

Sagital Suture, cross'd from one Parietal-Bone to the other, as far as the Coronal Suture on that side opposite to the Wound; another had gone cross the Coronal Bone; and the third was on the Parietal Bone on the side of the Wound, pretty near the *Sutura Squamosa*; but what is most singular, is that none of these Fissures did reach that, upon which the Trepan had been applied. An *Empyema* was found in the *Thorax*, and a considerable Impoſthume in the Liver.

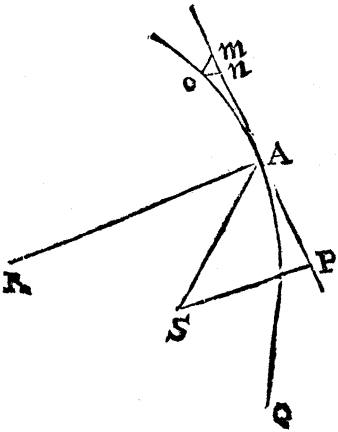
II. *Jo. Keill ex Æde Christi Oxoniensis, A. M. Epistola ad Clarissimum Virum Edmundum Halleium Geometriæ Professorem Savilianum, de Legibus Virium Centripetarum.*

HAUD oblitus es, uti arbitror, Vir Clarissime, te cum nuper esses Oxonii, Theorema, quo Lex vis centripetæ, *Quantitatibus finitis* exhiberi possit, mecum communicasse: Quod Theorema tibi monstravit Egregius Mathematicus D. Abrahamus De Moivre, Dixitque Dominum Isaacum Newtonum, Theorema huic simile prius Invenisse. Cum autem ejus demonstratio perfacilis sit, Eam, itemque alia de eadem re cogitata, non possum tibi non impertire. Etsi minime dubitem, quin, si idem argumentum pertractare libuisset, tu acerrimo quo polles ingenij acumine, rem omnem penitus exhausisse potuisses.

T H E O.

T H E O R E M A.

Si corpus Urgente vi Centripetâ in curva aliqua moveatur; Erit vis illa in quovis curvæ puncto, in ratione composita ex directâ ratione distantie corporis à centro virium, & reciproca ratione Cubi perpendicularis à Centro in rectam in eodem puncto Curvam Tangentem demissa, ducti in Radium Curvaturæ quem ibi obtinet curva.

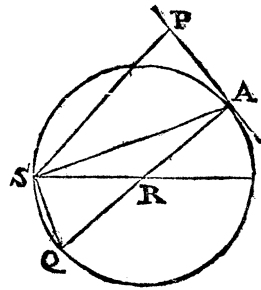


Sit QAO Curva qualibet à mobili urgente vi centripeta ad punctum S tendente descripta. Sitque AO arcus in minimo quovis tempore percurfus, Pm ejus tangens, AR Radius circuli æquicurvi, hoc est cujus Peripheriæ pars minima cum Arcu AO coincidat. Et fit SP recta a puncto S in tangentem perpendiculariter demissa; Ducantur Om ad SA & On ad SP Parallelæ. Et exponat Om vim

qua mobile in A urgetur versus S . Vis qua perpendiculariter à tangente recedit corpus, erit ut On , id est vis tendens versus R & faciens ut mobile, eadem qua prius velocitate latum, describet circulum æquicurvum arcui AO erit ad vim tendentem versus S , qua corpus in curva AO movetur, ut On ad Om , vel ob æquiangula triangula ut SP ad SA . Sed corporum in circulis latorum vires centripetæ sunt ut quadrata velocitatum applicata ad Radios; per Corol. Theorem. 4. Princip. Newtoni.

Est vero velocitas reciproce ut SP , five directe ut $\frac{1}{SP}$
 adeoque quadratum velocitat. erit ut $\frac{1}{SP^2}$: vis igitur ut On ,
 five vis qua in circulo æquicurvo moveri potest corpus,
 erit ut $\frac{1}{SP^2 \times AR}$: Ostensum autem est, esse SP ad SA
 ut vis tendens versus R , qua corpus in circulo æquicur-
 vo moveri potest, ad vim tendentem versus S : sed est vis
 tendens versus R ut $\frac{1}{SP^2 \times AR}$, adeoque cum sit
 $SP : SA :: \frac{1}{SP^2 \times AR} : \frac{SA}{SP^3 \times AR}$ erit vis tendens
 versus S , ut $\frac{SA}{SP^3 \times AR}$. *Q. E. D.*

Cor Si curva QAO fit circulus, erit vis centripeta tendens
 versus S , ut $\frac{SA}{SP^3}$. Adeoque si
 vis centripeta tendat ad S pun-
 ctum in circumferentia situm,
 erit [per 32 terti] ang. PAS
 $=$ ang. AQS ; adeoque ob si-
 militudinem triangulorum ASP , ASQ ,
 erit $AQ : AS :: AS : SP$:

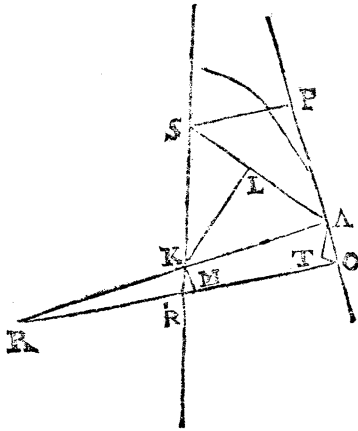


unde $SP = \frac{AS^2}{AQ}$ & $SP^3 = \frac{AS^6}{AQ^3}$ unde $\frac{SA}{SP^3} =$
 $\frac{SA \times AQ^3}{AS^6} = \frac{AQ^3}{AS^5}$, hoc est, ob datum AQ , erit vis
 reciproce ut AS^5 .

ac proinde $AR = \frac{4 AK^3}{L^2}$. Eodem prorsus ratiocinio

Invenietur Radius Curvature in Hyperbola æqualis

$$\frac{4 AK^3}{L^2} = \frac{L \times SA^3}{2 SP^3}$$



In Parabola vero facili-
or est calculus. Nam ob
datam subnormalem, est
 Kk semper = AT = Fluxi-
oni Axis; & triangula
 KkM , ATO , SPA , AKL ,
æquiangula, unde KM :
 Kk :: AP , SA , item est
 AT vel Kk : AO :: AP : SA ,
unde KM : AO :: AP^2
: SA^2 :: $SA^2 - SP^2$: SA^2 ::
unde erit SP^2 : SA^2 :: AO
- KM : AO :: AK : AR ,
ac proinde $AR = \frac{SA^2 \times AK}{SP^2}$;

sed est $AL = \frac{1}{2}$ lateris Recti = $\frac{1}{2}L$, & AK : AL :: SA : SP ,
quare erit $\frac{L^2 \times SA}{2 AK} = SP$, & $SP^2 = \frac{L^2 \times SA^2}{4 AK^2}$, quare e-

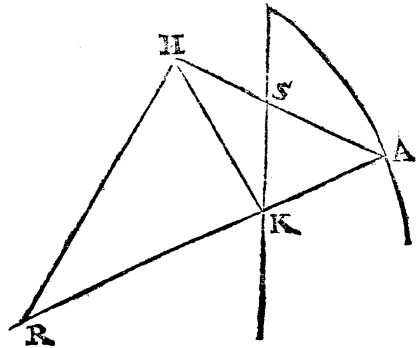
rit $AR = \frac{4 AK^3}{L^2}$, vel quoniam est, $AK = \frac{L \times SA}{2 SP}$,

erit $AR = \frac{L \times SA^3}{2 SP^3}$.

Atque ex his facillima oritur constructio, pro determi-
nando Radio curvatureæ in quavis Sectione Conica. Sit
enim AK perpendicularis in Sectionem occurrens Axi in
 K , ex K super AK erigatur perpendicularis HK , cum
 AS producta concurrens in H . Ex H erigatur super
 AH , perpendicularis HR , erit AR radius curvatureæ.

In

In Parabola paulo simplicior adhuc evadit constructio. Nam quoniam ex natura Parabolæ est $SA = SK$, & ang. AKH rectus, erit S centrum circuli per AKH transeuntis, unde invenitur Radius curvaturæ producendo SA in H ; ut $SH=SA$, & in H erigendo perpendiculararem HR ; Et R erit centrum circuli osculantis Parabolam in A .



Vis Centripeta tendens ad focum Sectionis Conicæ in qua corpus movetur, est reciproce proportionalis quadrato distantia. Nam quoniam

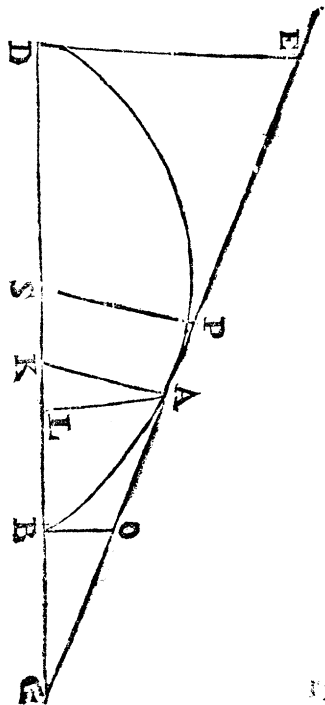
$$AR = \frac{L \times SA^3}{2 SP^3} \text{ erit } \frac{SA}{SP^3 \times AR}$$

$$= \frac{SA \times 2 SP^3}{SP^3 \times L \times SA^3} = \frac{2}{L \times SA^2}$$

hoc est ob datam $\frac{2}{L}$ erit vis

centripeta ut $\frac{1}{SA^2}$.

Sit Ellipsis BAD quam tangit in A recta GE . Sitque SP per centrum Ellipsis & KA per contactum, transeuntis, perpendicularares in tangentem. Erit $SP \times KA =$ quartæ parti figuræ Axis seu = quadrato semiaxis mino-



vis = BO × DE. Nam ob æquiangula triang. G B O,
G L A, G A K, G P S & G D E,

$$SP : SG :: BO : GO$$

$$SG : DG :: BG : LG :: GO : GA$$

$$DG : DE :: GA : AK,$$

unde SP : DE :: BO : AK; & SP × AK = DE × BO
= $\frac{1}{2}$ L × S B.

Hinc si Mobile moveatur in Ellipfi, vi centripeta ten-
dente ad centrum Ellipfis, erit vis illa directe ut distan-
tia; Nam est $\frac{SP^3 \times 4 AK^3}{L^2} =$ dati quantitati. Quia

est SP × AK quantitas data. Vis igitur, ut $\frac{SA}{SP^3 \times AR}$,
erit ut SA distantia.

In figura tertia Demissa ab altero umbilico F: in Tan-
gentem Perpendiculari FI. Ob æquiangula Triangula
S A P, F A I, erit SA : SP :: FA : FI = $\frac{SP \times FA}{SA}$,

unde erit SP × FI = $\frac{SP^2 \times FA}{SA} =$ quadrato femiaxis
minoris: unde si Axis major vocetur b , minor autem $2d$,
erit $SP^2 = \frac{d^2 SA}{b - SA}$ & $SP = \frac{d SA^{\frac{1}{2}}}{\sqrt{b - SA}}$.

In Hyperbola autem est $SP = \frac{d SA^{\frac{1}{2}}}{\sqrt{b + SA}}$.

In Parabola est $SP = \sqrt{d SA}$, posito ejus latere recto
= $4d$.

Quoniam est TA² : TO² :: AP² : SP² :: SA₂
— SP² : SP² :: SA² — $\frac{d^2 SA}{b - SA}$: $\frac{d^2 SA}{b - SA}$:: SA — $\frac{d^2}{b - SA}$
 $\frac{d^2}{b - SA}$:: $b SA - SA^2 - d^2$: d^2 , erit $\sqrt{b SA - SA^2 - d^2}$

: d

$d :: TA : TO$ cumque fit $TA = SA$, erit $TO =$
 $\frac{dSA}{SA}$

$$\sqrt{bSA - SA^2 - d^2}.$$

Sit jam QAO . Quælibet
 curva, cujus arcus minimus fit
 AO , tangentes in punctis A &
 O , AP , $O\rho$. Radius Curva-
 turæ AR , Perpendiculares in
 tangentes sint SP , $S\rho$, erit
 $\frac{SA \times TA}{fP} = AR$. Nam ob

equiangularia triangu-
 la est

$fP : AO :: PA : RA$
 & $AO : TA :: SA : PA$;
 unde ex æquo erit $fP : TA$
 vel $SA :: SA : RA$, est ve-

ro $fP = S\rho$, quare erit $RA = \frac{SA \times SA}{S\rho}$.

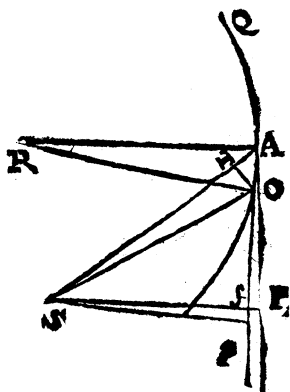
Hinc si distantia SA , in suam Fluxionem ducatur, &
 dividatur per Fluxionem perpendicularis, habebitur ra-
 dius Curvaturæ; Quo Theoremate facile determinatur
 Curvatura in Radialibus curvis. Exempli Gratia.
 Sit AQ , Spiralis Nautica; quoniam angulus SAP
 datur, ratio quoque SA ad $S\rho$ dabitur; fit illa ratio

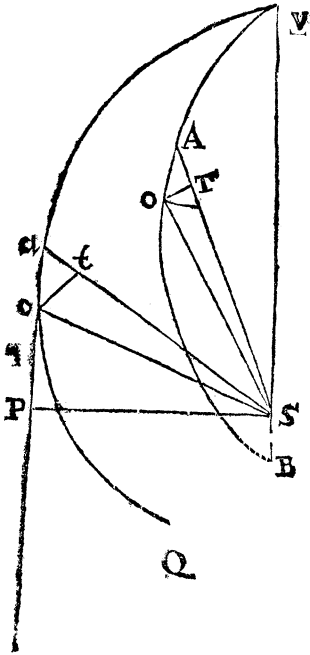
$$a \text{ ad } b, \text{ erit } SP = \frac{bSA}{a} \text{ \& } S\rho = \frac{b\dot{S}A}{a} \text{ \& } AR = \frac{SAS\dot{A}}{S\rho}$$

$= \frac{aSA}{b}$, unde facile constabit, Spiralis Nauticæ Evo-
 lutam esse eandem Spiralem, in alia positione.

$$\text{Quoniam } AR = \frac{SAS\dot{A}}{S\rho}, \text{ erit } \frac{SA}{S\rho \times AR} = \frac{S\rho}{S\rho \times SA}$$

Atque hinc rursus, ex data relatione SA ad $S\rho$, facile
 invenietur lex vis centripetæ.





Exemplum. Sit $V A B$ Ellipsis cujus focus S , Axis major $V B = b$, Axis minor $= 2 d$, latus Rectum $= 2 R$. Sitque $V a Q$ alia curva, ita ad hanc relata, ut sit perpetuo angulus $V S A$ angulo $V S a$ proportionalis, & sit $S a = S A$. Quæritur lex vis centripetæ tendentis ad S , qua corpus in curva $V a Q$ moveri potest.

Quoniam ang. $V S A$ est ad $V S a$, in data ratione; horum angulorum incrementa erunt in eadem ratione, fitque ea ratio

$$m \text{ ad } n; \text{ unde erit } o t = \frac{n \times O T}{m}.$$

$$\text{Eft autem } O T = \frac{d S \dot{A}}{\sqrt{b S A - S A^2 - d^2}}$$

$$\text{unde erit } o t = \frac{n d S \dot{A}}{m \sqrt{b S A - S A^2 - d^2}}.$$

Quoniam autem est $S A^2 + S P^2 : S P^2 :: t a^2 + o t^2 : o t^2$

$$\therefore S \dot{A}^2 + \frac{n^2 d^2 S \dot{A}^2}{m^2 b S A - S A^2 - d^2} : \frac{n^2 d^2 S^2}{m^2 b S A - S A^2 - d^2}$$

$$:: 1 + \frac{n^2 d^2}{m^2 \times b S A - S A^2 - d^2} : \frac{n^2 d^2}{m^2 \times b S A - S A^2 - d^2} ::$$

$$m^2 b S A - m^2 S A^2 - m^2 d^2 + n^2 d^2 : n^2 d^2, \text{ unde erit}$$

$$\sqrt{m^2 b S A - m^2 S A^2 - m^2 d^2 + n^2 d^2} : n d :: S A :$$

$$S P, \text{ \& } S \cdot P = \frac{n d S A}{\sqrt{m^2 b S A - m^2 S A^2 - m^2 d^2 + n^2 d^2}}.$$

Cujus ut habeatur fluxio pro $m^2 b S A - m^2 S A^2 -$

$m^2 d^2 + n^2 d^2$. Scribatur x & erit $SP = \frac{n d S A}{\sqrt{x}}$,

& $SP^3 = \frac{n^3 d^3 S A^3}{x^{\frac{3}{2}}}$; & est $\dot{x} = m^2 b S \dot{A} - 2 m^2 S A \dot{S} \dot{A}$,

& $S \dot{P} = n d S \dot{A} \times x^{-\frac{1}{2}} - \frac{1}{2} \frac{n A S A \dot{x}}{x^{\frac{3}{2}}}$, & redu-

cendo partes ad eundem denominatorem; erit $S \dot{P} = \frac{n d S \dot{A} x - \frac{1}{2} n d S A \dot{x}}{x^{\frac{3}{2}}}$. Et in numeratore loco, x &

$x^{\frac{3}{2}}$, ponendo ipsorum valores, & ordinando fit $SP = \frac{n d S A \times \frac{1}{2} m^2 b S A - m^2 d^2 + n^2 d^2}{x^{\frac{3}{2}}}$, unde erit $\frac{S \dot{P}}{SP^3 \times S \dot{A}}$

$= \frac{\frac{1}{2} m^2 b S A - m^2 d^2 + n^2 d^2}{n^2 d^2 S A^3}$. Sed est $\frac{S \dot{P}}{SP^3 \times S \dot{A}}$,

ut vis centripeta, quare erit vis, ut $\frac{m^2 b S A - m^2 d^2 + n^2 d^2}{n^2 d^2 S A^3}$

vel ob datam $n^2 d^2$ in denominatore erit vis, ut $\frac{\frac{1}{2} m^2 b S A - m^2 d^2 + n^2 d^2}{S A^3}$, vel loco d^2 ponendo $\frac{b R}{2}$,

erit vis ut $\frac{\frac{1}{2} m^2 b S A - \frac{1}{2} m^2 b R + \frac{1}{2} n^2 b R}{S A^3}$, seu ob

datam $\frac{b}{2}$, ut $\frac{m^2 S A - R m^2 + R n^2}{S A^3} = \frac{m^2}{S A^2} +$

$\frac{R n^2 - R m^2}{S A^3}$. Quæ omnia exacte coincidunt, cum iis

quæ à Domino Newtono de vi centripeta corporis in eadem curva moti, traduntur, in *Prop. 44. Princip.*

Quoniam vis Centripeta tendens ad punctum S, qua urgente corpus in curva moveri potest, est semper ut

$\frac{S \dot{P}}{SP^3 \times S \dot{A}}$; hinc ex data lege vis Centripetæ, Inveniri potest

potest ratio SA ad SP , ac proinde per methodum Tangentium Inversam, exhiberi potest Curva quæ data vi Centripeta describi possit.

Sit verbi gratia Vis reciproce ut distantiae Dignitas quælibet m , hoc est, sit $\frac{S\dot{P}}{SP^3 \times SA} = \frac{b}{a^2 SA^m}$, erit $\frac{S\dot{P}}{SP^3}$

$= \frac{b S\dot{A}}{a^2 SA^m}$, & capiendo harum fluxionum fluentes; erit

$$\frac{1}{2} SP^{-2} = b \frac{SA^{1-m} + e}{m-1 \times a^2}, \text{ unde erit } \frac{\frac{m-1}{2} \times a^2}{b SA^{1-m} + e} =$$

SP^2 , & multiplicando tam numeratorem, quam denominatorem fractionis, per SA^{m-1} ; & loco $\frac{m-1}{2} a^2$ ponendo d^2 , fit

$$\frac{d^2 SA^{m-1}}{b + e SA^{m-1}} = SP^2; \text{ quare erit } SP =$$

$$\frac{d \sqrt{SA^{m-1}}}{\sqrt{b + e SA^{m-1}}}$$

$$\frac{\text{Quod si quantitas constans } e \text{ fit nihilo æqualis erit } SP}{\sqrt{SA^{m-1}}}$$

Adeoque si vis reciproce ut distantiae quadratum, po-

ni potest $SP = \frac{\sqrt{d^2 SA}}{\sqrt{b}}$, & curva erit parabola cujus

latus rectum est $\frac{4d^2}{b}$, vel potest esse $SP = d \times \frac{\sqrt{SA}}{\sqrt{b - SA}}$,

& curva erit Ellipsis vel denique potest esse $SP = d \times$

$\frac{\sqrt{SA}}{\sqrt{b \times SA}}$, & curva evadit Hyperbola.

Si vis fit reciproce ut distantia cubus supponi potest, ut $S P$ fit $= \frac{d S A}{b}$, & curva fit spiralis Nautica, vel fieri potest ut fit $S P = \frac{d S A}{\sqrt{b - e S A}}$, & Curva erit eadem.

cum eâ cujus constructionem à sectore hyperbolæ petit Dominus Newtonus; vel potest esse $S P = \frac{d S A}{\sqrt{b + e S A}}$,

& ejus Curvæ constructionem per Sectores Ellipticos tradit idem Newtonus, *Cor. 3. Prop. 1. lib. 1. Princip*

Si vis centripeta fit reciproce ut distantia; ratio inter $S A$ & $S P$, æquatione Algebraica definiri nequit, Curva tamen per Logarithmicam vel per quadraturam Hyperbolæ construitur, fit enim $S P = \frac{d}{\sqrt{b - L S A}}$, ubi $L S A$

designat Logarithmum ipsius $S A$.

Hæc omnia sequuntur ex celebratissimâ nunc dierum Fluxionum Arithmeticâ, quam sine omni dubio Primus Invenit Dominus Newtonus, ut cui libet ejus Epistolas à Wallisio editas legenti, facile constabit, eadem tamen Arithmetica postea mutatis nomine & notationis modo; à Domino Leibnitio in Actis Eruditorum edita est.

Moveatur jam corpus in Curva $Q A O$, vide *fig. 1.* urgente vi centripeta tendente ad S ; & Celeritas corporis in A dicatur C ; celeritas autem qua corpus urgente eadem vi centripeta, in eadem distantia, in circulo moveri potest, dicatur c . Constat ex Theoremate primo, quod si $S A$ exponat vim Centripetam tendentem ad S ; vis Centripeta tendens ad R , qua urgente, corpus cum celeritate C , circulum cujus radius est $A R$ describet; per $S P$ exponetur. Corporum autem circulos describentium, vires Centripetæ sunt ut velocitatum quadrata ad circulorum radios applicata, quare erit $S P : S A ::$

$\frac{C^2}{AR} : \frac{c^2}{SA}$, unde erit $SP \times AR : SA^2 :: C^2 : c^2$ & $C : c :: \sqrt{SP \times AR} : SA$.

Si SP cum SA coincidat, ut fit in figurarum verticibus erit $C : c :: \sqrt{AR} : \sqrt{SA}$. Quod si curva fit Sectio Conica AR , radius curvaturæ in ejus vertice est æqualis dimidio lateris recti $= \frac{1}{2}L$, ac proinde erit velocitas corporis in vertice Sectionis, ad velocitatem corporis in eadem distantia circulum describentis, in dimidiata ratione lateris recti, ad distantiam illam duplicatam.

Quoniam est $AR = \frac{SA \times S\dot{A}}{S\dot{P}}$, erit $C^2 : c^2 :: \frac{SP \times SA \times S\dot{A}}{S\dot{P}} : SA^2 :: \frac{SP \times S\dot{A}}{S\dot{P}} : SA :: SP \times S\dot{A} : SA \times S\dot{P}$, adeoque ex data relatione SP ad SA , dabitur ratio C ad c , Exempli Gratia. Si vis fit reciproce

ut distantia dignatas m , hoc est fit $\frac{S\dot{P}}{SP^3 \times S\dot{A}} = \frac{b}{a^2 SA^m}$;

& erit $S\dot{P} = \frac{b SP^3 \times S\dot{A}}{a^2 SA^m}$, adeoque erit $C^2 : c^2 ::$

$SP \times S\dot{A} : \frac{b SP^3 \times SA \times S\dot{A}}{a^2 SA^m} :: a^2 SA^{m-1} : b SP^2$.

Unde si ponatur $SP^2 = \frac{d^2 SA^{m-1}}{b} = \frac{m-1}{2} \frac{a^2 SA^{m-1}}{b}$,

erit $C^2 : c^2 :: a^2 SA^{m-1} : \frac{m-1}{2} a^2 SA^{m-1} :: m-1 : 2$

ac proinde erit $C : c :: \sqrt{\frac{m-1}{2}} : \sqrt{m-1}$.

Quod si ponatur $SP^2 = \frac{d^2 SA^{m-1}}{b-e SA^{m-1}} = \frac{m-1}{2} \frac{a^2 SA^{m-1}}{b-e SA^{m-1}}$

fiet C^2 ad c^2 , ut $a^2 SA^{m-1}$ ad $\frac{m-1}{2} \frac{a^2 b SA^{m-1}}{b-e SA^{m-1}}$, hoc est

ut $b - e S A^{m-1}$ ad $\frac{m-1}{2} b$, sed est ratio $b - e S A^{m-1}$, ad $\frac{m-1}{2} \times b$, minor ratione b ad $\frac{m-1}{2} b$, seu ratione 2 ad $m - 1$, unde erit C ad c in minore ratione quam est $\sqrt{2}$ ad $\sqrt{m - 1}$.

Similiter, si capiatur $SP = \frac{d_2 S A^{m-1}}{b + e S A^{m-1}}$, inveniatur esse C ad c in majore ratione quam est $\sqrt{2}$ ad $\sqrt{m - 1}$.

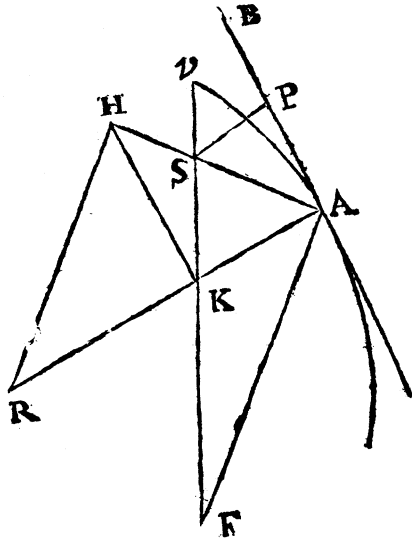
Cor. Si corpus in Parabola moveatur, & vis Centripeta tendat ad focum S, erit velocitas corporis, ad velocitatem corporis in eadem distantia, circulum describentis ubique ut $\sqrt{2}$ ad 1, nam in eo casu est $m = 2$ & $m - 1 = 1$. Velocitas corporis in Ellipsi est ad velocitatem corporis, in circulo ad eandem distantiam moti, in minore ratione quam $\sqrt{2}$ ad 1. Velocitas in Hyperbola est ad velocitatem in circulo in majore ratione, quam $\sqrt{2}$ ad 1.

Si Corpus in Spirali Nautica deferatur, est ejus velocitas ubique æqualis velocitati corporis in eadem distantia circulum describentis nam in eo casu est $m = 3$ & $m - 1 = 2$.

P R O B L E M A.

Posito quod vis Centripeta (cujus-quantitas absoluta nota est,) sit reciproce ut distantie quadratum & projiciatur corpus secundam datam rectam cum data velocitate. Invenire curvam in qua movetur corpus.

Projiciatur Corpus secundum datam rectam AB , cum data velocitate C . Et quoniam quantitas absoluta vis centripetæ nota est, dabitur inde velocitas qua corpus possit circulum ad distantiam SA describere urgente eadem vi; est enim æqualis ei quæ acquiritur, dum corpus vi illâ uniformiter applicata urgente, cadit per $\frac{1}{2} SA$. Sit illa velocitas c . Ex A in AB ,



erigatur perpendicularis AK , & in ea Capiatur AR , quarta proportionalis ipsis $c^2 C^2$ & $\frac{SA^2}{SP}$ & erit AR , ra-

dius curvaturæ in A . Ex R in AS demittatur perpendicularis RH & ex H in AR perpendicularis HK , & ducta recta SK , dabit axis positionem; Fiat angulus $FAK =$ angulo SAK . Et si FA sit ad SK Parallela figura in qua movetur corpus erit Parabola. Si autem Axi SK occurrat in F ; & puncta S & F , cadant ad eandem partem puncti K , figura erit Hyperbola; sin ad contrarias partes cadant puncta S & F , erit figura Ellipsis, unde focus S & F & Axe = $SA + FA$ describetur sectio, in qua corpus movebitur.