

Mathematics for natural sciences I**Exercise sheet 26****Warm-up-exercises**

EXERCISE 26.1. Determine the partial fraction decomposition of

$$\frac{3X^5 + 4X^4 - 2X^2 + 5X - 6}{X^3}.$$

EXERCISE 26.2. Determine the coefficients in the partial fraction decomposition of Example 26.5 by replacing X with some numbers.

EXERCISE 26.3. Determine the complex and the real partial fraction decomposition of

$$\frac{1}{X^2(X^2 + 1)}.$$

EXERCISE 26.4. Determine the complex partial fraction decomposition of

$$\frac{1}{X^3 - 1}.$$

EXERCISE 26.5. Determine the complex and the real partial fraction decomposition of

$$\frac{1}{X^3(X - 1)^3}.$$

EXERCISE 26.6. Determine the complex and the real partial fraction decomposition of

$$\frac{X^3 + 4X^2 + 7}{X^2 - X - 2}.$$

EXERCISE 26.7. Determine the complex and the real partial fraction decomposition of

$$\frac{X^7 + X^4 - 5X + 3}{X^8 + X^6 - X^4 - X^2}.$$

EXERCISE 26.8. Determine an antiderivative of the function

$$\frac{1}{x^2 + 5}.$$

EXERCISE 26.9. Determine an antiderivative of the function

$$\frac{1}{x^2 - 5}.$$

EXERCISE 26.10. Determine an antiderivative of the function

$$\frac{1}{2x^2 + x - 1}.$$

EXERCISE 26.11. Determine an antiderivative of the function

$$\frac{5x^3 + 4x - 3}{x^2 + 1}$$

through partial fraction decomposition.

EXERCISE 26.12. We consider the function

$$f : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R}, x \longmapsto \frac{x^5 + 3x^3 - 2x^2 + x - 1}{(x - 1)^2(x^2 + 1)}.$$

- a) Determine the real partial fraction decomposition of f .
- b) Determine an antiderivative of f for $x > 1$.

EXERCISE 26.13. Find a representation of the rational number $1/60$ as a sum of rational numbers, such that every denominator is a power of a prime number.

Hand-in-exercises

EXERCISE 26.14. (4 points)

Write the rational function

$$\frac{2x^3 - 4x^2 + 5x - 1}{4x + 3}$$

in the new variables $u = 4x + 3$. Compute the antiderivative through the real partial fraction decomposition and through the substitution $u = 4x + 3$.

EXERCISE 26.15. (4 points)

Determine the complex and the real partial fraction decomposition of

$$\frac{1}{X^4 - 1}.$$

EXERCISE 26.16. (4 points)

Determine the complex and the real partial fraction decomposition of

$$\frac{1}{X(X-1)(X-2)(X-3)}.$$

EXERCISE 26.17. (4 points)

Determine an antiderivative of the function

$$\frac{1}{1+x^4}.$$

EXERCISE 26.18. (5 points)

Determine an antiderivative of the function

$$\frac{3x-5}{(x^2+2x+7)^2}.$$

EXERCISE 26.19. (1 point)

Determine an antiderivative of the function

$$\frac{7x^6 - 18x^5 + 8x^3 - 9x^2 + 2}{x^7 - 3x^6 + 2x^4 - 3x^3 + 2x - 5}.$$