

II. *A Letter to Dr. Mead, Coll. Med. Lond. & Soc. Reg. S. concerning an Experiment, whereby it has been attempted to shew the falsity of the common Opinion, in relation to the force of Bodies in Motion.* By Henry Pemberton, M. D. R. S. S.

S I R,

PERusing the Learned *Polenus's* Tract *de Castellis*, you were pleased to send me; I have found in it several curious Experiments, among which I reckon that of letting Globes of equal Magnitude, but of different weights, fall upon a yielding Substance, as Tallow, Wax, Clay or the like, from heights reciprocally proportional to the weights of the Globes. This Experiment engaged in particular my Attention, as it is brought with design to overturn one of the first Principles establish'd in Natural Philosophy. And the Knowledge I have of your great Esteem for that part of Science, emboldens me to trouble you with my Thoughts upon this Experiment; for I cannot by any means admit of the Deduction that is drawn from thence, that because the Globes make in this Experiment equal Impressions in the yielding Substance, therefore they strike upon it with equal force: whereby it is attempted to prove the Assertion of Mr. *Leibnitz*, that the force of the same Body in descending is proportional to the height from whence it falls; or, in all Motion, proportional to the Square of the Velocity, and not proportional to the Velocity it self, as is commonly thought. On the contrary, I think this very Experiment

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ment proves the great unreasonableness of Mr, *Leibnitz's* Notion.

I am surpriz'd, that so careful a Writer as *Polenus* appears to be, from the accuracy wherewith he delivers his Experiments, should not rather suspect his reasoning in an intricate Case, than thus contradict a Principle in Philosophy, that has been directly prov'd by a multitude of Experiments, in particular by those Sir *Isaac Newton* recommends for that purpose (a) ; and that is moreover abundantly establish'd by its exact Agreement with all Observations ; as being the Principle upon which all appearances, hitherto observed in the motion of Bodies, are accounted for by just and undeniable Arguments ; and we shall find on Inquiry, that the present Case comes also under the same Rule.

As the use of Experiments in Natural Philosophy is to discover the Causes of Things, by exhibiting more simple Effects of those Causes, than occur in the ordinary course of Nature ; so for this end it is necessary, that our Argumentation upon Experiments be perfectly just, otherwise they may lead us into Errors. The first thing necessary for making right Deductions from an Experiment, is to determine the proper use thereof ; which I think in this before us is not rightly understood. Certainly this Experiment of *Polenus* is much more fit to inform us of the Law, by which these yielding Substances resist the motion of Bodies striking upon them, than to shew the forces, with which Bodies strike ; for whatever those forces be, the Effects must be very different, according to the Difference there may be in the Rule observed by such resistance.

(a) Princip. Philos. Nat. p. 19.

Now this Experiment shews, that if two Globes in Motion bear against equal Portions of the yielding Substance, the Opposition, that Substance makes to the Motion of the Globes, will be the same in both, however different the Velocities be, with which they move. This I shall demonstrate as follows.

Let *A* and *B* be two Globes, equal in Magnitude, but of different Weights, which are equally immersed into a yielding Substance. Suppose the Velocities, with which they move in their present Situation, to be reciprocally in the Subduplicate *ratio* of the Weights of the Globes; that is, let the *ratio* of the Weight of the Globe *A* to the Weight of the Globe *B*, be Duplicate of the *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. Since therefore the *ratio* of the quantity of Motion in the Globe *A*, or of the force with which it moves, to the quantity of Motion in the Globe *B*, or to the force with which that Globe moves, is compounded of the *ratio* of the Weight of the Globe *A*, to the Weight of the Globe *B*, and of the *ratio* of the Velocity of the Globe *A*, to the Velocity of the other Globe *B*, the force, with which the Globe *A* moves, is to the force, with which the Globe *B* moves, as the Velocity of this Globe *B*, to the Velocity of the other Globe *A*. But if the same Opposition be made to the Motion of the Globes, when they bear upon equal Portions of the yielding Substance, the Effect of that Opposition, while the Globes enter farther into the Substance by equal Spaces, will be proportional to the time, in which the Globes are moving those Spaces, or in which the Opposition is made, if we consider those Spaces while nascent or in their first Origine; the Effect therefore of this Opposition will be reciprocally proportional to the Velocity of each Globe: namely, the momentaneous loss of force in the Globe

A will be to the momentaneous loss of force in the Globe *B*, as the Velocity of the Globe *B*, to the Velocity of the Globe *A*; and the whole force of the Globe *A* has been found to bear the same *ratio* to the whole force of the Globe *B*; consequently these Globes, while they penetrate equal Spaces into the Substance, lose parts of their force, which bear the same proportion to the whole: and therefore, if their Velocities be at any time reciprocally in the Subduplicate *ratio* of their Weights, so that the forces or degrees of Motion, with which they move, be reciprocally proportional to their Velocities, the forces, with which they press into the yielding Substance, at equal Indentures made in the Substance, will continue in the same Proportion; and therefore upon the Theory of Resistance here supposed, when the whole Force and Motion of both these Globes is entirely lost, they will be plunged into the Substance at equal Depths.

Now whereas in the Experiment of *Polenus*, the Globes, falling from Heights reciprocally proportional to their Weights, strike upon the yielding Substance with Velocities reciprocally in the Subduplicate proportion of their Weights, and the Effect is in all Cases found to be, what is here deduced from the Theory of Resistance, I have proposed; it is a sufficient Confirmation of the Truth of this Theory.

Only here, Sir, I ought to observe to you, That I have supposed the Globes to be stopt by the whole Resistance of the Substance, they move against; although in strictness they are stopt only by the excess of that Resistance above the Action of Gravity upon them. But I have neglected the Consideration of the Action of Gravity, that being but small in Proportion to the Resistance, as will appear from the Globes being much more speedily stopt by this Resistance, than they

they would be by the Action of Gravity, if its force were applied upwards; for by that force alone, the Globes would not be stopt, till they had measured Spaces equal to the heights above the resisting Substance, from whence they fell; which heights bear a great Proportion to the depths, the Globes in this Experiment are immerfed into the yielding Substance, as I have found upon trial.

Thus, if I mistake not, may be removed the Difficulty attending this Experiment. But as Mr. *Leibnitz's* Opinion is deduced from it by Means of this Axiom, that Effects are proportional to their Causes; so that here the Effects being thought the same, the Causes are concluded to be so likewise; it will not be amiss to mention in this place an Experiment, where this Axiom may be more justly applied, than it can be in our present case, from which Experiment the received Opinion may be proved. This Experiment is mentioned by *Pol-nus* from *Merfennus*, though tried by him somewhat rudely; but has been often since made in the following manner. To one end of the Beam of a Ballance is hung a Weight, and from a proper height is let fall upon the other extremity of the Beam another Weight, which by striking thereon shall raise the end, to which the Weight is suspended, to such a height, as is just sufficient to set free a certain Spring. If then a different Weight be hung in the room of the former; the height, from whence the falling Weight must descend, in order to raise the Extremity of the Ballance, to which this other Weight is suspended, to the same height as before, that is, high enough to set the forementioned Spring at liberty, is found to be such, that the Velocity, with which the falling Weight strikes upon the Ballance, in this latter Case, will be to its former Velocity, as the latter Weight
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to the former, (a) except only that, agreeably to what *Mersennus* himself remarked, when the Weight descends from great heights, an Altitude somewhat greater than this Rule implies is required to raise the other Weight as much as is desired. But whether the bending of the Arm of the Ballance, when acted upon with a great force, or whether any increase of Friction in this Case, occasion the Irregularity here mentioned, we need not strictly enquire; for this Irregularity is yet less reconcilable with the new Opinion, than the regular Effects of the Experiment. Hence therefore we may see, that the very Method of reasoning, which being applied erroneously, is supposed to prove Mr. *Leibnitz's* Sentiment concerning the force of Bodies in Motion, will, when justly used, confirm the other Opinion in relation to that Matter.

But as I have asserted in the beginning of this Letter, that the very Experiment of *Polenus* is not only reconcilable to the common Doctrine of Motion, as I have now demonstrated; but even that it does it self make manifest the great unreasonableness, if not the absolute Absurdity, of Mr. *Leibnitz's* Opinion; it remains that I briefly make proof of this.

If two Globes *A* and *B*, of equal Magnitude but of different Weights, striking on a yielding Substance with equal force, in every Case lose all their Motion at equal Depths, it is necessary that at all times, during their Motion, they lose equal Degrees of force, when they bear upon equal Portions of the Substance, in entering equal Spaces into the Substance. This will be easily seen from what has before been said. Now whereas Mr. *Leibnitz* supposes the power of Gravity to

(a) *Vid.* 's *Gravesande* *Physic. Elem.* Tom. I. p. 39.

give to the same falling Body Degrees of force proportional to the height from whence it falls; according to his Opinion, by the power of Gravity, equal Degrees of force are added in the descent of the same Body through equal Spaces; and in different Bodies descending through equal Spaces, the Degrees of force added will be as the quantity of Matter, or as the weight of each Body. Therefore while the Globes *A* and *B* penetrate equal nascent Spaces into the yielding Substance, by the Action of Gravity, were not that Action overcome by the Resistance of that Substance, additional Degrees of force would be communicated in such proportion, that the force added to the Globe *A*, would be to the force added to the Globe *B*, as the weight of the Globe *A*, to the weight of the Globe *B*, or in the Duplicate *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. But since the Globes lose the same Degrees of force in entering equal Nascent Spaces into the yielding Substance, the Effect of the Opposition made by this Substance to the Motion of the Globes, during the time of their passing through such Nascent Spaces, will be both the taking from them that same Degree of force, and moreover the additional force, which would otherwise have been given them by their own Gravity. But farther, the Opposition made to the motion of the Globe *A*, to the Opposition made to the motion of the Globe *B*, will be in the *ratio* compounded of the *ratio* of the Effect of the Opposition, the Substance makes to the motion of the Globe *A*, to the Effect of the Opposition, the Substance makes to the motion of the Globe *B*, and of the *ratio* of the Time, in which the Opposition is made against the latter Globe, to the Time in which it is made against the former; which latter *ratio* is the same

same with the *ratio* of the Velocity of the Globe *A*, to the Velocity of the Globe *B*. But since it is shewn, that the Effect of the Opposition made by the yielding Substance to these Globes is two-fold, and that one part of the Effect of the Opposition made to the motion of the Globe *A*, is equal to one part of the Effect of the Opposition made to the motion of the Globe *B*; and that another part of the Effect of the Opposition made to the motion of the Globe *A*, to another part of the Effect of the Opposition made to the motion of the Globe *B*, is in the Duplicate *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. One part of the Opposition it self made to the motion of the Globe *A*, will be to one part of the Opposition against the motion of the Globe *B*, as the Velocity of the Globe *A*, to the Velocity of the Globe *B*, and another part of the Opposition to the motion of the Globe *A*, to another part of the Opposition to the motion of the Globe *B*, will be as the Velocity of the Globe *B*, to the Velocity of the Globe *A*. So that when the Globes bear upon equal Portions of the yielding Substance, the Opposition to their Motion will be in part as the Velocity of the Globes, and in part reciprocally as their Velocity. Hence, because the resisting Substance is of an uniform Texture, the Opposition to the Motion of either of the Globes in its present Situation, and when moving with its present Velocity, will be to the Opposition it would meet with in the same Situation, if it moved with any other Velocity, partly as the present Velocity to that other Velocity, and partly as that other Velocity to the present. But by that part of the Opposition made against the motion of the Globe, which is directly as the Velocity, the Globe can never be wholly stoppt; for upon the stopping of the Globe, that part of the Opposition

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tion to its Motion will likewise totally cease, and consequently the Globe's Weight will carry it further down, unless the other part of the Opposition against its Motion prevent it: But I say again, neither can this latter part of the Opposition made to its Motion be ever great enough to stop the Globe; for the Degree of this Opposition being reciprocally as the Velocity of the Globe, when the motion of the Globe is wholly taken away, it will become infinitely greater, than at any time, while the Globe is in Motion; so that when the Globe should be stoppt by this part of the Opposition made to its Motion, the Opposition to the Globe's Motion will become infinitely great; insomuch, that no Degree of force whatever could be able to impel the Globe further into the Substance, but this can never come to pass. Besides, it is not necessary to apply any such refined Argument against this part of the Resistance; it would be alone sufficient to consider, how unreasonable a Supposition it is, that a Resistance should increase, when the Velocity of the resisted Body decreases.

Thus may this Experiment be made use of to invalidate that very Opinion it is brought to support. But another use may likewise be made thereof: For it will serve to illustrate what the great Sir *Isaac Newton* has more than once hinted, that the Resistance of Fluids, which arises from the Tenacity of their Parts, decreases in a less Proportion than the Velocity of the resisted Body decreases (*a*); for as this Resistance bears a great Analogy to the Resistance of the yielding Substances we have been here treating of, so we have found, that the Resistance of these Substances does

(*a*) *Vid.* Philos. Nat. Princip. Math. Prop. 52. lib. 2. in Schol. Opticks. Qu. 28. p. 339, 340.

not much depend upon the Velocity of the Body, against which the Resistance is applied.

And thus, Sir, we may see how all Experience conspires in confirming and setting forth that stupendous force of reasoning, which has enabled our great Philosopher most surprizingly to search out, and distinguish the Springs of Natural Operations; a Work infinitely more difficult to accomplish than even the great Improvements he has made in pure Mathematicks, which were previously necessary in order to his succeeding in his Searches after the Knowledge of Nature; for in this last Pursuit he has given Proof, not only of a more unbounded Invention, than is required in the subtillest Geometrical Speculations; but has also there discovered the greatest Discernment, and most consummate Judgment; since in his Philosophical Writings, he has never been once imposed on by an Hypothesis, nor by any other of the various Fallacies, which my Lord Bacon in his *Novum Organon*, has reckon'd up as the Causes, that had hindred the Improvement of the true Philosophy.

But here I shall put an end to this long Letter; for the Freedom of which, I know, I need make no Apology to you, Sir, of whose great Candour I have for some Years past been a constant Witness; and as I have frequently admired, how amidst the vast Employments of your Profession, you should find time to pursue with so great Success such various sorts of Learning; so I have as often been delighted to observe with what Benignity you receive those, who have made any the least part of useful Knowledge their Study.

I am, Sir, &c.

H. P.

POST-SCRIPT.

ABout a Week after I had sent you the Letter, containing my Observations on *Polenus's* Experiment, I had the good Fortune to hear an excellent and learned Friend of yours, to whom you had been pleased to shew my Letter, give a very curious and weighty Argument to confirm Sir *Isaac Newton's* Sentiment in relation to the Resistance of Fluids, which I had deduced from the above mentioned Experiment; and as this very much pleased me, I shall here endeavour to send you an Account of it in the following manner:

Suppose pieces of fine Silk, or the like thin Substance, extended in Parallel Planes, and fixt at small Distances from each other. Suppose then a Globe to strike perpendicularly against the middle of the outermost of the Silks, and by breaking through them to lose part of its Motion. If the pieces of Silk be of equal Strength, the same Degree of force will be required to break each of them; but the Time, in which each piece of Silk resists; will be so much shorter as the Globe is swifter; and the loss of Motion in the Globe consequent upon its breaking through each Silk, and surmounting the Resistance thereof, will be proportional to the Time, in which the Silk opposes itself to the Globe's Motion; insomuch that the Globe by the Resistance of any one piece of Silk will lose so much less of its Motion as it is swifter. But on the other Hand, by how much swifter the Globe moves,

fo many more of the Silks it will break through in a given Space of Time; whence the number of the Silks, which oppose themselves to the Motion of the Globe in a given Time, being reciprocally proportional to the Effect of each Silk upon the Globe, the Resistance made to the Globe by these Silks, or the loss of Motion, the Globe undergoes by them in a given Time, will be always the same.

Now if the Tenacity of the Parts of Fluids observes the same Rule as the Cohesion of the Parts of these Silks; namely, That a certain Degree of force is required to separate and disunite the adhering Particles, the Resistance arising from the Tenacity of Fluids must observe the same Rule as the Resistance of the Silks, and therefore in a given Time the loss of Motion, a Body undergoes in a Fluid by the Tenacity of its Parts, will in all Degrees of Velocity be the same; or in fewer Words, that part of the Resistance of Fluids, which arises from the Cohesion of their Parts, will be Uniform.
