

Mtg 1: Tue, 5 Jan 10

Mtg \nearrow $\frac{11-1}{}$ page

user: egm 6241. s10

Numerical Int. (Atkinson, p. 249)

$f: \underbrace{[a, b]}_{\text{domain}} \rightarrow \underbrace{\mathbb{R}}_{\text{range}}$ $\xleftarrow{A.}$ set of real numb.

$$I := \int_a^b f(x) dx \equiv I(f) \quad (1)$$

equal by defn $\begin{matrix} \swarrow \\ \text{def} \\ \Delta \end{matrix}$

(2) Approx. $f(\cdot)$ by $f_n(\cdot) \Rightarrow f \approx f_n$
usually a poly.

$$(3) I(f) \approx I_n := \int_a^b f_n(x) dx = I(f_n)$$

(4) Prop. of $f - f_n$: $\|f - f_n\|_\infty \rightarrow 0$ as $n \rightarrow \infty$

$$\|g\|_\infty = \max_x |g(x)| \quad \left. \begin{array}{l} \text{conv. of} \\ f_n \text{ to } f. \end{array} \right\} \text{infinity norm of } g$$

Conv. of num. int.

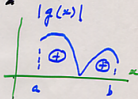
11-2

$$E_n(f) := I(f) - I(f_n) = \int_a^b [f(x) - f_n(x)] dx \quad (1)$$

$$|E_n(f)| \leq \int_a^b |f(x) - f_n(x)| dx \\ \leq (b-a) \|f - f_n\|_\infty \quad (2)$$

Note:

$$\left| \int_a^b g(x) dx \right| \leq \int_a^b |g(x)| dx \quad (3)$$



$$(4) \|g\|_\infty = \max_x |g(x)| \Rightarrow |g(x)| \leq \|g\|_\infty \\ \text{"for all" } \rightarrow \forall x \in [a, b] \\ \text{"belongs to"}$$

$$\int_a^b |g(x)| dx \leq \int_a^b \underbrace{\|g\|_\infty}_{\text{const}} dx \quad \ll-3$$
$$= \|g\|_\infty \underbrace{\int_a^b dx}_{(b-a)}$$