Prof. Dr. H. Brenner **Osnabrück WS 2011/2012**

Mathematics for natural sciences I

Exercise sheet 5

Warm-up-exercises

Exercise 5.1. M, P, S and T are the members of one family. In this case M is three times as old as S and T together, M is older than P and S is older than T, moreover the age difference between S and T is twice as large as the difference between M and P. Furthermore P is seven times as old as T and the sum of the ages of all family members is equal to the paternal grandmother's age, that is 83.

a) Set up a linear system of equations that expresses the conditions described.

b) Solve this system of equations.

Exercise 5.2. Kevin pays 2,50 € for a winter bunch of flowers with 3 snowdrops and 4 mistletoes and Jennifer pays 2,30 € for a bunch with 5 snowdrops and 2 mistletoes. How much does a bunch with one snowdrop and 11 mistletoes cost?

Exercise 5.3. We look at a clock with hour and minute hands. Now it is 6 o'clock, so that both hands have opposite directions. When will the hands have opposite directions again?

Exercise 5.4. Find a polynomial

$$f = a + bX + cX^2$$

with $a, b, c \in \mathbb{R}$, such that the following conditions hold.

$$f(-1) = 2,\ f(1) = 0,\ f(3) = 5\,.$$

Exercise 5.5. Find a polynomial

$$f = a + bX + cX^2 + dX^3$$

with $a, b, c, d \in \mathbb{R}$, such that the following conditions hold.

$$f(0) = 1,\ f(1) = 2,\ f(2) = 0,\ f(-1) = 1\,.$$

Exercise 5.6. Exhibit a linear equation for the straight line in $\mathbb{R}^2$, which runs through the two points $(2, 3)$ and $(5, -7)$.

Before the next tasks, we recall the concept of secant.

On the subset $T \subseteq \mathbb{R}$ it is given a function

$$f : T \longrightarrow \mathbb{R}$$

and two points $a, b \in T$, the straight line through $(a, f(a))$ and $(b, f(b))$ is called *secant* of f to a and b.

EXERCISE 5.7. Determine an equation of the secant of the function

$$\mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto -x^3 + x^2 + 2,$$

to the points 3 and 4.

EXERCISE 5.8. Determine a linear equation for the plane in $\mathbb{R}^3$, where the three points

$$(1, 0, 0), (0, 1, 2) \text{ and } (2, 3, 4)$$

lie.

EXERCISE 5.9. Given a complex number $z = a + bi \neq 0$, find its inverse complex number with the help of a real system of linear equations with two variables and two equations.

EXERCISE 5.10. Solve over the complex numbers the linear system of equations

$$\begin{array}{cccccc} ix & +y & +(2-i)z & = & 2 \\ & 7y & +2iz & = & -1+3i \\ & & (2-5i)z & = & 1\,. \end{array}$$

EXERCISE 5.11. Let K be the field with two elements of Example 2.3. Solve in K the inhomogeneous linear system

$$\begin{array}{cccccc} x & +y & & = & 1 \\ & y & +z & = & 0 \\ x & +y & +z & = & 0\,. \end{array}$$

EXERCISE 5.12. Show with an example that the linear system given by three equations I, II, III is not equivalent to the linear system given by the three equations I-II, I-III, II-III.

Hand-in-exercises

EXERCISE 5.13. (4 points)

Solve the following system of inhomogeneous linear equations.

$$\begin{array}{rcl} x+2y+3z+4w &=& 1 \\ 2x+3y+4z+5w &=& 7 \\ x \quad +z \quad &=& 9 \\ x+5y+5z+w &=& 0\,. \end{array}$$

EXERCISE 5.14. (3 points)

Consider in $\mathbb{R}^3$ the two planes

$E = \{(x,y,z) \in \mathbb{R}^3 |\, 3x+4y+5z = 2\}$ and $F = \{(x,y,z) \in \mathbb{R}^3 |\, 2x-y+3z = -1\}$.

Determine the intersection line $E \cap F$.

EXERCISE 5.15. (3 points)

Determine a linear equation for the plane in $\mathbb{R}^3$, where the three points

$$(1,0,2), (4,-3,2) \text{ and } (2,1,-1)$$

lie.

EXERCISE 5.16. (3 points)

Find a polynomial

$$f = a + bX + cX^2$$

with $a, b, c \in \mathbb{C}$, such that the following conditions hold.

$$f(i) = 1,\ f(1) = 1+i,\ f(1-2i) = -i\,.$$

EXERCISE 5.17. (4 points)

We consider the linear system

$$\begin{array}{rrrcr} 2x & -ay & & = & -2 \\ ax & & +3z & = & 3 \\ -\frac{1}{3}x & +y & +z & = & 2 \end{array}$$

over the real numbers, depending on the parameter $a \in \mathbb{R}$. For which a does the system of equations have no solution, one solution or infinitely many solutions?