frac{\text{highest signal frequency}}{\text{bandwidth } B}$$

Min, Max Condition on f_s (1)



$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{k} \le f_s \le \frac{2f_c - B}{k-1}$$

$$m + 1 = k$$

min f_s **max f_s**

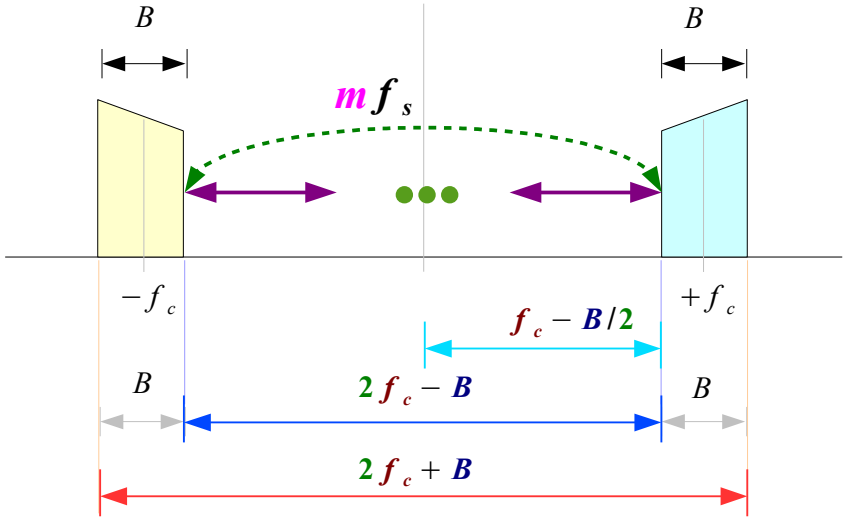
$$\frac{2f_H}{k} \le f_s \le \frac{2f_L}{m}$$

$$k = 2 \quad f_H \le f_s \le 2f_L \quad m = 1$$

$$k = 3 \quad \frac{2}{3}f_H \le f_s \le f_L \quad m = 2$$

$$k = 4 \quad \frac{1}{2}f_H \le f_s \le \frac{2}{3}f_L \quad m = 3$$

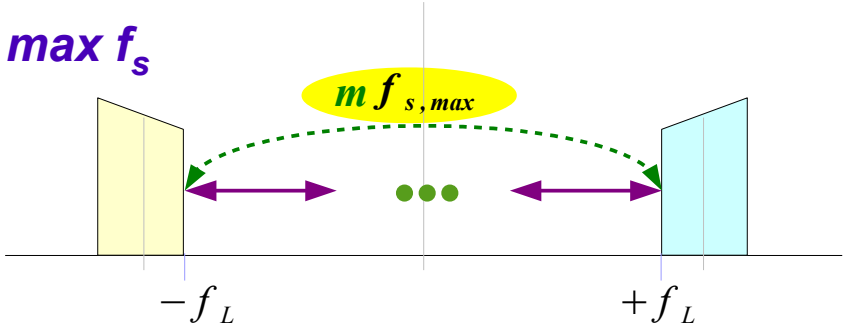
Min, Max Condition on f_s (2)



$$\frac{2f_H}{k} \le f_s \le \frac{2f_L}{m}$$

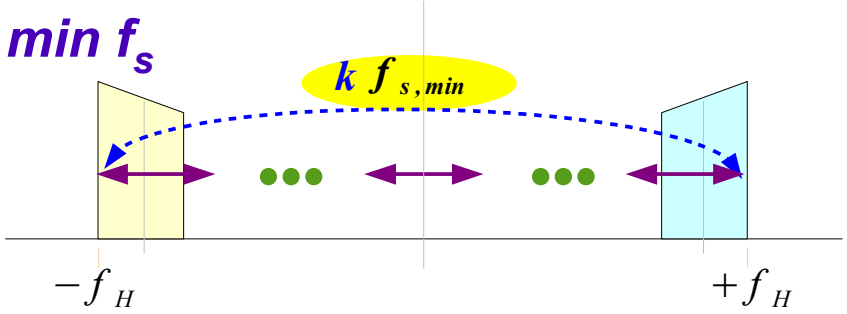
min f_s max f_s

$$k = m + 1$$



m represents how many f_s are in $2f_c - B$ in max f_s

$$\max f_s = \frac{2f_c - B}{m} = \frac{2f_L}{m}$$



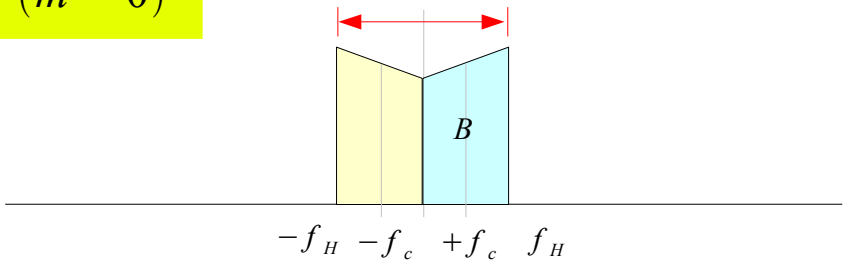
k represents how many f_s are in $2f_c + B$ in min f_s

$$\min f_s = \frac{2f_c + B}{k} = \frac{2f_H}{k}$$

Example k=1 (m=0)

$k = 1$
$(m = 0)$

$f_H = f_c + B/2 = 1B$



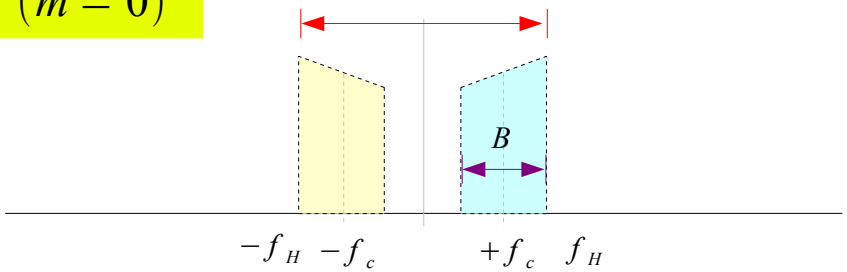
$R = f_H / B = 1$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = 2$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = +\infty$$

$k = 1$
$(m = 0)$

$f_H = f_c + B/2 = 1.5B$



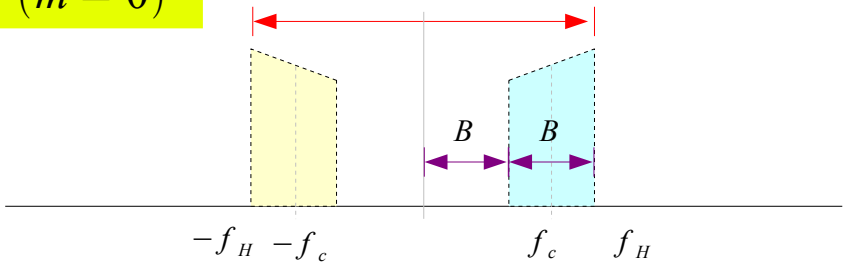
$R = f_H / B = 1.5$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = 3$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = +\infty$$

$k = 1$
$(m = 0)$

$f_H = f_c + B/2 = 2B$

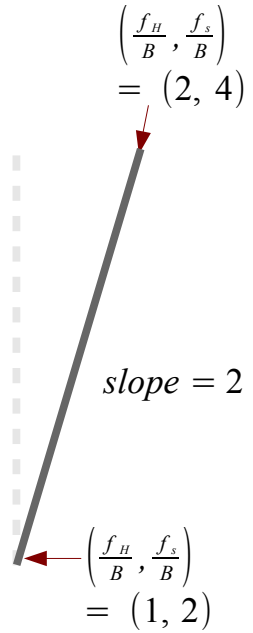


$R = f_H / B = 2$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = 4$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1)B} = +\infty$$

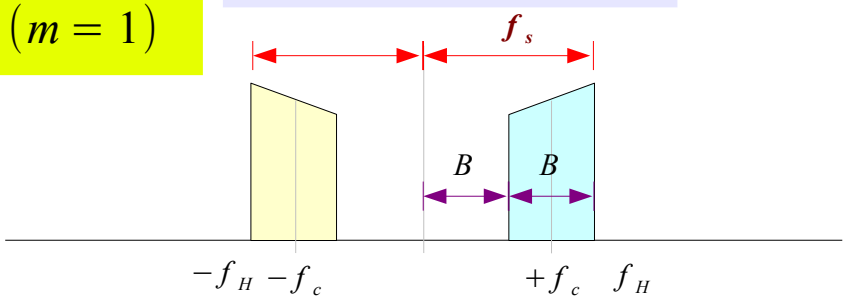
$R \in [1,\ 2]$



Example k=2 (m=1)

$k = 2$
$(m = 1)$

$f_H = f_c + B/2 = 2B$

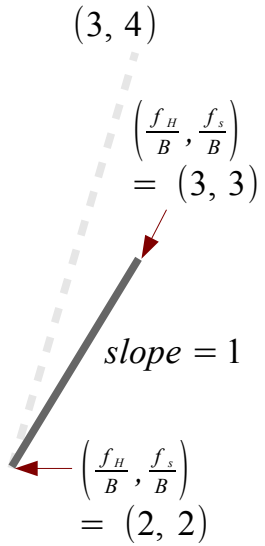


$R = f_H / B = 2$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2$$

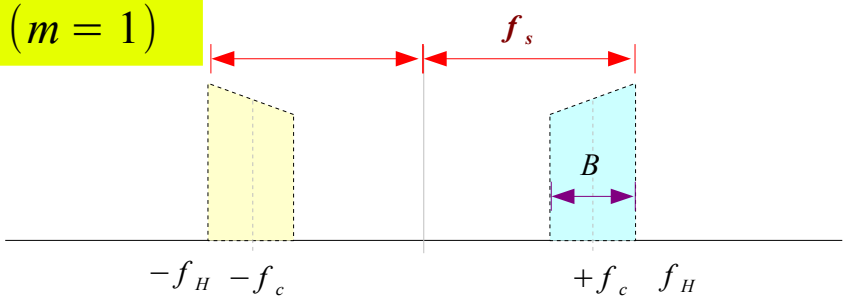
$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 2$$

$R \in [2,\ 3]$



$k = 2$
$(m = 1)$

$f_H = f_c + B/2 = 2.5B$



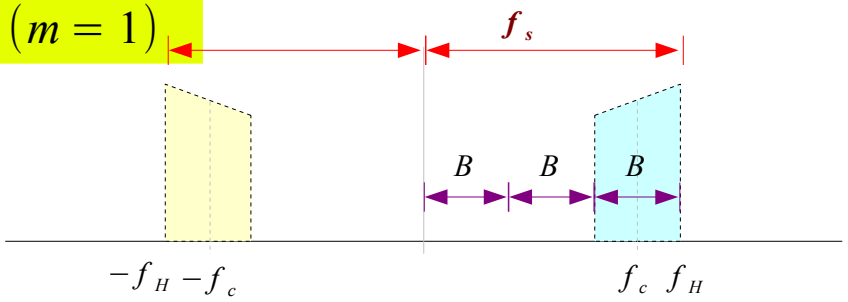
$R = f_H / B = 2.5$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2.5$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 3$$

$k = 2$
$(m = 1)$

$f_H = f_c + B/2 = 3B$



$R = f_H / B = 3$

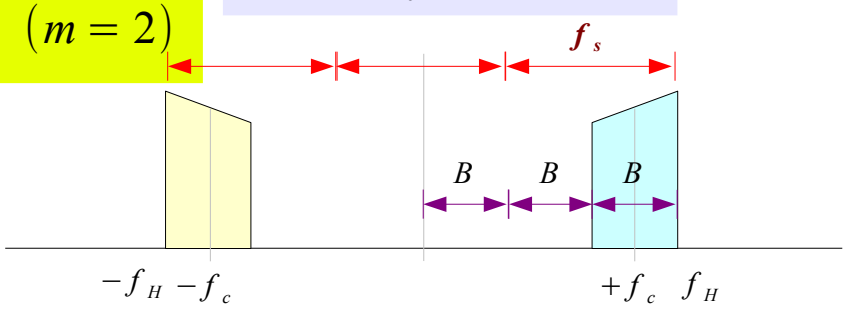
$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 3$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 4$$

Example k=3 (m=2)

$k = 3$
$(m = 2)$

$f_H = f_c + B/2 = 3B$



$R = f_H / B = 3$

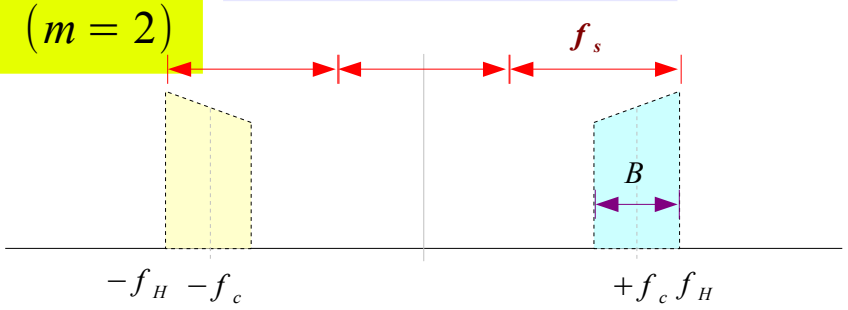
$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = 2$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 2$$

$R \in [3,\ 4]$

$k = 3$
$(m = 2)$

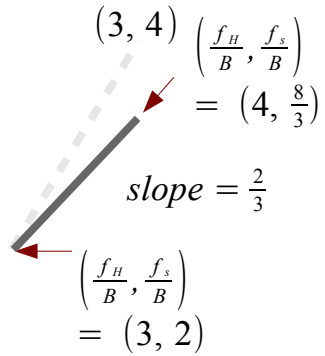
$f_H = f_c + B/2 = 3.5B$



$R = f_H / B = 3.5$

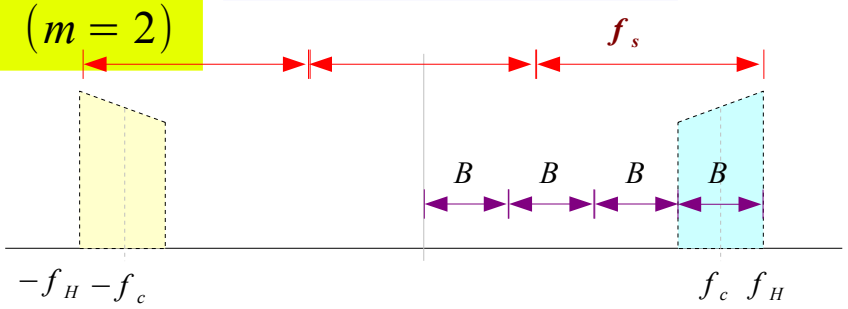
$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \frac{7}{3}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 3$$



$k = 3$
$(m = 2)$

$f_H = f_c + B/2 = 4B$

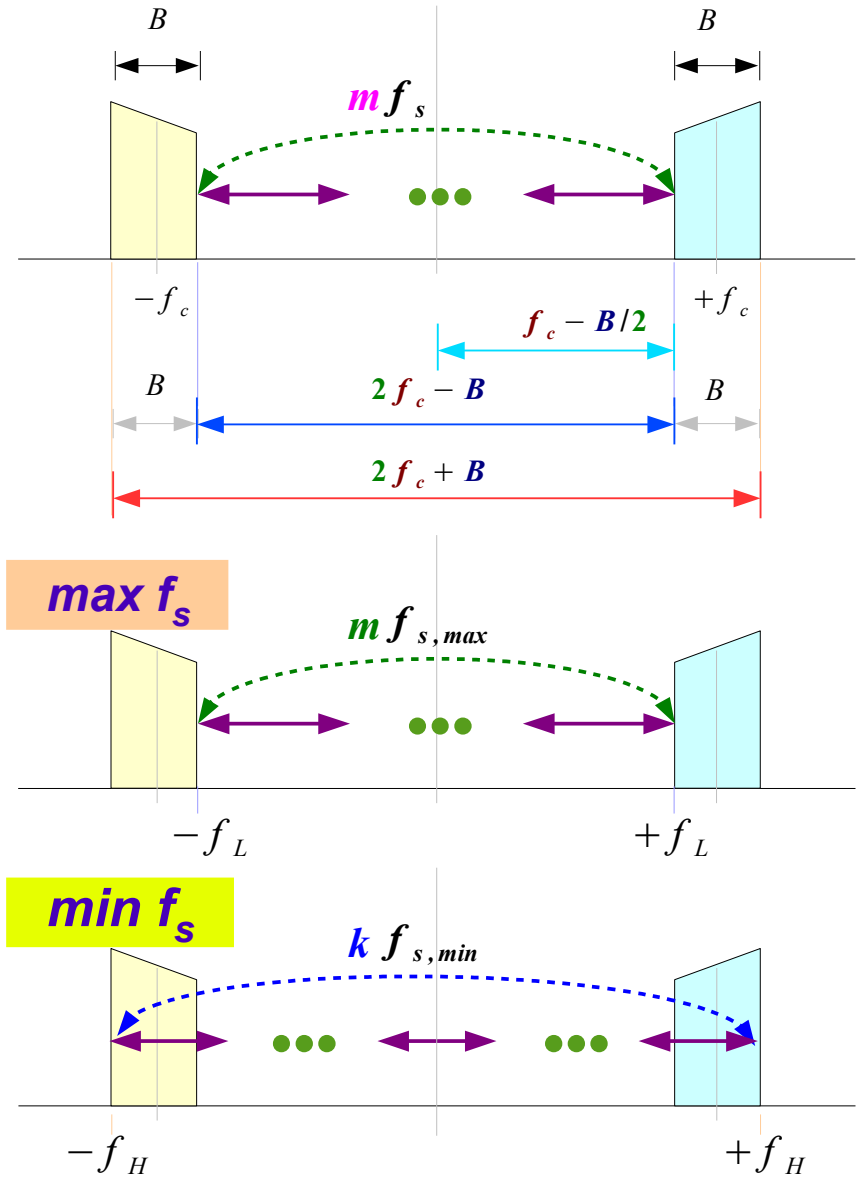


$R = f_H / B = 4$

$$\frac{f_{s,min}}{B} = \frac{2 f_H}{k B} = \frac{8}{3}$$

$$\frac{f_{s,max}}{B} = \frac{2(f_H - B)}{(k-1) B} = 4$$

Min, Max Condition on f_s (2)



$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{k} \le f_s \le \frac{2f_c - B}{k-1}$$

$$m + 1 = k$$

min f_s max f_s

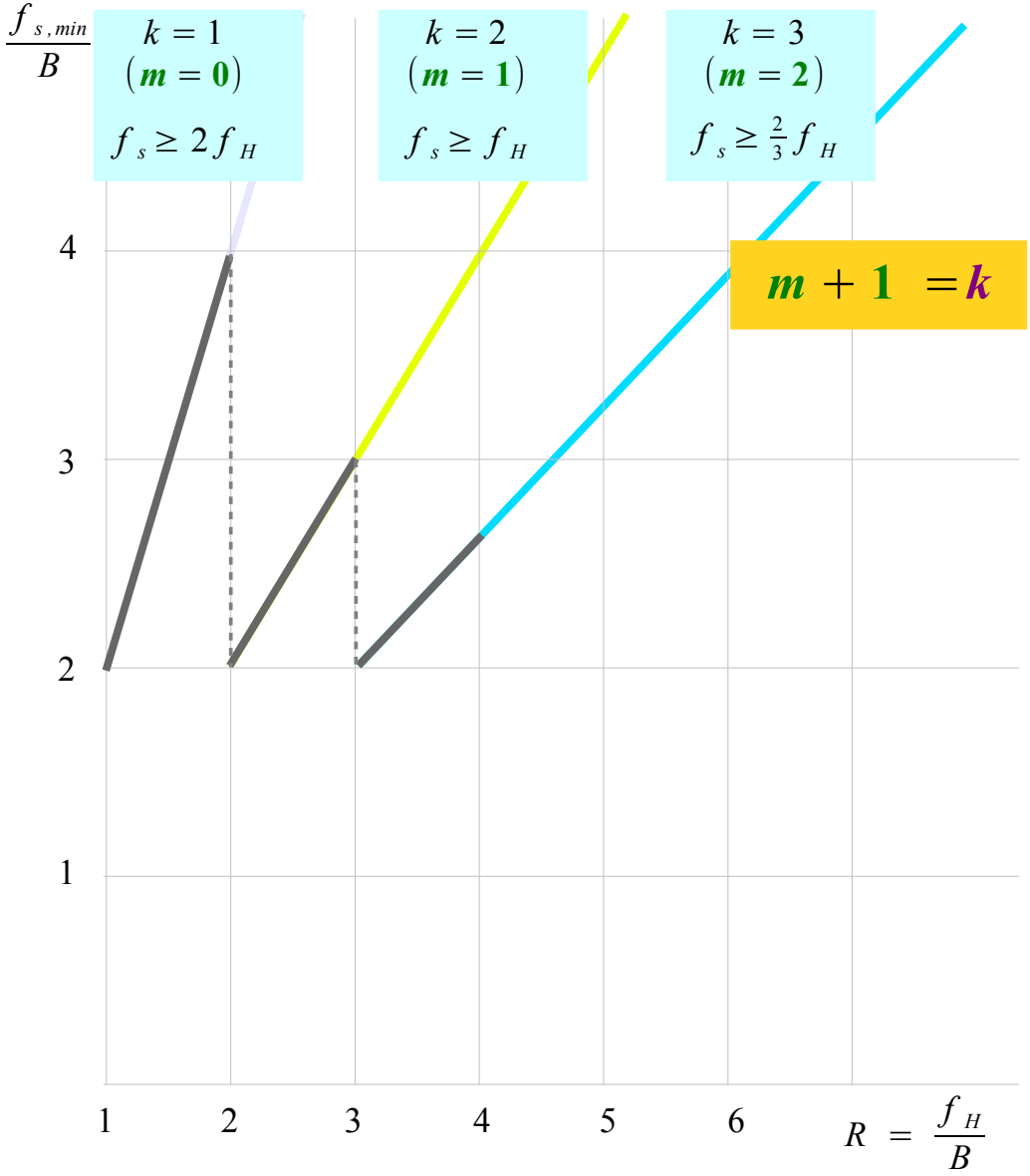
$$\frac{2f_H}{k} \le f_s \le \frac{2f_L}{m}$$

$$k = 2 \quad f_H \le f_s \le 2f_L \quad m = 1$$

$$k = 3 \quad \frac{2}{3}f_H \le f_s \le f_L \quad m = 2$$

$$k = 4 \quad \frac{1}{2}f_H \le f_s \le \frac{2}{3}f_L \quad m = 3$$

Min Max f_s Plot (1)

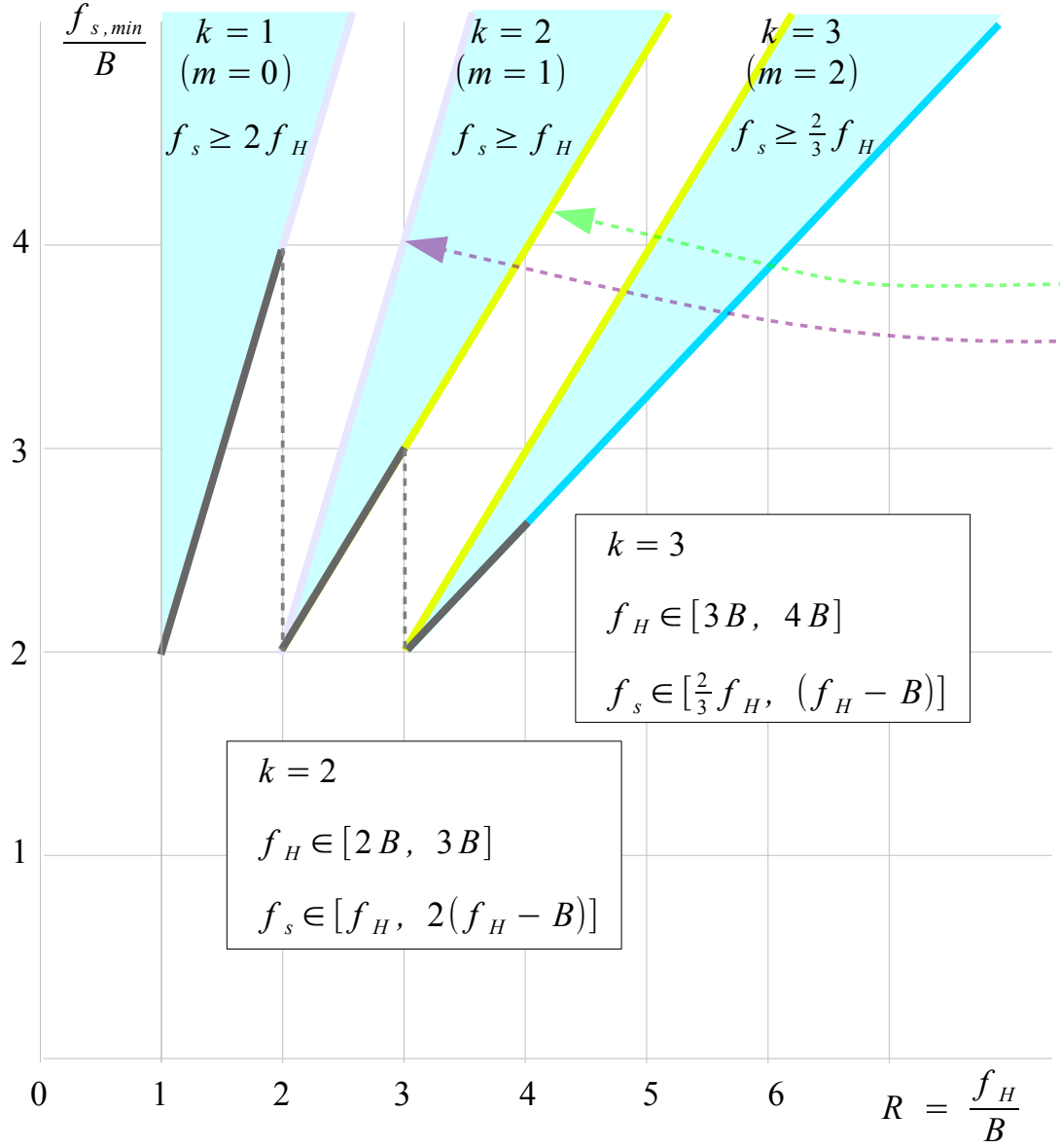


$$\frac{2 f_c + B}{m + 1} \leq f_s \leq \frac{2 f_c - B}{m}$$

$$\frac{2 f_c + B}{k} \leq f_s \leq \frac{2 f_c - B}{k - 1}$$

$$\frac{2 f_H}{k} \leq f_s \leq \frac{2 f_L}{m}$$

Min Max f_s Plot (2)



$$\frac{2 f_H}{k} \leq f_s \leq \frac{2 f_L}{m}$$

Min f_s **Max f_s**

$k = 2$	$f_H \leq f_s \leq 2 f_L$	
$k = 3$	$\frac{2}{3} f_H \leq f_s \leq f_L$	

Min f_s		**Max f_s**
$y = 1(x-2)+2$	$k = 2$	$y = 2(x-2)+2$
$y = x$		$y = 2x-2$
$y = \frac{2}{3}(x-3)+2$	$k = 3$	$y = 1(x-3)+2$
$y = \frac{2}{3}x$		$y = x-1$

Range of f_s (1)

For a given m

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

Nyquist Criterion

$$2B \le f_s$$

$f_c = 20\ MHz$

$B = 5\ MHz$

	min f_s		max f_s	Optimum Sampling Frequency
$m=1$	$\frac{2\cdot 20+5}{1+1} = 22.5 \le f_s \le$		$\frac{2\cdot 20-5}{1} = 35$	$f_s = 22.5\ MHz \quad (10 \le f_s)$
$m=2$	$\frac{2\cdot 20+5}{2+1} = 15 \le f_s \le$		$\frac{2\cdot 20-5}{2} = 17.5$	$f_s = 17.5\ MHz \quad (10 \le f_s)$
$m=3$	$\frac{2\cdot 20+5}{3+1} = 11.25 \le f_s \le$		$\frac{2\cdot 20-5}{3} = 11.67$	$f_s = 11.25\ MHz \quad (10 \le f_s)$
$m=4$	$\frac{2\cdot 20+5}{4+1} = 9 \ge$		$\frac{2\cdot 20-5}{4} = 8.75$	X
$m=5$	$\frac{2\cdot 20+5}{5+1} = 7.5 \ge$		$\frac{2\cdot 20-5}{5} = 7.0$	X

Range of f_s (2)

For a given m

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

Nyquist Criterion

$$2B \le f_s$$

$f_c = 20\ MHz$

$B = 5\ MHz$

$$\frac{2f_H}{k} \le f_s \le \frac{2f_L}{m}$$

min f_s max f_s

$f_H = f_c + B/2 = 22.5\ MHz$

$f_L = f_c - B/2 = 17.5\ MHz$

$k=2$ $m=1$	$f_H \le f_s \le 2f_L$	$22.5 \le f_s \le 35$
$k=3$ $m=2$	$\frac{2}{3}f_H \le f_s \le f_L$	$15.0 \le f_s \le 17.5$
$k=4$ $m=3$	$\frac{1}{2}f_H \le f_s \le \frac{2}{3}f_L$	$11.2 \le f_s \le 11.67$
$k=5$ $m=4$	$\frac{2}{5}f_H \le f_s \le \frac{1}{2}f_L$	$9.0 \le f_s \le 8.75$
$k=6$ $m=5$	$\frac{1}{3}f_H \le f_s \le \frac{2}{5}f_L$	$7.5 \le f_s \le 7.0$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997