I. *An Account of the Cause of the late remarkable Appearance of the Planet* Venus, *seen this Summer, for many Days together, in the Day time.* By Edm. Halley, *R. S. Secr.*

IT may justly be reckoned one of the principal Uses of the Mathematical Sciences, that they are in many Cases able to prevent the Superstition of the unskilful Vulgar; and by shewing the genuine Causes of rare Appearances, to deliver them from the vain apprehensions they are apt to entertain of what they call *Prodigies*; which sometimes, by the Artifices of designing Men, have been made use of to very evil purposes.

Of this kind was the late Appearance of *Venus* in the Day time, generally taken notice of about *London* and elsewhere; and by some reckoned to be Prodigious. This put me upon the enquiry, how it came to pass that at that time the *Planet* should be so plainly seen by Day, whereas she rarely shews her self so, unless to those who know exactly where to look for her. To resolve this, the following Problem arose, *viz.* To find the Situation of the *Planet* in respect of the *Earth*, when the *Area* of the illuminated part of her Disk is a *Maximum*.

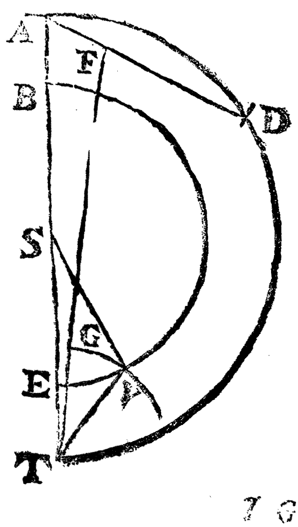
To investigate this *Maximum*, I found it requisite to assume the following *Lemmata*. I. That the visible *Areas* of the Disk of the same *Planet*, at differing Distances, are always reciprocally as the Squares of those Distances; which is evident from the first Principles of *Opticks*. II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the illuminated Part thereof, as the Diameter of a Circle to the Versed-Sine of the exteriour Angle at the *Planet*, in the Triangle at whose Angles are the *Sun*, *Earth*, and *Planet*. III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the excess of the Square of the Sum of the Sides above the Square of the Base, as the Diameter is to the Versed-Sine of the

the Complement of the contained Angle to a Semicircle, which I call the exteriour Angle:. This is a new *Theorem* of good uſe in *Trigonometry*, and eaſily proved from the 12*th* and 13*th* of the II. *Elem. Euclid.*

This premiſed, putting m for the Diſtance of the *Sun* and *Earth*, and n for that of the *Sun* and *Venus*, and x for the Diſtance of the *Earth* and *Venus*, or the third Side of the Triangle which we ſeek; by the third *Lemma*, $4nx$, will be to the exceſs of the Square of $n+x$ above the Square of m, as the *Area* of the whole Disk of *Venus* to the *Area* of the part illuminated; and by the firſt *Lemma*, the *Area*'s of her whole Disk are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn+2nx+xx-mm}{4nx^3}$. will in all Caſes be proportional to the *Area* of the illuminated part.

Now that this ſhould be a *Maximum*, it is required that the Fluxion thereof be equal to o, or that the Negative parts thereof be equal to the Affirmative, that is, that $2n\dot{x}+2x\dot{x} \times 4nx^3 = 12nx^2\dot{x} \times \overline{nn+2nx+xx-mm}$; and dividing all by $4nx^2\dot{x}$, the Equation becomes $2nx+2xx = 3nn+6nx+3xx-3mm$. Conſequently $3nn+4nx+xx=3mm$, and therefore $x=\sqrt{3mm+nn}-2n$.

From hence a ready and not inelegant Geometrical Conſtruction (if I may be allowed to ſay ſo) becomes obvious; for with the Center S and Radius $ST=m$, deſcribe the Semicircle TDA; and with the ſame Center and Radius $SE=n$, the Semicircle EVB; which two Semicircles ſhall repreſent the Orbs of the *Earth* and *Venus*. Make the chord AD equal to the Radius ST, and from D towards A, lay off $DF=SE$; draw TF, and thereon place $FG=BE=2n$, and with the Center T and *Radius*



TG deſcribe the arch *GV*, cutting the Semicircle *BVE* in *V*; and draw the lines *SV*, *TV*: I ſay the Triangle *STV* is Similar to that at whoſe Angles are the *Sun*, *Earth* and *Venus*, at the time when the *Area* of the enlightned part of that Planet's Disk, as ſeen from the *Earth*, is greateſt. How this Geometrical Effection follows from the Equation is too evident to need repetition.

In conſequence of this Solution, I find this *Maximum* always to happen, when the Planet is about forty Degrees diſtant from the *Sun*; and the times thereof, about the middle between her greateſt Elongations on both ſides from him, and her retrograde Conjunctions with him; when little more than a quarter of her viſible Disk is luminous, and reſembling the Moon of about five Days old; and notwithſtanding that her Diameter is at that time but 50 Seconds, yet ſhe ſhines with ſo ſtrong a Beam, as to ſurpaſs the united light of all the fixt Stars that appear with her, and caſts a very ſtrong Shade on the Horizontal plain whereon they all ſhine: an irrefragable Argument to prove that the Disks of the fixt Stars are unconceivably ſmall, and next to nothing; ſince ſhining with a native Light, ſo many of them do not equal the reflex Light of one quarter of a Disk of leſs than a Minute Diameter.

In this ſituation *Venus* was found in *July* laſt, on the tenth Day, about which time, when the *Sun* grew low, ſhe was very plainly ſeen in the Day time, for many Days together; as ſhe might have been in the Mornings, about the latter end of *September*. But this, ariſing from the Cauſes we have now ſhewn, is nothing uncommon; for every eighth Year it returns again, ſo that the Planet may be ſeen on the ſame Day of the Month and Hour, very nearly in the ſame place; as all acquainted with the Heavenly Motions muſt know.

Laſtly, it may not be amiſs to note that the Equation $x = \sqrt{3mm + nn} - 2n$ has a Limit; for if n be equal to $\frac{1}{4}m$, the point *V* will fall on *B*; and the whole Disk of a Planet at that diſtance from the Sun would be the *Maximum*, *viz.* when in its ſuperior Conjunction with the *Sun*. And the like if n were leſs than $\frac{1}{4}m$; the Arch *GV* in ſuch Caſe not interſecting the Semicircle *BE*.