

Anti-aliasing Prefilter (6B)

-
-

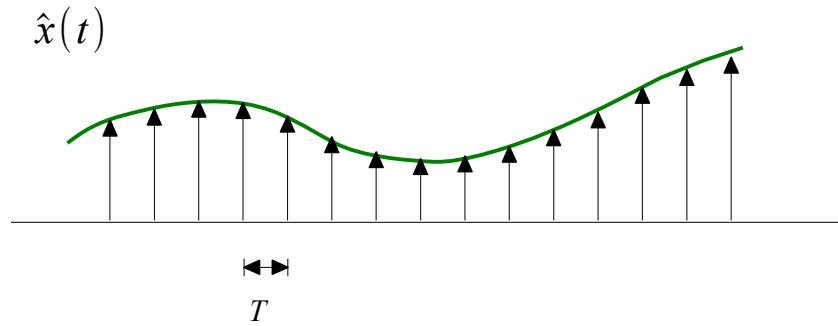
Copyright (c) 2012 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.

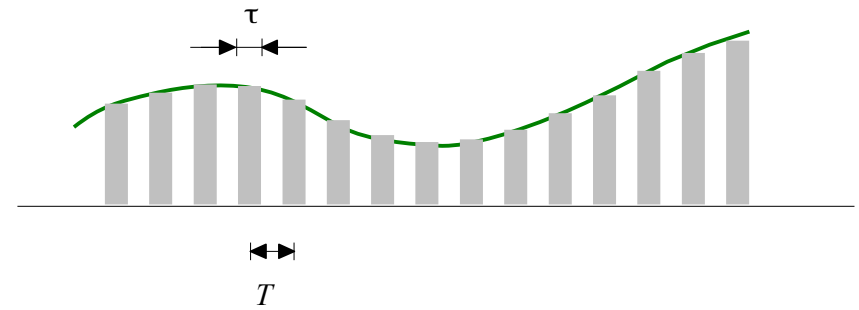
This document was produced by using OpenOffice and Octave.

Sampler



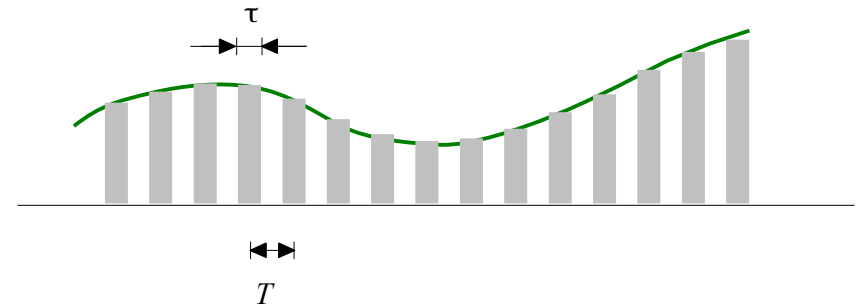
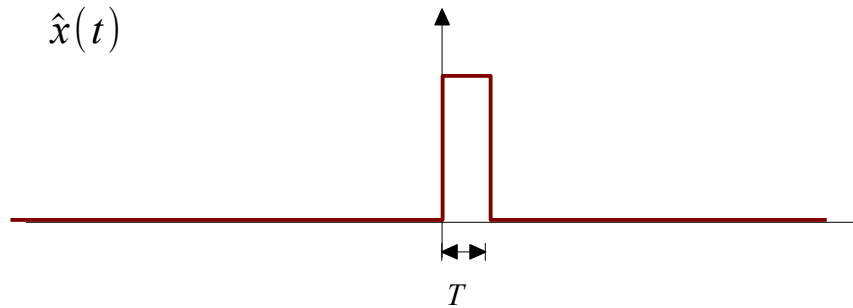
$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t) e^{-j2\pi ft} dt$$



$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT) p(t-nT)$$

Zero Order Hold (ZOH)



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT) p(t-nT)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t) e^{-j2\pi ft} dt$$

Square Wave CTFT

Continuous Time Fourier Series

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=-\infty}^{\infty} C_k e^{+jk\omega_0 t}$$

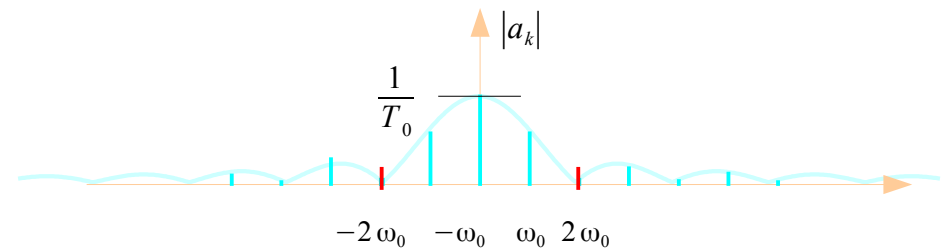
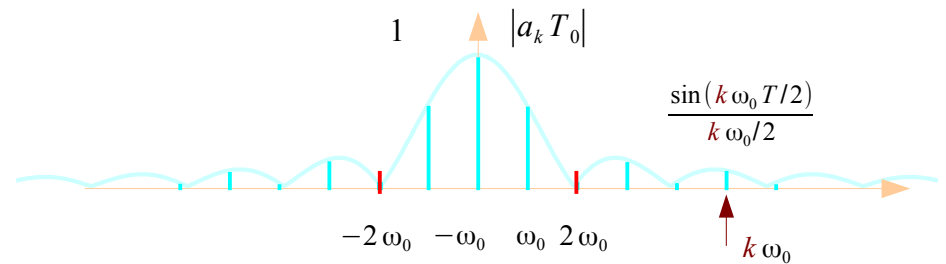
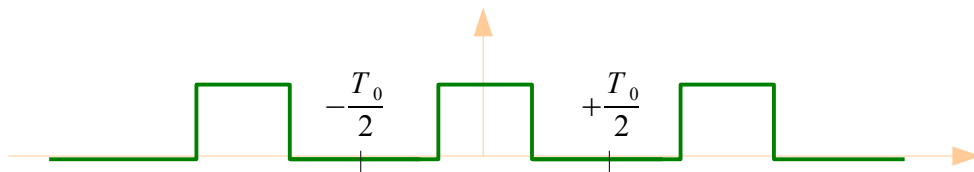
$$C_k = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jk\omega_0 t} dt$$

$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jk\omega_0 t} dt$$

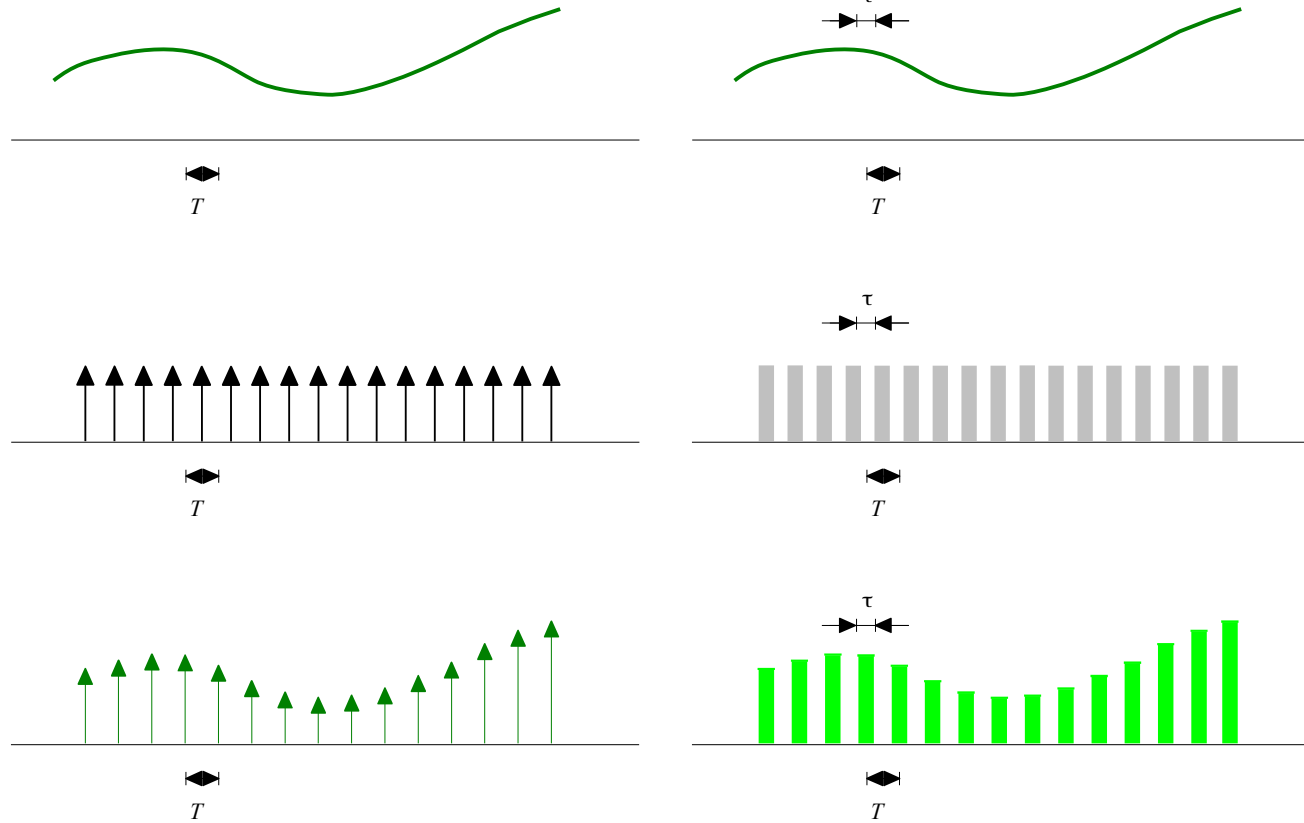
$$= \int_{-T_0/2}^{+T_0/2} e^{-jk\omega_0 t} dt = \left[\frac{-1}{jk\omega_0} e^{-jk\omega_0 t} \right]_{-T_0/2}^{+T_0/2}$$

$$= \frac{e^{-jk\omega_0 T_0/2} - e^{+jk\omega_0 T_0/2}}{jk\omega_0} = \frac{\sin(k\omega_0 T_0/2)}{k\omega_0/2}$$

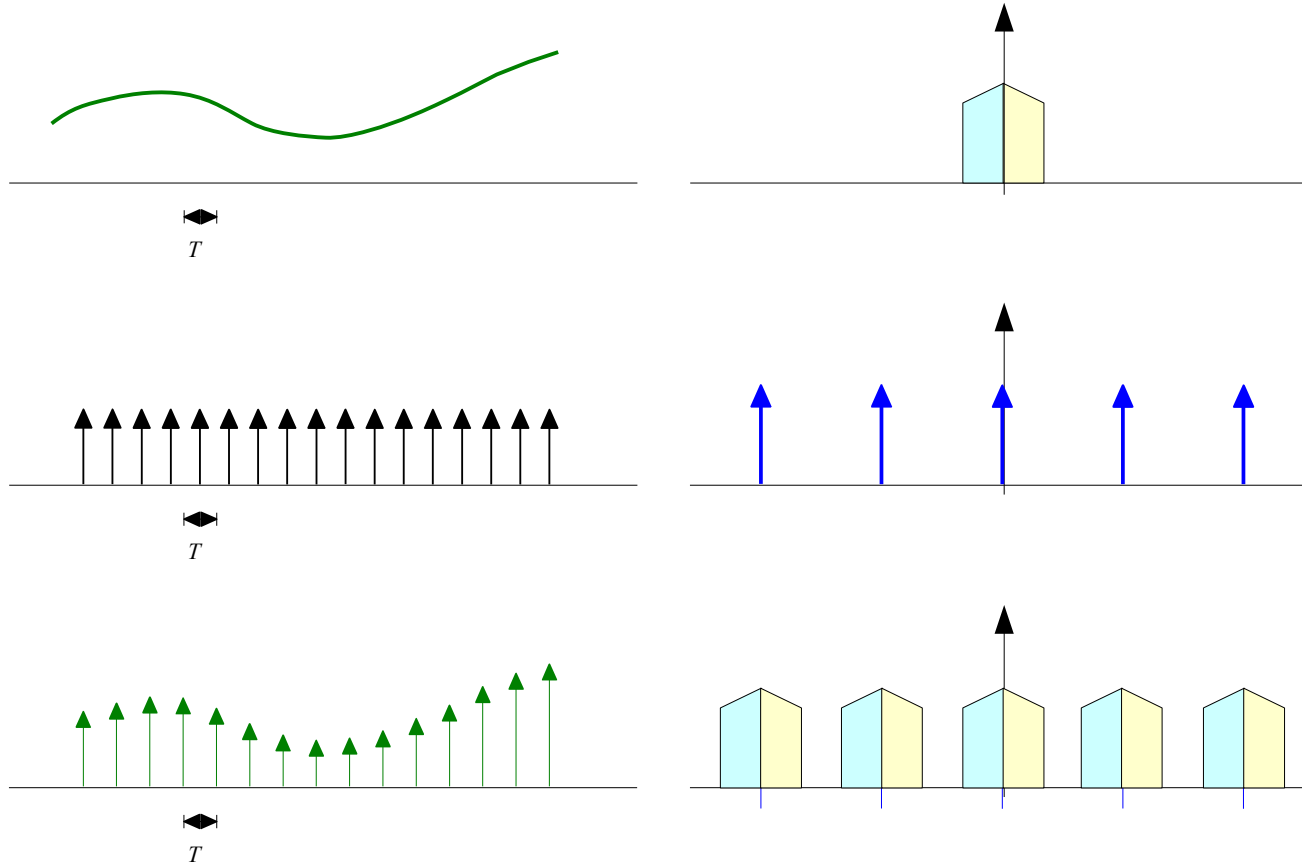
$$\omega_0 = \frac{2\pi}{T_0} \quad \text{Fundamental Frequency}$$



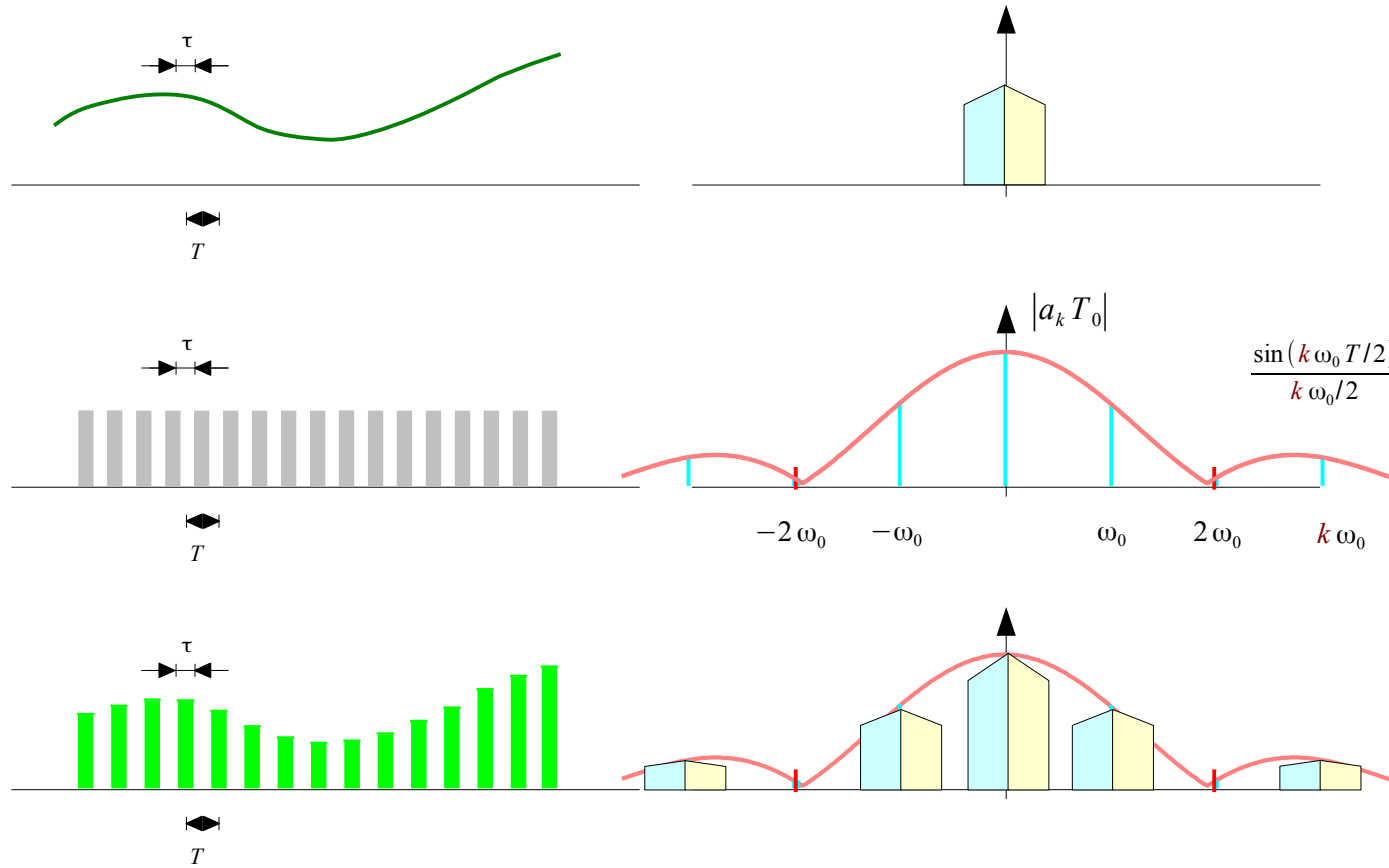
Sampler



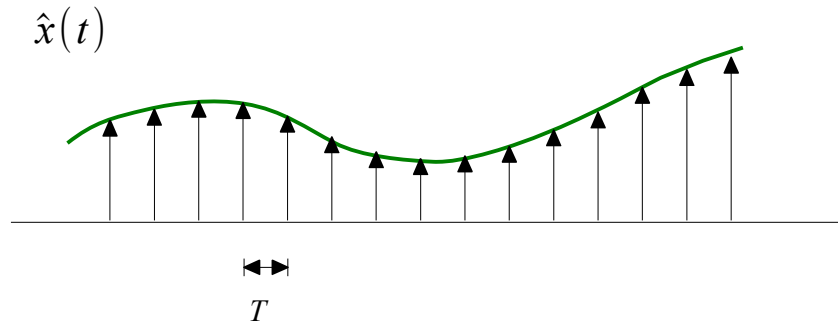
Sampler



Sampler



Discrete Time Fourier Transform

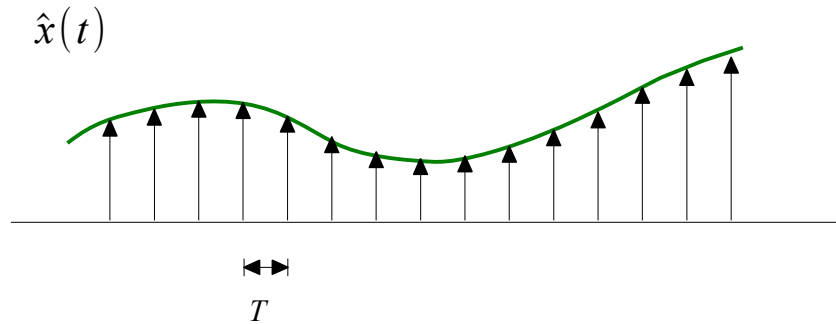


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

$$\begin{aligned} \hat{X}(f) &= \int_{-\infty}^{+\infty} \hat{x}(t) e^{-j2\pi f t} dt \\ &= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT) e^{-j2\pi f t} dt \\ &= \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} dt \end{aligned}$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} dt$$

Fourier Series



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

$$x(nT) = \frac{1}{f_s} \int_{f_s - f_s/2}^{+f_s/2} \hat{X}(f) e^{+j2\pi f T n} df$$

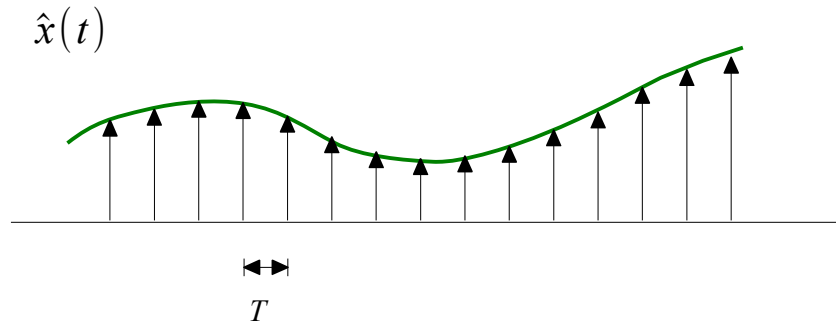
$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} dt$$

$$= \int_{-\pi}^{+\pi} \hat{X}(\omega) e^{+j\omega n} \frac{d\omega}{2\pi}$$

$$\omega = 2\pi f / f_s$$

$$\frac{d\omega}{2\pi} = \frac{df}{f_s}$$

Numerical Approximation



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} dt$$

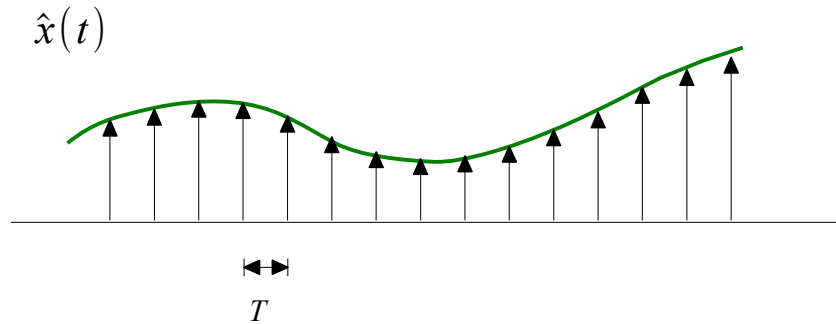
$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{+j2\pi f t} dt$$

$$\approx \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} \cdot T$$

$$X(f) \approx T \hat{X}(f)$$

$$X(f) = \lim_{T \rightarrow 0} T \hat{X}(f)$$

Spectrum Replication (1)



$$\begin{aligned}\hat{x}(t) &= \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT) \\ &= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)s(t)\end{aligned}$$

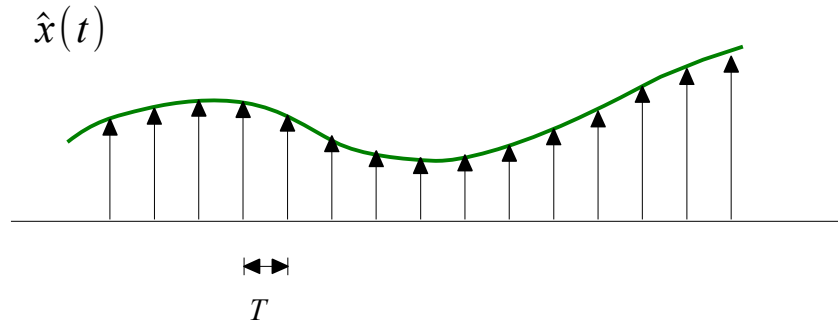
$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} e^{+j2\pi m f_s t}$$

$$\hat{x}(t) = x(t)s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t) e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-m f_s)$$

Spectrum Replication (2)



$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j2\pi f T n} dt$$

$$\begin{aligned} \hat{x}(t) &= \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT) \\ &= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)s(t) \end{aligned}$$

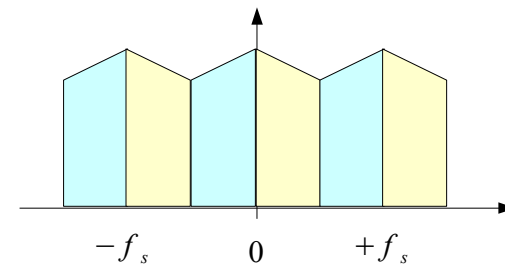
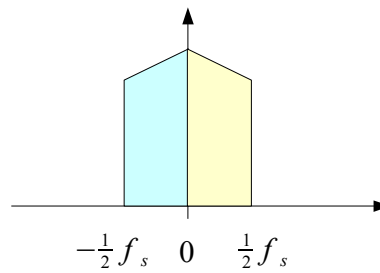
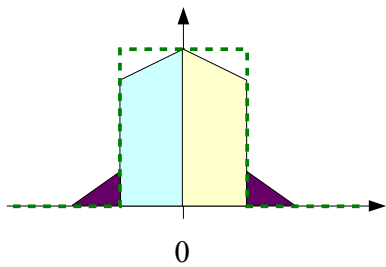
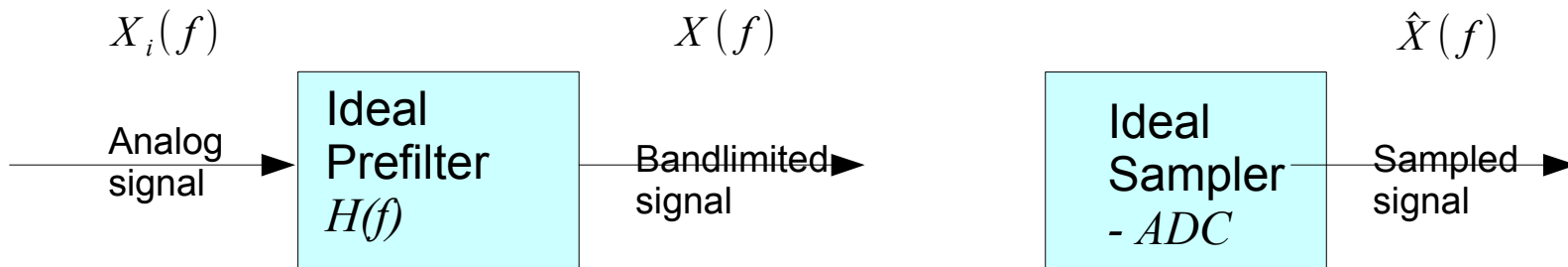
$$\begin{aligned} \hat{X}(f) &= \int_{-\infty}^{+\infty} X(f-f')S(f') df' \\ &= \frac{1}{T} \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f-f')\delta(f'-mf_s) df' \end{aligned}$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} e^{+j2\pi m f_s t}$$

$$S(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta(f-mf_s)$$

$$\hat{x}(t) = x(t)s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t) e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-mf_s)$$



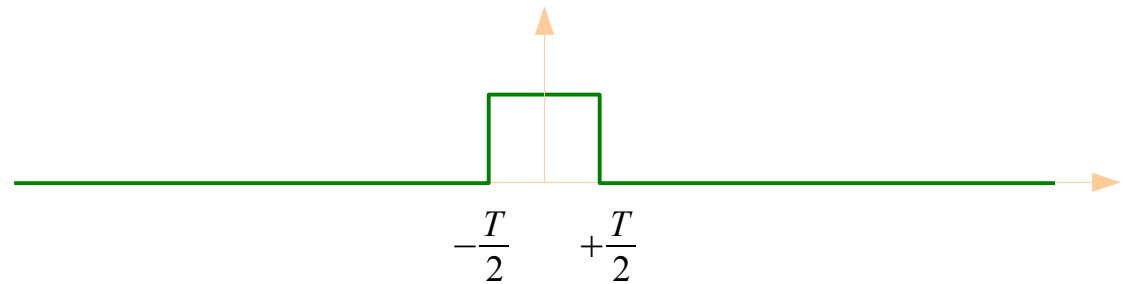
$\frac{2}{4}f_s$ $\frac{3}{4}f_s$ f_s

CTFT and CTFS (1)

Continuous Time Fourier Transform

Aperiodic Continuous Time Signal

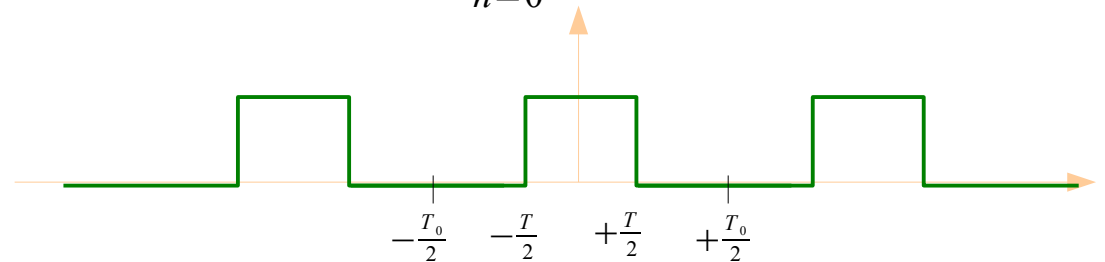
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$



Continuous Time Fourier Series

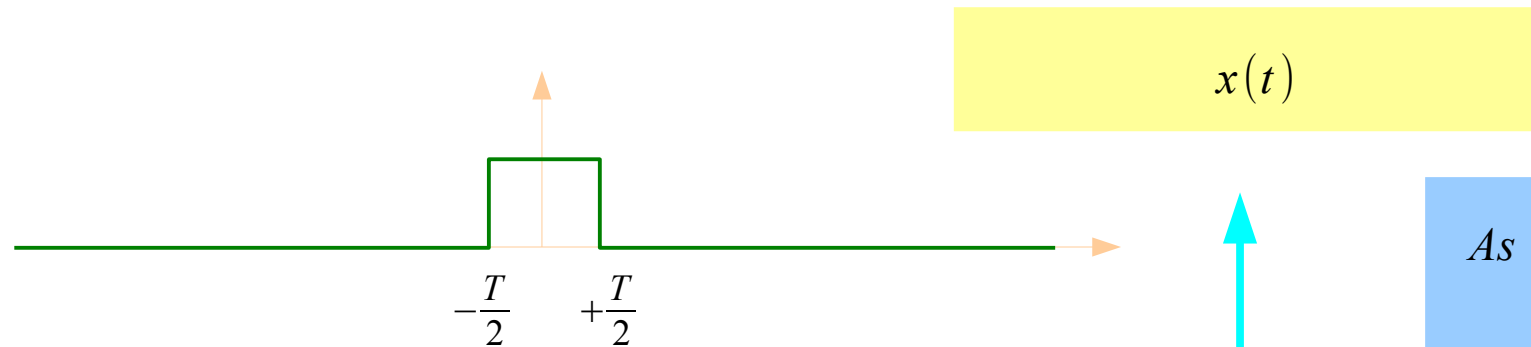
Periodic Continuous Time Signal

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=-\infty}^{\infty} C_k e^{+jk\omega_0 t}$$

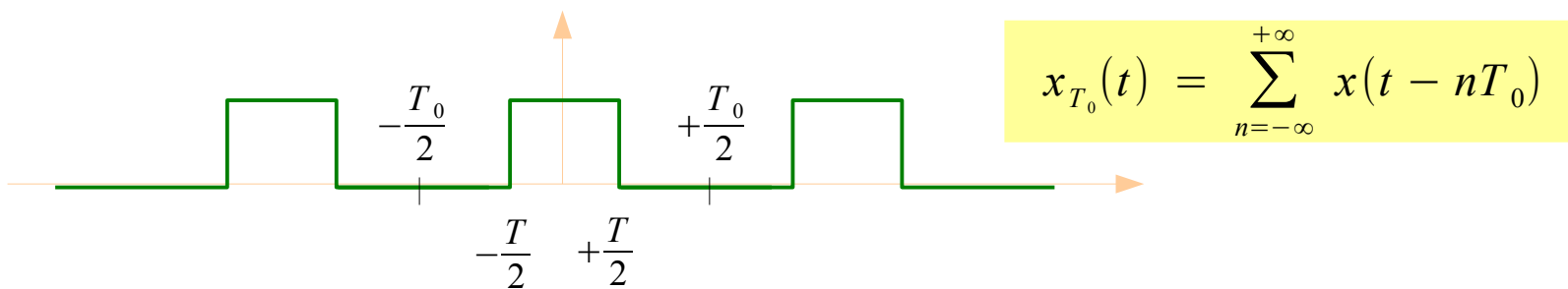


CTFT and CTFS (2)

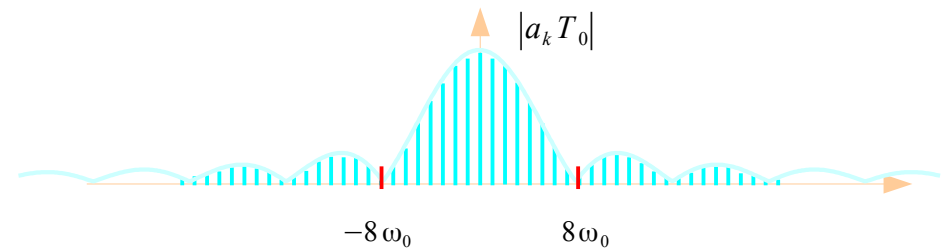
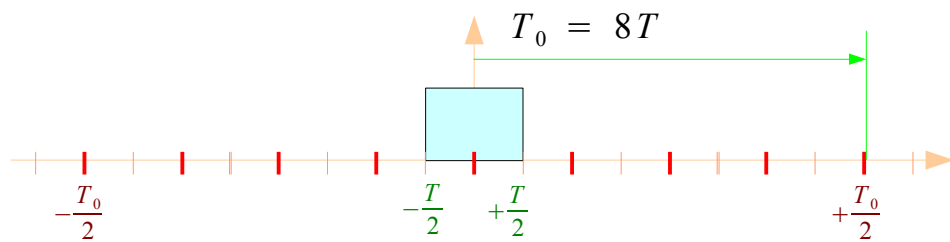
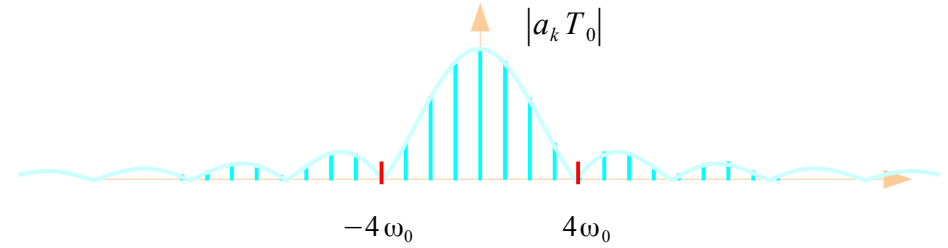
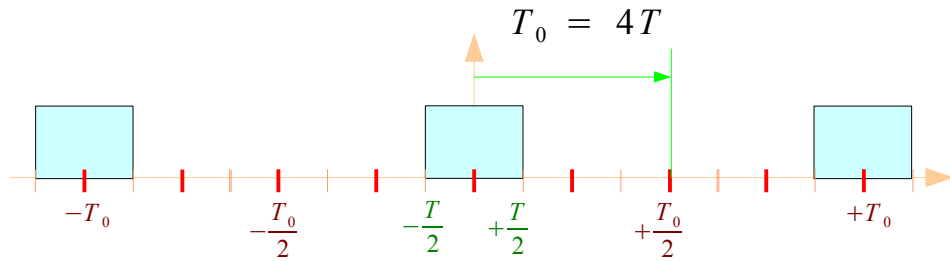
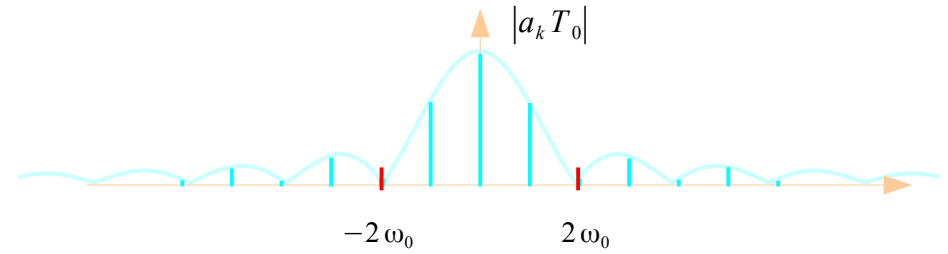
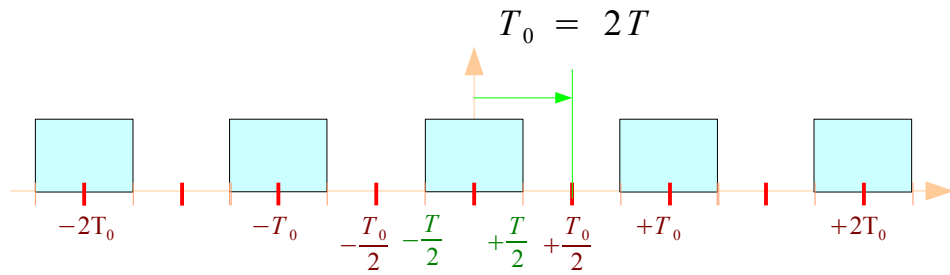
Aperiodic Continuous Time Signal Continuous Time Fourier Transform



Periodic Continuous Time Signal Continuous Time Fourier Series



CTFT and CTFS (3)



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997
- [5] AVR121: Enhancing ADC resolution by oversampling
- [6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp