

Gamma Function Formulas

فرمولهای تابع گاما

By Hamid Naderi Yeganeh

حمید نادری یگانه

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad x > 0$$

$$\Gamma(x) = \lim_{m \rightarrow \infty} \frac{m!}{(x+1) \cdots (x+m)} m^{x-1}$$

$$\frac{1}{\Gamma(x)} = x e^{\gamma x} \prod_{m=1}^{\infty} e^{\frac{-x}{m}} \left(1 + \frac{x}{m}\right)$$

$$\Gamma(x+1) = x\Gamma(x)$$

$$\Gamma(x+1) = x!$$

$$\lim_{k \rightarrow \infty} \frac{\sqrt{2\pi k} k^k e^{-k}}{k!} = 1$$

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x} \quad 0 < x < 1$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{1.3.5 \cdots (2n-1)}{2^n} \sqrt{\pi}$$

$$\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-2)^n}{1.3.5 \cdots (2n-1)} \sqrt{\pi}$$

$$2^{2x-1} \Gamma(x) \Gamma\left(x + \frac{1}{2}\right) = \sqrt{\pi} \Gamma(2x)$$

$$\prod_{k=0}^{n-1} \Gamma\left(x + \frac{k}{n}\right) = n^{\frac{1}{2}-nx} (2\pi)^{\frac{n-1}{2}} \Gamma(nx)$$

$$\log \Gamma(1+x) = -\log(1+x) + x - \gamma x + \sum_{k=2}^{\infty} \frac{(-x)^k}{k} (\zeta(k) - 1) \quad |x| < 2$$

$$\int_0^1 \log \Gamma(x) dx = \frac{1}{2} \log(2\pi)$$

$$\Gamma(x+1) = \sqrt{2\pi x} x^x e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} + \dots\right)$$

$$\Gamma'(1) = \int_0^{\infty} e^{-x} \log x dx$$

$$\Gamma'(1) = -\gamma$$

$$|\Gamma(ix)|^2 = \frac{\pi}{x \sinh \pi x}$$

$$\Gamma(x) = \frac{\int_0^\infty \frac{t^{x-1}}{e^t-1} dt}{\zeta(x)} \quad x > 1$$

$$\frac{\Gamma'(x)}{\Gamma(x)} = -\gamma + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{x+k-1} \right)$$