bule of *Air* as *B* has been above a larger as *A*, which Globule has not risen upward to *C*, and so to *D*, but been thrust downwards to *A*, whence it was distant about two hair's breadths, and immediately upon touching united therewith. I have likewise observed, that a little Air-bubble as *G*, loosening itself from the straw, when a larger Bubble, as *F*. was underneath it, has there rested immovable in the Liquor, when at the same time other much smaller Bubbles have risen to the top thereof. The reason of the standing still of the Bubble *G*, I suppose was from a double motion it is impelled to, the one upwards from its being specifically higher than the Liquor, the other downwards, by which it was protruded, to joyn with the other larger Bubble *F*. Tho' I have seen several Effects of Sympathy, if we may so call it, yet I never saw any so manifest as this, of the descending of a Bubble contrary to its levity, to unite with another.

IV. *An Instance of the Excellence of the Modern* ALGEBRA, *in the Resolution of the Problem of finding the* Foci *of Optick Glasses universally.* *By* E. Halley, *S. R. S.*

THE Excellence of the *Modern Geometry* is in nothing more evident, than in those full and adequate Solutions it gives to Problems; representing all the possible Cases at one view, and in one general Theorem many times comprehending whole Sciences; which deduced at length into Propositions, and demonstrated after the manner of the *Ancients*, might well become the Subjects of large Treatises: For whatsoever Theorem solves the most complicated Problem of the kind, does with a due Reduction reach all the subordinate Cases. Of this

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I now design to give a notable Instance in the Doctrine of *Dioptricks*.

This Dioptrick Problem is that of finding the *Focus* of any sort of *Lens*, exposed either to Converging, Diverging or parallel Rays of Light, proceeding from, or tending to a given Point in the *Axis* of the *Lens*, be the *ratio* of *Refraction* what it will, according to the nature of the Transparent Material whereof the *Lens* is formed, and also with allowance for the thickness of the *Lens* between the *Vertices* of the two Spherical Segments. This Problem being solved in one Case, *mutatis mutandis* will exhibit Theorems for all the possible Cases, whether the *Lens* be *Double-Convex* or *Double-Concave*, *Plano-Convex* or *Plano-Concave*, or *Convexo-Concave*, which sort are usually called *Menisci*. But this is only to be understood of those Beams which are nearest to the *Axis* of the *Lens*, so as to occasion no sensible difference by their Inclination thereto; and the *Focus* here formed is by *Dioptrick Writers* commonly called the principal *Focus*, being that of use in *Telescopes* and *Microscopes*.

Let then (in Fig. 18.) BEβ be a double Convex *Lens*, C the Center of the Segment EB, and K the Center of the Segment Eβ, Bβ the thickness of the *Lens*, D a point in the *Axis* of the *Lens*; and it is required to find the point F, at which the Beams proceeding from the point D, are collected therein, the *ratio* of Refraction being as m to n. Let the distance of the object DB = DA = d (the point A being supposed the same with B, but taken at a distance therefrom, to prevent the coincidence of so many Lines) the *Radius* of the Segment towards the Object CB or CA = r, and the *Radius* of the Segment from the Object Kβ or Kα = ρ, and let Bβ the thickness of the *Lens* be = t, and then let the Sine of the Angle of incidence DAG be to the sine of the refracted Angle HAG or CAϕ as m to n; And in very small Angles the Angles them-

themſelves will be in the ſame proportion; whence it will follow that,

As d to r, ſo the Angle at C to the Angle at D, and $d+r$ will be as the Angle of incidence GAD; and again as m to n, ſo $d+r$ to $\frac{dn+rn}{m}$, which will be as the Angle GAH = CAφ. This being taken from ACD which is as d, will leave $\frac{\overline{m-n}\,d-nr}{m}$ analogous to the Angle AφD, and the ſides being in this caſe proportional to the Angles they ſubtend, it will follow, that as the Angle AφD is to the Angle ADφ, ſo is the ſide AD or BD to Aφ or Bφ: that is Bφ will be $=\frac{mdr}{\overline{m-n}\,d-nr}$ which ſhews in what point the beams proceeding from D would be collected by means of the firſt Refraction; but if nr cannot be ſubſtracted from $\overline{m-n}\,d$, it follows that the Beams after Refraction do ſtill paſs on diverging, and the point φ is on the ſame ſide of the *Lens* beyond D. But if nr be equal to $\overline{m-n}\,d$ then they proceed parallel to the *Axis*, and the point φ is infinitely diſtant.

The point φ being found as before, and Bφ—Bβ being given, which we will call δ, it follows by a proceſs like the former, that βF or the focal diſtance ſought, is equal to $\frac{\delta\rho n}{\overline{m-n}\,\delta+m\rho}=f$. And in the room of δ ſubſtituting Bφ—Bβ $=\frac{mdr}{\overline{m-n}\,d-nr}-t$, putting p for $\frac{n}{m-n}$ after due reduction this following Equation will ariſe,

$$\frac{mpdr\rho-nd\rho t+npr\rho t}{mdr+md\varrho-mpr\varrho-\overline{m-n}\,dt+nrt}=f.$$

Which Theorem, however it may ſeem operoſe, is is not so, considering the great number of *data* that enter the Question, and that one half of the terms arise from our taking in the thickness of the *Lens*, which in most cases can produce no great effect, however it was necessary to consider it, to make our Rule perfect. If therefore the *Lens* consist of *Glass*, whose Refraction is as 3 to 2 'twill be $\frac{6drp - 2dpt + 4rpt}{3dr + 3dp - 6rp - dt + 2rt} = f.$ If of *Water*, whose Refraction is as 4 to 3 the Theorem will stand thus $\frac{12drp - 3dpt + 9rpt.}{4dr + 4dp - 12rp - dt + 3rt} = f.$ If it could be made of *Diamant*, whose Refraction is as 5 to 2, it would be $\frac{\frac{10}{3}drp - 2dpt + \frac{4}{3}rpt}{5dr + 5dp - \frac{10}{3}rp - 3dt + 2rt} = f.$ And this is the universal Rule for the *foci* of double Convex Glasses exposed to Diverging Rays. But if the thickness of the *Lens* be rejected as not sensible, the Rule will be much shorter, *viz.*

$$\frac{pdrp}{dr + dp - prp} = f, \text{ or in Glass } \frac{2drp}{dr + dp - 2rp} = f.$$

all the terms wherein t is found being omitted, as equal to nothing. In this case, if d be so small, as that $2rp$ exceed $dr + dp$, then will it be $-f$, or the *focus* will be Negative, which shews that the Beams after both Refractions still proceed Diverging.

To bring this to the other Cases, as of Converging Beams, or of Concave Glasses, the Rule is ever composed of the same terms, only changing the signs of + and —; for the distance of the point of Concourse of converging Beams, from the point B, or the first surface of the *Lens*, I call a negative distance or $-d$; and the Radius of a Concave *Lens* I call a negative Radius or $-r$ if it be the first surface, and $-p$, if it be the second surface. Let then converging Beams fall on a double Convex

vex of Glaſs, and the Theorem will ſtand thus $\frac{-2dr\rho}{-dr-d\rho-2r\rho}=+f$. which ſhews that in this caſe the *Focus* is always affirmative.

If the *Lens* were a *Meniſcus* of Glaſs, expoſed to diverging Beams, the Rule is $\frac{-2dr\rho}{-dr+d\rho+2r\rho}=f$. which is affirmative when $2r\rho$ is leſs than $dr-d\rho$, otherwiſe negative: But in the caſe of converging Beams falling on the ſame *Meniſcus*, 'twill be $\frac{+2dr\rho}{+dr-d\rho+2r\rho}=f$. and it will be $+f$ whilſt $d\rho-dr$ is leſs than $2r\rho$, but if it be greater than $2r\rho$, it will always be found negative or $-f$. If the *Lens* be double Concave, the *focus* of converging Beams is negative, where it was affirmative in the caſe of diverging Beams on a double Convex, *viz.* $\frac{-2dr\rho}{+dr+d\rho-2r\rho}=f$, which is affirmative only when $2r\rho$ exceeds $dr+d\rho$: But diverging Beams paſſing a double Concave have always a negative *focus*, *viz.* $\frac{-2dr\rho}{+dr+d\rho+2r\rho}=-f$.

The Theorems for Converging Beams are principally of uſe to determine the *focus* reſulting from any ſort of *Lens* placed in a Teleſcope, between the *focus* of the Object-glaſs and the Glaſs it ſelf; the diſtance between the ſaid *focus* of the Object-glaſs and the interpoſed *Lens* being made $=-d$.

I here ſuppoſe my Reader acquainted with the Rules of Analytical Multiplication and Diviſion, as that $+$ multiplied by $+$ makes the product $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, ſo dividing $+$ by $+$ makes the Quote $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, which will be neceſſary to be underſtood in the preceding Exampſes.

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In case the Beams are parallel, as coming from an infinite distance, (which is supposed in the case of Telescopes) then will d be supposed infinite, and in the Theorem $\frac{p\,d\,\rho\,r}{d\,r + d\,\rho - p\,r\,\rho}$ the Term $p\,r\,\rho$ vanishes, as being finite, which is no part of the other infinite terms and dividing the remainder by the infinite part d, the Theorem will stand thus $\frac{p\,\rho\,r}{r+\rho} = f$, or in Glass, $\frac{2\,r\,\rho}{r+\varrho} = f$.

In case the *Lens* were *Plano-Convex* exposed to diverging Beams, instead of $\frac{p\,d\,\rho\,r}{d\,r + d\,\varrho - p\,r\,\varrho}$, r being infinite, it will be $\frac{p\,d\,\varrho}{d - p\,\varrho} = f$. or $\frac{2\,d\,\varrho}{d - 2\,\varrho} = f$, if the *Lens* be Glass.

If the *Lens* be Double-Convex, and r be equal to ϱ, as being formed of Segments of equal Spheres, then will $\frac{p\,d\,\varrho\,r}{d\,r + d\,\varrho - p\,r\,\varrho}$ be reduced to $\frac{p\,d\,r}{2\,d - p\,r} = f$; and in case d be infinite, then it will yet be farther contracted to $\frac{1}{2}\,p\,r$, and p being $= \frac{n}{m-n}$ the focal distance in Glass will be $= r$, in Water $1\frac{1}{2}\,r$, but in Diamint $\frac{1}{3}\,r$.

I am sensible that these Examples are too much for the compleat Analyst, though I fear too little for the less Skilful, it being very hard, if possible, in such matters, so to write as to give satisfaction to both; or to please the one, and instruct the other. But this may suffice to shew the extent of our Theorem, and how easy a Reduction adapts any one case to all the rest.

Nor is this only uſeful to diſcover the *focus* from the other propoſed *data*, but from the *focus* given, we may thereby determine the diſtance of the Object, or from the *focus* and diſtance given, we may find of what Sphere it is requiſite to take another Segment, to make any given Segment of another Sphere caſt the beams from the diſtance *d* to the *focus* *f*. As likewiſe from the *Lens*, *focus*, and diſtance given, to find the *ratio* of Refraction, or of *m* to *n*, requiſite to anſwer thoſe *data*. All which it is obvious, are fully determined from the Equation we have hitherto uſed, *viz.* $p\,d\,\rho\,r = d\,r\,f + d\,\rho\,f - p\,r\,\rho\,f$, for to find *d* the Theorem is

$$\frac{p\,r\,\rho\,f}{r\,f + \rho\,f - p\,\rho\,r} = d,$$ the diſtance of the Object.

For ρ the Rule is $$\frac{d\,r\,f}{p\,d\,r + d\,f + p\,r\,f} = \rho$$

But for *p* will be $\dfrac{d\,r\,f + d\,\rho\,f}{d\,\rho\,r + f\,\rho\,r} = p$, which latter determines the *ratio* of Refraction, *m* being to *n* as $1 + p$ to *p*.

I ſhall not expatiate on theſe Particulars, but leave them for the exerciſe of thoſe that are deſirous to be informed in Optical Matters, which I am bold to ſay are comprehended in theſe three Rule, as fully as the moſt inquiſitive can deſire them, and in all poſſible caſes; regard being had to the Signs + and −, as in the former caſes of finding the *focus*. I ſhall only ſhew two conſiderable uſes of them; the one to find the diſtance whereat an Object, being placed ſhall by a given *Lens* be repreſented in a *Species* as large as the Object it ſelf, which may be of ſingular uſe, in drawing Faces, and other things in their true Magnitude, by tranſmitting the *Species* by a Glaſs into a dark Room, which will not only give the true Figure and Shades, but even the Colours themſelves, almoſt as vivid as the Life. In this

caſe

case d is equal to f, and substituting d for f in the Equation, we shall have $p\,d\,r\,\varrho = d\,d\,r \maltese d\,d\,\varrho - d\,p\,\varrho\,r$, and dividing all by d. $p\,r\,\varrho = d\,r + d\,\varrho - p\,r\,\rho$, that is $\frac{2\,p\,r\,\varrho}{r \maltese \varrho} = d$; but if the two Convexities be of the same Sphere so as $r = \varrho$ then will the distance be $= p\,r$, that is, if the *Lens* be Glass $= 2\,r$, so that if an Object be placed at the Diameter of the Sphere distant, in this case the *focus* will be as far within as the Object is without, and the *Species* represented thereby will be as big as the Life; but if it were a *Plano-Convex*, the same distance will be $= 2\,p\,r$, or in Glass to four times the *Radius* of the Convexity; but of this method I may perhaps entertain the Curious in some other Transaction, and shew how to magnifie or diminish an Object in any proportion assigned, (which yet will be obvious enough from what is here delivered) as likewise how to erect the Object which in this method is represented inverted.

A second use is to find what Convexity or Concavity is required, to make a vastly distant Object be represented at a given *focus*, after the one surface of the *Lens* is formed; which is but a Corollary of our Theorem for finding ϱ, having p, d, r and f given; for d being infinite, that Rule becomes $\frac{r\,f}{p\,r - f} = \varrho$, that is in Glass $\frac{r\,f}{2\,r - f} = \rho$, whence if f be greater than $2\,r$, ϱ becomes Negative, and $\frac{r\,f}{f - 2\,r}$ is the *Radius* of the Concave sought.

Those that are wholly to begin with this Dioptrical Science cannot do better than to read with Attention a late Treatise of Dioptricks, published by *W. Molineux*, Esq; R. S. S. who has at large shewn the Nature of

Optick Glaſſes, and the Conſtruction and Uſe of Microſcopes and Teleſcopes; and though ſome nicely Critical have endeavoured to ſpy faults, and to traduce the Book, yet having long ſince examined it with care, I affirm, that if I can judge, it hath but two things that with any Colour may be call'd faults; the one an over-careful acknowledgment of every Trifle the Author had received from other; and the other, that he labours to make eaſie this curious Subject, ſo little underſtood by moſt, in a manner perhaps too familiar for the *Learned Critick*, and which demonſtrates that it was writ *cum animo docendi*, both which require but very little Friendſhip or good Nature in the Reader, to paſs for Vertues in an Author.

But to return to our firſt Theorem, which accounting for the thickneſs of the *Lens*, we will here again reſume, *viz.*

$$\frac{mpdr\rho - nd\rho t + npr\rho t}{mdr \pm md\varrho - mpr\varrho - \overline{m-n}\,dt + nrt} = f.$$

And let it be required to find the *focus* where a whole Sphere will collect the Beams proceeding from an Object at the diſtance d; Here t is equal to $2r$ and $r = \varrho$. And after due Reduction the Theorem will ſtand thus, $\frac{mpdr - 2ndr \pm 2nprr}{2nd \pm 2nr - mpr} = f$, but if d be infinite, it is contracted to $\frac{mpr}{2n} - r = \frac{2n - m}{2m - 2n} r = f$, wherefore a Sphere of Glaſs collects the Suns Beams at half the Semi-diameter of the Sphere without it, and a Sphere of Water at a whole Semi-diameter. But if the *ratio* of Refraction m to n be as 2 to 1, the *focus* falls on the oppoſite ſurface of the Sphere, but if it be of greater inequality it falls within.

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Another Example ſhall be when a Hemiſphere *is* expoſed to parallel Rays, that is d and ϱ being infinite, and $t = r$, and after due Reduction the Theorem reſults $\frac{nn}{mm - mn} r = f$. That is, in Glaſs it is at $\frac{4}{3} r$, in Water at $\frac{9}{4} r$, but if the Hemiſphere were Diamant, it would collect the Beams at $\frac{4}{15}$ of the *Radius* beyond the Center.

Laſtly, As to the effect of turning the two ſides of a *Lens* towards an Object; it is evident, that if the thickneſs of the *Lens* be very ſmall, ſo as that you neglect it, or account $t = 0$, then in all caſes the *focus* of the ſame *Lens*, to whatſoever Beams, will be the ſame, without any difference upon the turning the *Lens*: But if you are ſo Curious as to conſider the thickneſs, (which is ſeldom worth accounting for) in the caſe of parallel Rays falling on a *Plano-Convex* of Glaſs, if the plain ſide be towards the Object, t does occaſion no difference, but the focal diſtance $f = 2 r$. But when the Convex ſide is towards the Object, it is contracted to $2 r - \frac{2}{3} t$, ſo that the *focus* is nearer by $\frac{2}{3} t$. If the *Lens* be double Convex the difference is leſs; if a *Meniſcus* greater. If the Convexity on both ſides be equal, the focal length is about $\frac{1}{6} t$ ſhorter than when $t = 0$. In a *Meniſcus* the Concave ſide towards the Object encreaſes the focal length, but the Convex towards the Object diminiſhes it. A General Rule for the difference ariſing on turning the *Lens*, where the *Focus* is Affirmative, is this $\frac{2rt - 2\varrho t}{3r + 3\varrho - t}$, for double Convexes of differing Spheres. But for *Meniſci* the ſame difference becomes $\frac{2rt + 2\varrho t}{3r - 3\varrho + t}$; of which I need give no other demonſtration, but that by a due Reduction it will ſo follow from what is premiſed, as will the Theorems for all ſorts of Problems relating to the *foci* of Optick Glaſſes.

V. *An*

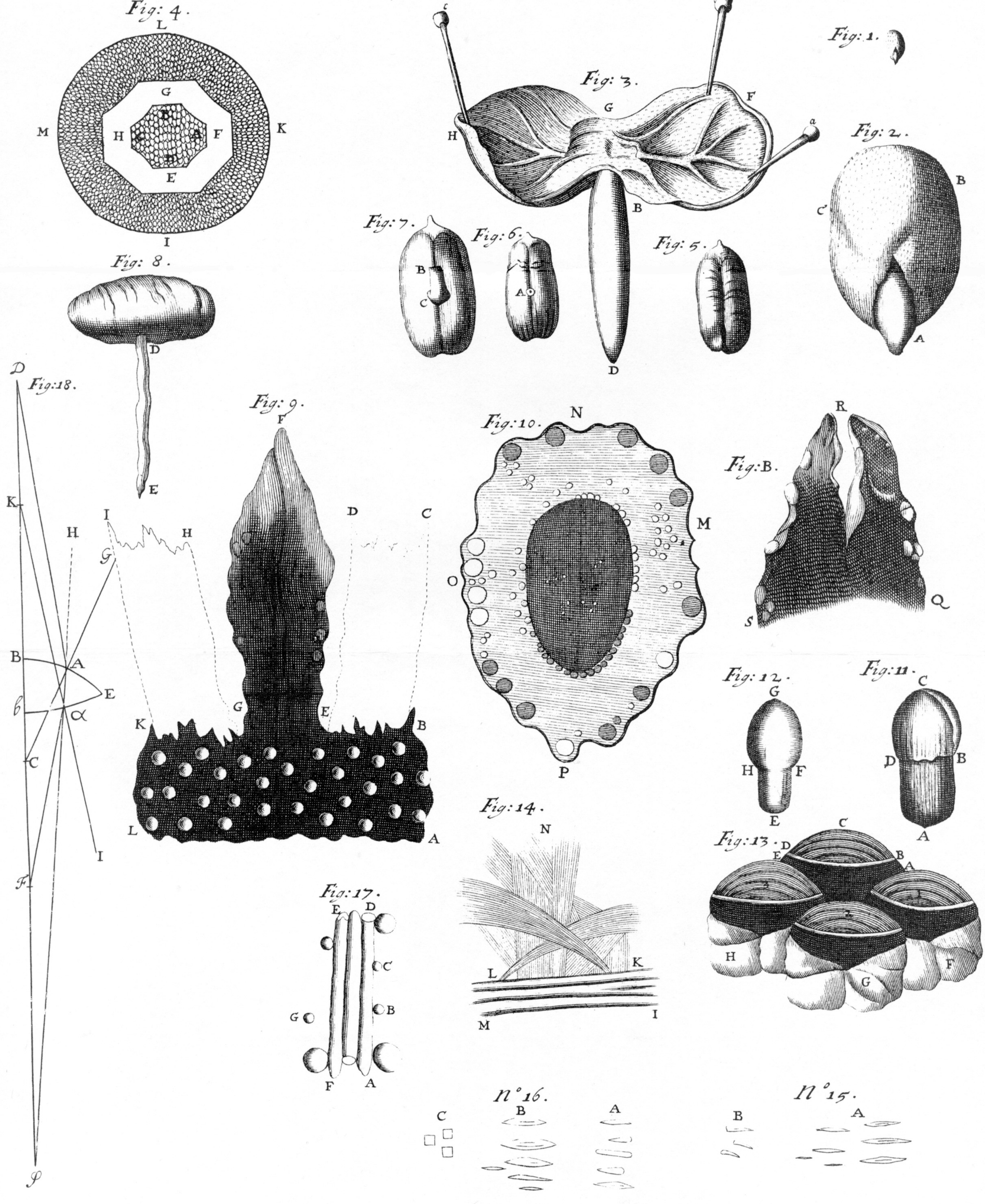
Fig: 1.
Fig: 2.
Fig: 3.
Fig: 4.
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Fig: 6.
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Fig: 10.
Fig: B.
Fig: 11.
Fig: 12.
Fig: 13.
Fig: 14.
Fig: 17.
Fig: 18.
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N° 15.