

# Trigonometry (4A)

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- Trigonometric Identities
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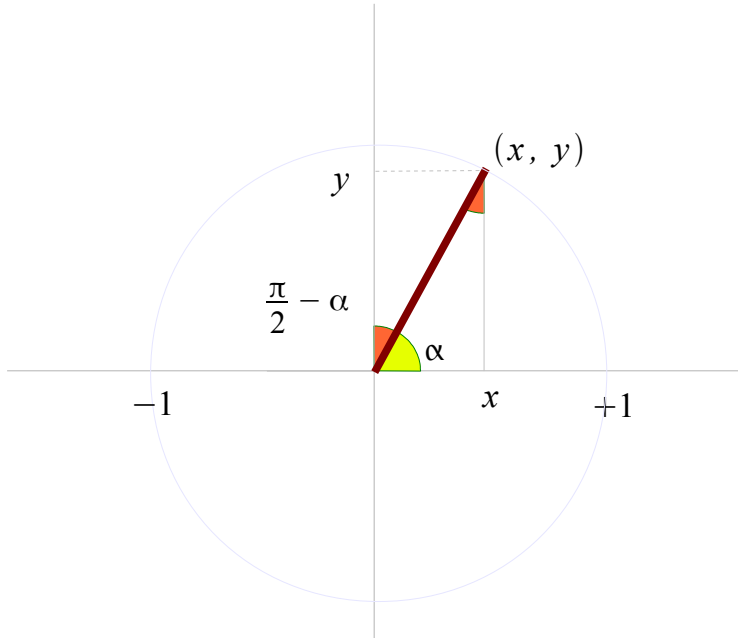
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# Co-function Identities



$$\sin \alpha = y \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos \alpha = x \Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan \alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

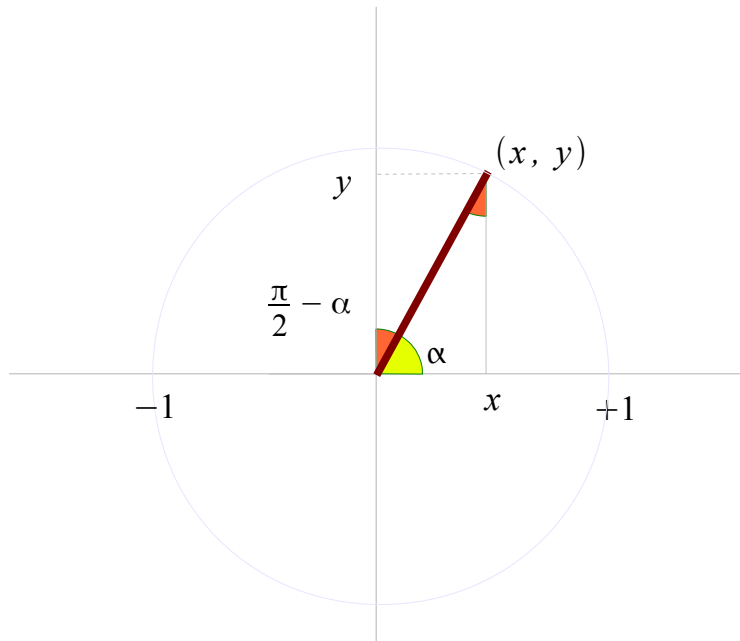
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

$$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec \alpha$$

$$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc \alpha$$

# Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

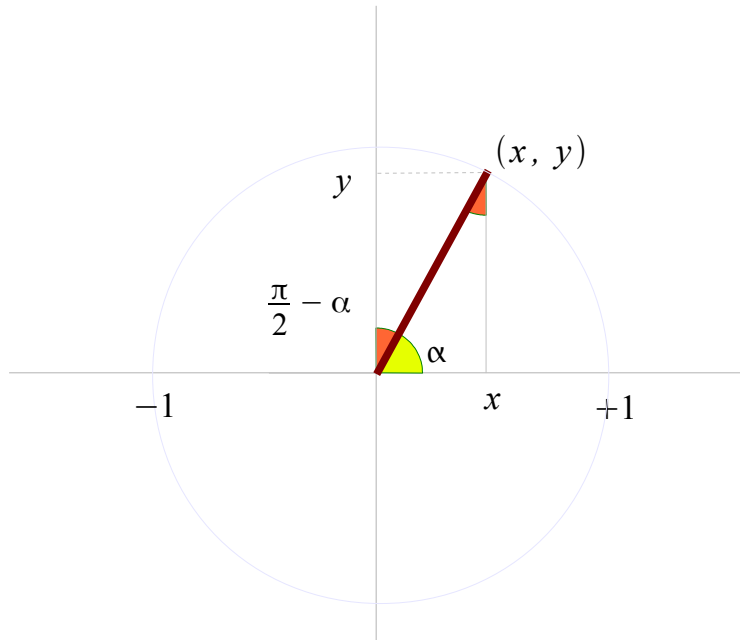
$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = 1$$

$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$$\begin{array}{l} \sin(60^\circ) = \frac{\sqrt{3}}{2} \\ \cos(60^\circ) = \frac{1}{2} \end{array} \quad \times \quad \begin{array}{l} \sin(30^\circ) = \frac{1}{2} \\ \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

# Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

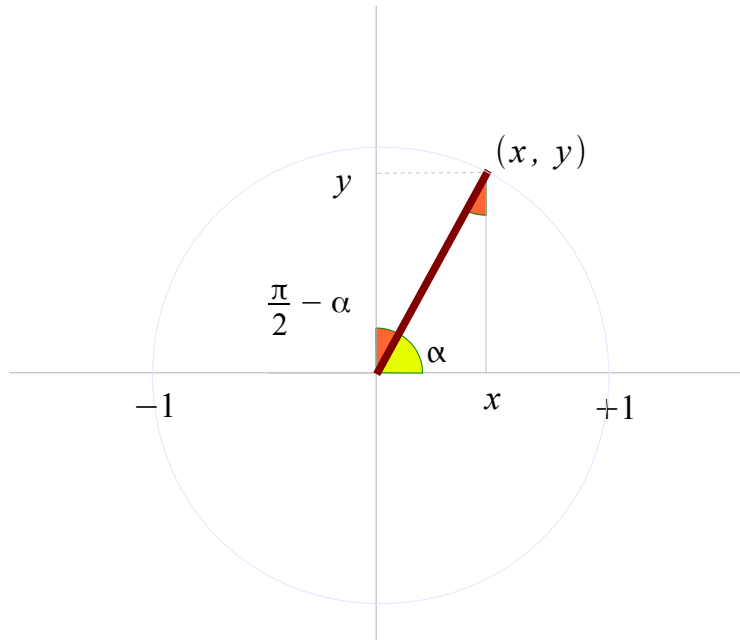
$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \text{—————} \quad \sin(30^\circ) = \frac{1}{2}$$

$$\cos(60^\circ) = \frac{1}{2} \quad \text{—————} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

# Angle Sum and Difference Identities (3)



$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ + 60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = +\infty$$

$$\tan(30^\circ - 60^\circ) = -\frac{1}{\sqrt{3}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \underline{\hspace{2cm}} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \underline{\hspace{2cm}} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = -\frac{1}{\sqrt{3}}$$

## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] Blitzer, R. "Algebra & Trigonometry." 3rd ed, Prentice Hall
- [4] Smith, R. T., Minton, R. B. "Calculus: Concepts & Connections," Mc Graw Hill
- [5] 홍성대, "기본/실력 수학의 정석,"성지출판