The partial fractions are not so tough as we think with the traditional methods.
Just follow us and and fell the fun with Partial Fractions.
Let us consider few problems.

- **Example 1.**

$$\frac{x^2-3x+2}{(x-5)^3}=\frac{A}{x-5}+\frac{B}{(x-5)^2}+\frac{C}{(x-5)^3} \qquad [1]$$

$$\Rightarrow x^2-3x+2=A(x-5)^2+B(x-5)+C \qquad [2]$$

Put $x=5$ in equation [2] to get

$$\Rightarrow 25-3\times 5+2=A\times 0+B\times 0+C$$

$$\Rightarrow C=12$$

Differentiate equation [2] with respect to x

$$\Rightarrow 2x-3=2A(x-5)+B \qquad [3]$$

Put $x=5$ in equation [3] to get

$$\Rightarrow 2\times 5-3=2A\times 0+B$$

$$\Rightarrow B=7$$

Again Differentiate equation [3] with respect to x

$$\Rightarrow 2=2A \qquad [4]$$

$$\Rightarrow A=1$$

Now

$$\frac{x^2-3x+2}{(x-5)^3}=\frac{1}{x-5}+\frac{7}{(x-5)^2}+\frac{12}{(x-5)^3}$$

- **Example 2.**

$$\frac{x^3-3x}{(x^2+2x-1)(x-5)^2}=\frac{Ax+B}{(x^2+2x-1)}+\frac{C}{(x-5)}+\frac{D}{(x-5)^2} \qquad [1]$$

$$\Rightarrow x^3-3x=(Ax+B)(x-5)^2+C(x^2+2x-1)(x-5)+D(x^2+2x-1) \qquad [2]$$

put $x=5$ in equation [2] to get

$$5^3-3\times 5=D(5^2+2\times 5-1)\Rightarrow 110=D(34)$$

$$\Rightarrow D=\frac{55}{17}$$

Differentiate equation [2] with respect to x

$$\Rightarrow 3x^2-3=A(x-5)^2+2(Ax+B)(x-5)+C\{(2x+2)(x-5)+(x^2+2x-1)\}+D(2x+2) \qquad [3]$$

put $x=5$ in equation[3]

$$3\times 5^2-3=A\times 0+B\times 0+C\{0+(5^2+2\times 5-1)\}+\frac{55}{17}(2\times 5+2) \qquad \left(\text{using } D=\frac{55}{17}\right)$$

$$\Rightarrow 72 = 34\,C + \frac{660}{17}$$

$$\Rightarrow C = \frac{282}{289}$$

Differentiate equation [3] with respect to x

$$6\,x = 2\,A\,(x-5) + 2\,A\,(x-5) + 2\,(Ax+B) + C\,\{2\,(x-5) + (2\,x+2) + (2\,x+2)\} + 2\,D$$

$$6\,x = 4\,A\,(x-5) + 2\,(Ax+B) + C\,\{2\,(x-5) + 4\,(x+1)\} + 2\,D \qquad [4]$$

put $x = 5$ in equation [4]

$$\Rightarrow 30 = 10\,A + 2\,B + C\,(24) + 2\,D = 10\,A + B + \frac{282}{289}\times 24 + 2\times\frac{55}{17}$$

$$\Rightarrow \frac{32}{289} = 10\,A + 2\,B \qquad \Rightarrow \frac{16}{289} = 5\,A + B \qquad [5]$$

Differentiate equation [4] with respect to x

$$6 = 6\,A + 6\,C \quad \Rightarrow \quad 1 = A + \frac{282}{289} \qquad \text{(using value of C)} \qquad [6]$$

$$\Rightarrow A = \frac{7}{289}$$

Use value of A in [5] to get the value of B as below

$$\frac{16}{289} = 5\,A + B = 5\times\frac{7}{289} + B$$

$$\Rightarrow B = -\frac{19}{289}$$

Finally

$$\frac{x^3 - 3\,x}{\left(x^2 + 2\,x - 1\right)\,(x-5)^2} = \frac{\frac{7}{289}\,x - \frac{19}{289}}{\left(x^2 + 2\,x - 1\right)} + \frac{\frac{282}{289}}{(x-5)} + \frac{\frac{55}{17}}{(x-5)^2}$$

Just use this method and enjoy the fun of partial fraction.

Regards -

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