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FLORIDA STATE UNIVERSITY

IMPACT STUDIES ON PURE METALS

By
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A Thesis
Submitted to the Graduate Council of Florida State University in partial fulfillment of the requirements for the degree of Master of Science.

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Thanks are due also to the several companies who supplied the pure metal samples. A list of these companies may be found in Appendix III.

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INTRODUCTION

The first qualitative studies of the simple two-body impact problem were carried out by Sir Isaac Newton in the seventeenth century and published in the Principia in 1686.[1] In 1668 Sir Christopher Wren had demonstrated before the Royal Society of London that the total momentum of a system of two impacting bodies is a conserved quantity.[2] Soon afterwards, Newton realized that an additional relation was needed in order to solve uniquely for the motion of the two bodies after impact, assuming their previous motion to be known. His experiments led him to formulate the empirical equation

$$|v_r| = e|v_a| \qquad (1)$$

which, together with the equation for the conservation of momentum, determines the motion completely after collision. In the above equation v_a and v_r are the relative velocity of the two bodies before and after impact, while e is an empirical constant. Three restrictions are made for equation (1) to apply to an impact problem. These are:

1. v_a and v_r must lie on a common straight line which is normal to the surfaces of the two bodies at the point of impact.

2. This line must also pass through the mass

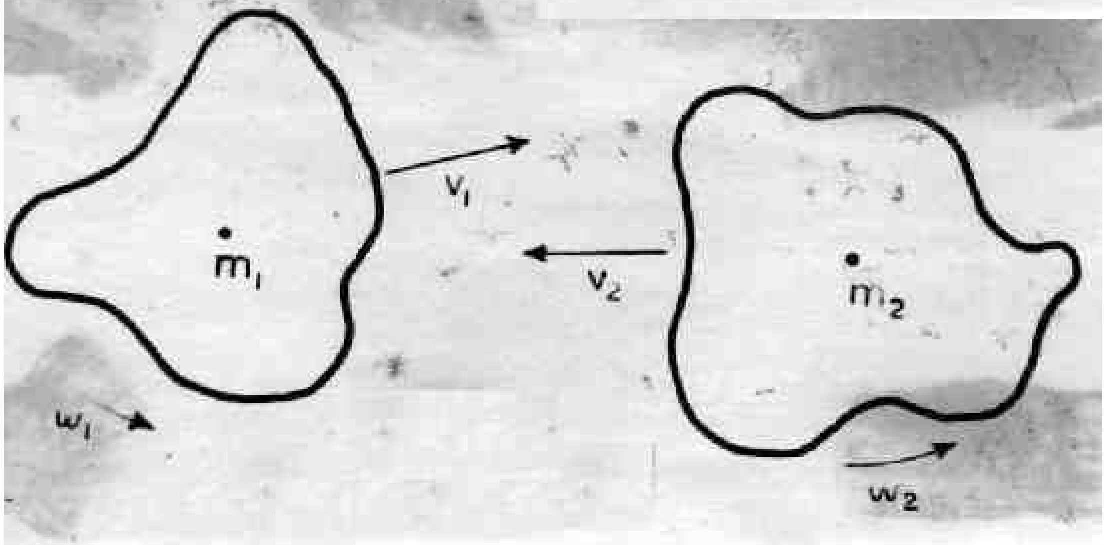


Fig. 1. General Two-Body Impact Problem

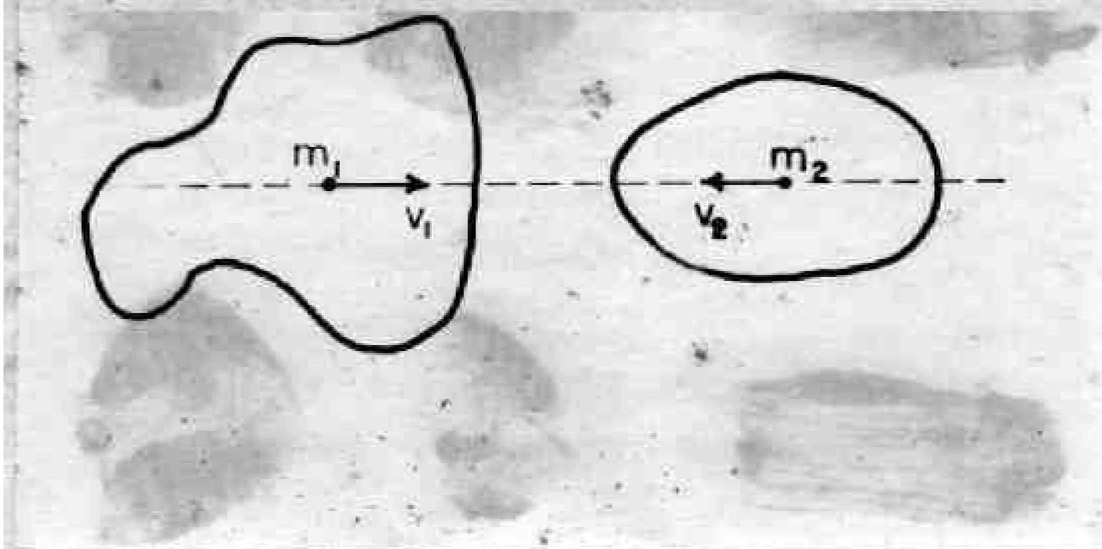
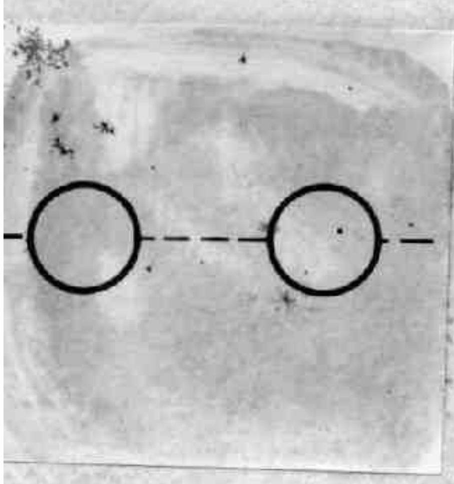
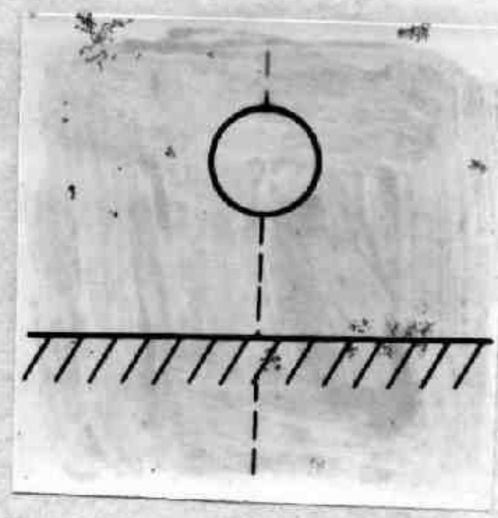


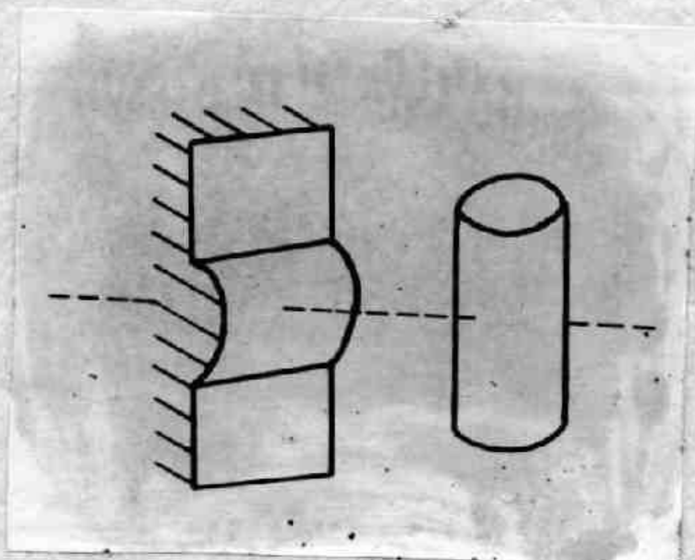
Fig. 2. Restricted Two-Body Impact Problem



sphere on sphere



sphere on infinite wall



cylinder on infinite cylinder

Fig. 3. Examples of Restricted Problem

center of each body.

3. The bodies must have no angular momenta.

Figure 1 shows the general (two dimensional) impact problem; Figure 2 shows the restricted problem. In this paper we shall be concerned exclusively with systems conforming to the restricted problem, in particular with the examples shown in Figure 3.

Newton called the term e in equation (1) the " coefficient of restitution." Recently, however, the word " resiliency" has been suggested for this term as being less cumbersome and more descriptive than the original expression,[3] and it will be so denoted in this paper. Newton concluded, on the basis of his experiments with impacting suspended spheres, that e is a constant of material, ranging between the values zero and one. Putty is a good example of a material with zero resiliency; impacting bodies of this material stick together after collision (v_r equals zero). A resiliency of one is approached by materials such as hardened steel or ivory.

Newton had decided that the resiliency was independent of the relative approach velocity v_a. Later investigators, however, found that some dependency does exist. Hodgkinson, in 1834,[4] and Vincent, in 1900,[5] found a slow decrease in e with increasing v_a. Another variation was discovered in 1918, when C. V. Raman in India attacked the problem.[6] Lord Rayleigh, in 1906, had demonstrated that the energy lost in impact to vibrations in the impacting bodies was, in the case

Page 5

of sphere on sphere, quite negligible.[7] In 1916, Banerji had shown that the energy lost in the production of sound waves and other vibrations to the surrounding medium was also inappreciable.[8] With these things in mind, Raman postulated that the energy lost (sphere-sphere) was due mainly to "stresses occurring during impact exceeding the limits within which elastic recovery is immediate and perfect." Reasoning from this, he predicted that the resiliency should approach unity as v_a approaches a critical v_A^0 under which the impact is so soft that no irreversible stresses occur. A rough calculation of v_a^0 using a model of impact developed by Hertz,[9] shows that it is of the order of 0.5 cm/sec for hardened steel and somewhat under this for softer materials.

Raman performed a series of experiments on various metals, confirming these predictions. Figures 4, 5, 6, and 7 show the curves of e vs v_a for the materials he tested. These curves, as they appear here, are photographed directly from his original article in the Physical Review. Figure 8 shows a curve of e vs v_s obtained by J. P. Andrews in some confirmatory work done in 1929-1931.[10]

The purpose of the present investigation was to verify and extend the work of Raman, using as specimens metals of high purity. Samples of pure metals were obtained from various companies, a list of which appears in Appendix III.

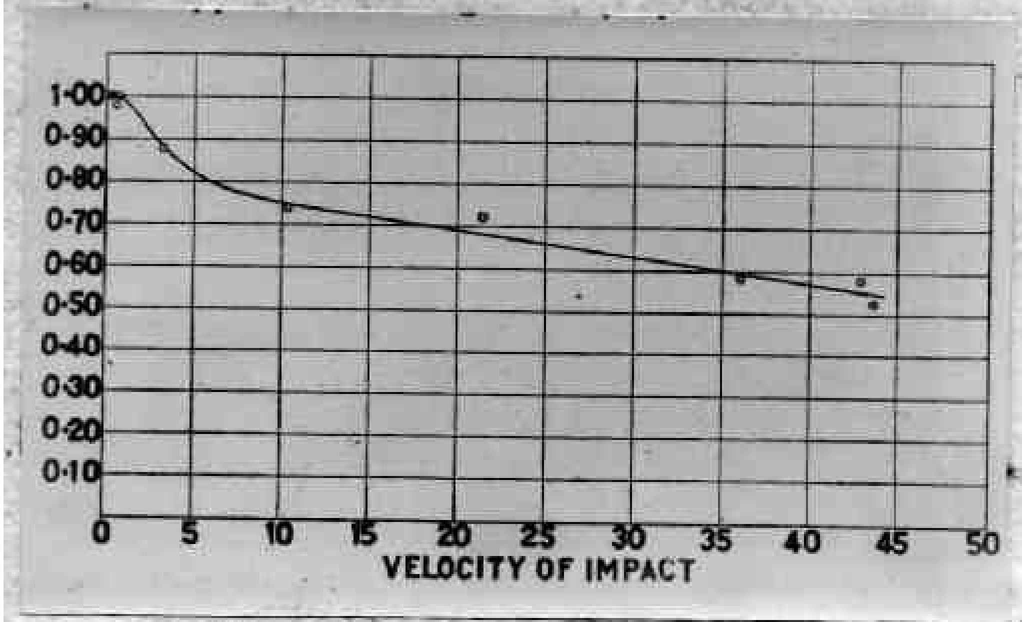


Fig. 4. Raman's Resiliency Curve for Brass

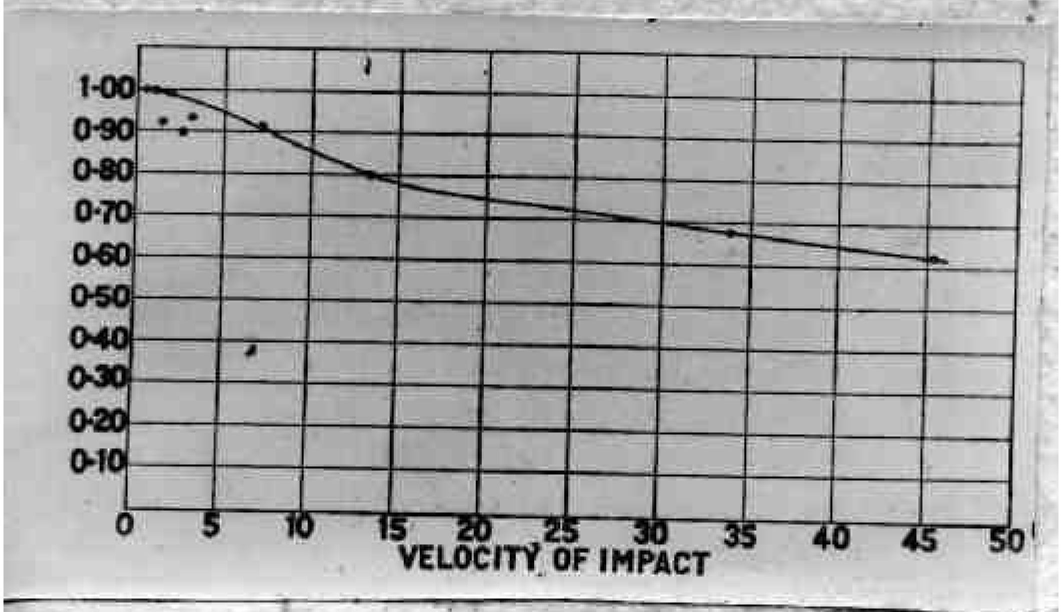


Fig. 5. Raman's Resiliency Curve for Aluminum

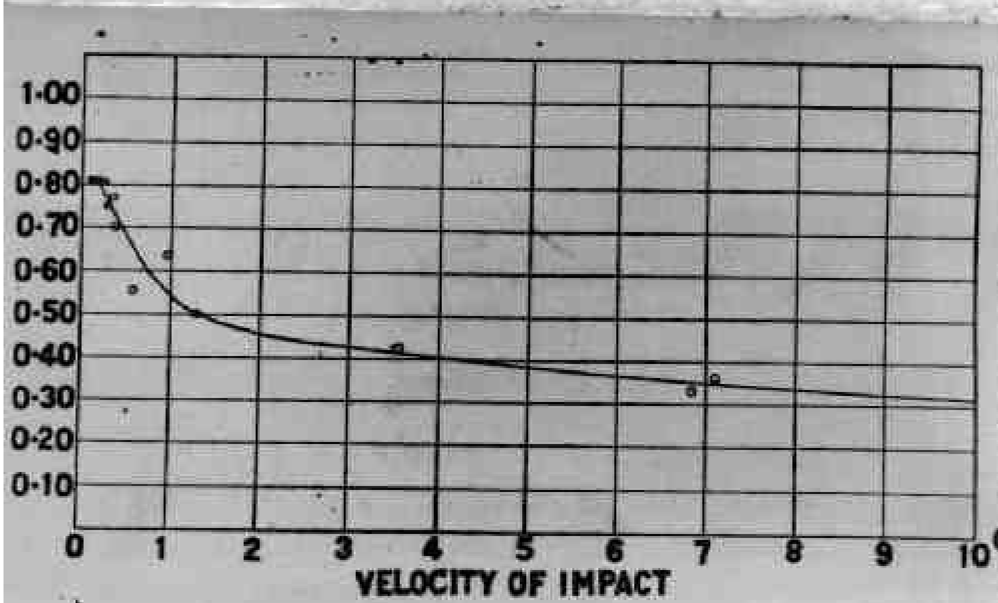


Fig. 6. Raman's Resiliency Curve for Lead

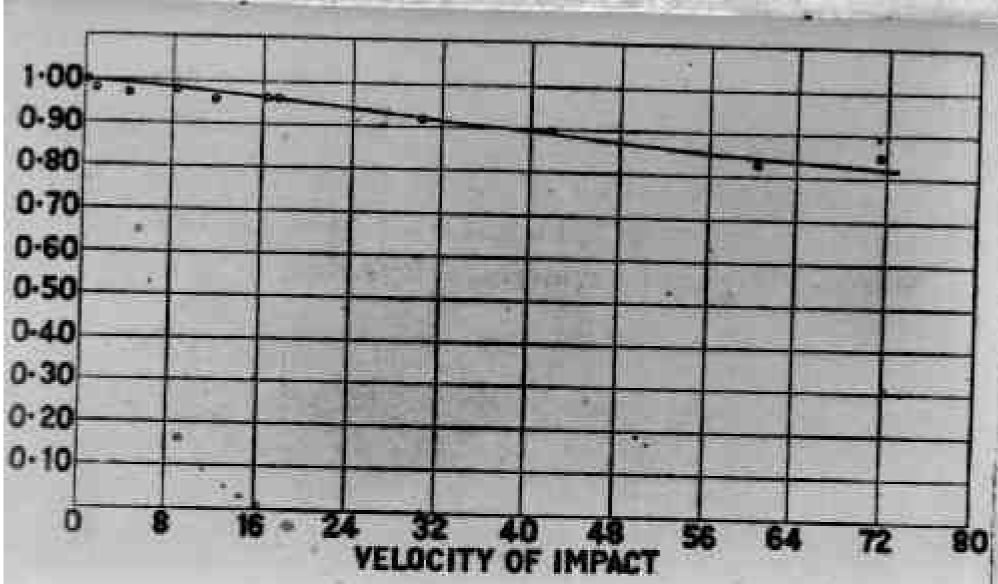


Fig. 7. Raman's Resiliency Curve for Hard Bronze

INSTRUMENTATION

Description of the Impacting Bodies

In previous investigations of the properties of the restricted impact problem, the usual apparatus has involved suspended spheres or spheres bouncing on a semi-infinite wall, such as a steel anvil.[11] The experimental setup which has been devised for this investigation is somewhat different. It consists (Figures 9 and 10) of a heavy (100 pound) cast-iron disk with sections cut away to provide striking faces. This disk, the " impactor," rotated by an electric variable-speed motor, impacts against a light test specimen, pendularly suspended and initially at rest. By comparison with this test specimen the impactor approximates a semi-infinite wall to better than 0.5% (Appendix I). The remainder of the apparatus consists of instruments used for measuring the velocities of these two bodies.

On the striking face of the impactor a cylindrical segment of the metal under test is rigidly mounted, its axis horizontal. The test specimen consists of a suspended cylinder of the metal with its axis vertical. Smaller cylinders extend from each end of the main cylinder, the top one machined to the exact diameter of 0.1873 inches, for use in one of the velocity measuring devices to be described later. The lower

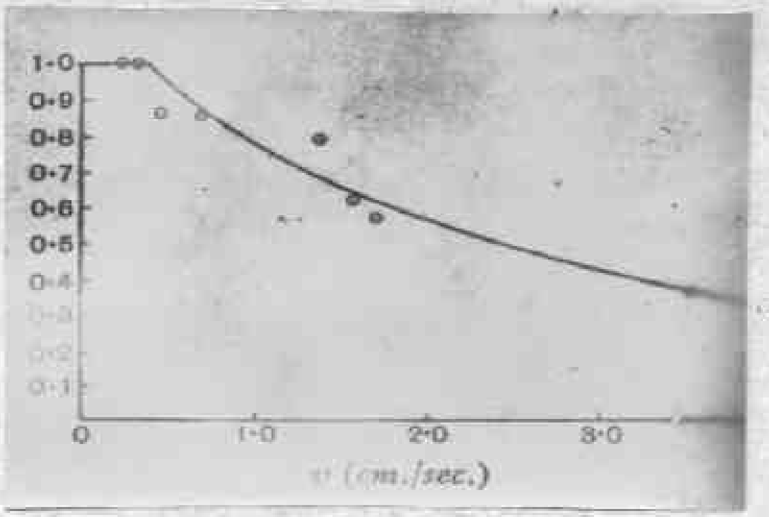


Fig. 8. Andrews' Resiliency Curve for Tin

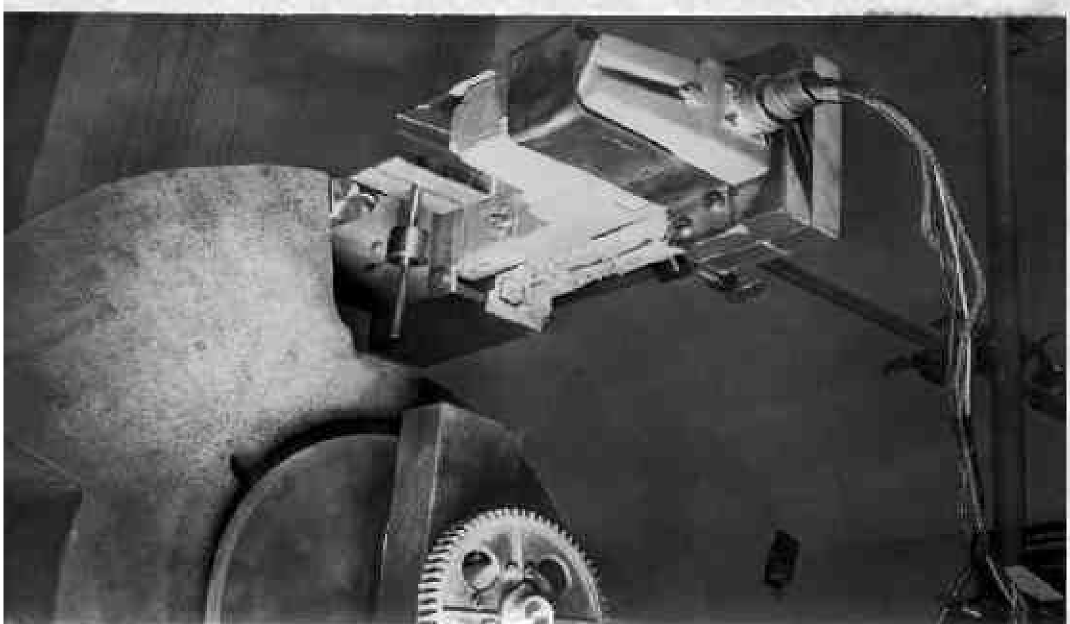


Fig. 9. Photograph of the Apperatus .

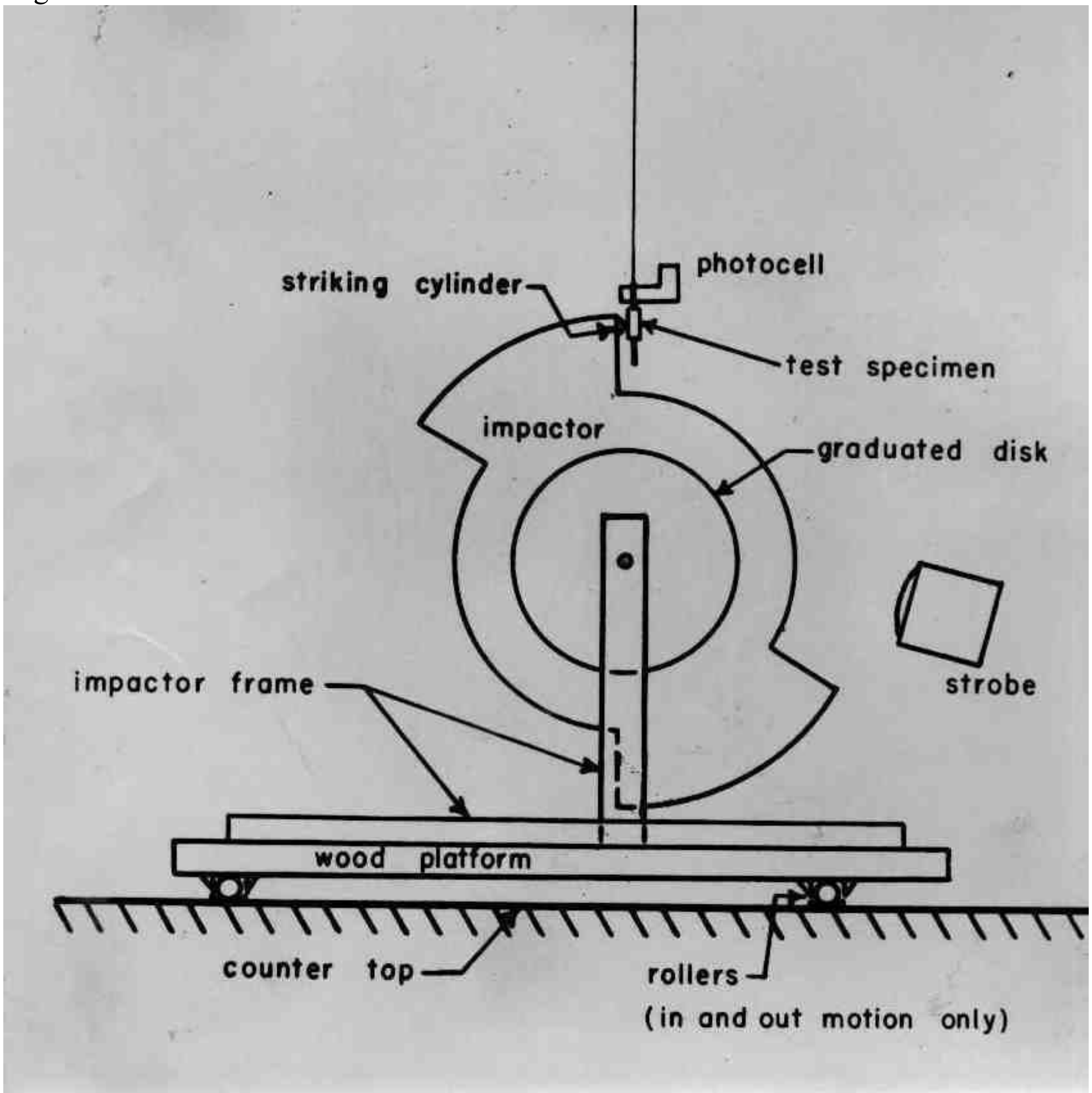


Fig. 10. Diagram of the Apparatus

extension counterbalances the top, keeping the specimen symmetrical so that central impact does not cause a wobbling motion.

The impactor disk measures about 50 centimeters in diameter and is 5 centimeters thick. The striking cylinders mounted on the impactor were 6 centimeters long, had radii of about 10 centimeters, and were from 1 to 3 centimeters thick (striking surface to flat back). They were mounted by means of two 10-24 bolts 1 and 3/8 inches apart. The striking surfaces were between these bolts. The test specimens were about 10 centimeters long. The main cylindrical body was from 2 to 8 centimeters long and had a diameter of 2 to 3 centimeters in the various specimens. The small cylindrical extensions were symmetrical with the main cylinders and of varying lengths to make up 10 centimeter specimens.

Description of the Velocity Measuring Devices

The velocity U of the striking surfaces of the impactor given by

$$U = R\omega \qquad (2)$$

where R is the radial distance of the surface from the axis of the impactor and ω is the angular velocity of the impactor. R was originally determined (24.08 cm) when the apparatus was constructed by measuring from the center of the impactor to the central position of the striking plate mounting threads. The angular velocity was measured for each individual impact by "stopping" one end of the driving motor shaft with a stroboscopic light. The gear ratio between the motor and the impactor being known (ratios of 1:1000 and 1:1600 were used), ω could be calculated from the reading of the stroboscope. An alternative method which, however, turned out not to be as convenient, was to observe the rim of a graduated brass disk (500 divisions) turning with the impactor through a low power microscope with the stroboscopic light. This process was helpful, however, in ascertaining whether the impactor was turning smoothly or not.

The measurement of the velocity V of the test specimen after impact was somewhat more complicated. With the motion imparted by impact, the upper cylindrical extension of the specimen passed through the focal area of a photocell beam (Figure 11). The output from the photocell circuit (Figure 12) was fed into a Berkeley Model 5510 Universal Counter and Timer used as a time interval meter, which measured the length of time for the output of the photocell circuit to go between two preset voltages.

The output voltage of the photocell circuit was measured as a function of the distance of penetration into the focal area of the light beam of an 0.1873 inch diameter cylinder by mounting such a cylinder on a carriage and pushing it through the beam with a micrometer head (Figure 13). For this procedure, the photocell was taken

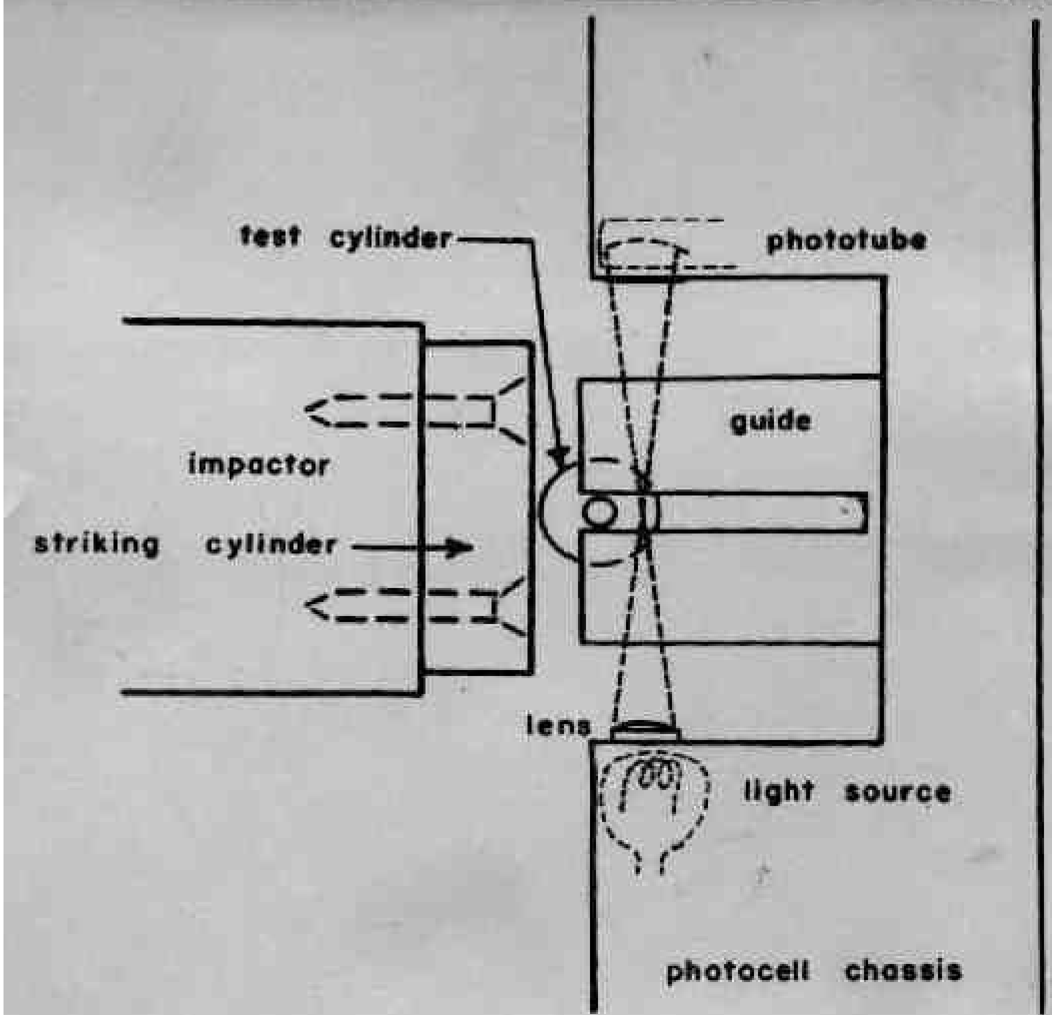


Fig. 11. Top view diagram of the apparatus

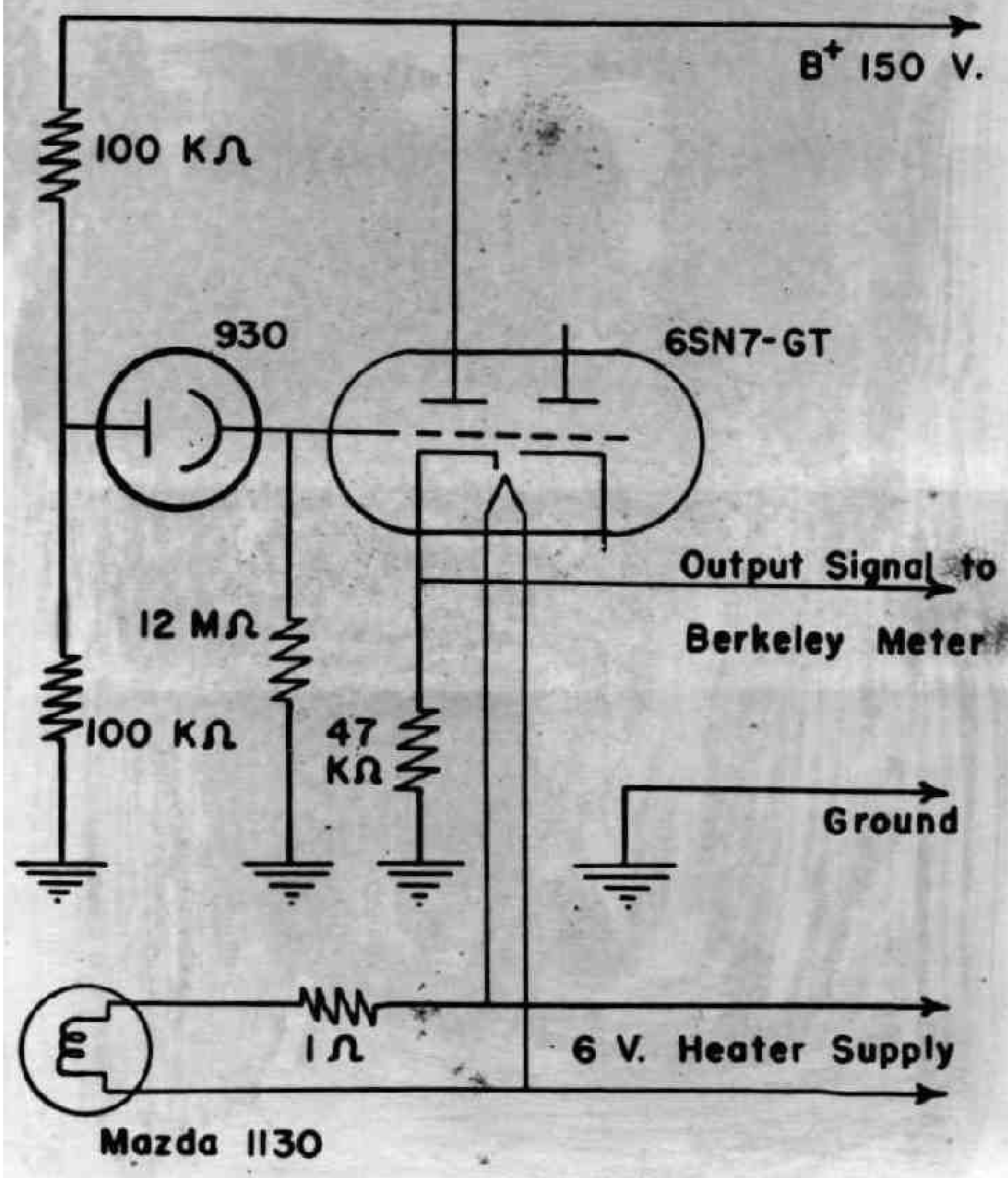


Fig. 12. Photocell Circuit

Fig. 13. Photocell position for the measurement of L'

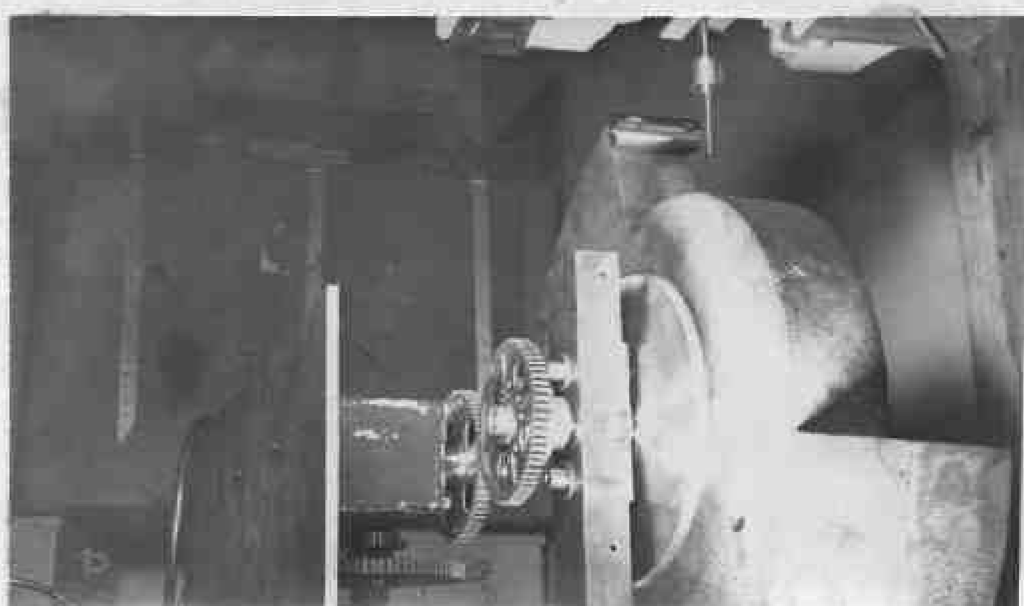


Fig. 14. Photograph of apparatus, motor in place

from its mounting above the impactor and placed on the counter top. From the curve obtained by this procedure, the distance between any two voltage settings could be determined and this distance, the effective path length L', when divided by the time of passing T shown by the Berkeley instrument, gave a velocity V_m from which V was calculated using two correction factors discussed in Appendix II.

The resiliency e was then calculated from U and V as follows

$$|v_a| = U \qquad\qquad |v_r| = V - U \qquad (3)$$

$$e = \frac{|v_r|}{|v_a|} = \frac{V - U}{U} = \frac{V}{U} - 1 \qquad (4)$$

RESULTS OF THE INVESTIGATION

Materials Tested

In all, seven metals were investigated. These metals, and their resiliencies at $v_a = U = 5\frac{1}{2}$ cm/sec (where the resiliency curves had flattened out) are tabulated below.

TABLE I

MATERIALS TESTED

Metal	Resiliency
Molybdenum	0.12 ± 0.10
Copper	0.30 ± 0.06
Cadmium	0.30 ± 0.08
Titanium	0.35 ± 0.04
Magnesium	0.36 ± 0.04
Nickel	0.50 ± 0.05
Carbon Steel	0.70 ± 0.08

All of these, except the carbon steel, were in at least a "commercially pure" state.

All of the specimens and striking plates were lathe turned, polished with emery cloth and crocus cloth, and cleaned with carbon tetrachloride and alcohol before testing. No attempt was made to evaluate the effect of work-hardening of the specimens due to machining.

Data on the composition of these metals may be found in Appendix III.

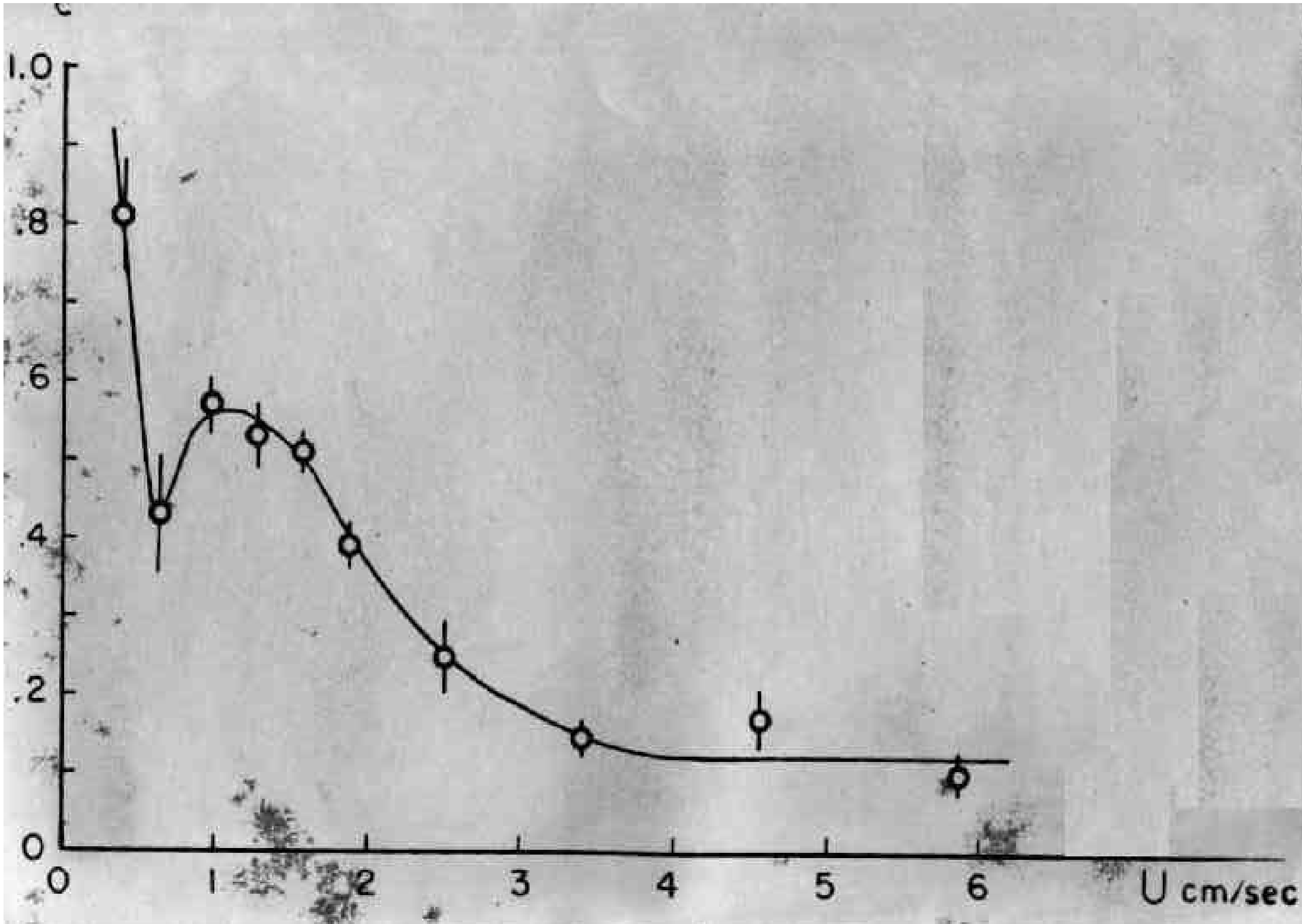


Fig. 15. Resiliency Curve for Molybdenum

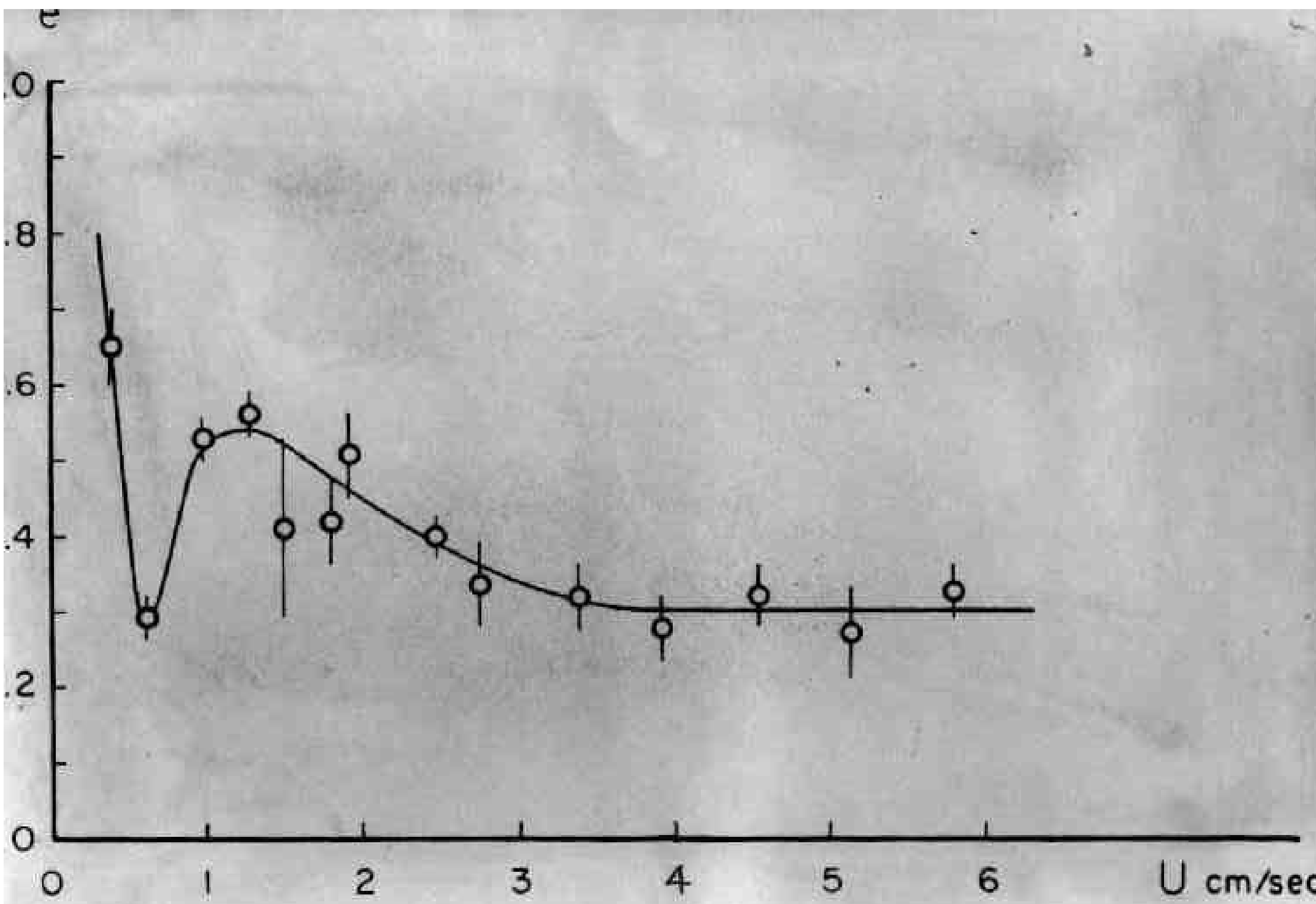


Fig. 16. Resiliency Curve for Copper

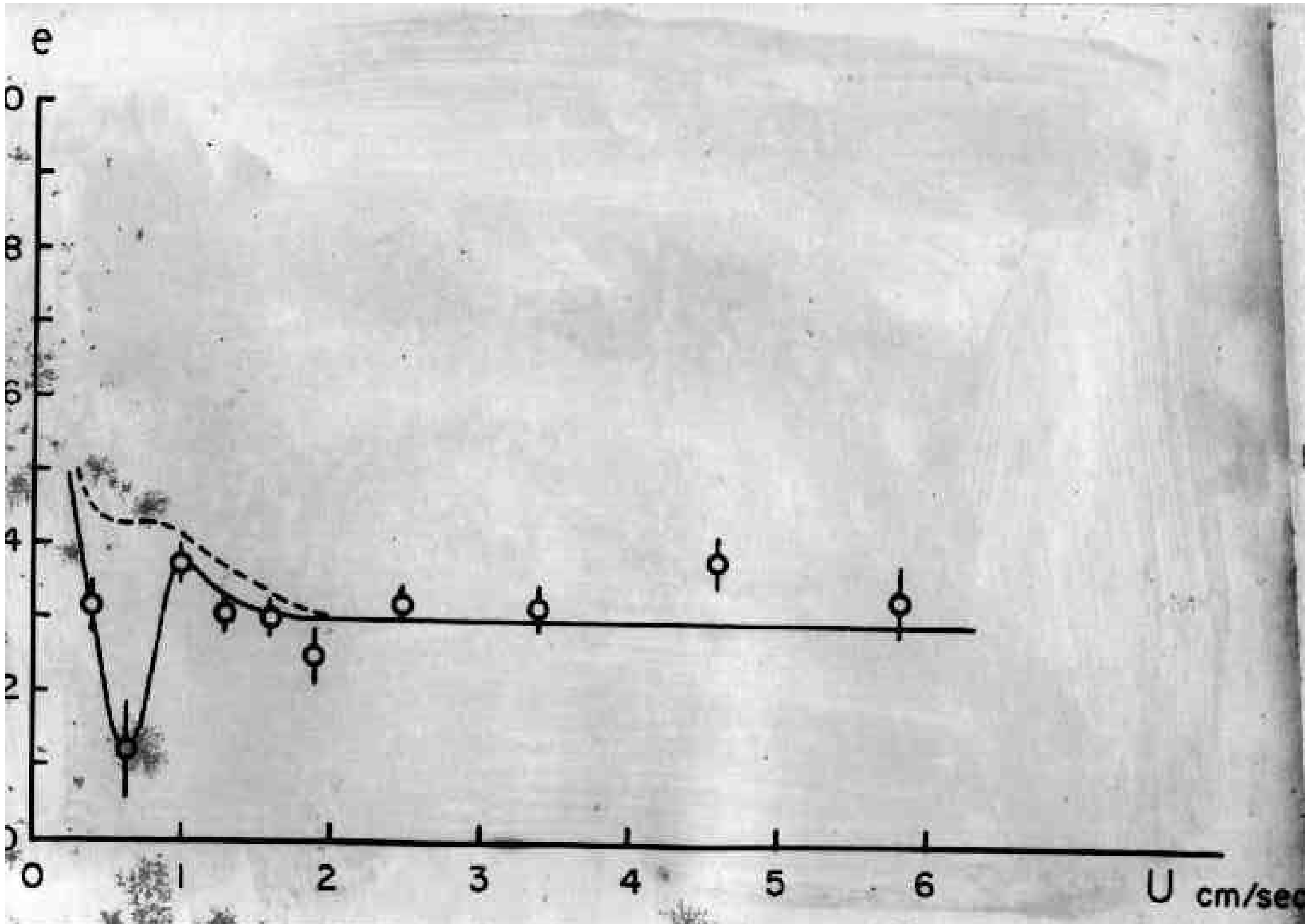


Fig. 17. Resiliency Curve for Cadmium

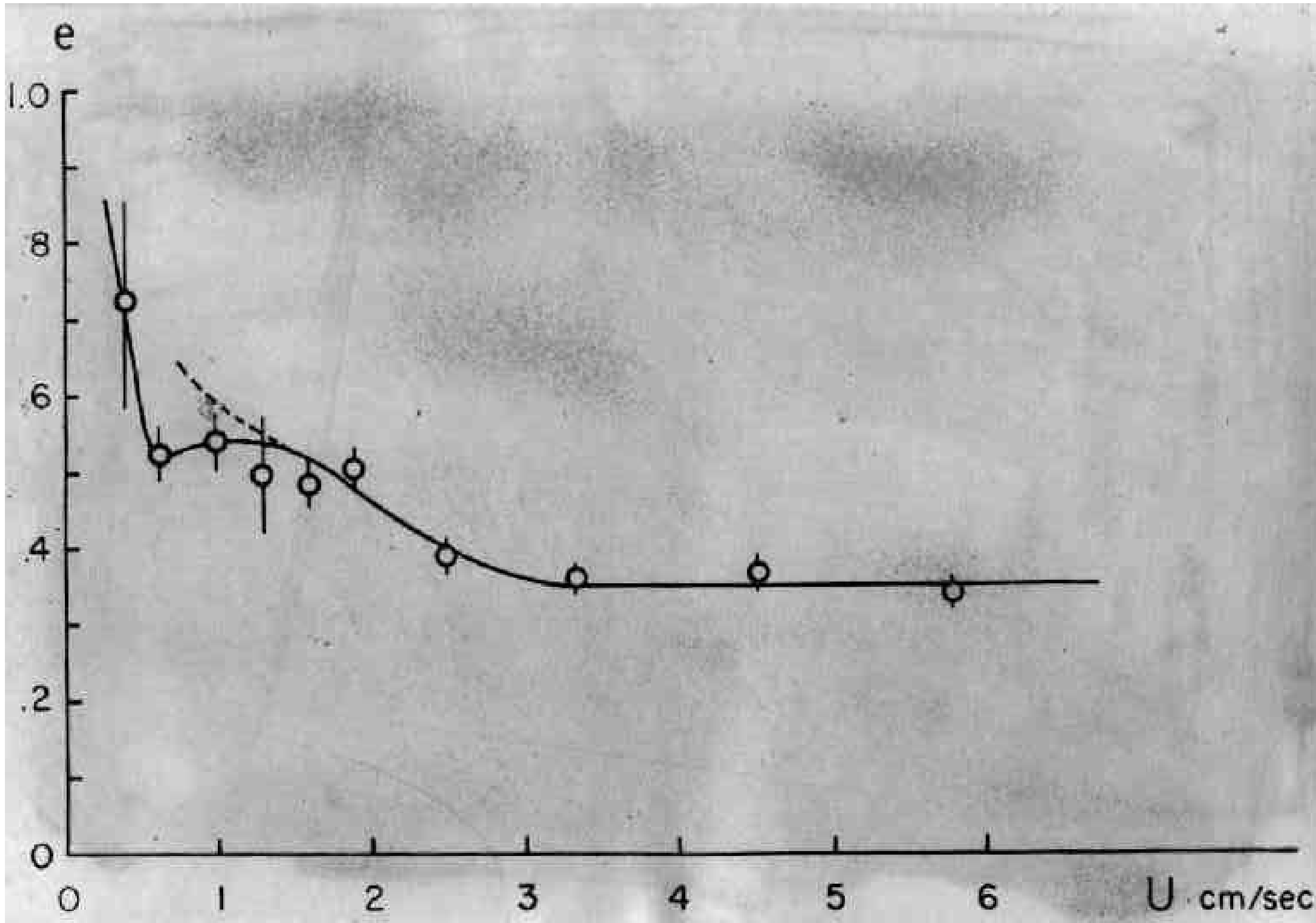


Fig. 18. Resiliency Curve for Titanium

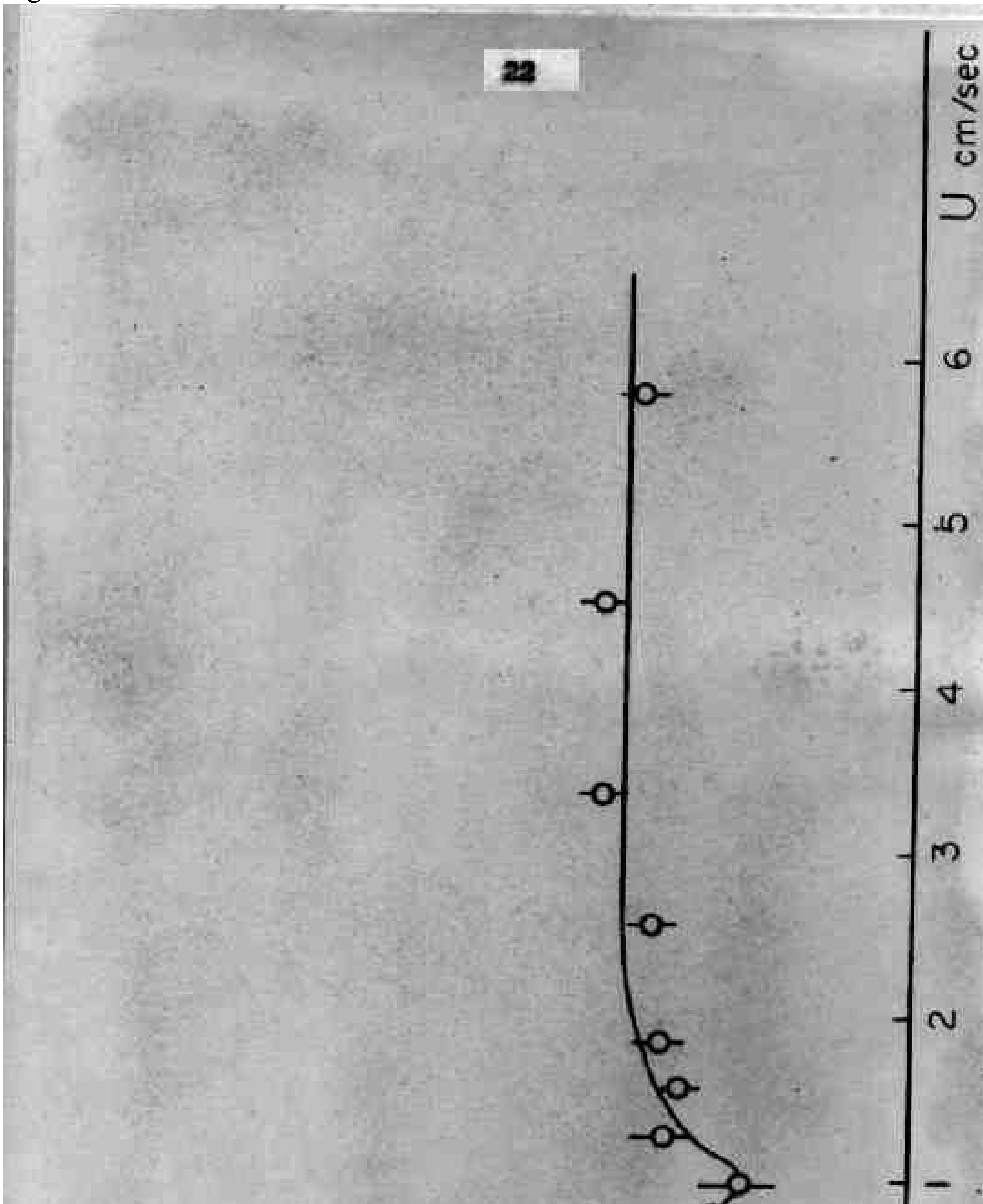


Fig. 19. Resiliency Curve for Magnesium

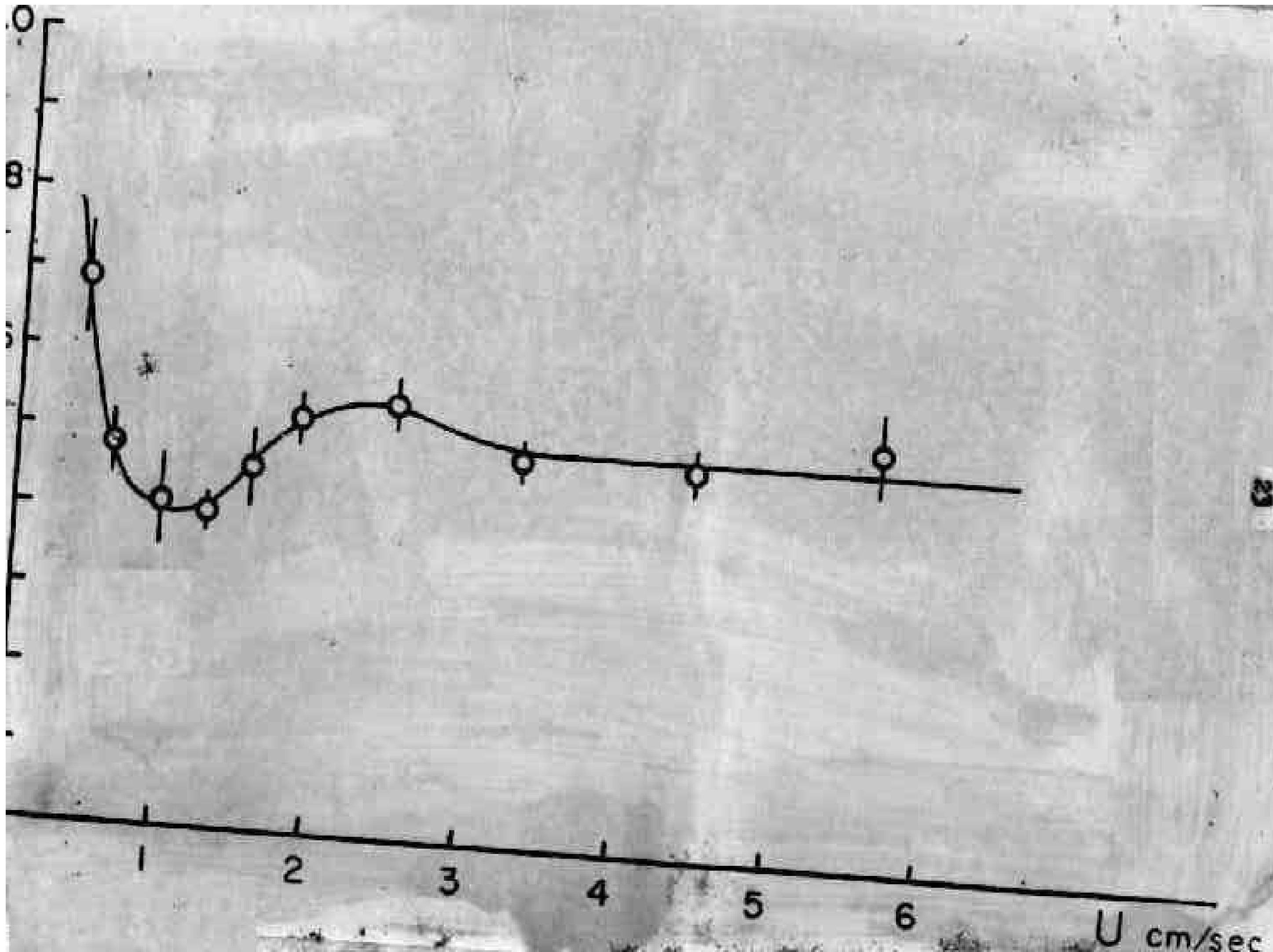


Fig. 20. Resiliency Curve for Nickel

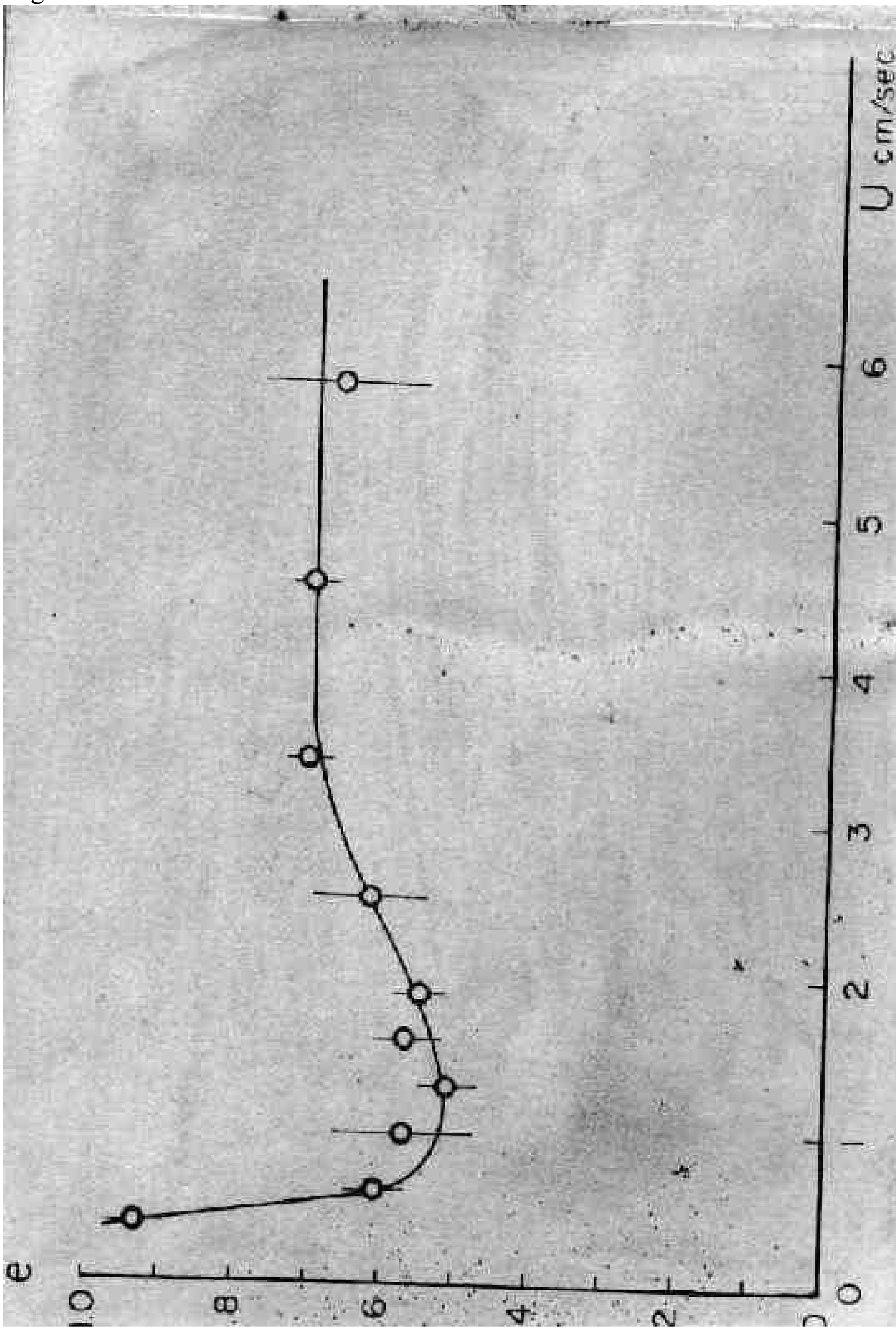


Fig. 21. Resiliency Curve for Carbon Steel

General Results

Figures 15 through 21 show the results of the investigation. The deviation indicated by the straight line extending vertically from each data circle represent the s. d. of the particular set of measurements. From 4 to 33 measurements (an average of 14) were taken for each point shown. At least two separate runs were made on each specimen except the carbon steel. Except for cadmium and titanium, it appeared to make no difference whether a fresh surface or a previously struck surface was impacted. For these two metals, a dashed curve indicates approximately the results of impacts on an "old" surface, one which had previously been struck with the velocity U = 5.8 cm/sec. In both of these cases, such data was much more widely scattered than the data for fresh surfaces.

The results, in all cases, bear out the expectation of a resiliency rise toward unity at low velocities. The dip appearing in all the curves between U = 0.4 and 1.0 has not been previously reported, although some indication of such a dip may be seen in two of Raman's curves (lead and aluminum) and in Andrews' curve. One possible explanation of this dip is that it is caused by a vibrational resonance effect in the system, superimposed over the rise toward unity expected from Raman's hypothesis. A rough calculation on this shows that the time involved for an elastic wave to traverse the impactor disk and return is of the same order of magnitude as the total contact time of impact, as reported by Andrews. A second explanation, to be investigated in further work, is that the dip is caused by going through a critical region of "internal friction" occuring during impact. According to this interpretation, at velocities of impact below the critical range, the duration of contact is so long and the heating effect so small that the impact takes place practically isothermally; the elastic constants of the metal, particularly the bulk modulus, remain the same during the compression and expansion of the metal and no mechanical energy is dissipated. For large velocities of impact, the time of impact becomes short and the net irreversible flow of heat from the region of local heating negligible, so that the metal experiences an adiabatic compression and expansion and again no mechanical energy is dissipated. But somewhere in between these two cases, there exists a velocity range where the duration of contact is moderately long and the heating effect appreciable. In this case the compression takes place at higher temperatures than the expansion and a net heat outflow occurs. To this energy dissipation corresponds the dip in the resiliency curves. Approximate calculations[12] made on this effect indicate it occurs in the general velocity region shown by the curves.

SOURCES OF ERROR

There were four sources for possible error in the measurement of the striking plate velocity U. These were:

1. The measurement of R. This may be neglected since the original determination of R was accurate to better than one part in 250 (one millimeter in 25 centimeters) and since the impact point was always positioned to within at least a millimeter of the center of the striking plate.

2. The reading of the stroboscope. This error may also be neglected, since the strobe was kept calibrated (the a-c line frequency was the standard) to within $\frac{1}{2}$ of 1% at all times.

3. The possible variations of the motor speed. This was powered by a battery bank, a known source of very constant voltage. No significant variations turned up; if any had, the strobe would have revealed them.

4. The "jerkiness" of the impactor motion due to backlash and friction in the gears and bearings. This was eliminated as far as possible by having the impactor out of balance at the impact position, and exerting by hand a steady torque on the impactor just before and during the impact period. Observations on the graduated brass disk (see instrumentation) showed that this procedure

did much to cut down on the variations, reducing them to a maximum of two percent.

In the measurement of V, the following were possible sources of error:

1. The Berkeley time interval meter. The accompanying instruction and maintenance manual for this machine lists three separate sources of error inherent in the instrument. These are the counting limitation (±1 in the last significant figure) of the instrument, oscillator drift, and "noise" on the input triggering signals. The first of these, since the times recorded were all four figure numbers or larger, is entirely negligible. The second, since the oscillator was crystal controlled, was limited to an error of .0001% or less, again negligible. Finally, the noise on the input signals was kept to a minimum by using only battery supplied power for the photocell circuit. This stability was checked, of course, with no variations observed at any time. Hence, it is assumed that no noise was introduced beyond the random thermal background, and the possible error for this source is also negligible.

2. Variations in the effective path length L' as the specimen traveled to one side or the other of the guide slot. Deviations of this kind were taken into consideration when L' was measured, by pushing the specimen carriage discussed in the instrumentation section to one side or the other of the guide slot. The error observed by this procedure was about 1%.

3. Possible wobbling motion of the specimen caused by off-center impacts. The apparatus was always kept adjusted so that no wobbling motion was apparent; however, due to the extremely small distances involved ($L' = 0.04$ cm), a minute wobble which may have occurred would make a difference. The estimated error due to this cause is two per cent. This is estimated from the data of a run made with the impacts definitely ofr center.

4. Inhomogeneities in the specimens and striking plates. Andrews remarked on this as being probably a large indeterminate source of error. It seems reasonable to attribute to this cause variations in the data larger than expected from consideration of the previously discussed errors.

The total of the random errors in U and V discussed above, excluding the ones occasioned by the possible inhomogeneities of the metals, show that the quantity V/U should have variations of about four per cent. The percentage variations in e, however, are much larger, the amount depending on the magnitude of e. For example, if $V/U = 1.5 \pm 5\%$, then $e = V/U - 1 = 0.5 \pm 15\%$. If e is near one, its error is about twice that of V/U; if it is near 0.1, its error is 11 times as large. This state of affairs seems to be an inherent property of the apparatus, and no way short of a major change could be thought of to change it.

APPENDIX I

CALCULATIONS ON THE "INFINITENESS" OF THE IMPACTOR

The degree to which the impactor approximates an infinite wall is considered to be the same as the degree to which the velocity U of the point of impact is changed (δU) by the impact. In this calculation the following symbols are defined:

M = mass of the impactor

m = mass of the test specimen

U = speed of the impactor striking face

V = speed of the specimen after impact

ω = angular velocity of the impactor

$\delta\omega$ = change in ω due to impact

δp = momentum change of the test specimen (= mV)

R = radial distance to the striking surface

I = moment of inertia of the impactor

A fundamental equation relates the loss of angular momentum by the impactor to the gain of angular momentum by test specimen, all the momenta being taken about the axle of the impactor. In terms of the symbols defined above it is

$$I\delta\omega = R\delta p = RmV \qquad (3)$$

Rearranging terms, we have

$$\frac{\delta\omega}{\omega} = \frac{RmV}{I\omega} \qquad (4)$$

But $\omega = U/R$; $\delta\omega = \delta U/R$, so

$$\frac{\delta U}{U} = \frac{R^2 mV}{IU} \qquad (5)$$

Rearranging (1), we find that $V/U = e + 1$; thus finally we have

$$\frac{\delta U}{U} = \frac{R^2 m}{I}(e + 1) \qquad (6)$$

The moment of inertia I of the impactor was computed using the elementary formula for the inertia of a disk by decomposing the calculation into parts corresponding to the different radii. In terms of the mass M of the impactor, it is

$$I = 180M \text{ gm cm}^2 \qquad (7)$$

Approximate values for R, m, and M are

$R = 24$ cm $\qquad m = 30$ gms $\qquad M = 40{,}000$ gms

Putting these values and (7) into (6), we arrive at

$$\frac{\delta U}{U} = 0.0024\ (e + 1) \qquad (8)$$

The error in U is then

$$0.24 (e + 1)$$

which, since e is necessarily equal to or less than unity, is less than $\frac{1}{2}$ of 1%.

APPENDIX II

DERIVATION OF THE FORMULA FOR V

We start with the velocity V_m measured by the photocell circuit

$$V_m = L'/T \tag{9}$$

where L' is the effective path length (see instrumentation section) and T is the time measurement shown by the Berkeley instrument. Two correction factors are applied to V_m to obtain V.

I. The first correction takes into consideration the fact that the specimen must move a finite distance after impact before entering the section of the photocell beam where the time measurement is taken. This distance, x_o, was measured in a similar manner to the effective path length L', by finding with the micrometer head setup previously discussed the distance between a photocell circuit output of 60 volts and the first of the preset voltages which started the time measurement. During the experimentation, the specimen and photocell were always aligned so that at rest the specimen was just enough in the light beam to give a reading of 60 volts (uninterrupted output was 67 volts). It is assumed that V_m is the actual velocity of the specimen when it has gone a distance from rest

$$x_q = x_o + \frac{1}{2}L' \qquad (10)$$

The justification for this assumption lies in the following considerations:

1. V_m, since the velocity is a monotonic decreasing function of time, is the actual velocity somewhere in the measurement interval, that is, between x_o and $x_o + L'$. It is more likely to be near $x_o + \frac{1}{2}L'$ than near either extreme.

2. x_o is more than four times as large as L'. Thus x_q cannot be in error by more than 20% and by reason 1 above is undoubtedly much less than that.

3. The total correction is a small one, ranging from about 15% for $V_m = 0.45$ to less than 1% for $V_m > 1.8$ cm/sec. Thus the error possible at the lowest velocity is only 3% or less, and rapidly becomes negligible as V_m increases.

With this assumption, we then use the energy equation to determine v, the velocity of the specimen at the point and time of impact.

$$\frac{1}{2}mV_m^2 = \frac{1}{2}mv^2 - mgh \qquad (11)$$

where h is the vertical rise of the specimen in traveling a distance x_q. Rearranging terms, canceling where possible, we have

$$v^2 = V_m^2 + 2gh \qquad (12)$$

Letting L = the length of the pendulum thread, we find by elementary geometrical considerations

$$h \cong \frac{x_d^2}{2L} \qquad (h \ll L) \qquad (13)$$

so that (12) becomes

$$v^2 = v_m^{\ 2} + \frac{g}{L} x_d^2 \qquad (14)$$

If this equation is expanded in a binomial series, we obtain the approximate expression

$$v = v_m + \frac{1}{2}(g/L)x_d^2/v_m \qquad (15)$$

where the next correction term, $-(g/L)^2 x_d^4/8v_m^3$, is less than 1% for all v_m greater than 0.5 cm/sec. In only one run, with cadmium, were there values of v_m less than 0.5. For these values the above correction term was taken into account. It amounted to about 1 percent.

II. The second correction to be applied arises from the fact that the velocity measured is not that of the center of the specimen but that of a point five centimeters above the center. This gives us

$$V = vL/(L - 5) \qquad (16)$$

Combining this with (15), we have, finally,

$$V = \frac{L}{L-5}\left\{v_m + \frac{gx_d^2}{2Lv_m}\right\} = 1.026\left\{\frac{L'}{T} + \frac{0.0311}{L'}T\right\} \qquad (17)$$

APPENDIX III

Following is the available data on the purity of each metal tested. The magnesium and copper data came from the ASME Handbook *Metals Properties*;[13] the data on the carbon steel from the *Machinery's Handbook*.[14] All other data came from correspondence with the supplying companies.

Molybdenum

supplied by the Battelle Memorial Institute
Columbus, Ohio

C	.005 %
O	.003
Fe	.02
Other	.005

This was commercial molybdenum, produced by Westinghouse. It was in a "fibered" (cold-worked condition and was expected to be quite anisotropic in its properties.

Copper

supplied by the American Brass Company
Waterbury 20, Connecticut

Ag	.10 %
O	.01 to .07
Pb	.005

Two grades of copper were sent, electrolytic tough pitch and OFHC (oxygen free high conductivity). The first contained an impurity cuprous oxide (CuO) which the second did not. Both had the additional impurities listed above. Specimens of both were tested and both followed the same curve of e vs U. However, the OFHC copper data appeared to be somewhat less scattered.

Cadmium

supplied by the American Zinc, Lead
and Smelting Company
East St. Louis, Illinois

Zn	.057 %
Pb	.01
In	.001
Tl	.003
Cu	.002
Sn	.0006
Ag	.0009
Ni	.003
Fe	.0002

The specimens made from this cadmium were first recast in a brass mold before machining. The impurity content is believed to have been increased by this process.

Titanium

supplied by the Mallory-Sharon Titanium Corporation
Niles, Ohio

C	.05 %
N	.028
Fe	.31
O	.1
Mn	"trace"

The material, before machining, had been annealed at 1300° F for one hour and air cooled. The ASTM grain size was 9/9.5.

Magnesium

supplied by the Metallurgical Laboratories
Dow Chemical Company
Midland, Michigan

Cu	.02 %
Ni	.001
Other	.18

The only information from the company on this metal said the sample was "pure Mg, F temper".

Nickel

supplied by the International Nickel Company, Inc.
Research Laboratory
Bayonne, New Jersey

Cu	.04 %
Si	.06
S	.005
Fe	.110
Mn	.24
C	.05
Other	.025

Carbon Steel

supplied from the physics shop, F.S.U.

C	1.10 to 1.25 %
Mn	.15 to .35
P	.025
S	.025
Si	.25

This metal came from standard commercial grade drill rod stock, not generally considered to be a "hardened" steel.

Several other companies which were contacted in our search for pure metals sent us samples which could not be used either because of their smallness or because of the difficulty in machining. These companies, and the metals in question, were

Climax Molybdenum Company Detroit, Michigan	Molybdenum

Anaconda Copper Mining Company Great Falls, Montana	copper, zinc cadmium, indium
American Smelting and Refining Company South Plainfield, New Jersey	copper, lead indium
Chase Copper and Brass Company Waterbury 20, Connecticut	copper
Sullivan Mining Company Kellogg, Idaho	zinc, cadmium
United States Metals Refining Company Carteret, New Jersey	copper
The Shepherd Chemical Company Cincinnati 12, Ohio	cobalt
Calumet and Hecla, Inc. Calumet, Michigan	native copper
Electro Metallurgical Company Niagara Falls, New York	chromium
Fairmont Aluminum Company Fairmont, West Virginia	aluminum
Aluminum Company of America New Kensington, Pennsylvania	aluminum
Rem-Cru Titanium, Inc. Midland, Pennsylvania	titanium

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