2.9.1

Show $|E_1| \leq \frac{(b-a)^3}{12} \cdot M_2 = \frac{h^3}{12} M_2 \qquad h = b-a$

$h = b-a$

Starting with $|E_n| \leq \frac{M_{n+1}}{(n+1)!} \int_a^b |q_{n+1}(x)| dx$

for $n=1$ $\quad q_2 = (x-a)(x-b)$

note $M_{n+1} := f^{(n+1)}(\xi)$

$$|E_1| \leq \frac{M_{n+1}}{(n+1)!} \int_a^b |\overset{\geq 0}{(x-a)}\overset{\leq 0}{(x-b)}| dx$$

$$= \frac{M_2}{2} \int_a^b \overset{\geq 0}{(x-a)} \overset{\geq 0}{(b-x)} dx$$

$$= \frac{M_2}{2} \int_a^b (-x^2 + bx - ab + ax) dx \qquad (*)$$

$$= \frac{M_2}{2} \left(-\frac{x^3}{3} + \frac{bx^2}{2} - abx + \frac{ax^2}{2} \right) \Big|_a^b$$

$$= \left(-\frac{b^3}{3} + \frac{b^3}{2} - ab^2 + \frac{ab^2}{2} + \frac{a^3}{3} - \frac{ba^2}{2} + a^2b - \frac{a^3}{2} \right) \frac{M_2}{2}$$

$$\frac{1}{6} \Big(\underbrace{(3b^3 - 2b^3)}_{b^3} + \underbrace{(2a^3 - 3a^3)}_{-a^3} + \underbrace{(3ab^2 - 6ab^2)}_{-3b^2a} + \underbrace{(6a^2b - 3a^2b)}_{+3a^2b} \Big) \cdot \frac{M_2}{2}$$

note $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 \rightarrow |E_1| = \frac{1}{6}(b-a)^3 \frac{M_2}{2} = \frac{h^3 M_2}{12}$