DFT Analysis (5B)

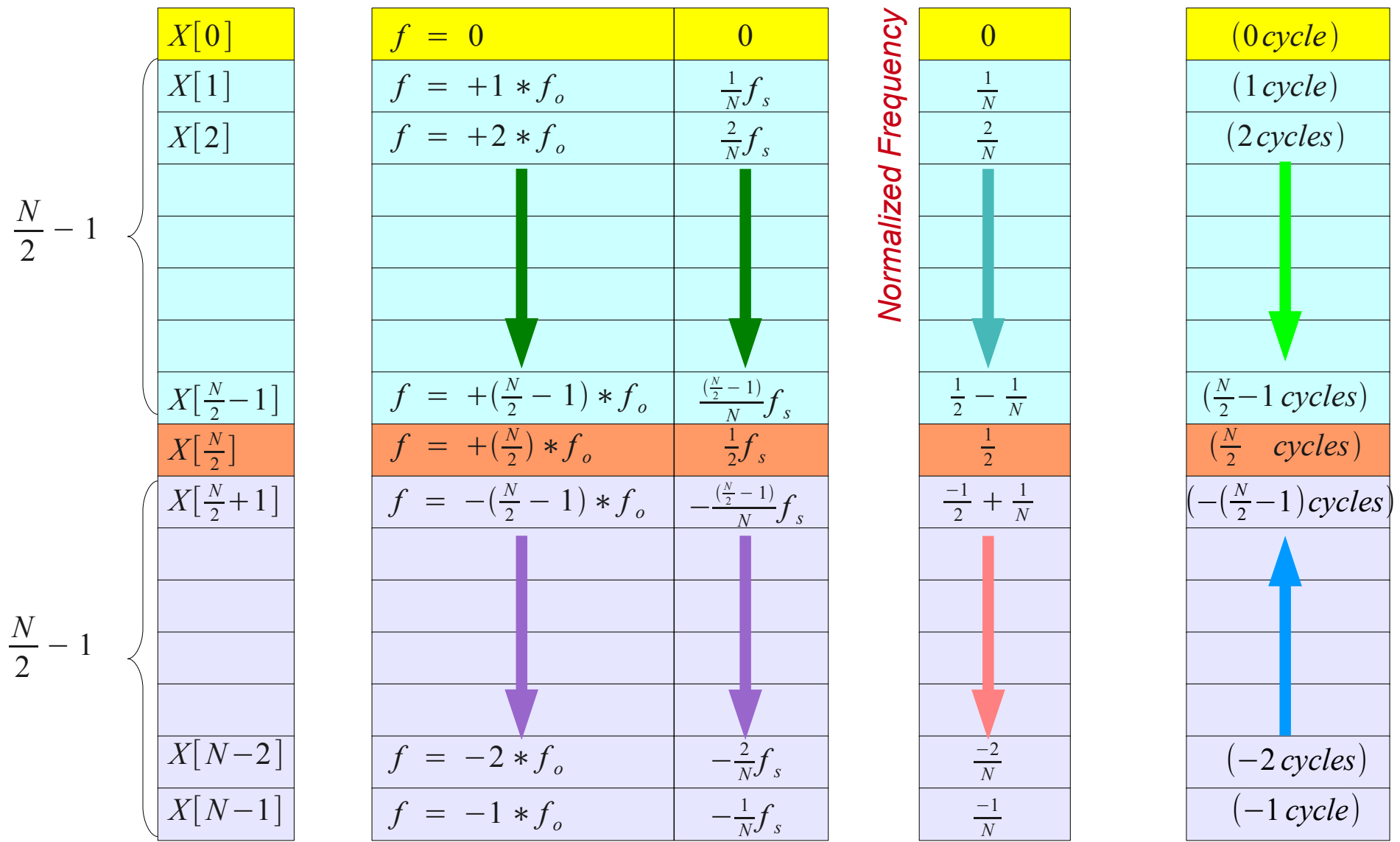Young Won Lim
5/18/11

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Frequency View of a DFT Matrix

row 0	$f = 0$	$(0\,cycle)$	0
row 1	$f = -1 * f_o$	$(-1\,cycle)$	$\frac{-1}{N}$
row 2	$f = -2 * f_o$	$(-2\,cycles)$	$\frac{-2}{N}$
row $\left(\frac{N}{2}-1\right)$	$f = -\left(\frac{N}{2}-1\right) * f_o$	$\left(-\left(\frac{N}{2}-1\right)cycles\right)$	$\frac{-1}{2} + \frac{1}{N}$
row $\left(\frac{N}{2}\right)$	$f = -\left(\frac{N}{2}\right) * f_o$	$\left(\frac{N}{2}\quad cycles\right)$	$\frac{-1}{2}$
row $\left(\frac{N}{2}+1\right)$	$f = +\left(\frac{N}{2}-1\right) * f_o$	$\left(\frac{N}{2}-1\,cycles\right)$	$\frac{1}{2} - \frac{1}{N}$
row $N-2$	$f = +2 * f_o$	$(2\,cycles)$	$\frac{2}{N}$
row $N-1$	$f = +1 * f_o$	$(1\,cycle)$	$\frac{1}{N}$

$\dfrac{N}{2} - 1$

$\dfrac{N}{2} - 1$

Normalized Frequency

$f_o = \dfrac{f_s}{N}$
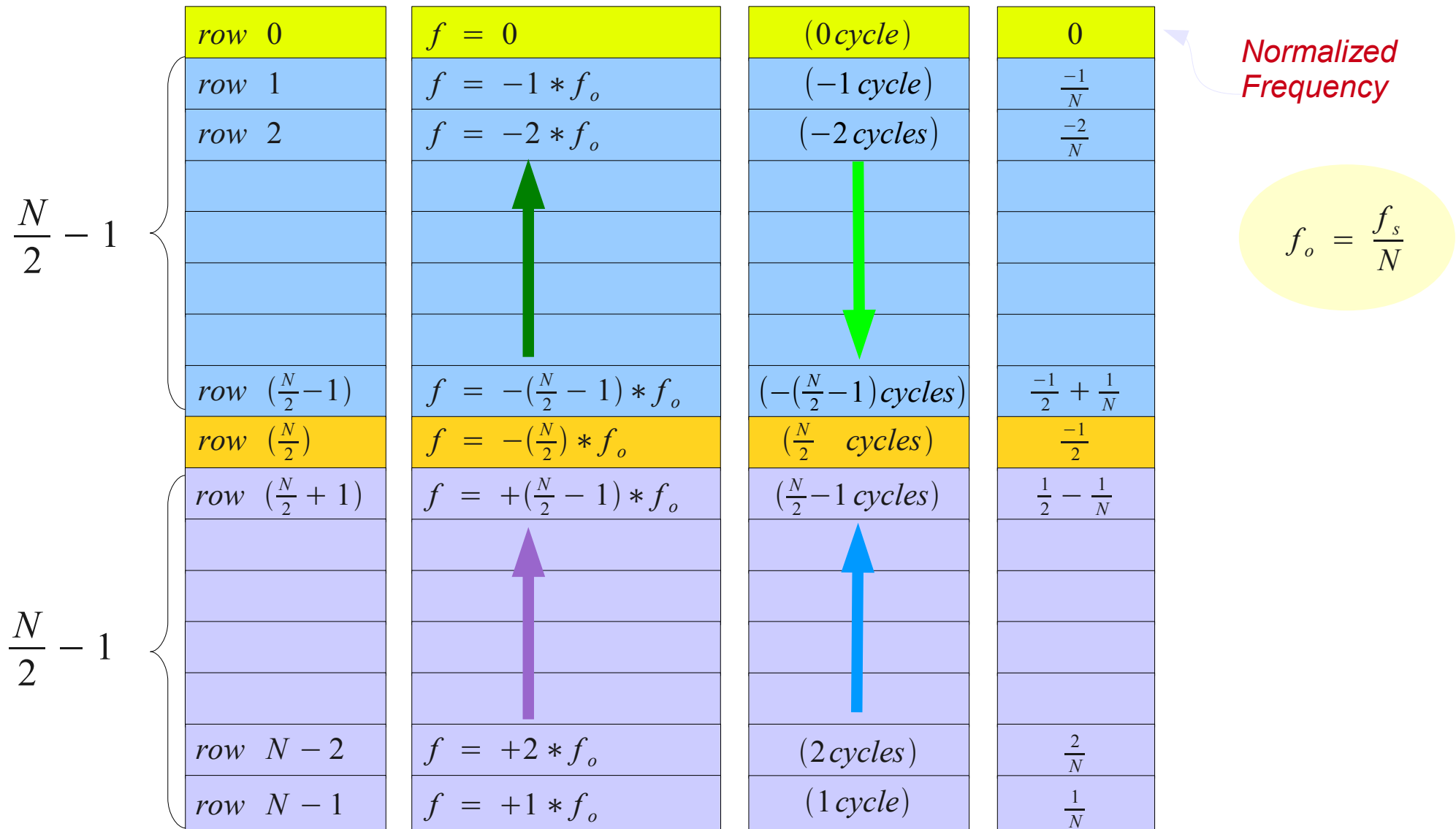
Frequency View of a X[i] Vector

			Normalized Frequency	
$X[0]$	$f = 0$	0	0	$(0\,cycle)$
$X[1]$	$f = +1 * f_o$	$\frac{1}{N}f_s$	$\frac{1}{N}$	$(1\,cycle)$
$X[2]$	$f = +2 * f_o$	$\frac{2}{N}f_s$	$\frac{2}{N}$	$(2\,cycles)$
$X[\frac{N}{2}-1]$	$f = +(\frac{N}{2}-1) * f_o$	$\frac{(\frac{N}{2}-1)}{N}f_s$	$\frac{1}{2}-\frac{1}{N}$	$(\frac{N}{2}-1\,cycles)$
$X[\frac{N}{2}]$	$f = +(\frac{N}{2}) * f_o$	$\frac{1}{2}f_s$	$\frac{1}{2}$	$(\frac{N}{2}\quad cycles)$
$X[\frac{N}{2}+1]$	$f = -(\frac{N}{2}-1) * f_o$	$-\frac{(\frac{N}{2}-1)}{N}f_s$	$\frac{-1}{2}+\frac{1}{N}$	$(-(\frac{N}{2}-1)\,cycles)$
$X[N-2]$	$f = -2 * f_o$	$-\frac{2}{N}f_s$	$\frac{-2}{N}$	$(-2\,cycles)$
$X[N-1]$	$f = -1 * f_o$	$-\frac{1}{N}f_s$	$\frac{-1}{N}$	$(-1\,cycle)$

$\frac{N}{2}-1$

$\frac{N}{2}-1$

$$\frac{f_s}{4}$$

$$\left[0, \quad \frac{f_s}{2}\right]$$

4-pt DFT

$$T = 4\tau$$

$$f_s = \frac{1}{\tau}$$

$$\frac{f'_s}{8}$$

$$= \frac{f_s}{4}$$

$$\left[0, \quad f_s\right]$$

8-pt DFT

$$\tau/2$$

$$T = 8\tau$$

$$f'_s = \frac{2}{\tau}$$

$$= 2f_s$$

$$\dfrac{f_s}{8}$$

$$f_s = \dfrac{1}{\tau}$$

τ

=

8-pt DFT

$$\left[0, \quad \dfrac{f_s}{2}\right]$$

$$2\mathrm{T} \ = \ 8\tau$$
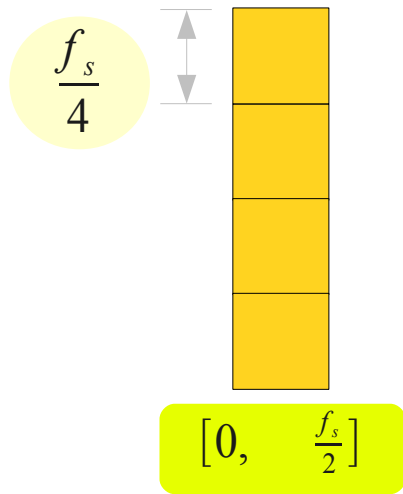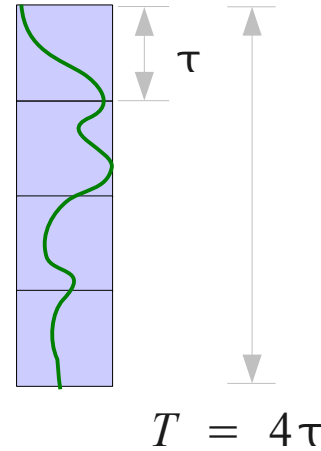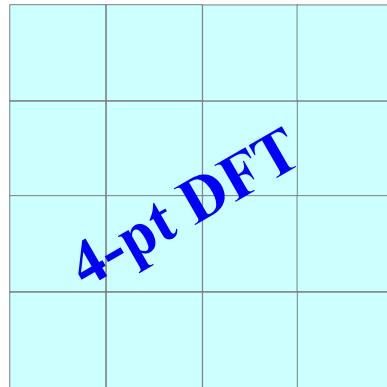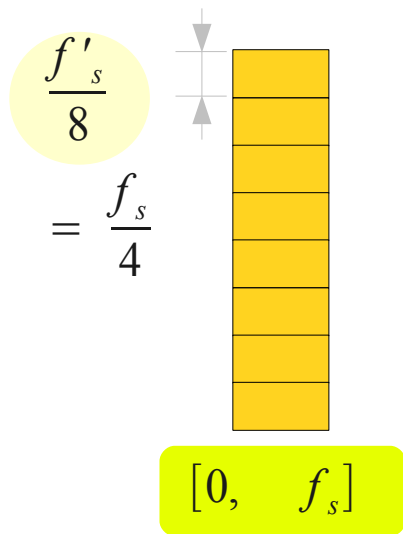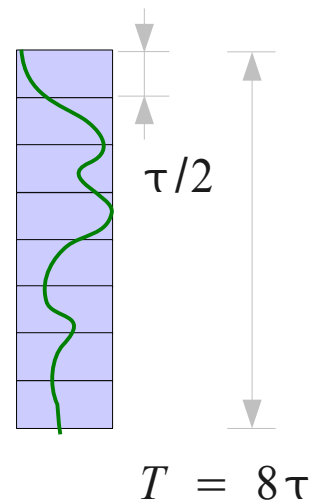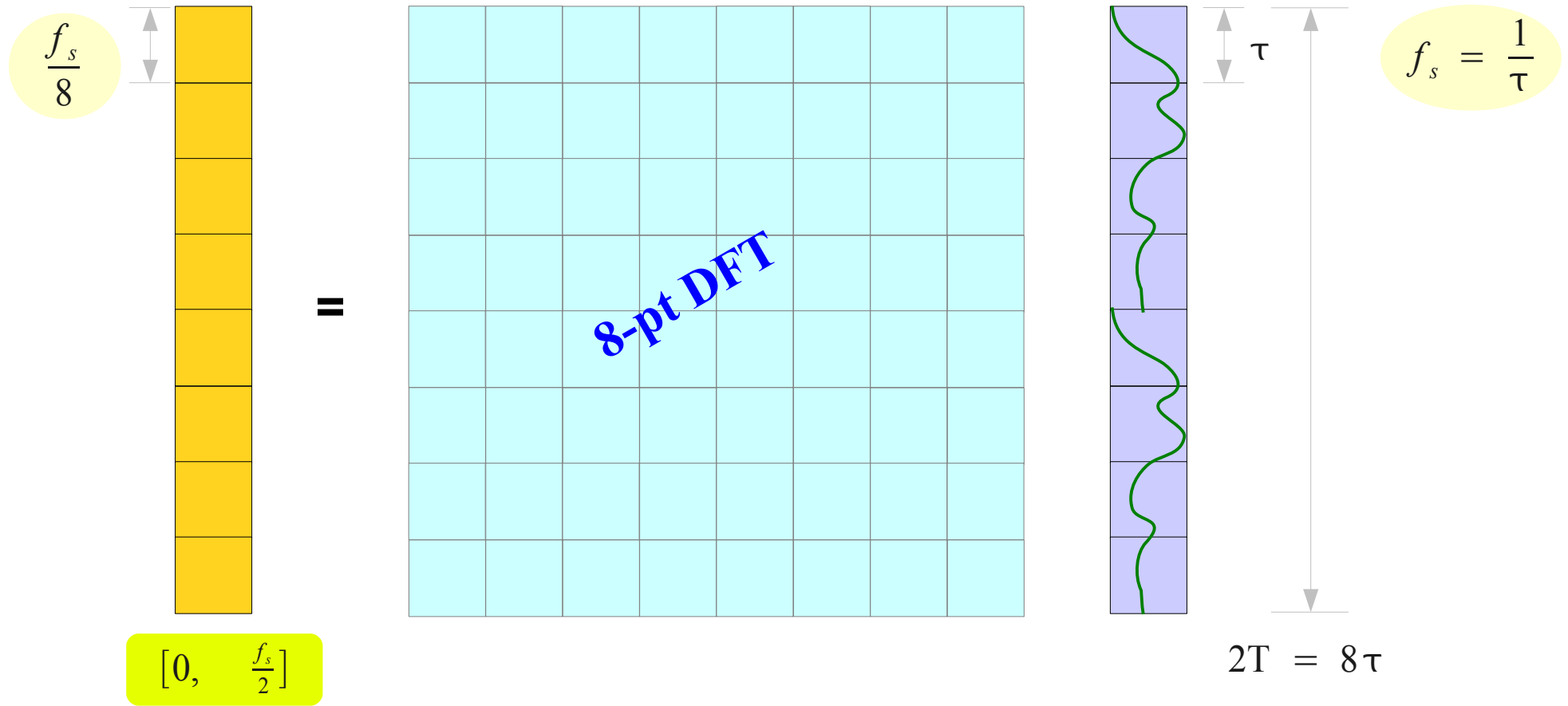
Data Truncation
Frequency Resolution
Zero Padding
Periodogram
Spectral Plot

Amplitude spectrum in quantity peak
Phase spectrum in radians
Amplitude spectrum in volts rms
Phase spectrum in degrees
Power spectrum

Signals without discontinuity
Signals with discontinuity

Sampling frequency is not an integer multiple
of the FFT length

8

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann