INTERVALLIC THEORY

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1 Theoretical Basis: Intervals

The name interval basically means distance, and, more specifically, in music is the term for the distance between any two notes. It follows, then, that intervals appear in chords, scales and that, by themselves, can be used for melodic or harmonic playing. Consider a Major triad, which is spelt 1-3-5. If we were to plot these on the neck of a guitar for a given key, we would find a chord shape. However, from 1 to 3 is an interval, as is from 3 to 5. Also, from 1 to 5 is another interval. To reiterate, this section will concern itself with the distances between any two notes.

But what can we say of intervals? As well as being either harmonic (played simultaneously), or melodic (played sequentially), they have identifiers known as type and quality. Type will be taken to be a numerical value. Quality will refer to the differing distances for one type. Such characteristics will allow us to talk about any chord (or, more generally, arpeggio) or scale uniquely in terms of their 'flavours'.

Intervallic number representations generally cover two octaves. Thus, types will be

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

however, the student will note that while there are 24 notes in two octaves of music, there are only 15 numbers occurring above. This is where we come to the characteristic of quality. In other words, there will be more of some types and less of others. The qualities which we will regularly encounter are

doubly diminished
diminished
minor
Major
perfect
augmented
doubly augmented

Note, then, that there are seven qualities. Given that there are 15 types in two octaves, and that 7×15 is 105, it follows that an interval of some specific type and quality will be the equivalent of another specific type and quality. This will become clearer as we progress, however, we can mention one example from the outset, and that is the equivalence of a sharpened fourth ($\sharp4$) and a flattened fifth ($\flat5$). Note that each is of a different type and quality although they are enharmonically equivalent.

We can comment further to the extent that the different qualities of intervals give rise to sounds which are either pleasing to the ear or dissonant. For example, in general, Major and perfect intervals should sound easy to the ear and natural. The others, on the other hand, may leave the ear wishing for another interval to resolve the tension that they can create.

Having dispensed with the preliminaries, let us get down to intervals and their characteristics.

1.1 Intervallic Types and Qualities

It should be mentioned from the outset that by altering an existing interval, we arrive at another interval. This simply means playing a note and lengthening or contracting the distance to the next note. This is where qualities become important. We will talk about how a quality changes while the type remains the same. As a precursor to this discussion, let us look at the more elementary intervals. Naturally the first is never altered, and thus has only one quality. For the distance from the first to the other 24 notes we arrive at the following list,

$1-\flat2$	minor 2nd	(m2)	$1-\flat9$	minor 9th	(m9)
$1-2$	Major 2nd	(M2)	$1-9$	Major 9th	(M9)
$1-\flat3$	minor 3rd	(m3)	$1-\flat10$	minor 10th	(m10)
$1-3$	Major 3rd	(M3)	$1-10$	Major 10th	(M10)
$1-4$	perfect 4th	(p4)	$1-11$	perfect 11th	(p11)
$1-\sharp4$	augmented 4th	(A4)	$1-\sharp11$	augmented 11th	(A4)
$1-\flat5$	diminished 5th	(d5)	$1-\flat12$	diminished 12th	(d5)
$1-5$	perfect 5th	(p5)	$1-12$	perfect 12th	(p5)
$1-\flat6$	minor 6th	(m6)	$1-\flat13$	minor 13th	(m6)
$1-6$	Major 6th	(M6)	$1-13$	Major 13th	(M6)
$1-\flat7$	minor 7th	(m7)	$1-\flat14$	minor 14th	(m7)
$1-7$	Major 7th	(M7)	$1-14$	Major 14th	(M7)
$1-8$	perfect 8th (Unison)	(p8)	$1-15$	perfect 15th (Unison)	(p15)

Note that the interval $1-\sharp4$ corresponds to the same distance as $1-\flat5$ as does $1-\sharp11$ to $1-\flat12$, that is, they are enharmonic equivalents. Note that all qualities are present in this list as well. The combinations in parentheses for each interval are simply the shorthand notation commonly used to denote intervals. For example, 'm' means minor type, 'M' means Major type, etc.

Before discussing the alterations to existing intervals and, therefore, their qualities, we should say something of why this is important. Firstly, let us consider the diatonic Major scale. Most of you will already know that the spelling of this scale in one octave is

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

The astute reader will note that the distances from the first to each of the subsequent notes is

$1-2$	Major 2nd
$1-3$	Major 3rd
$1-4$	perfect 4th
$1-5$	perfect 5th
$1-6$	Major 6th
$1-7$	Major 7th
$1-8$	(perfect) octave

In other words, the Major scale is made up only of Major and perfect intervals. This is where the name 'diatonic' comes from. Now, for example, we question what would happen if we took this scale and decided flatten its seventh twice and its sixth once. This would result in

$1-2$	Major 2nd
$1-3$	Major 3rd
$1-4$	perfect 4th
$1-5$	perfect 5th
$1-\flat 6$	minor 6th
$1-\flat\flat 7$	??
$1-8$	(perfect) octave

At first glance, it would appear that flattening the seventh twice makes it equivalent to a Major sixth, however, we already have a sixth, and it is of a minor quality. In the key of C the minor sixth would be $A^{\flat}$, while the double flattened seventh would appear to be A. However, this is not the case, it is in fact $B^{\flat\flat}$ (which is enharmonically equivalent to A). For conventional reasons, seven note scales are such that some quality of the second, third, fourth, fifth, sixth and seventh are all present, and this prevents the type of doubling up on types as could occur in this example.

This example helps provide the motivation behind how the quality of intervals is named once an alteration to an existing interval is made, and there are some general rules about how they are named once an alteration has been made. We have the following chain for minor intervals,

$$\text{diminished} \quad \xleftarrow{\flat} \quad \text{minor} \quad \xrightarrow{\sharp} \quad \text{Major} \tag{1}$$

I.e., when a minor interval is sharpened, it becomes a Major interval. Additionally, when it is flattened it becomes diminished. The following chain applies to Major intervals,

$$\text{minor} \quad \xleftarrow{\flat} \quad \text{Major} \quad \xrightarrow{\sharp} \quad \text{augmented} \tag{2}$$

This brings us back to the example when the seventh was flattened twice. Recall that we started with a Major interval. Flattening this once leads to a minor interval and the further flattening of this minor interval leads to a diminished interval, as follows from the application of relation (2) and then applying relation (1) to the result. Thus we can conclude that

$1-2$	Major 2nd
$1-3$	Major 3rd
$1-4$	perfect 4th
$1-5$	perfect 5th
$1-\flat 6$	minor 6th
$1-\flat\flat 7$	diminished 7th
$1-8$	(perfect) octave

We now examine the chain of implications for perfect intervals,

$$\text{diminished} \quad \xleftarrow{\flat} \quad \text{perfect} \quad \xrightarrow{\sharp} \quad \text{augmented} \tag{3}$$

For example, a diminished fourth interval is equivalent to a Major third interval and an augmented fifth interval is equivalent to a minor sixth interval. The final two chains which apply to intervals are those for diminished and augmented intervals. That is,

$$\text{doubly diminished} \quad \xleftarrow{\flat} \quad \text{diminished} \quad \xrightarrow{\sharp} \quad \text{minor/perfect} \tag{4}$$

and

$$\text{Major/perfect} \quad \xleftarrow{\flat} \quad \text{augmented} \quad \xrightarrow{\sharp} \quad \text{doubly augmented} \tag{5}$$

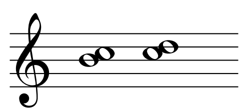
Having covered the basic properties of intervals, let us now look at them in practice.

2 Representations of Intervals

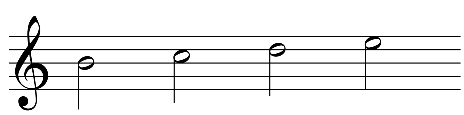
We now provide a systematic exposition of intervals of various qualities and types. We start with the second type intervals and their qualities.

2.1 Second Intervals

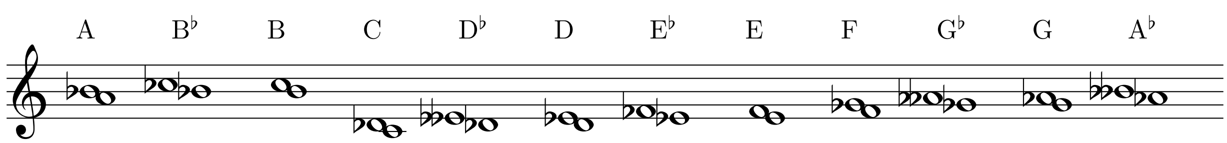
When we view second type intervals on a musical staff, we find that if the first is on one of the lines of the staff, the second will be between that line and the next line up. Similarly, if the second is between two lines on the staff, the second will be on the top one of those two lines. We stress that this occurs regardless of quality. In general, evenly numbered intervals will result in one note on a line and the other between two lines. Odd numbered intervals are such that both notes will fall on a line, or both will fall in between two lines. For example,



are (harmonic) second intervals of some quality, though we don't identify the quality without the presence of sharps or flats. Melodic second intervals may appear as in the following examples,

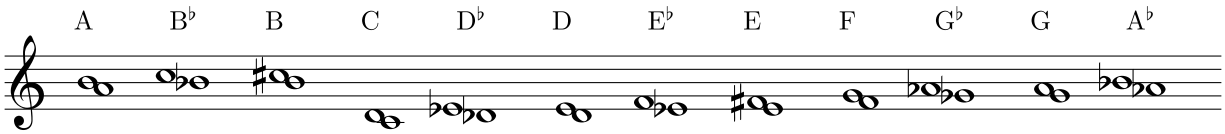


Having explained the structural appearance of second intervals on a musical staff, let us examine them in each key in a given octave. For (harmonic) minor second intervals in each key we have,



Minor 2nd Intervals

while harmonic Major second intervals would appear as



Major 2nd Intervals

We now examine how second type intervals may appear on the neck of the guitar. First we have minor second intervals. These can appear anywhere on the neck of the guitar, where 1 represents

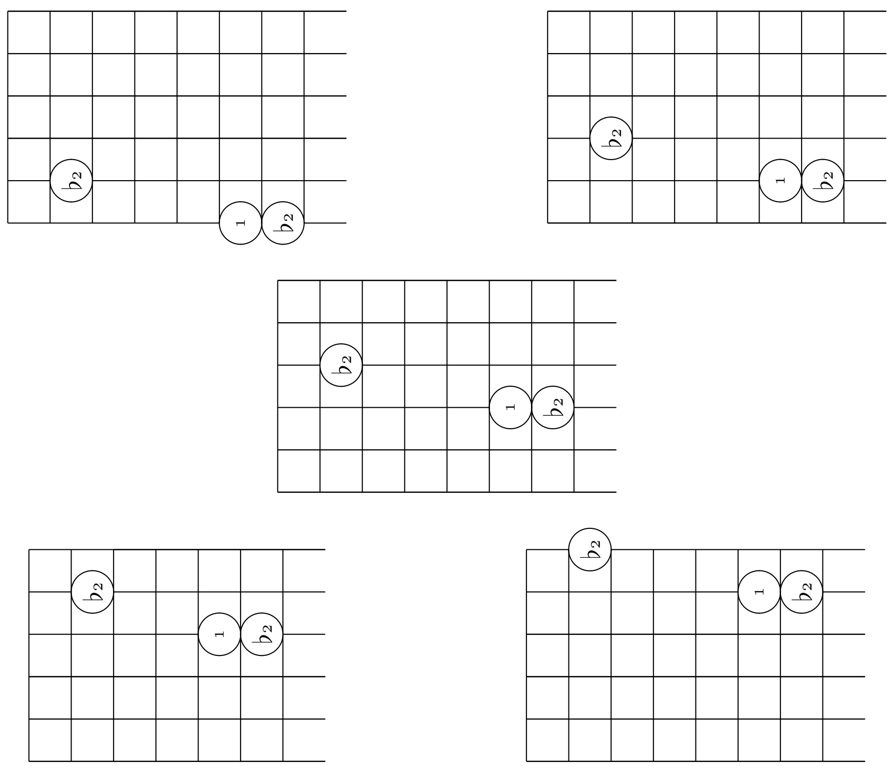


Table 1: Minor Second Intervals

the root note. Next we have Major second intervals,

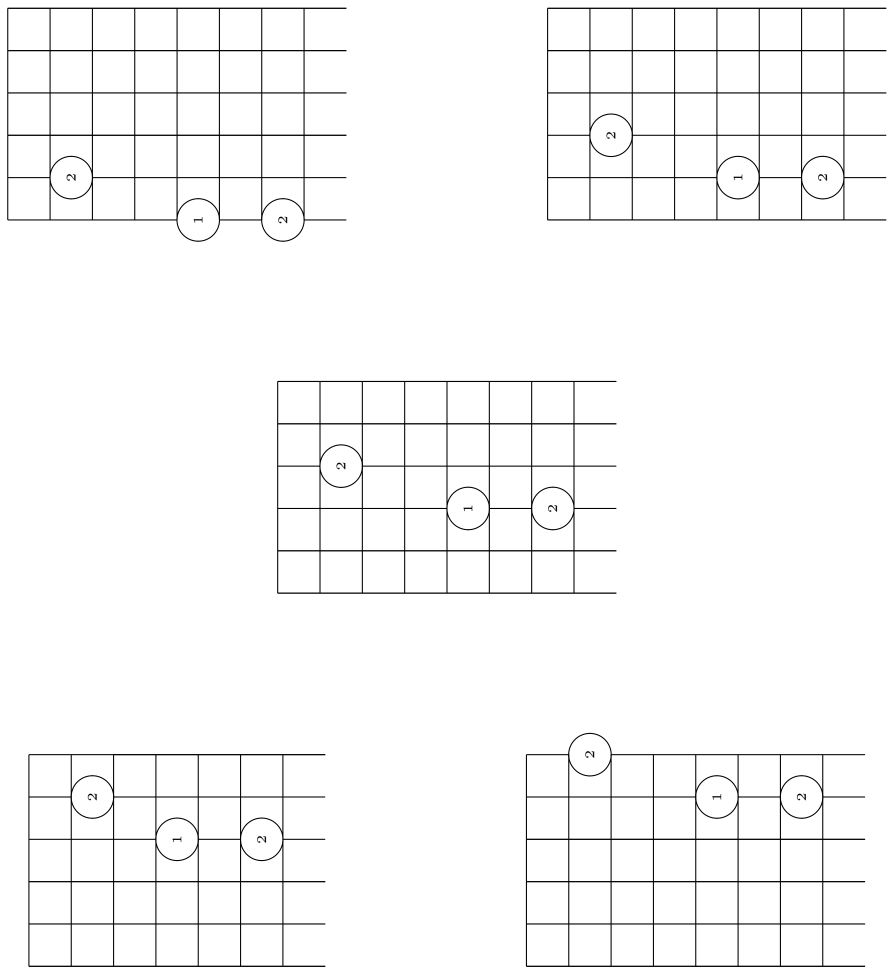
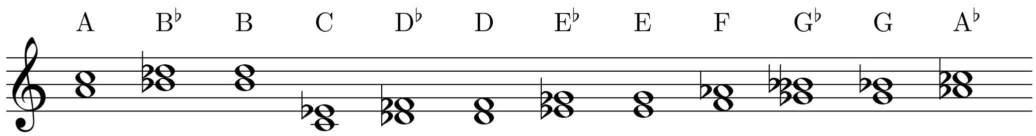


Table 2: Major Second Intervals

2.2 Third Type Intervals

Third type intervals appear as a distance of three or four frets (depending on the quality of the interval) on one string. Naturally, intervals that are played on one string only, must be melodic. There are, however, equivalents across two adjacent string for those which would like to play harmonically as well. Let us look at how minor thirds might appear on a musical staff.



Minor 3rd Intervals

On the neck of the guitar, general minor third intervals would appear as

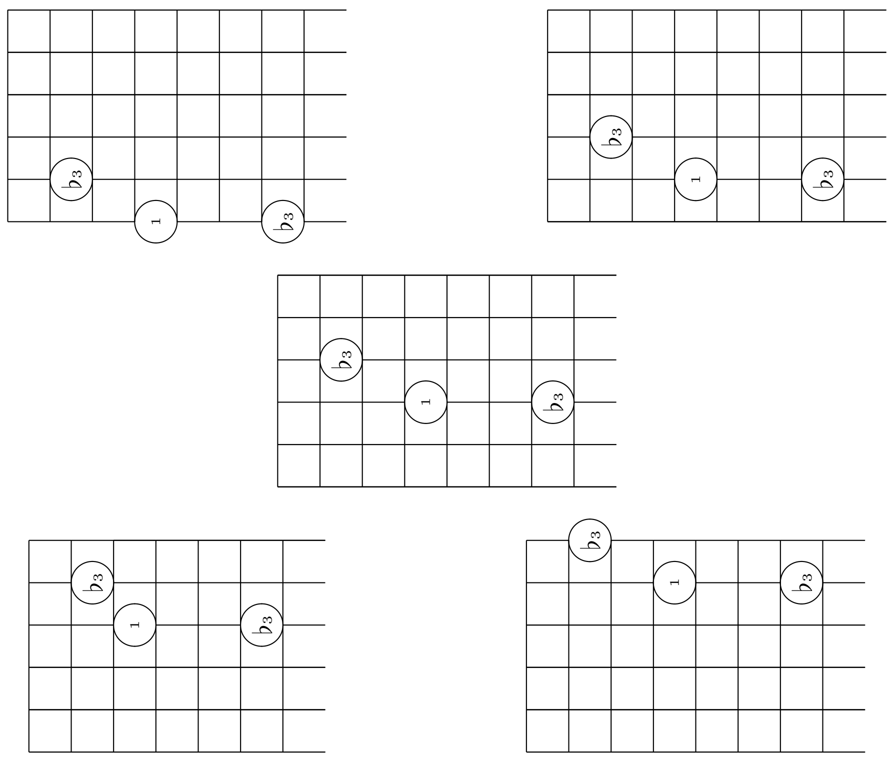
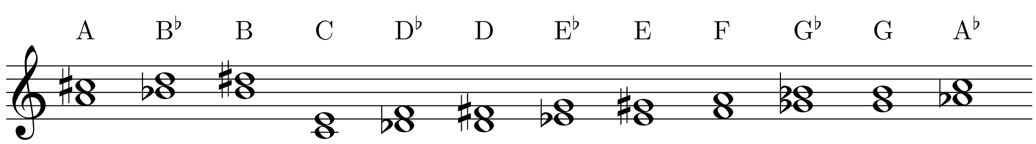


Table 3: Minor Third Intervals

Next we examine how Major thirds may appear on a musical staff,



Major 3rd Intervals

For the fretboard, we have, as was the case for second type intervals and minor third intervals, several representations for Major third intervals. Once again,these include melodic intervals played on one string as well as third intervals on adjacent strings which can be played either melodically or harmonically. To wit we have

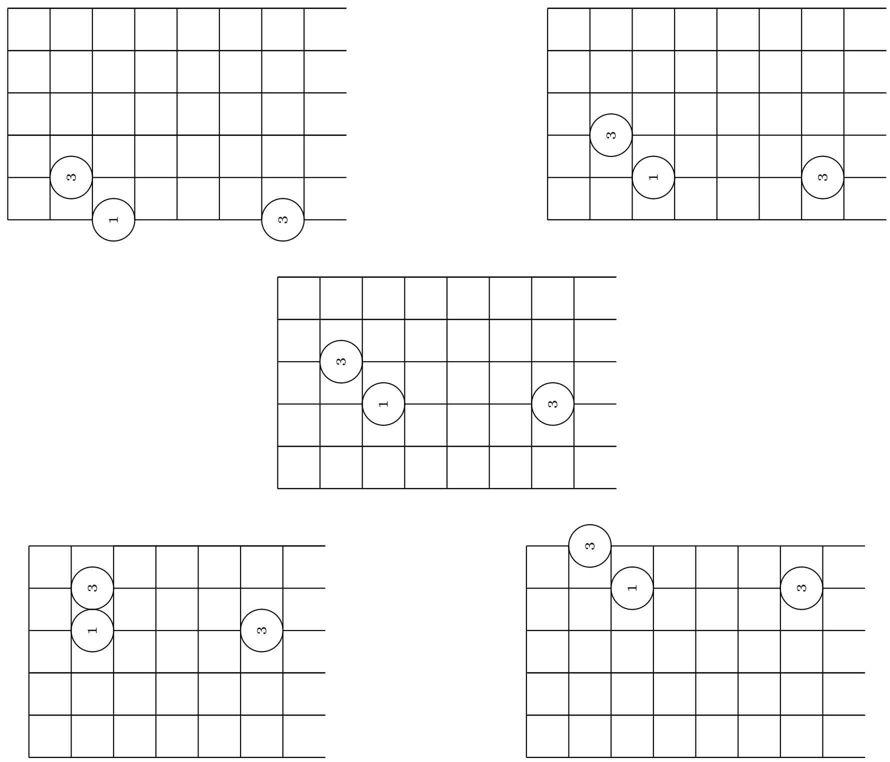


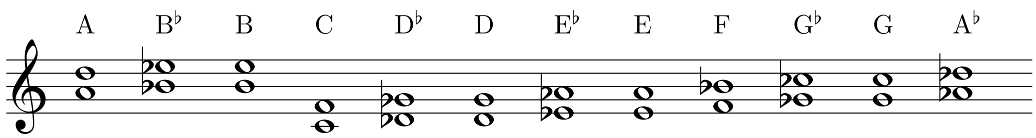
Table 4: Major Third Intervals

Having covered seconds and thirds, we now move to examine fourths. There are two of these which we will cover in detail; perfect fourths and augmented fourths.

2.3 Fourth Type Intervals

Recall from our initial discussions that fourths are naturally perfect in terms of quality. We also have diminished fourths if the perfect fourth is flattened, and these are enharmonically equivalent to Major thirds. In addition, if we are to sharpen the perfect fourth, we arrive at an augmented fourth. While it is true that there are doubly diminished and doubly augmented fourths as well, we consider that these are enharmonically equivalent to other intervals, including the minor third shown in the previous subsection. To that end, we only present perfect and augmented fourths in this subsection.

On a musical staff, perfect fourths can appear as



Perfect 4th Intervals

and, on the neck of the guitar, would look like

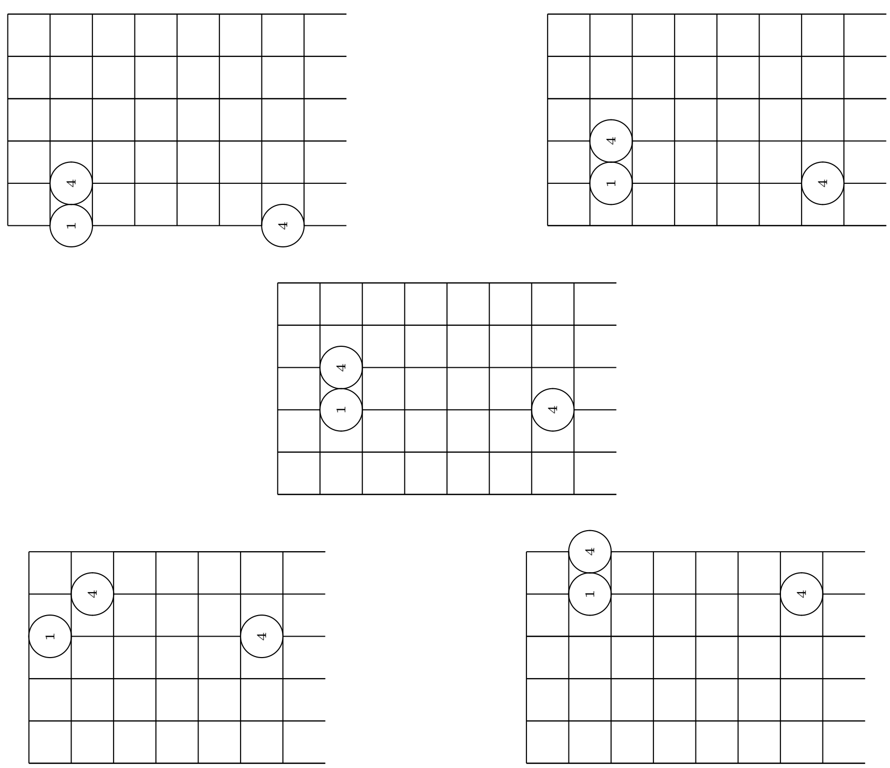
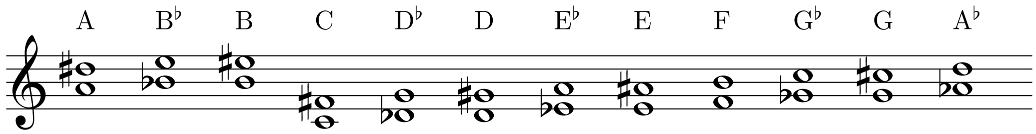


Table 5: Perfect Fourth Intervals

For augmented fourths, we have the following musical staff,



Augmented 4th Intervals

We now examine these on the neck of the guitar. Raising the fourth on the fretboard diagrams provided in Table 5 leads to the following representations.

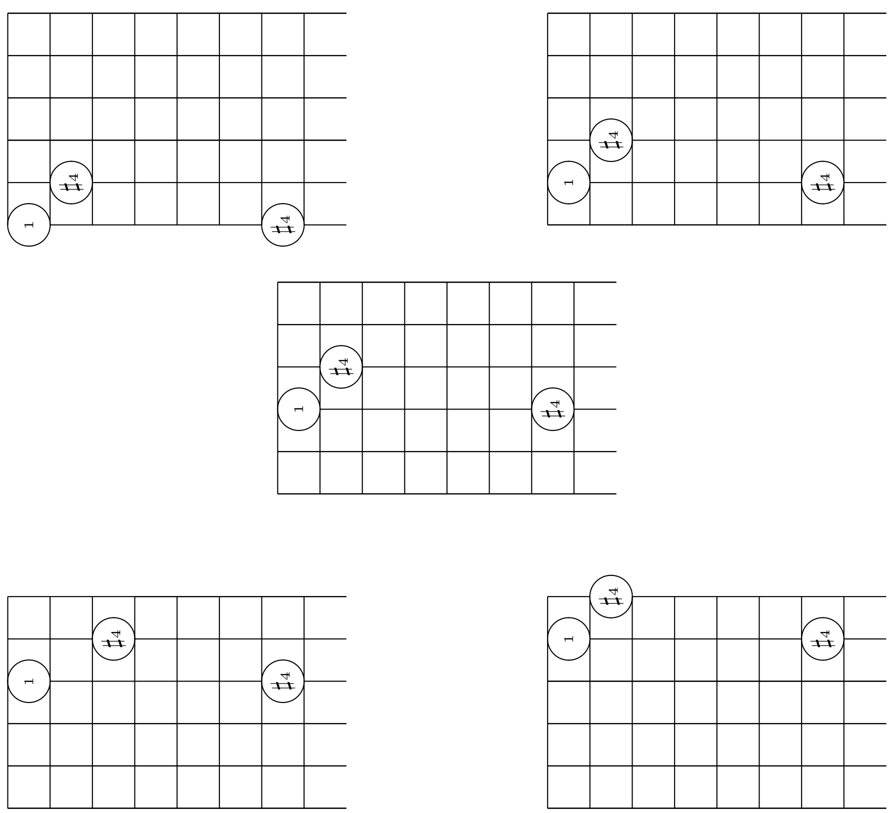
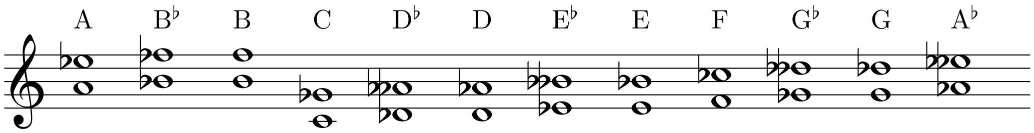


Table 6: Augmented Fourth Intervals

Next, we move to fifths, and while there is an enharmonic equivalence between augmented fourths and diminished fifths, thus leading to the same intervallic shapes on the fretboard, we will present them for completeness.

2.4 Fifth Type Intervals

While a diminished fifth interval is enharmonically equivalent to an augmented fourth, it is natural to present them as different intervals despite the fact that they share the same positions on the fretboard. The motivation for doing so stems from the construction of the seven modes of the diatonic Major scale. For example, we may have a mode which has a perfect fourth and a diminished fifth along with its other constituents. Similarly, we may have an augmented fourth and a perfect fifth, as is the case for the fourth mode of the Major scale, otherwise known as the Lydian scale. The clear distinction between enharmonic equivalents, therefore, is more obvious on the musical staff than it is on the fretboard. Here is the musical staff for diminished fifth intervals.



Diminished 5th Intervals

In other words, while being enharmonically equivalent to an augmented fourth interval, the appearance is different. In particular, when it relates to modes of the Major scale, the Locrian mode has perfect fourth and diminished fifth intervals (it obviously has other notes as well). Therefore, the first fourth and fifth intervals on one part of the neck for this scale may appear as,

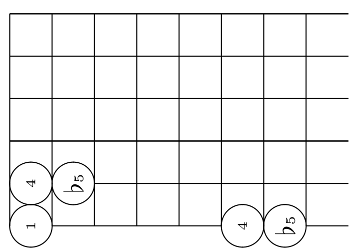
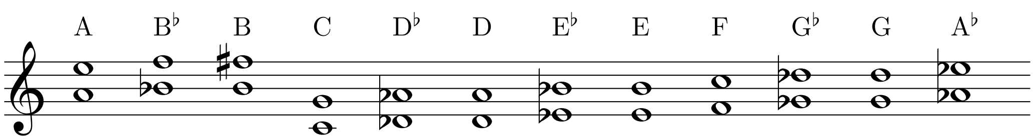


Table 7: An Example of the Distinction Between Fourths and Fifths

Next, when it comes to perfect fifth intervals, we have the following musical staff.
We now look at the fretboard for diminished fifth intervals,



Perfect 5th Intervals

which appear on the fretboard according to the following diagrams,

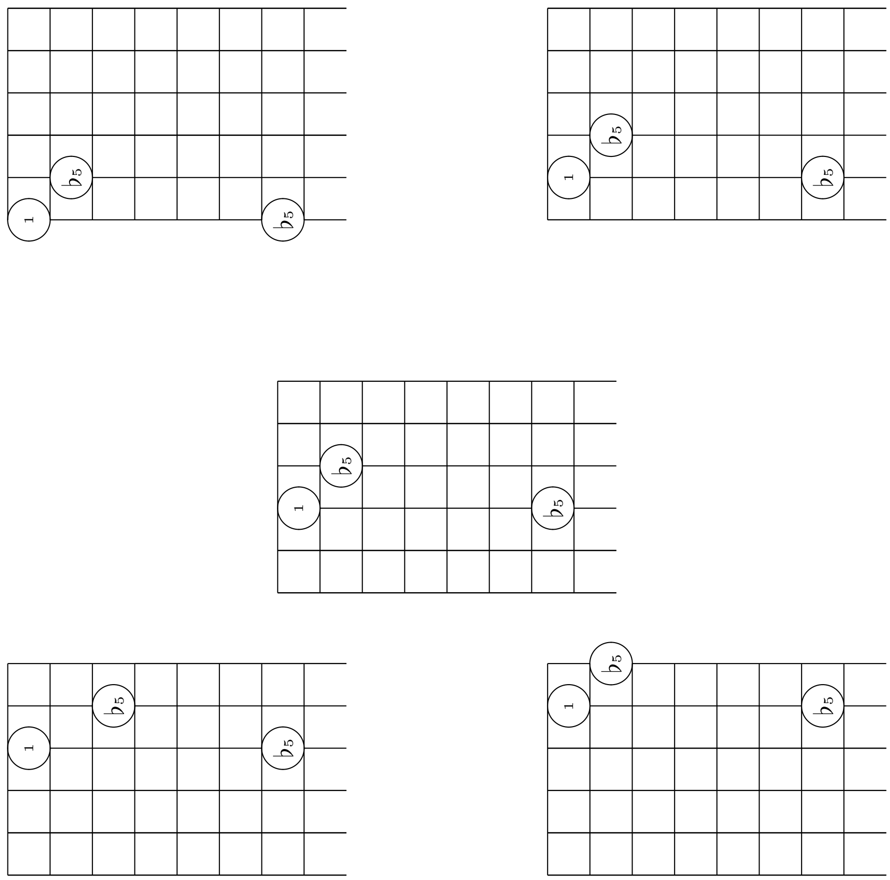
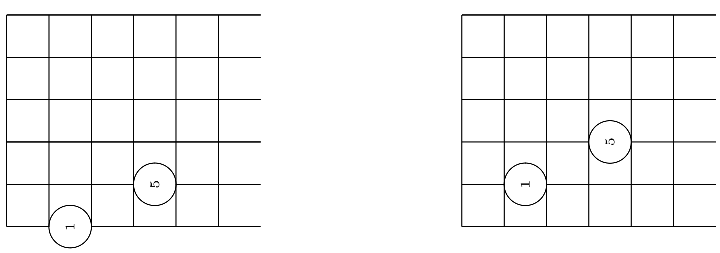


Table 8: Diminished Fifth Intervals



As an aside, let us note that when these are played simultaneously, i.e, as harmonic intervals, they represent ‘power chords’, which are common in rock and heavy metal music. This basically

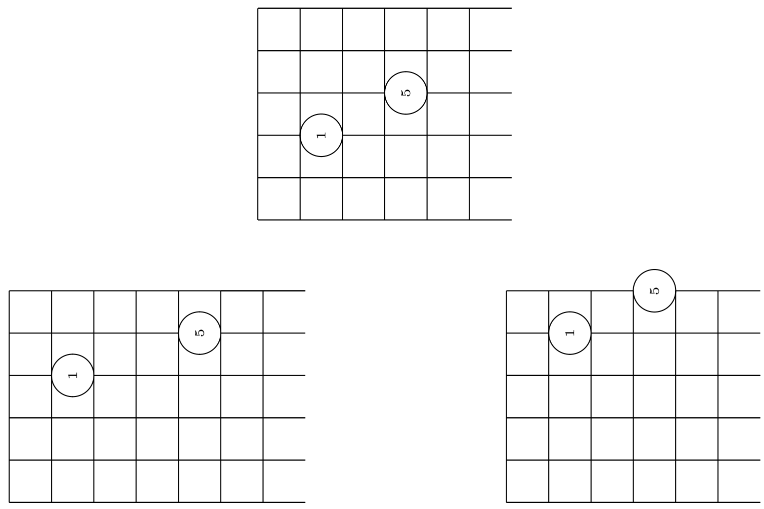


Table 9: Perfect Fifth Intervals

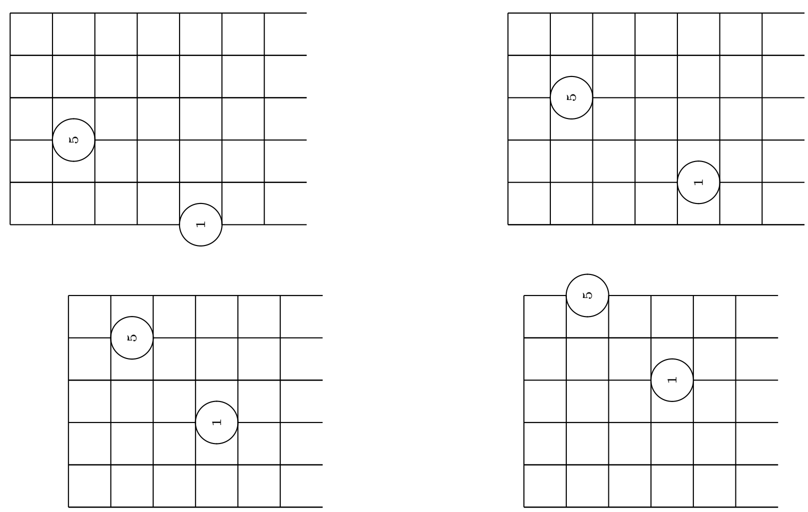
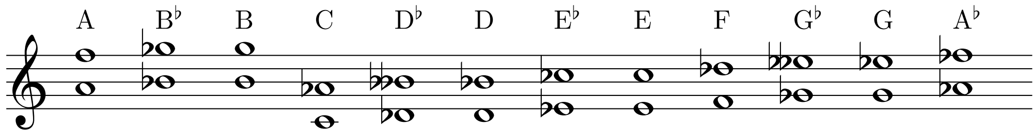


Table 10: Perfect Fifth Intervals - Alternate Representations

means that they are fifth ‘chords’ with no quality of third to distinguish them as either minor or Major. Equivalent representations, i.e., those across three strings non-adjacent, are

2.5 Sixth Type Intervals

We now come to sixth type intervals, which we recall are most naturally considered to minor or Major in terms of quality. Let us, then, look at the musical staff for minor sixth intervals.



Minor 6th Intervals

Fretboard diagrams which follow from the application of a minor sixth to the root take the form of,

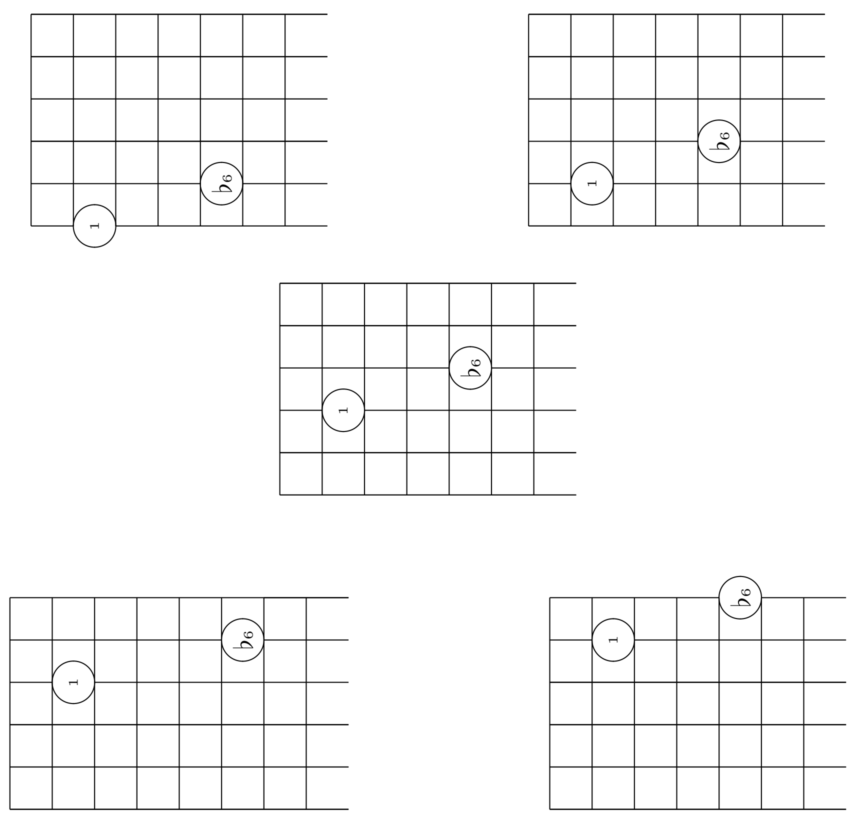


Table 11: Minor Sixth Intervals

or, alternatively, for non-adjacent strings, we have the representations

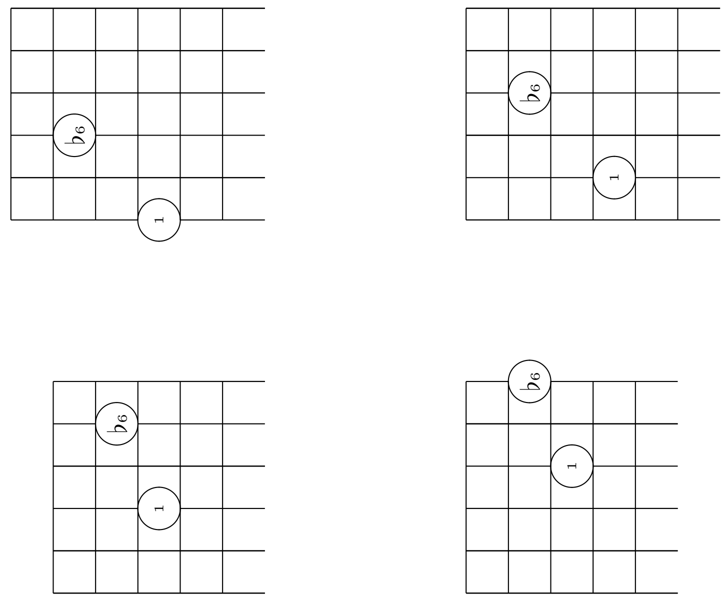
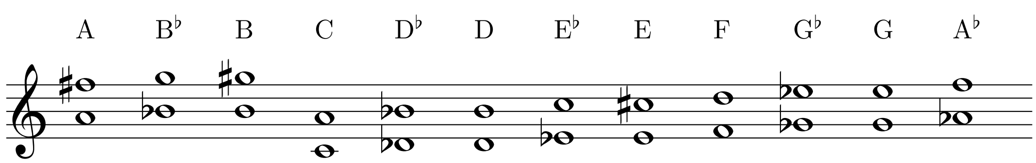


Table 12: Minor Sixth Intervals - Alternate Representation

When it comes to Major sixth intervals, we have, in one octave, the following staff.



Major 6th Intervals

On adjacent strings, our fretboard diagram for any octave becomes,

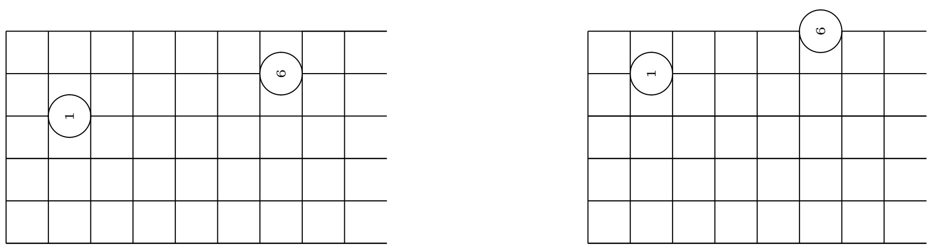
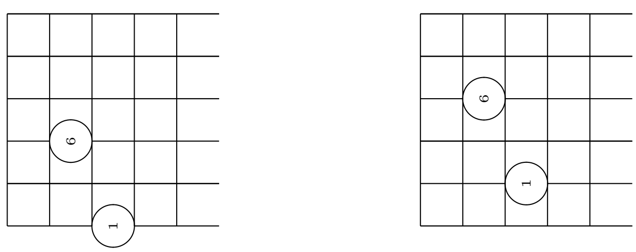


Table 13: Major Sixth Intervals

while for non-adjacent strings we have,



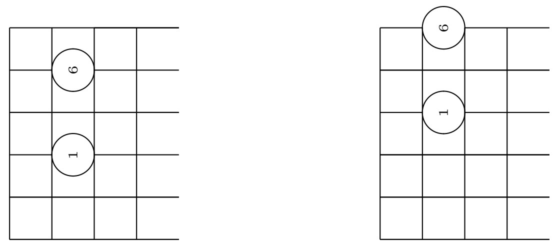
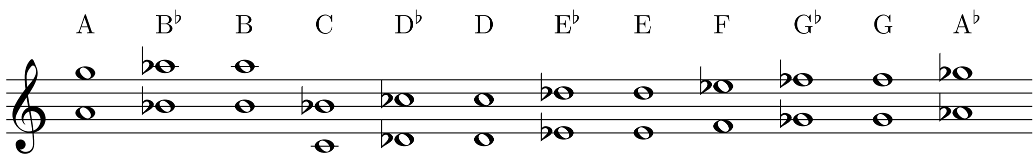


Table 14: Major Sixth Intervals - Alternate Representation

2.6 Seventh Type Intervals

We now examine the properties of seventh type intervals in the cases where these are either minor or Major. We start with the musical staff for minor intervals.



Minor 7th Intervals

Positions arising on the fretboard for various octaves of these are such that two adjacent string patterns are to spread out for normal guitar playing. With this in mind, we look to positions that are on non-adjacent strings. Firstly, we have,

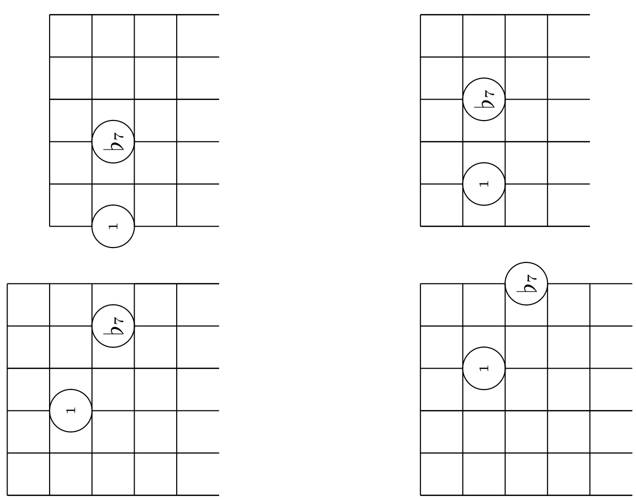


Table 15: Minor Seventh Intervals

Another representation is,

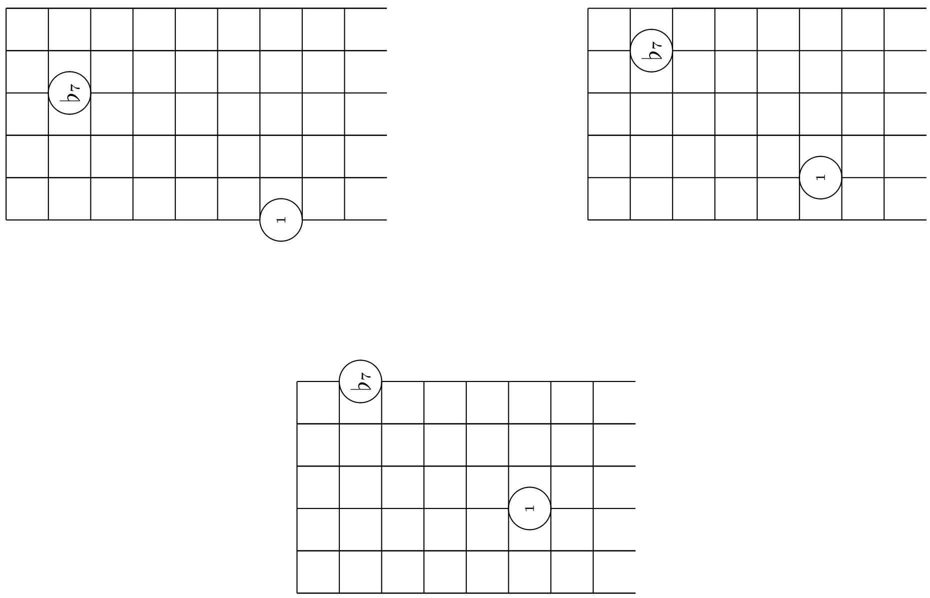
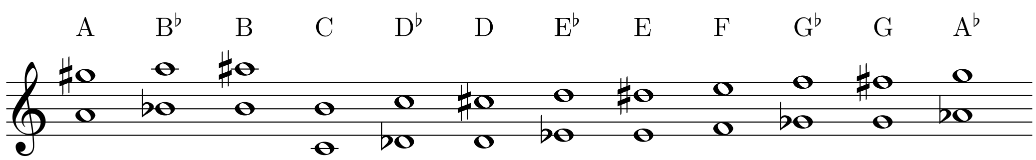


Table 16: Minor Seventh Intervals - Alternate Representation

In the case of Major seventh intervals, we have the following musical staff,



Major 7th Intervals

Yet again, as was the case with minor seventh intervals, we can arrive at two types of representations on the fretboard, i.e., ones where there are one string between the fretted notes and one where there are two strings between the fretted notes. The first of these is,

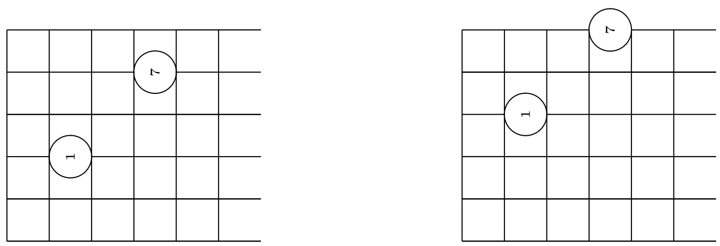


Table 17: Major Seventh Intervals

The second representation is, as before, across four strings. We have,

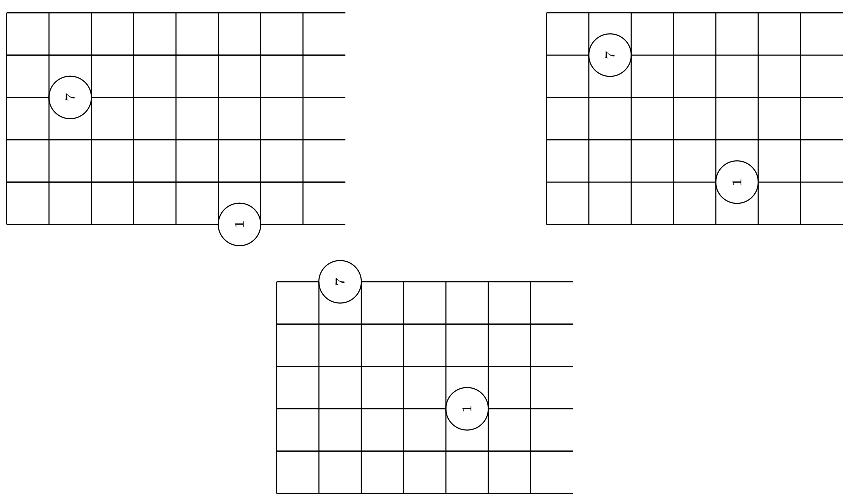
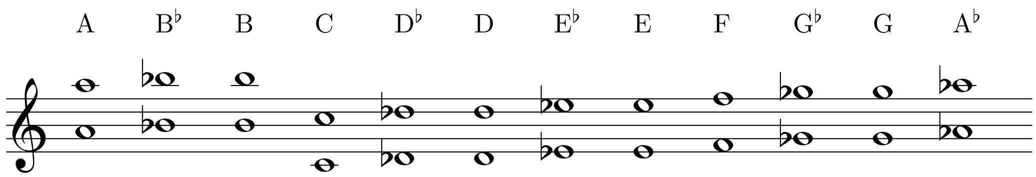


Table 18: Major Seventh Intervals - Alternate Representation

2.7 Perfect Eighth (Unison) Type Intervals

The final interval in the first octave to examine is that of the octave itself, i.e., the same note (the first) played twice in two different, but adjacent, octaves. For example,



Major 7th Intervals

Yet again, we have two representations. The first of these is,

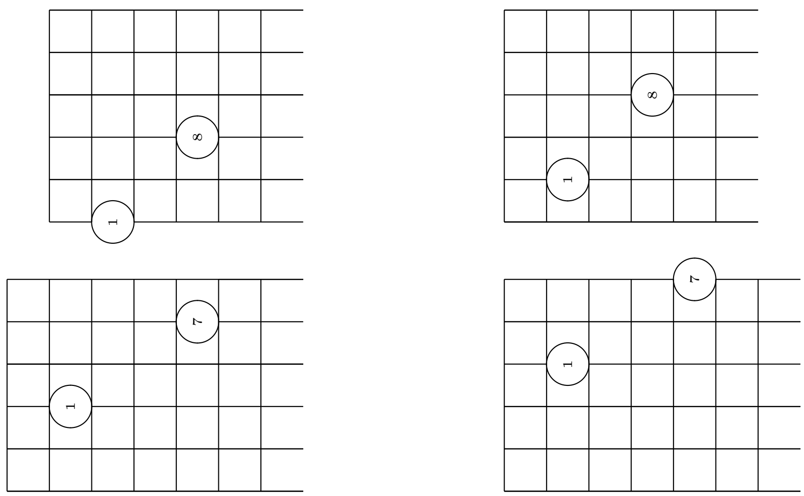
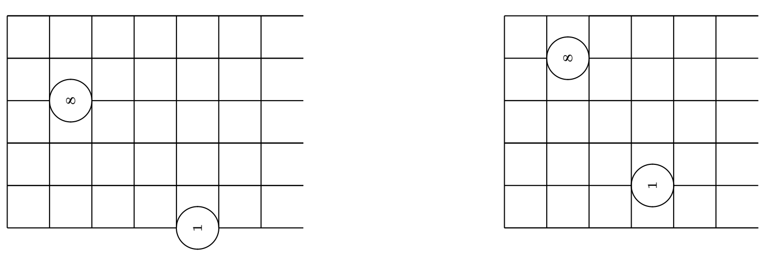


Table 19: Perfect Eighth Intervals

or, alternatively,



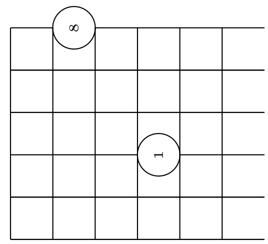
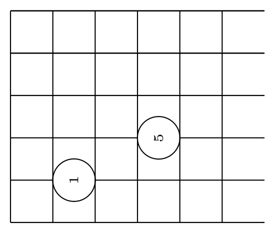
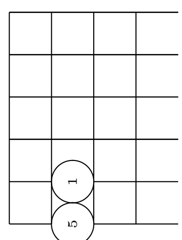


Table 20: Perfect Eighth Intervals - Alternate Representation

This concludes our exposition of intervals in the first octave. Before we examine intervals in the second octave in the next section, we look at the properties of inverting intervals in the first octave. We restrict the discussion to the first octave because the results there can be extended and carry over to the second also. To motivate the discussion, let us consider the following interval,



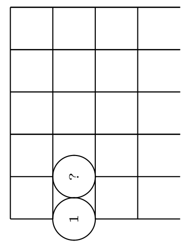
We know this interval to be a perfect fifth. But, we need to understand what happens if we take the fifth and put it in the bass position, thus arriving at This is what as known as inverting the



interval, i.e., the first remains the first, but the position of the second constituent of the interval is now in a different position. So, what if we were to call the fifth in the bass position the 'new' first? We would arrive at the following diagram,

where a question mark denotes the note (to be determined) in relation to the new first. Our study so far would tell us that this is a perfect fourth interval (see Table 5). In other words, we took a perfect fifth interval, and, upon inversion, it became a perfect fourth. We note that four+five=nine. This motivates the list of rules which follow when intervals are inverted;

- The original interval type once inverted becomes a new interval type, where the old and new types *always* sum to nine. For example, if we take a third interval and invert it then it becomes a sixth (3+6=9). Similarly, if we take a seventh and invert it, we arrive at a second (7+2=9). The sum is always nine. This takes care of what happens to types when we invert intervals. We now discuss the implications of inversion on quality.



- doubly diminished intervals become doubly augmented,
- diminished intervals become augmented,
- minor intervals become Major intervals,
- Major intervals become minor intervals, and, as was shown in the motivating example,
- perfect intervals *remain* perfect.

With regard to the set of rules applying to qualities of intervals, we comment firstly on the last rule. Perfect intervals remain perfect (as was the case with the motivating example), and this is why they are so-named: They remain of the same quality under inversion. For the other rules, let's look at some examples; if we were to play a minor third interval and invert it, we already know that its type will be sixth. The rules state that minor intervals become Major intervals, and thus, the inversion of a minor third is a Major sixth. Similarly, if we are to invert a Major seventh, it will become a minor second.

3 Beyond the First Octave: 9^{ths}, 11^{ths} and 13^{ths}

Recall from our early discussion that, in the second octave, we have intervals of type 9,10,11,12,13,14 and 15. This section will look at only a selection of these, and the interested reader is encouraged to derive the others as a completion of their own work.

In the sequel, we shall only examine 9th, 11ths and 13ths. The reason for restricting attention to these is the fact that various qualities of these types are useful in chord constructions. The others can appear in chords, also, but are less predominant. Because second octave intervals are simply first octave intervals played at an octave higher, they usually appear in the first octave at the very least. The reason that we omit the 15th from our discussions is that these are simply the first repeated two octaves above.

3.1 Ninth Type Intervals

It should be pointed out from the outset that these are simply second type intervals played in the *second* octave above the root. Because of this equivalence, we will examine them in terms of the second type intervals and the root. For example, what is the best way to view these intervals, given that there are generally a large number of strings between the two notes comprising the interval? In order to visualise such intervals, possibly the best approach for the student is to play a perfect eighth, i.e., an octave from the root. Then, while keeping the root intact, play a second type interval above the eighth. To help clarify this idea for the minor ninth interval, we have the following diagram.

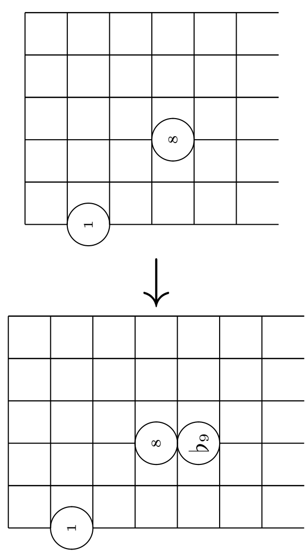
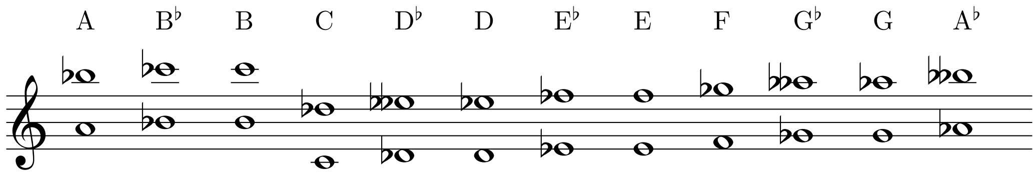


Table 21: Constructing Ninth Intervals From First Principles

Let us continue now with minor ninth intervals in general. The musical staff might appear as,



Minor 9th Intervals

Now for the fretboard patterns. We have the following table.

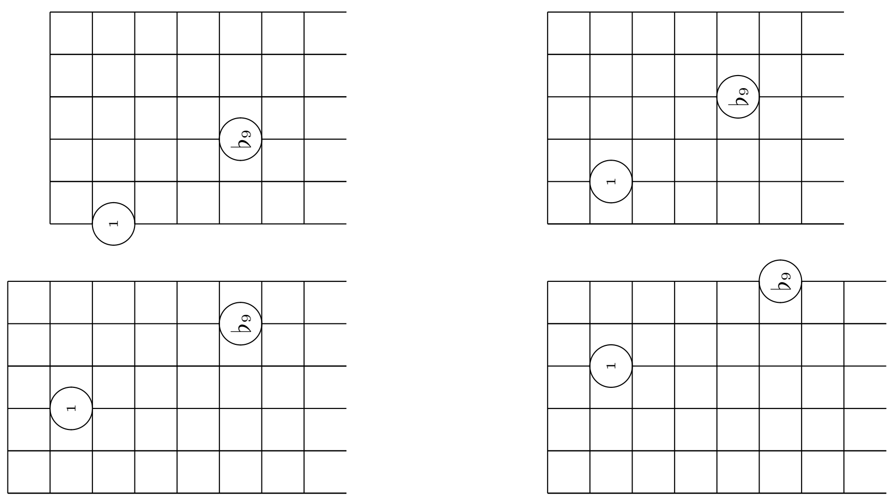
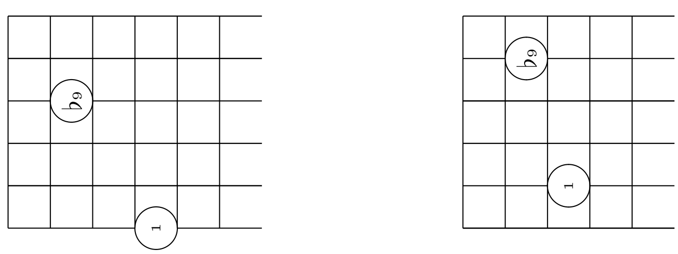


Table 22: Minor Ninth Intervals

or, alternatively,



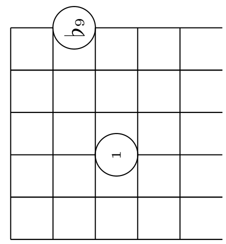
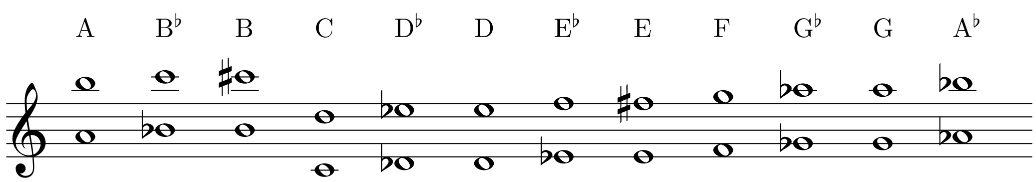


Table 23: Minor Ninth Intervals - Alternate Representation

For Major ninth intervals, we have the following musical staff,



Major 9th Intervals

The first fretboard representation of major ninth intervals is provided by the following diagrams,

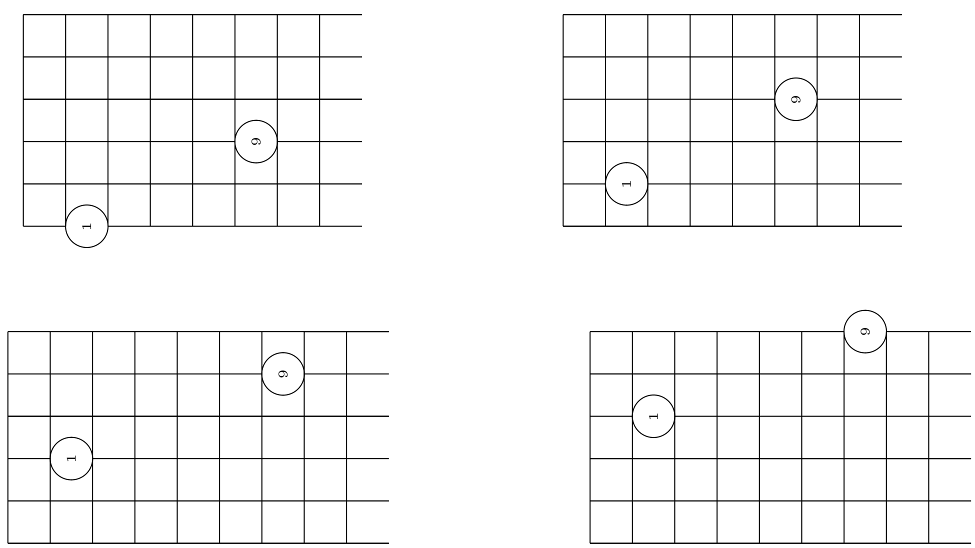


Table 24: Major Ninth Intervals

As an alternative to these, we also have

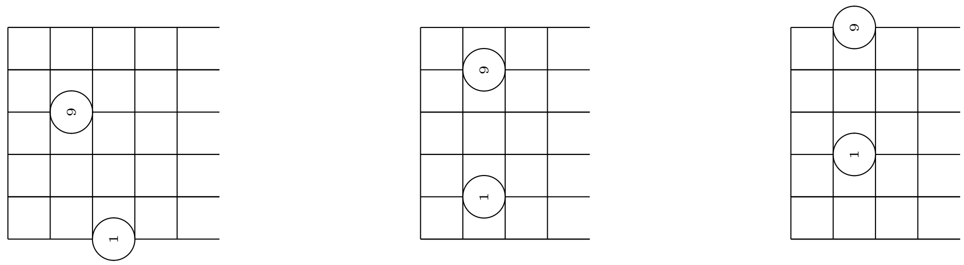
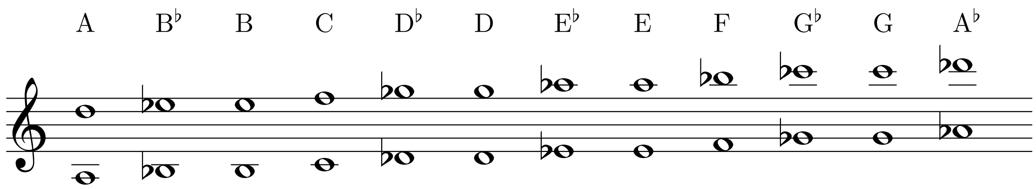


Table 25: Major Ninth Intervals - Alternate Representation

3.2 Eleventh Type Intervals

Eleventh type intervals are, as is the case of ninths, useful in the construction of extended chords. As was the case with fourths (the elevenths' first octave counterpart), we will examine both perfect and augmented elevenths. Starting with perfect elevenths, we have the following musical staff.



Perfect 11th Intervals

The first fretboard representation of perfect eleventh intervals is provided by the following diagrams,

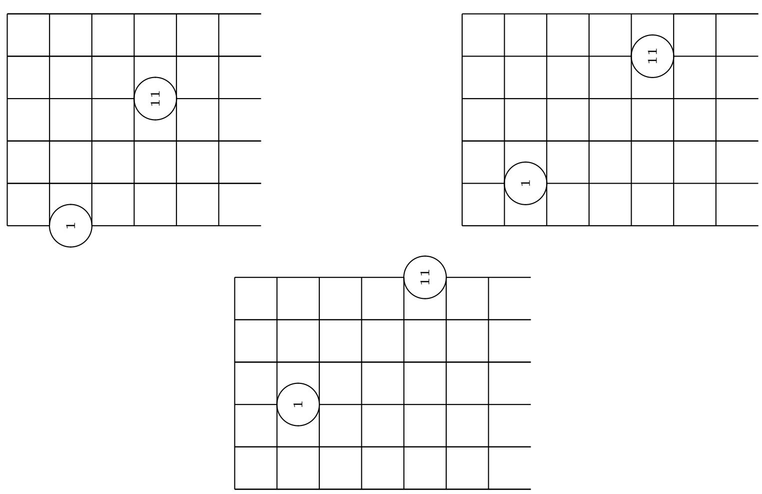


Table 26: Perfect Eleventh Intervals

These shapes have the alternate representation provided by,

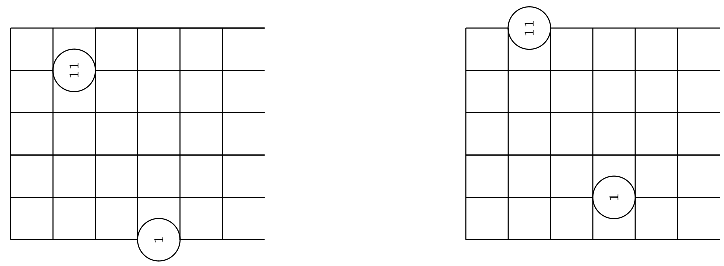
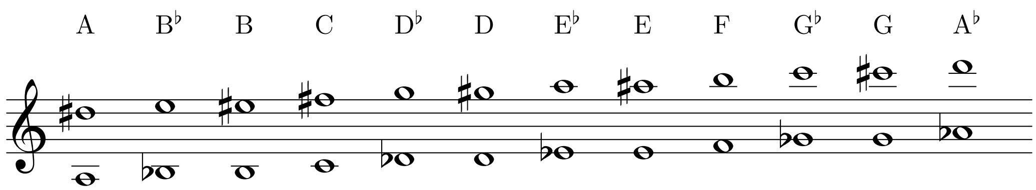


Table 27: Perfect Eleventh Intervals - Alternate Representation

We now look to close this subsection with the presentation of augmented eleventh intervals, which are enharmonically equivalent to diminished twelfth intervals. As has been custom with this document, we begin with the musical staff.



Augmented 11th Intervals

For the fretboard representations, we have,

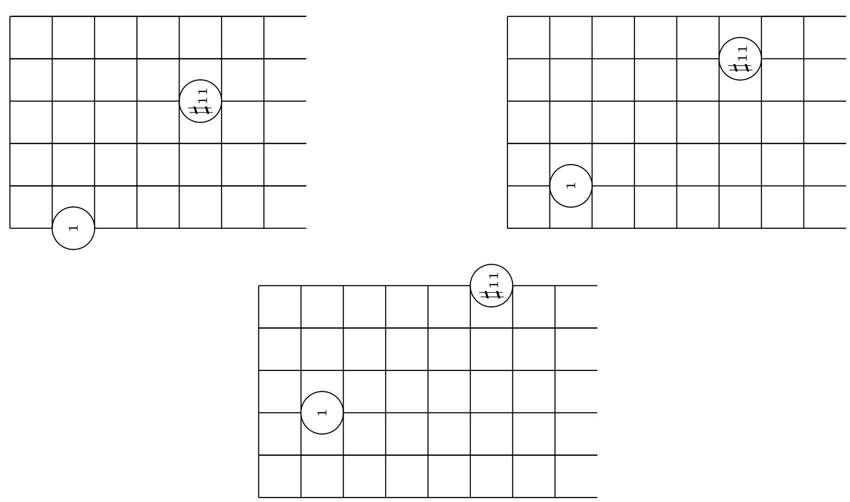


Table 28: Augmented Eleventh Intervals

and

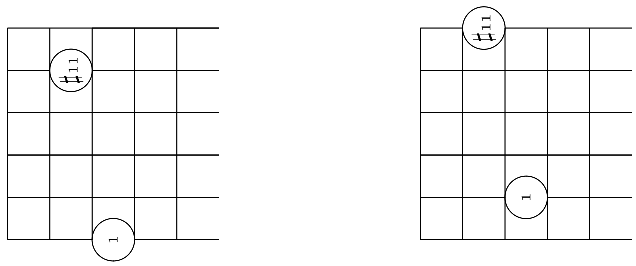
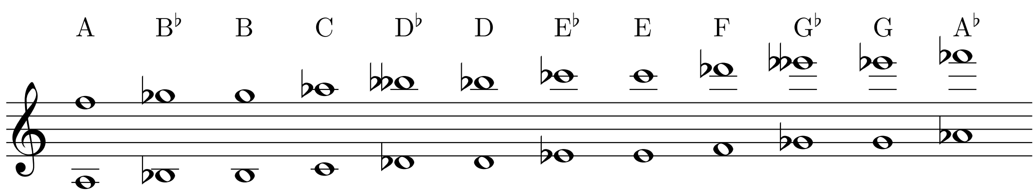


Table 29: Augmented Eleventh Intervals - Alternate Representation

3.3 Thirteenth Type Intervals

The last type of interval which we examine is the thirteenth type, which we recall is an octave above the sixth. On a musical staff, we have the following representations for minor thirteenths,



Minor 13th Intervals

We now examine the various fretboard positions. To begin with, we have,

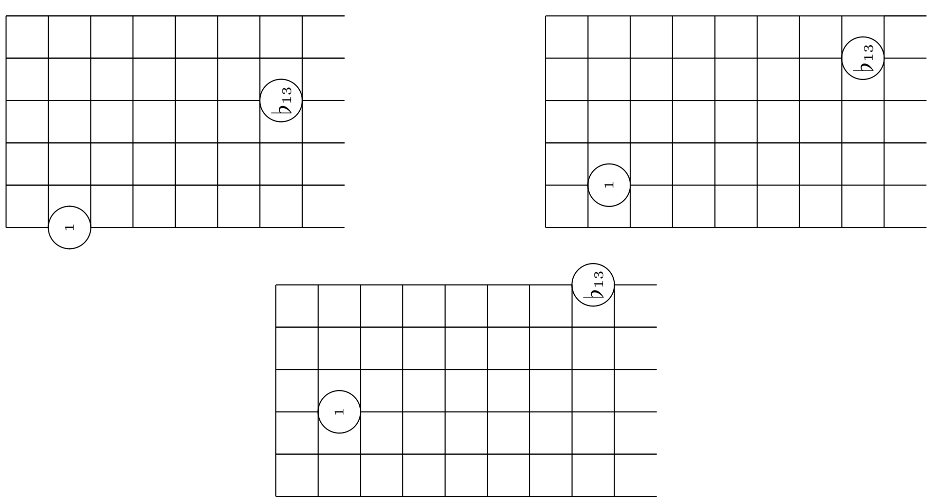


Table 30: Minor Thirteenth Intervals

and,

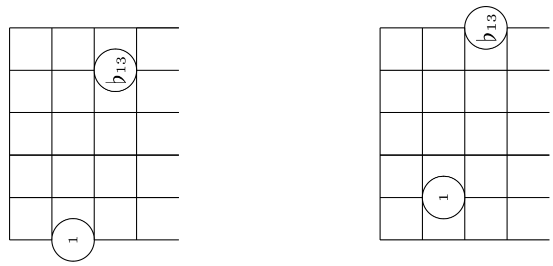


Table 31: Minor Thirteenth Intervals - Alternate Representations

and finally,

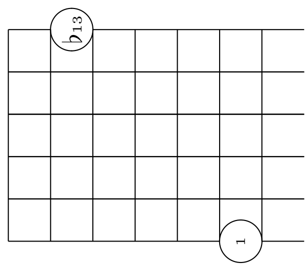
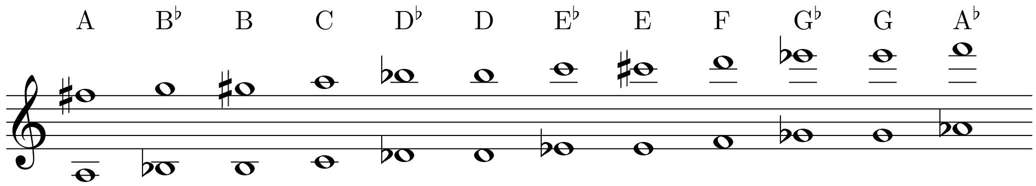


Table 32: Minor Thirteenth Intervals - Alternate Representation II

In the case of Major thirteenth intervals, we have,



Major 13th Intervals

along with the fretboard representations,

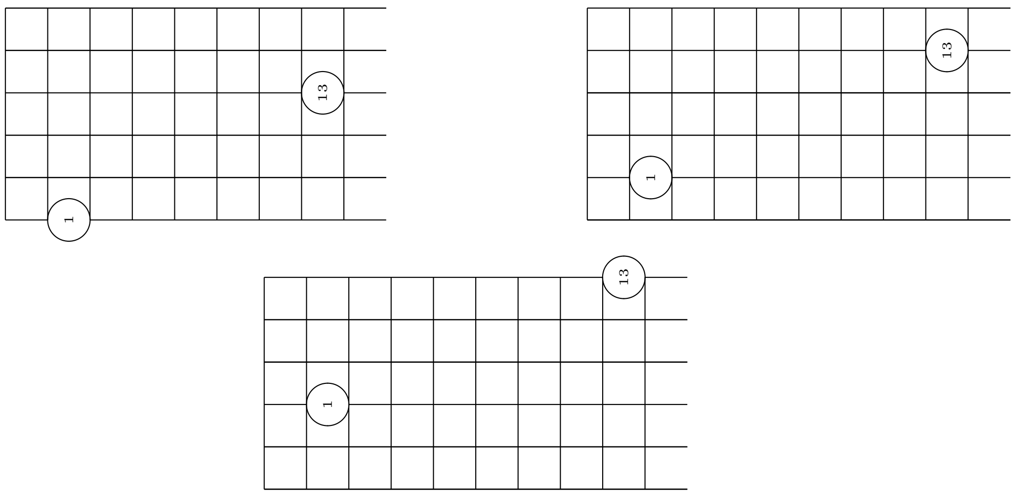


Table 33: Major Thirteenth Intervals

and,

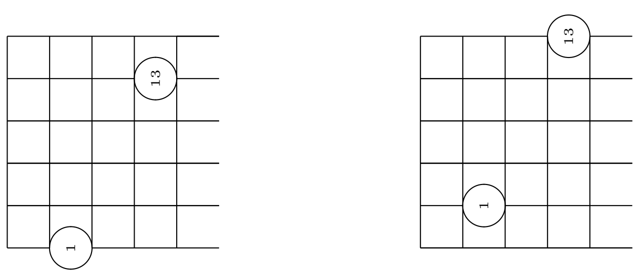


Table 34: Major Thirteenth Intervals - Alternate Representations

and finally,

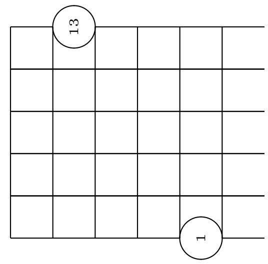
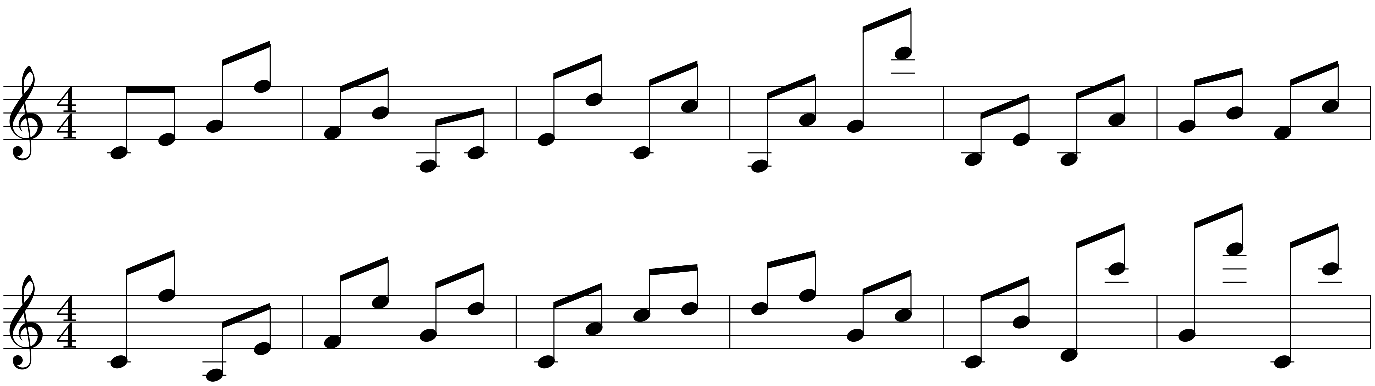


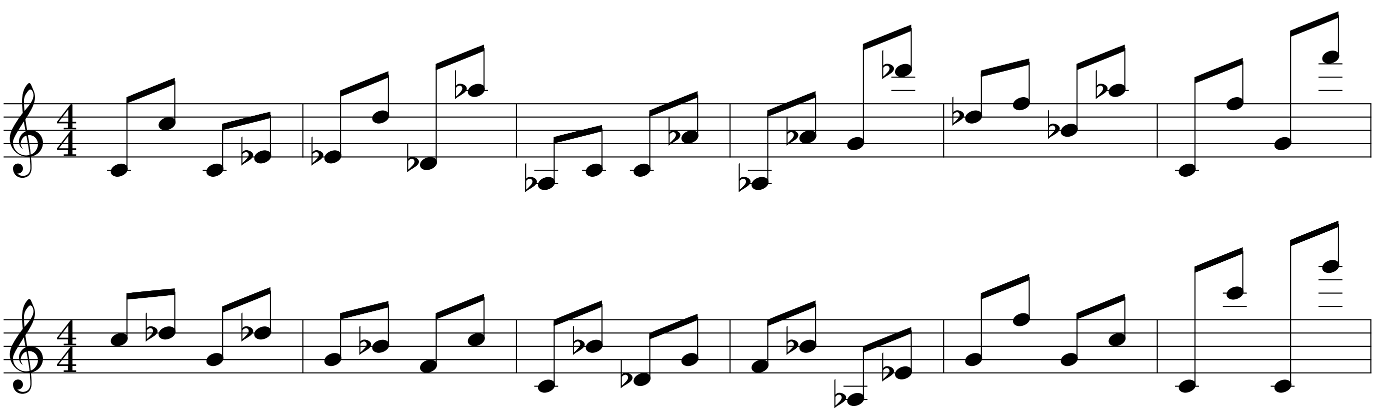
Table 35: Major Thirteenth Intervals - Alternate Representation II

4 Exercises

1. Identify the type and quality of these melodic intervals in the key of C:



2. Identify the type and quality of these melodic intervals in the key of C Phrygian:



3. Pick a key that you like. Then,
   a) play all the seconds in that key in one octave,
   b) play all the thirds in that key in a single octave,
   c) play all the fourths in that key in a single octave,
   d) play all the fifths in that key in a single octave,
   e) play all the sixths in that key in a single octave,
   f) play all the sevenths in that key in a single octave.
4. Repeat exercise 3 in a different octave.
5. Play different thirds on one string at a time only in a single key.

6. Play different fifths on one string at a time only in a single key.

7. Invert the intervals in exercise 1. Then apply the rules of inversion to identify the new intervals.