

Porro, cum Angulum sic, ut dictum est, definiverat, p. 67; subjungit, p. 68. *Quodsi magnitudines illa sint dua lineæ, comprehensus ab iis angulus, Planus vocabitur*: quasi quidem de *Triangulis sphericis* nil unquam inaudiverit; nec alius esse possit superficialis angulus, quam in *Plano*.

Adhæc, illud *duarum pluriumve*, de *Lineis* non tuto dicitur. *Trium* enim linearum concursus, non angulum, sed angulos saltem duos, constituunt; non enim lineæ plures duabus ad unum superficialem angulum constituendum concurrunt. Item, cum p. 67. Angulum in genere per *duarum pluriumve*, &c. definiverat; Angulum p. 68. *una vel pluribus* superficiebus comprehensum ait (& unâ quidem angulum verticalem Coni comprehensum,) quasi quidem *una*, fuerit, *dua vel plures*.

Insuper, quid demum illud est, quod per *brevissimam distantiam* insinuatum vult? Quippe in ipso concursus puncto, *Nulla est distantia*; extra illud, *nulla minima*: nulla utique assignari poterit, qua non sit minor: sed revera tota hæc, quam de *Angulo* notionem concipit, est parum sana. Definendus utique est non per *distantiam* seu *remotionem*, sed per *Inclinationem*. quod ex *Euclidis* definitione didicisset.

Deniq; (ne multus nunc sim) p. 171. in duabus his Quadraticarum æquationum formulis $aa - ca + dd = 0$, & $aa + ca + dd = 0$; utramque radicem *affirmativam* esse pronunciat. quod omnino secus est. Et quidem in priore, Radix utraque Affirmativa; sed in posteriore, Negativa utraque.

Atque hæc quidem, ex multis pauca, si non sufficiant, ut ex ungue Leonem æstimes, plura facile congerentur. Num autem hos *Incuria*, an *Inscitia*, errores fuderit (prout ipse *pag. ult.* distinguit) non determino. Vale.

Hæc Dn. Wallisus epistola una; cui postea submisit alteram, 18. *Julii* ad me scriptam, quam istoc mense, ob alia, non licebat typis committere; nec quidem licet hoc ipso: ne scilicet hæc schedulas, publicationi veteriorum, idque imprimis sermone *Anglico*, destinatas, disceptationibus *Latinis* compleamus. Proxima occasione, quæ idem *Author* porro notanda invenit vel in *unico primo* Capite *Synopses Laurentiana*, Lectori (cum particularia flagitet Dn. Du Laurens) ob oculos sistemus.

An Account of Two Books.

I. *R. de GRAAF Med. D. de VIRORUM ORGANIS GENERATIONI INSERVIENTIBUS, &c.* Ludg. Bat. 1668. in 12°.

This Treatise was promised by the Author in a printed *Epistle* of his, which we gave an account of in *April* last, *Num. 34.* p. 663. There being at the same time publisht a *Prodromus* of *Job. Van Horne*, suspecting, that the *Observations* of *De Graef* were much the same with his upon this Subject; we do now upon the perusal of this Book, find chiefly these considerable Differences between them.

p. 663.

First, the said *Van Horne* makes the *Spermatick Artery* in man to goe to the *Testicles* in a winding, but *De Graaf*, in a streight way.

Secondly, the former affirms, that the *vasa deferentia* have no communication with the *vesiculae seminales*; but the latter maintains, and demonstrateth it to the Ey, there is so great a commerce betwixt them, *ut semen dum à Testibus per vasa differentia affluens in Urethram effluere nequit, propter carunculam clausam; necessariò insuat in Vesiculas, in iisque pro futuro coiture reservetur.*

Thirdly, the former is of opinion, *triplicem esse materiam seminis*; but *De Graaf* will have but *one only*; answering the Arguments, used both by *Van Horne* and *Dr. Wharton* to prove that *triplicity*.

But that, which *De Graaf* much insists on in this Book, is, to shew what is the *true Substance* of the *Testicles*, and to vindicate the Discovery thereof to himself; affirming positively, that no man, before him ever knew the truth

of it. * For the making out of which, he first denyeth, that the *Testes* are *glandulous*, or *pultaceous*; and then affirms that their substance is nothing else

* See the Letter of Doctor Tim. Clark, N. 35. p. 681.

but a *Congeries minutissimorum vasculorum semen conficiendam*, *qua si absque ruptione dissoluta sibi invicem adnecteretur, facile viginti ulnarum longitudinem excederent.* Which he affirms, he can prove by ocular Demonstration.

Then he sheweth, how the seminal vessels pass *à Testibus ad Epididymides*, vid. not by *one Trunck* (as *Dr. Highmore* thinks) but by 6. or 7. small *ductus*'s; assigning the cause, why *Doctor Highmore* did not see them.

Further he examines, *An semen in testibus conficiatur; utrum ex Sanguine vel ex Lympha: quomodo elaboretur, crassescat, lactescat: qua via à Testibus ad Urethram excurrat.*

Moreover he endeavours to prove, *Vesiculas seminales ordinatas esse non seminis generationi, sed receptioni & asservationi.*

He also observeth concerning the *seminal matter*, that 'tis composed *ex duplici materia*, which after *Aristotle*, he calls *λίρον αεζυματικόν και ὄγκον περιμαλινόν*, considering this twofold matter like *Dough* and *Ferment*, this infecting and quickning that, and the grosser part being; a conservatory and vehicle to that, which is most elaborate.

When he examines the *Penis*, he taketh notice, *Omnes ha. Te-*

nis Anatomicos perperam assignasse usum musculorum Penis, quos Erectores appellant; Eorum quippe provinciam non esse, Penem erigere, & dilatare Urethram, cum omnis Musculi actio sit contractio, quæ extensioni contraria est; eos potius Penem versus interiora retrahere quam erigere: Interim, hosce Penis Musculos, coarctando corpora nervosa circa eorum exortum, materiam seminalem versus Penis partem anteriorem propellere, atque hac ratione corporum nervosorum distensione erectionem augere.

Before we conclude this Account, we cannot but take notice, that the Author occasionally inserts in this Book divers curious and remarkable Examples and Observations; some whereof are.

1. Concerning those, that are born, either *absque Testibus*; or, *cum Testiculo uno*; or, *cum tribus, idque hæreditario per aliquot familias, admodum fecundas.*

2. About the *situs præternaturalis Testiculorum, generationis tamen virtutem non impediens.*

3. Concerning *lactescent Bloud* in a man living at *Delft* in *Holland*, whose Bloud alwayes turn'd into Milk, when let out either by venæ-sections, or by bleeding at the Nose, or by a wound. V. pag. 84, 85. Compare *Numb. 6.* pag. 100, 117, 118. and *Numb. 8.* pag. 139. of these *Transactions.*

4. Concerning the strange alteration made in Femals, *ab Aura seminali*: quod confirmat exemplo felis, diu sugentis (idque ad integram fere sui nutritionem) lac mammarum caniculæ, per aliquot annos à coitu prohibita; deinceps vero, postquam catella admiserat canem, nunquam ab eo tempore lac ex mammis ejus exsugere volentis.

5. About a strange *Hæmorrhagy per Penem*, which amounted to 14. pound, in a Porter of 52. years old, falling down with a heavy load upon a board, laid over a ditch, and so turning about, when the said porter trod upon it, that it cast him down upon its edge, turn'd between his legs; yet the Patient by the skill and care of our Author recover'd.

6. Various Observations of Clysters and Suppositories, cast up by Vomits, p. 195, 196.

7. Several wayes of performing unbloody dissections of Animals, p. 228, 229, &c.

II. LOGARITHMOTECNIA NICOLAI MERCATORIS.

Concerning which we shall here deliver the account of the Fudicious Dr. I. Wallis, given in a Letter to the Lord Vis-count Brouncker, as follows ;

Incidebam heri (Illustrissime Domine) in D. Mercatoris Logarithmotecniam, nuper editam. Quæ ita mihi placuit, ut non prius dimiserim quàm perlegissem totam. Et quamquam pauca quædam, Phraseologiam quod spectat seu loquendi formulas nonnullas, mutata mallet; sunt tamen ipsa sensu suo sana: Eaque quæ superstruitur Doctrina, Logarithmos expedite atque subtiliter construendi, perspicue satis atque ingeniose traditur.

Quæ huic subjungitur Quadratura Hyperbolæ, elegans admodum est atque ingeniosa. Nempe ad hunc sensum. V.Fig. 1.

Postquam in Hyperbola MBF, (cujus Asymptotæ AN, AH, ad angulum rectum coeunt) ostenderit, prop. 14, Rectangula BIA, FHA, spA, &c. (ductis BI, FH, sp, &c, parallelis Asymptotæ AN,) invicem esse æqualia; adeoque latera habere reciproce proportionalia; (quæ nota est Hyperbolæ proprietas:) Positis AI = BI = 1, & HI = a: ostendit, prop. 15.

FH = 1 / (1+a) (Nempe propter analogiam AH. AI :: BI. FH. hoc est.

1+a. I.: 1. 1 / (1+a). Sed & (quod Dividendo 1, per 1+a ostenditur,)

1 / (1+a) = 1 - a + a^2 - a^3 + a^4 &c.] (continuatis deinceps, ipsius a potestibus, alternatim negatis & affirmatis.)

1+a
-a
-a -a^2
+a^2
+a^2 +a^3
-a^3
-a^3 a^4
+a^4
&c.

Cumque hoc perinde obtineat, ubicunque ultra punctum I, ponatur H. Positis, ut prius AI = 1; hujusque continuatione qualibet, ut Ir = A; quæ intelligatur in æquales partes innumeras dividi, quarum quælibet, ut Ip, pq, &c. dicatur a; adeoque Ip, Iq, &c, sint a, 2a, 3a, &c. usque ad A: Quæ his respondent rectæ ps, qt, &c. usque ad ru, (spatium BI ru completes) sunt,

$$\begin{aligned}
 1 &- a + a^2 - a^3 + a^4 \&c. \\
 1 &- 2a + 4a^2 - 8a^3 + 16a^4 \&c. \\
 1 &- 3a + 9a^2 - 27a^3 + 81a^4 \&c. \\
 &\& \text{ sic deinceps usque ad} \\
 1 &- A + A^2 - A^3 + A^4 \&c.
 \end{aligned}$$

Cum itaque sint

$$\begin{aligned}
 1 + 1 + 1 \&c. \text{ (usque ad ultimum)} &= A \\
 a + 2a + 3a \&c. \text{ (usque ad } A) &= \frac{1}{2}A^2 \\
 a^2 + 4a^2 + 9a^2 \&c. \text{ (usque ad } A^2) &= \frac{1}{3}A^3 \\
 a^3 + 8a^3 + 27a^3 \&c. \text{ (usque ad } A^3) &= \frac{1}{4}A^4 \\
 &\& \text{ sic deinceps :}
 \end{aligned}$$

(quod ostendit ille prop. 16, estque à me alibi demonstratum:) Recte colligit, prop. 17. Expositum spatium Hyperbolicum $B I r u = A - \frac{1}{2}A^2 + \frac{1}{3}A^3 - \frac{1}{4}A^4 + \frac{1}{5}A^5, \&c.$ Adeoque si (assignato, ipsi $A = I r$, valore suo in numeris, ut res postulaverit,) distribuatur ih duas classes $A, \frac{1}{2}A^2, \frac{1}{3}A^3, \&c.$ (potestates affirmatæ,) & $\frac{1}{2}A^2, \frac{1}{3}A^3, \&c.$ (potestates negatæ;) harumque Aggregatum, ex Aggregato illarum, subducatur; Residuum erit ipsum $B I r u$ spatium Hyperbolicum.

Nequis autem operam lusum iri existimet,, propter Addendorum seriem in utraque classe infinitam; adeoque non absolvendam: Hinc incommodo medelam (tacitus) adhibet: ponendo $A = 0, 2 I$, vel $A = 0, 2 I$, aliive fractioni decimali æqualem, adeoque minorem quam 1 : (Hoc est, sumpta $I r$ minore quam $A I = 1$.) Quo fit, ut posteriores ipsius A potestates tot gradibus infra Integrorum sedem descendant, ut merito negligi possint.

Exempli gratia; positis $A I = 1$, & $I r = 0, 2 I$. erit

$$\begin{array}{ll}
 A = 0, 2 I & \\
 \frac{1}{2}A^2 = 0, 003087 & \frac{1}{3}A^3 = 0, 02205 \\
 \frac{1}{3}A^3 = 0, 000081682 & \frac{1}{4}A^4 = 0, 000486202 \\
 \frac{1}{4}A^4 = 0, 000002572 & \frac{1}{5}A^5 = 0, 000014294 \\
 \frac{1}{5}A^5 = 0, 000000088 & \frac{1}{6}A^6 = 0, 000000472 \\
 \frac{1}{6}A^6 = 0, 000000003 & \frac{1}{7}A^7 = 0, 000000016
 \end{array}$$

$$+ 0, 213171345 - 0, 022550984 = 0, 190620361 = B I r u$$

Quæ est brevis Synopsis Quadraturæ suæ satis elegans.

Diffimulandum interim non est; si quis totius $B I H F$ spatii (cujus latus $I H$ longius intelligatur quam $A I$) quadraturam postulet; rem non ita feliciter successuram: propter medelam, quam modo diximus, malo minus sufficientem. Cum enim jam ponenda sit $A > 1$; manifestum est, posteriores ipsius potestates, altius in Integrorum sedes penetraturas, adeoque minime negligendas.

Huic autem incommodo, levi constructionis immutatione, facile subvenitur.

Vid. Fig. 1.

Cæteris utique ut prius constructis; Quadrandum exponatur $H F u r$ spatium

tium; (cujuscunque fuerit longitudinis A H; puta major minorve quam A I, vel huic æqualis: sumptoque ubivis inter A & H, puncto r; puta ultra citrave punctum I, vel in ipso I puncto:) Ponantur autem (non, ut prius A I = r, & I r = A: sed) A H = r; & H r = A; quæ intelligatur in æquales partes innumeras dividi, quarum quælibet sit a. Erunt itaque, post A H = r, reliquæ deinceps decrescentes r - a, r - 2a, r - 3a, &c. usque ad A r = r - A. Item, propter æqualia Rectangula F H A, u r A, B I A, &c. puta, = b²: Erit H F = $\frac{b^2}{r}$; reliquæque deinceps

$$\frac{b^2}{r-a}, \frac{b^2}{r-2a}, \frac{b^2}{r-3a}, \&c. \text{ usque ad } r u = \frac{b^2}{r-A} \text{ spatium H F u r complemen-}$$

tes. (Quæ omnia ostensa sunt, in mea *Aritmetica Infinitorum*, prop. 88, 94, 95.)

Factaque Divisione; reperietur

$$\frac{b^2}{r-a} = b^2 + b^2 a + b^2 a^2 + b^2 a^3$$

$$+ b^2 a^4, \&c.$$

Hoc est,

$$b^2 \text{ in } 1 + a + a^2 + a^3 + a^4, \&c.$$

(sumptis ipsius a potestatibus continue sequentibus affirmatis omnibus.) Cumque de reliquis idem sit judicium; erunt rectæ omnes, ipsis H F & r u interjectæ,

$$\left. \begin{array}{l} r-a)b^2(b^2 + b^2 a + b^2 a^2 + b^2 a^3 \&c. \\ \underline{b^2 - b^2 a} \\ + b^2 a \\ \underline{+ b^2 a - b^2 a^2} \\ + b^2 a^2 \\ \underline{+ b^2 a^2 - b^2 a^3} \\ + b^2 a^3 \\ \underline{+ b^2 a^3 - b^2 a^4} \\ + b^2 a^4 \\ \&c. \end{array} \right\}$$

$$\left. \begin{array}{l} 1 + a + a^2 + a^3 + a^4 \&c. \\ 1 + 2a + 4a^2 + 8a^3 + 16a^4 \&c. \\ 1 + 3a + 9a^2 + 27a^3 + 81a^4 \&c. \\ \&c. \text{ sic deinceps usque ad} \\ 1 + A + A^2 + A^3 + A^4 \&c. \end{array} \right\} \text{ in } b^2.$$

Omniumq; Aggregatū, $A + \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5 \&c.$ in $b^2 = F H r u.$ (per *Aritm. Infu.* prop. 64.)

Exempli gratia:
 Positis A H = r.
 H r = A = 0, 21
 A I = b = 0, 1
 Adeoque b² = 0, 01

Erunt

$$\begin{array}{l} A = 0, 21 \\ \frac{1}{2} A^2 = 0, 02205 \\ \frac{1}{3} A^3 = 0, 003087 \\ \frac{1}{4} A^4 = 0, 00048623 - \\ \frac{1}{5} A^5 = 0, 000081682 + \\ \frac{1}{6} A^6 = 0, 000014294 + \\ \frac{1}{7} A^7 = 0, 000002573 - \\ \frac{1}{8} A^8 = 0, 000000473 - \\ \frac{1}{9} A^9 = 0, 000000088 + \\ \frac{1}{10} A^{10} = 0, 0000000017 - \\ \frac{1}{11} A^{11} = 0, 000000003 + \end{array}$$

Horum summa = 0, 235722333

Ducta in b² = 0, 01

Exhibet = 0, 00235722333 = F H r u
 Qua-

Qualium 1. = ANGN $\left. \begin{array}{l} \text{Quadrato,} \\ \text{Rhomb,} \end{array} \right\}$ si angulus A sit $\left. \begin{array}{l} \text{Rectus.} \\ \text{Obliquus.} \end{array} \right\}$

Quæ quidem tam absoluta est tamque expedita Hyperbolæ quadratura, ut nesciam an melior sperari debeat.

Atque hæc sunt quæ hac de re scribenda duxi. Quæ si D. Mercatori impertiveris; non displicebit, credo, hæc sive Quadraturæ facta accessio.

Posse hæc ad *Logarithmorum* inventionem accommodari, non est quod moneam: Sed & ad *Summam Logarithmorum* inveniendam: (quam inquit ille prop. 19.) Nempe, Positis $AH=1$, $AI=IB=b$, (ut prius) planoque $BIHF=pl$. Erit $pl-b^2+b^3=BIps+BIqt+BIru, \&c.$ usque ad $BIHF$. Si autem non ab ipsa BI incipiatur; sed ultra citrave, puta à ps : Posita $pH=a$ & $psFH=pl$. erit (universaliter) $ps tq + psur \&c$ (usque ad $psFH$) $= pl-ab^2$: qualium 1, æquetur cubo ipsius AH .) Quod alias, si opus erit, demonstrabitur. Tu interim, Illustrissime Domine, Vale.

Oxon. d. 8. Julii, 1668.

Fig 1.

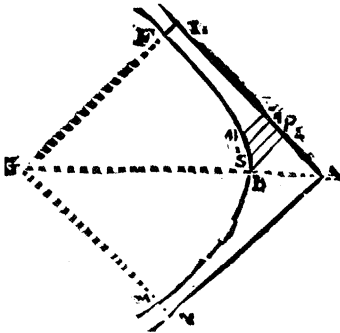
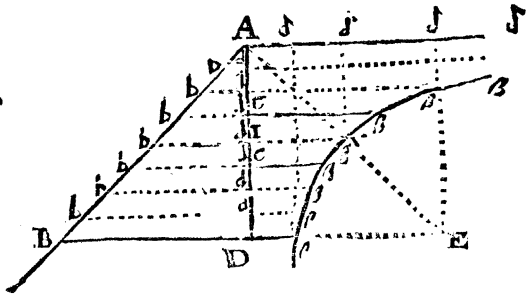


Fig 2.



The Demonstration

Promised at the end of the foregoing Letter, follows in another from the same Author to the same Noble Lord, thus;

Petis (Illustrissime Domine) per literas tuas Aug. 3. datas (quas hesternâ nocte accepi) ut demonstrare velim methodum meam, Logarithmorum summam inveniendi, quam literis meis Julii 8. datis, brevissime insinua-veram.

Quæ quidem, cum sit cum Ungularum doctrina (quam alibi trado) connexa; opus erit ut utramque simul exponam: sed & rem totam (quam D. Mercatoris

entoris figuræ & methodo quantum res ferebat accommodaveram) ad prin-
cipia mea revocatam ab origine repetam. V. Fig. 2.

Ostenſum eſt, in mea *Arithmetica Infinitorum*, prop. 95. Spatium Hyper-
bolicum $AD\beta\beta\delta$ (in infinitum continuatum à parte $\beta\delta$, ſed à parte $D\beta$ ubi-
vis terminatum,) Figuram eſſe quam ex *Primariorum Reciprociſ* conſtatam
appello, Prop. 88. definitam: Cujus nempe Ordinatum—applicatæ $d\beta$, $d\delta$,
ſint Primariſ (ſeu *Arithmetice* proportionalibus) db , db , (*Triangulum*
complementibus) adeoque ipſis dA , dA , (ſuis à vertice diſtantiis) Recipro-
ce Proportionales. Hoc eſt, (poſito $AD = d$; & rectangulo $AD\delta = b^2$;
particulifque infinite exiguis a , a , &c;) ſi à vertice $A\delta$ incipias $\frac{b^2}{0}$, $\frac{b^2}{a}$, $\frac{b^2}{2a}$

$\frac{b^2}{3a}$, &c. uſque ad $\frac{b^2}{d} = D\delta$: vel, ſi à baſe $D\beta$ incipias, $\frac{b^2}{d}$, $\frac{b^2}{d-a}$, $\frac{b^2}{d-2a}$

$\frac{b^2}{d-3a}$, &c. uſque ad $\frac{b^2}{d-d} = A\delta$ infinitæ, (nempe, ſi ad Verticem uſque
proceſſum continuaveris;) vel, uſque ad $\frac{b^2}{d-A} = C\beta$, (poſito $DC = A$),

ſi continuaveris uſque ad $C\delta$, ubiſvis intra $A\delta$ & $D\delta$ ſumptam. (Adeoque

omnium Aggregatum, $\frac{b^2}{d} + \frac{b^2}{d-a} + \frac{b^2}{d-2a} + \frac{b^2}{d-3a}$, &c, eſt ipſum

$DC\beta\delta$ planum.)

Maniſteſtum itaque eſt, (& ibidem prop. 94. oſtenſum) ſi intelligen-
tur ſingulæ $d\beta$, in ſuis à vertice diſtantiis Ad , ductæ; hoc eſt, $\frac{B^2}{a}$ in a , $\frac{B^2}{2a}$
in $2a$, (& ſic de reliquis;) erunt omnia rectangula $A d\beta$; hoc eſt, rectan-
gulum $d\beta$ momenta reſpectu $A\delta$, (intellige, facta ex magnitudine in diſtanti-
am ductâ;) ſeu plana ſemiquadrantalem Ungulam (cujus acies $A\delta$) com-
plementia, (eiſdem $d\beta$ rectis perpendiculariter inſiſtentia;) invicem æqualia.
Quippe ſingula $= b^2$. (Quorum cum unum ſit $AI V\delta$ quadratum, erit
 $AI = b$.)

Adeoque Totius $AD\beta\beta\delta$ (plani infiniti) ſeu omnium $d\beta$ il-
lud complementium, momentum reſpectu rectæ $A\delta$, (ut axis æquilibrii;) ſeu
Ungula ſemiquadrantalis eidem $AD\beta\beta\delta$ inſiſtens (aciem habens $A\delta$;) ſunt
totidem b^2 ; hoc eſt, $d b^2$. (Ungula magnitudinis finitæ plano infinitæ
magnitudinis inſiſtens.) Ejuſque pars plano $AC\beta\beta\delta$ inſiſtens (propter AC
 $= d - A$.) ſimiliter oſtendetur æqualis ipſi $d - A$ in b^2 . ductæ; hoc eſt,
 $d b^2 - A b^2$. Adeoque pars reliqua, ipſi $DC\beta\beta$ inſiſtens, æqualis ipſi $A b^2$.
Quod itaque eſt ejuſdem $DC\beta\beta$ momentum reſpectu $A\delta$.

Atque hoc momentum per plani $DC\beta\beta$ magnitudinem, puta per pl , divisum; exhibet plani distantiam Centri gravitatis ab $A\delta$, $\frac{ab^2}{pl}$: adeoque distantiam ejusdem a $D\beta$, $d - \frac{ab^2}{pl}$.

Hæc itaque à $D\beta$ distantia, in pl (plani magnitudinem) ducta; exhibet $dpl - Ab^2$ ejusdem $DC\beta\beta$ momentum respectu $D\beta$; seu Ungulam eidem $DC\beta\beta$ insistentem, cujus acies sit $D\beta$.

Hæc denique Ungula (cujus altitudo, in $D\beta$, nulla sit, sed, in $C\beta$, ipsi DC æqualis:) si ex planis ipsi $DC\beta\beta$ parallelis conflari intelligatur; eunt ea, $CD\beta\beta$, $Cd\beta\beta$, & sic deinceps; hoc est, aggregatum omnium $Cd\beta\beta$, $Cd\beta\beta$, usque ad $CD\epsilon\epsilon$.

Sunt autem ea plura (ut ex *Gregorii de Sancto Vincentio*, aliorumque post illum, doctrina constat) tanquam Logarithmi Arithmetice proportionalium Cd , Cd , &c. usque ad CD ; (seu a , $2a$, $3a$, &c. usque ad A .) Ergo Ungula ipsa, est eorundem Aggregatum. Hoc est (posito $D = 1$.) $dpl - Ab^2 = pl - Ab^2$. Quod ostendendum erat.

$$\text{Porro; cum sit } \frac{b^2}{d-a} (= d\beta) = \frac{b^2}{d} + \frac{ab^2}{d^2} + \frac{a^2b^2}{d^3} + \frac{a^3b^2}{d^4} \&c$$

(Quod dividendo b^2 per $d - a$, patebit:) vel, posito $d = 1$, (quò ipsius d potestates omnes deleantur,) $b^2 + ab^2 + a^2b^2 + a^3b^2$ &c. seu $1 + a$

$$+ a^2 + a^3, \&c. \text{ in } b^2. \& \text{ similiter } \frac{b^2}{d-2a} = \frac{b^2}{d} + \frac{2ab^2}{d^2} + \frac{4a^2b^2}{d^3}$$

$$+ \frac{8a^3b^2}{d^4} \&c. = b^2 + 2ab^2 + 4a^2b^2 + 8a^3b^2 \&c. = b^2 \text{ in } 1$$

+ $2a + 4a^2 + 8a^3$, &c. & similiter in reliquis:

$$\text{Erunt omnes } d\beta, \text{ (spatium } DC\beta\beta \text{ complectentes,)} \left. \begin{array}{l} 1 + a + a^2 + a^3 + a^4 \&c. \\ 1 + 2a + 4a^2 + 8a^3 + 16a^4 \&c. \\ 1 + 3a + 9a^2 + 27a^3 + 81a^4 \&c. \end{array} \right\} \text{ in } b^2.$$

$$\text{Adeoque; (per Arithm. Infin. Prop. 64.) omnium Aggregatum, seu ipsum } DC\beta\beta \text{ spatium, erit } \left. \begin{array}{l} \& \text{ sic deinceps usque ad} \\ 1 + A + A^2 + A^3 + A^4 \&c. \end{array} \right\} A + \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5 \&c. \text{ in } b^2 = pl.$$

Quælium $I = ABE\beta$ Quadrato vel Rhombo

Ideoque, Plani $DC\beta\beta$ momentum respectu $D\beta$; seu semiquadrantis Ungula eidem insistentis cujus acies sit $D\beta$; seu planorum aggregatum ipsam constituentium; seu Logarithmorum summa quos ea representant, $dpl - Ab^2 = pl - Ab^2 = \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ in b^2 :

Quælium

Qualium Cubus (feu Rhombus solidus) A D E s fit 1.
 Si vero non ponatur $d=1$, sed cujuscunque magnitudinis: erit saltem

$$\frac{A}{d} + \frac{A^2}{2d^2} + \frac{A^3}{3d^3} + \frac{A^4}{4d^4} \&c. \text{ in } b^2. = pl.$$

Vel (posito $\frac{A}{d} = e$) erit $e + \frac{1}{2}e^2 + \frac{1}{3}e^3 + \frac{1}{4}e^4 \&c. \text{ in } b^2 = pl.$ Qualium

$d^2 = A D E$ Quadrato vel Rhombo.

Ungulaque (ut prius) $d pl = A b^2$. Qualium $d^3 = A D E$ Cubo, vel (si angulus A sit obliquus) Rhombo solido.

Cumque A (posito $d=1$) vel e (quicunque ponatur valor ipseus d) sit minor quam 1, (propter $A < d$) illius potestates posteriores ita continue decrescunt, ut tandem negligi possint; planique valor $pl.$ exhibeatur quantumlibet vero propinquus.

Atque hæc est, Illustrissime Domine, Methodi, quam innuebam, ex meis principiis deductio, & demonstratio brevis. Vale. Oxon. d. 5. Aug. 1668.

Some Illustration

Of the Logarithmotechnia of M. Mercator, who communicated it to the Publisher, as follows.

Si quorum in manus incidit Logarithmotechnia mea, non inviti, opinor, adspicient paucula hæc exempla, miram istius methodi facilitatem cum summa præcissione conjunctam ostendentia.

Exponentes	Unitatis ordo	Binarii ordo
1	I i	2
2	I o, 5	4
3	I o, 333333	8
4	I o, 25	16
5	I o, 2	32
6	I o, 166666	64
7	I o, 142857	128
8	I o, 125	256
9	I o, IIIIII	512
10	I o, I	1024

Duo sunt ordines tabellæ, prior unitatis, alter binarii propigo, quorum uterque denorum numerorum primorum Log - os producit, præter compositorum Log - os, qui & ipsi requiruntur.

(760)
Ex primo ordine

i
 .05
 033333333
 025
 02
 016666
 01428
 0125
 011
 01
 +10033534772
 - 502516792
 10536051564⁹₁₀
 9531017980¹⁰₁₁

i
 .05
 03333333
 025
 02
 +10000333353
 - 500025
 10050335853⁹⁹₁₀₀
 9950330853¹⁰⁰₁₀₁
 Parimodo ex eodem
 ordine procedunt ra-
 tiones ^{999 1000 9999}
_{10000 100000 1000000},
_{1000000 10000000 100000000}

Ex secundo ordine.

2
 2
 266666666
 4
 64
 1066666
 182857
 32
 5689
 1024
 186
 341
 630
 +20273255404
 - 2041099724

2
 2
 26666666
 4
 64
 10
 + 20002667306
 - 200040010
 20202707316⁹⁸₁₀₀
 19802627296¹⁰⁰₁₀₂
 Haud secus ex eo-
 dem ordine eliciuntur
 rationes ^{998 1000}
_{10000 100000 1000000},
_{9998 10000 99998},
_{100000 1000000 10000000},
_{1000000 10000000 100000000}, &c.

1		22314355128 ⁸ ₁₀	
2		18232155680 ¹⁰ ₁₂	
3	1 + 2	40546510808 ⁸ ₁₂	= $\frac{2}{3}$
4	ex pc. pag.	10536051564 ⁹ ₁₀	
5	2 + 4	28768207244 ⁹ ₁₂	= $\frac{2}{4}$
6	3 + 5	69314718052 ¹² ₁₄	= $\frac{1}{2}$ = L 2 rii
7	6 x 3	207944154156 ¹² ₁₈	= L 8 rii
8	1 + 7	230258509284 ¹ ₁₀	= L 10 rii
9	ex pc. pag.	9531017980 ¹⁰ ₁₁	
10	8 + 9	239789527264 ¹ ₁₁	= L 11 rii
11	3 + 6	109861228860 ¹ ₂	+ $\frac{2}{3}$ = L 3 rii

Simi-

Similes ordines à 3^{rio}, 4^{rio}, & quovis alio numero derivari possunt, suas quisque rationes exhibiturus.

Acquisito Log-o 10ⁱⁱⁱ, conficienda est statim tabella reducendorum Log-orum naturalium ad Tabulares, ut quævis ratio, simul ac inventa est, reducat ad mensuram tabularium; ita enim Log-i compositorum, quorum ope ad primorum Log-os descenditur, simul fient Tabulares absque reductione.

Fiat igitur, ut Log-us 10ⁱⁱⁱ non-tabularis 2302585, ad tabularem 10000000, ita 1, ad 4,3429448. Hic numerus bis, ter, quater & pluries sumptus constituit tabellam reducendorum Log-orum naturalium ad tabulares, quam hic subjectam vides.

1	043429448190
2	086858896380
3	130288341570
4	173717792761
5	217147240951
6	260576689141
7	304006137332
8	347435585522
9	390865033712

Hujus igitur ope tabellæ, rationis $\frac{98}{100}$ mensura naturalis 20202707316 reducitur ad ta-

bularem hoc modo :

2	086858896381
0	0
2	0868588964
0	0
2	08685890
7	3040061
0	0
7	30401
3	1303
1	043
6	26
<hr/>	
	87739243069

Tum à Log-o 100ⁱⁱⁱ 20000000000000
 auferatur ratio- 87739243069
 nis, $\frac{98}{100}$ mensura restat 19912260756031 = L 98
 unde ablato Log-2ⁱⁱ 3010299956640
 restat ————— 16901960800291 = L 99
 cujus semis ————— 8450980400145 = L 7
 Item rationis $\frac{100}{102}$ mensura naturalis 19802627296
 reducta, fit 86001717619.
 Ergo jã Log-o 100ⁱⁱⁱ 20000000000000
 adde rationis $\frac{100}{102}$ mensurã 86001717619
 fit ————— 20086001717619 = L 102
 unde ablato Log-o 6ⁱⁱⁱ 7781512503836
 restat ————— 12304489213783 = L 17

Hic tabula numerorum primorum egregium usum præstare potest.

Sed & ejusdem primi 17 Log-um absque ambage invenire datur, dicendo: 20. 17:: 10. 8 $\frac{1}{5}$; tum differentiæ inter 10 & 8 $\frac{1}{5}$ (nimirum 1 $\frac{1}{5}$) sumendo quadrati semissem, cubi trientem, &c. tractandoque istum ordinem, ut supra, inveniemus simul Log-os absolutorum 23, 197, 203, 1997, 2003, &c.

1	1,5	1,5	15
2	2,25	1,125	1125
3	3,375	1,125	1125
4	5,0625	1,265625	1265625
5	7,59375	1,51875	1518
6	11,390625	1,8984375	189
			22
			+15114040
			- 1137845
			16251885 ¹⁷ / ₁₀₀
			13976195 ²⁰ / ₁₀₀

Cæterum isthæc omnia, & longè plura ex prop. 13, 15, & 16 Logarithmotechniæ nostræ aptè derivantur, non magis considerando hyperbolam, quàm si ea nusquam in rerum natura extitisset. Quare frustra sunt, qui hyperbolam ad constructionem Logarithmorum vel hilum conferre autumant; imo Logarithmorum ope quadrare hyperbolam, verius est. Id quod exemplo ostendere haud pigebit. In diagrammate (Fig. 1.) sit AH 74305816 parium, qualium AI est 1, & oporteat invenire aream BIHF.

Opus est ad eam rem tabella subjecta, quæ continet Log-os naturales supra acquisitos, in priori columna ab 1 usque ad 9, in altera à 10 usque ad 1000000000.

1	0000000000	02,30258509299
2	69314718052	04,60517018599
3	109861228860	06,90775527898
4	138629436104	09,21034037198
5	160943791232	11,51292546497
6	179175946912	13,81551055796
7	194591014904	16,11809565096
8	207944154156	18,42068074395
9	219722457720	20,72326583695

Tum prima figura numeri dati semper distinguatur à sequentibus separatrie, hoc modo : 7,4305816, & ipsi primæ figuræ semper adjiciatur 1, ita constantur, hoc loco, 8. Quærenda est nunc rationis 8 ad 7,4305816 mensura naturalis. Id ut fiat commodius, dic : ut 8 ad 7,4305816; ita 1 ad 0,9288227, hunc quartum proportionalem aufer ab 1, reliquum 0,0711773 voco potestatem primam, quæ ducenda est in se ita, ut in factò idem numerus partium extet, qui erat in ipso 0,0711773; productum 0,0050662 est potestas secunda, quæ rursus ducatur in primam 0,0711773, ut idem numerus partium extet, prodit 0,0003606, quæ est tertia potestas; & eodem modo invenitur quarta 0,0000256, & quinta 0,0000018.

Deinde

Potestas

Potestas prima	0,0711773	} addantur
Et secundæ semis	25331	
Et tertiæ triens	1202	
Et quartæ quadrans	64	
Et quintæ pars quinta	4	

summa ————— 0,0738374 est mensura rationis 8 ad 7,4305816, eadem scilicet cum ratione 80000000 ad 74305816. Porro Log-us absoluti 80000000 facile acquiritur ex superiori tabella; cum enim index primæ figuræ numeri 80000000 sit 7, è regione 7ⁱⁱⁱ ex secunda columna excerpo Log-um absoluti 10000000 (hoc est unitatis septem cyphris affectæ).

qui reperitur	16,11809565	} addo
cui subscribo Log-um 8 ⁱⁱⁱ	2,07944154	
summa est Log-us absoluti 80000000	= 18,19753719	
ablata mensura rationis 80000000 ad 74305816	= 0,0738374	
restat Log-us absoluti 74305816	= 18,1236997, atque	

tanta est area B I H F.

Mantissæ loco accipe modum facilissimum quadrandi quamvis hyperbolæ partem per Log-os tabulares. Dati numeri 74305816 Log-us tabularis est 7,87102278, per superioris tabellæ columnam secundam reducendus ad naturalem, proditque eadem, quæ supra, area B I H F = 18,123699872.

Postremo, ne quis hæsitacioni locus restet, accipe, quo pacto ex Prop. 13, 15, 16. Logarithmot. calculum superiorem derivem.

Differentia terminorum rationem quamvis experimentium si concipiatur divisa in partes æquales innumeras; composita erit ratio tota extremorum terminorum ex innumeris ratiunculis terminorum à minimo ad maximum infinitissima parte ipsius differentiæ se mutuo excedentium. Sin iidem illi termini innumeri accipiantur pro mediis Arithmeticis aliorum terminorum simili parte infinitissima distantium; summa omnium ratiuncularum posterioribus hisce terminis intercedentium deficiet à tota ratione extremorum, non nisi semisse primæ & ultimæ ratiuncularum à prioribus terminis contentarum, id est, ratiuncula minori, quam quæ ullis numeris exprimi possit. Quare posito Maximo termino = 1, & parte infinitissima differentiæ = i, & mensura rationis minimæ itidem i; erit ut medium Arithmeticum terminorum rationis minimam proxime præcedentis, ad medium Arithmeticum terminorum ipsius minimæ; ita mensura minimæ, ad mensuram proxime majoris; hoc est:

$$\begin{array}{l}
 1 - i . 1 :: i . i + ii + i^3 + i^4 \&c. \text{ mensuræ ultimæ } \\
 1 - 2i . 1 :: i . i + 2ii + 4i^3 + 8i^4 \&c. \text{ penultimæ } \\
 1 - 3i . 1 :: i . i + 3ii + 9i^3 + 27i^4 \&c. \text{ antepenultimæ } \\
 \hline
 \text{fit summa ratiuncul.} = 3i + 6ii + 14i^3 + 36i^4 \&c. = \text{numero terminorum,}
 \end{array}$$

rum, plus summa eorundem terminorum, plus summa quadratorum ab iisdem, &c.

Sin minimus terminus ponatur $\equiv 1$, manentibus cæteris ut *supra*; evadit summa ratiuncularum $\equiv 3i - 6ii + 14i^3 - 36i^4$, &c.

Hinc data differentia terminorum $\equiv 0\frac{1}{2}$, erit numerus terminorum $\equiv 0\frac{1}{2}$, & per 16 Logarithmot. summa eorundem terminorum $\equiv 0,005$, & summa quadratorum $\equiv 0,000333$. At data differentia terminorum $\equiv 0\frac{10}{10}$; numerus terminorum est $\equiv 0,01$, & summa eorundem $\equiv 0,00005$, & summa quadratorum $\equiv 0,00000333$, &c.

Nota. Prop. IV. Logarithmot. Signa speciebus intercedentia debebant esse alternatim affirmata & negata: atque ubicunque, Lector offenderit *infinitissimam*, legat *infinitesimam*.

Errata.

Page 742. l. 25. put a comma after open'd, (which is material for the sense.) p. 749. l. 16. r. *idque*. *ibid.* l. 40. r. *magnitudinum*. p. 753. l. 20. r. — $a + a^2, — a^3$, p. 754. l. 19. r. *Huic*. p. 755. l. 11. r. $b^2 a^2 + b^2 a^3 + b^2 a^4$. *ibid.* l. 14. r. $+ a^2 + a^3$. p. 756. in Fig. 1. the letters appearing obscure, those that denote the small lines parallel to the Asymptote NA, are I B. *ps. qt. rx.* And the other capital letters are G F H. G B A. G M N.

In the S A V O Y,

Printed by T. N. for John Martyn, Printer to the Royal Society, and are to be sold at the Bell a little without Temple-Bar, 1668.