# Gamma Function Formulas

# فرمولهای تابع گاما

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$$\Gamma\left(x\right)=\int_{0}^{\infty}t^{x-1}e^{-t}dt\qquad x>0$$

$$\Gamma\left(x\right)=\lim_{m\rightarrow\infty}\frac{m!}{x\left(x+1\right)\cdots\left(x+m\right)}m^{x}$$

$$\frac{1}{\Gamma\left(x\right)}=xe^{\gamma x}\prod_{m=1}^{\infty}e^{\frac{-x}{m}}\left(1+\frac{x}{m}\right)$$

$$\Gamma\left(x+1\right)=x\Gamma\left(x\right)$$

$$\Gamma\left(x+1\right)=x!$$

$$\lim_{k\rightarrow\infty}\frac{\sqrt{2\pi k}\,k^{k}e^{-k}}{k!}=1$$

$$\Gamma\left(x\right)\Gamma\left(1-x\right)=\frac{\pi}{\sin\pi x}\qquad 0<x<1$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$$\Gamma\left(n+\frac{1}{2}\right)=\frac{1.3.5\cdots\left(2n-1\right)}{2^{n}}\sqrt{\pi}$$

$$\Gamma\left(-n+\frac{1}{2}\right)=\frac{\left(-2\right)^{n}}{1.3.5\cdots\left(2n-1\right)}\sqrt{\pi}$$

$$2^{2x-1}\Gamma\left(x\right)\Gamma\left(x+\frac{1}{2}\right)=\sqrt{\pi}\Gamma\left(2x\right)$$

$$\prod_{k=0}^{n-1}\Gamma\left(x+\frac{k}{n}\right)=n^{\frac{1}{2}-nx}\left(2\pi\right)^{\frac{n-1}{2}}\Gamma\left(nx\right)$$

$$\log\Gamma\left(1+x\right)=-\log\left(1+x\right)+x-\gamma x+\sum_{k=2}^{\infty}\frac{\left(-x\right)^{k}}{k}\left(\zeta\left(k\right)-1\right)\qquad\left|x\right|<2$$

$$\int_{0}^{1}\log\Gamma\left(x\right)dx=\frac{1}{2}\log\left(2\pi\right)$$

$$\Gamma\left(x+1\right)=\sqrt{2\pi x}\,x^{x}e^{-x}\left(1+\frac{1}{12x}+\frac{1}{288x^{2}}-\frac{139}{51840x^{3}}+\cdots\right)$$

$$\Gamma'\left(1\right)=\int_{0}^{\infty}e^{-x}\log x dx$$

$$\Gamma'\left(1\right)=-\gamma$$

$$\left|\Gamma\left(ix\right)\right|^{2}=\frac{\pi}{x\sinh\pi x}$$

$$\Gamma\left(x\right)=\frac{\int_{0}^{\infty}\frac{t^{x-1}}{e^{t}-1}dt}{\zeta\left(x\right)}\qquad x>1$$

$$\frac{\Gamma'\left(x\right)}{\Gamma\left(x\right)}=-\gamma+\sum_{k=1}^{\infty}\left(\frac{1}{k}-\frac{1}{x+k-1}\right)$$