

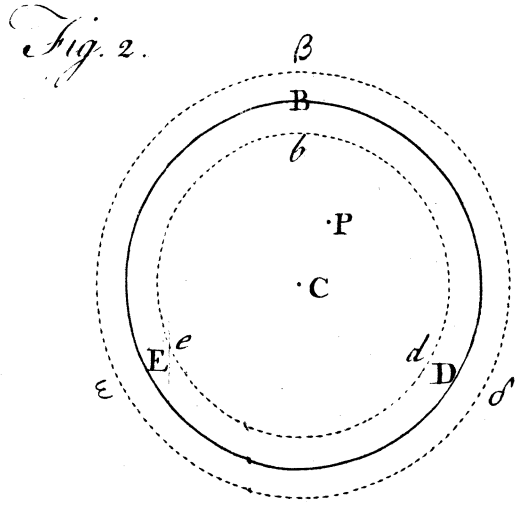
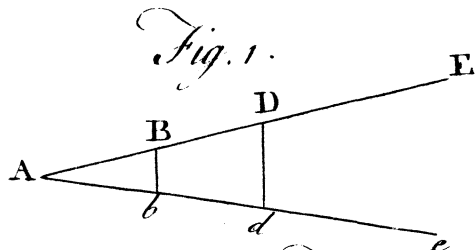
LV. *An Attempt to explain some of the principal Phænomena of Electricity, by Means of an elastic Fluid: By the Honourable Henry Cavendish, F. R. S.*

Read Dec. 19, 1771, and Jan. 9, 1772. **S**INCE I first wrote the following paper, I find that this way of accounting for the phænomena of electricity, is not new. Æpinus, in his *Tentamen Theoriæ electricitatis & magnetismi*, has made use of the same, or nearly the same hypothesis that I have; and the conclusions he draws from it, agree nearly with mine, as far as he goes. However, as I have carried the theory much farther than he has done, and have considered the subject in a different, and, I flatter myself, in a more accurate manner, I hope the Society will not think this paper unworthy their acceptance.

The method I propose to follow is, first, to lay down the hypothesis; next, to examine by strict mathematical reasoning, or at least, as strict reasoning as the nature of the subject will admit of, what consequences will flow from thence; and lastly, to examine how far these consequences agree with such experiments as have yet been made on this subject. In a future paper, I intend to give the result of some experiments I am making, with intent to examine still further the truth of this hypothesis, and to find out the law of the electric attraction and repulsion.

I

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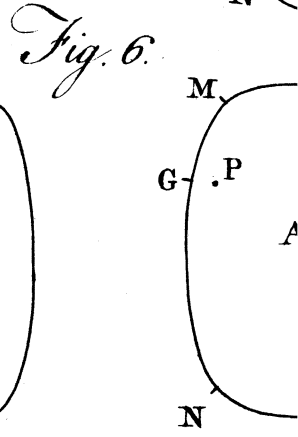
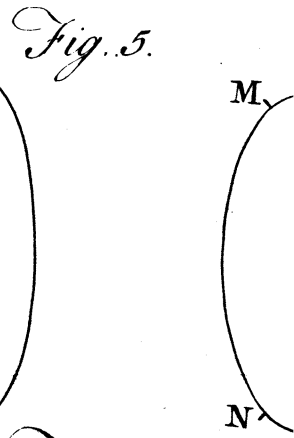
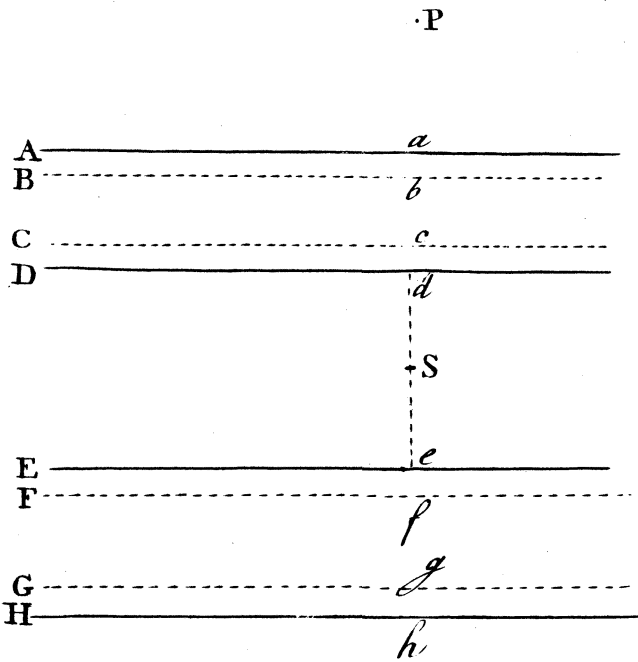
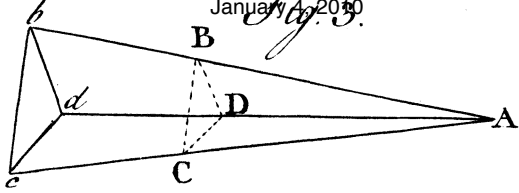


Fig. 7.

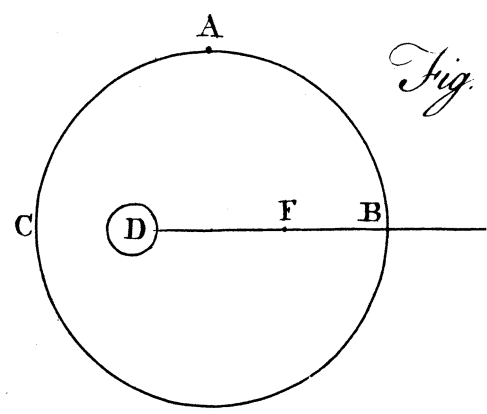
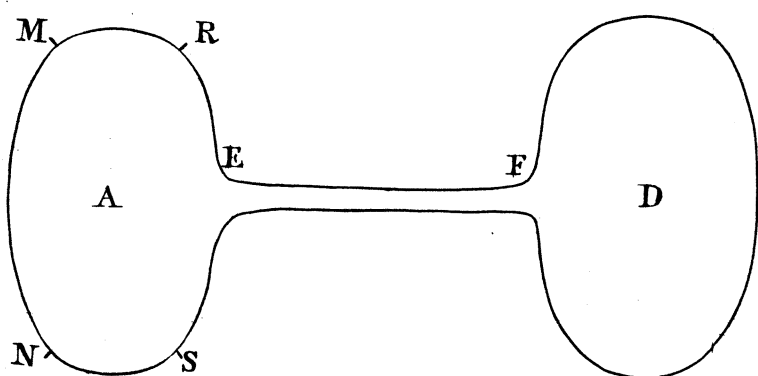
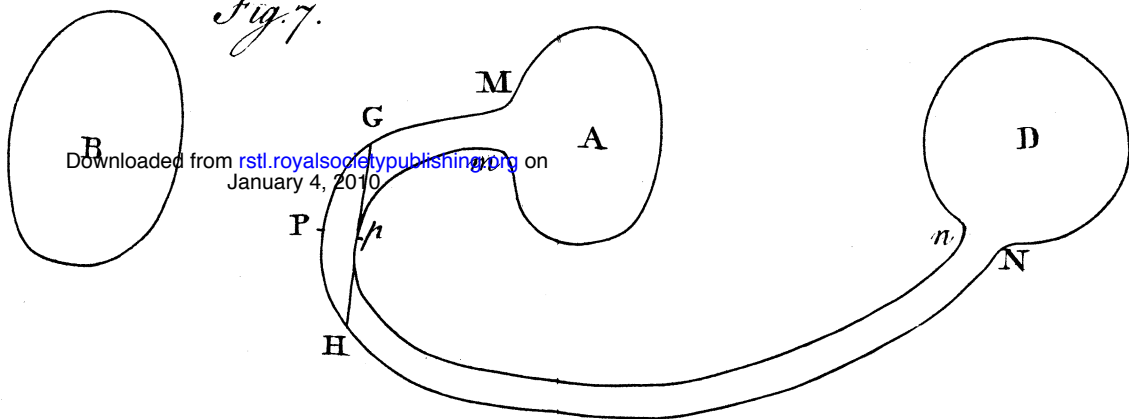


Fig.

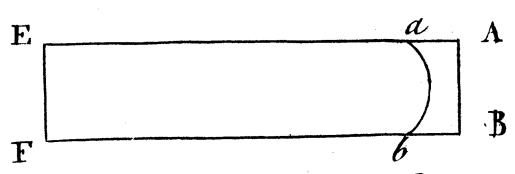
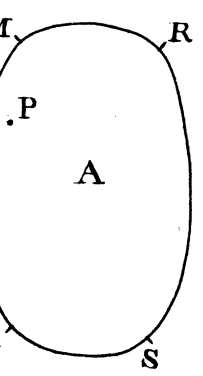


Fig. 8.

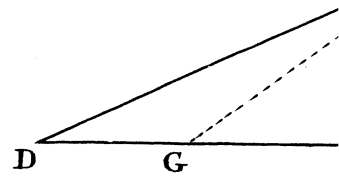
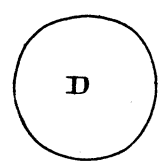


Fig. 10.

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Fig. 8.

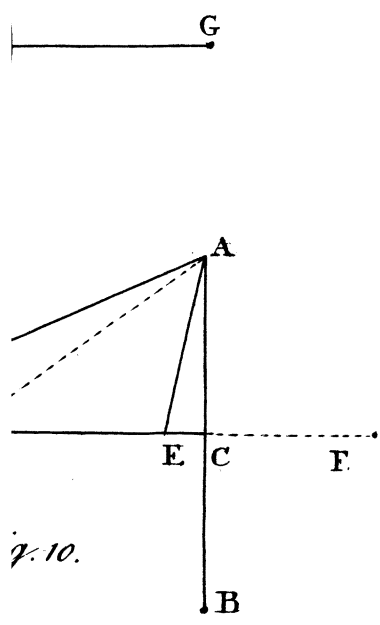


Fig. 10.

HYPOTHESIS.

THERE is a substance, which I call the electric fluid, the particles of which repel each other and attract the particles of all other matter, with a force inversely as some less power of the distance than the cube: the particles of all other matter also, repel each other, and attract those of the electric fluid, with a force varying according to the same power of the distances. Or, to express it more concisely, if you look upon the electric fluid as matter of a contrary kind to other matter, the particles of all matter, both those of the electric fluid and of other matter, repel particles of the same kind, and attract those of a contrary kind, with a force inversely as some less power of the distance than the cube.

For the future, I would be understood never to comprehend the electric fluid under the word matter, but only some other sort of matter.

It is indifferent whether you suppose all sorts of matter to be indued in an equal degree with the foregoing attraction and repulsion, or whether you suppose some sorts to be indued with it in a greater degree than others; but it is likely that the electric fluid is indued with this property in a much greater degree than other matter; for in all probability the weight of the electric fluid in any body bears but a very small proportion to the weight of the matter; but yet the force with which the electric fluid therein attracts any particle of matter must be equal to the force with which the matter therein

repels that particle; otherwise the body would appear electrical, as will be shewn hereafter.

To explain this hypothesis more fully, suppose that 1 grain of electric fluid attracts a particle of matter, at a given distance, with as much force as n grains of any matter, lead for instance, repel it: then will 1 grain of electric fluid repel a particle of electric fluid with as much force as n grains of lead attract it; and 1 grain of electric fluid will repel 1 grain of electric fluid with as much force as n grains of lead repel n grains of lead.

All bodies in their natural state, with regard to electricity, contain such a quantity of electric fluid interspersed between their particles, that the attraction of the electric fluid in any small part of the body on a given particle of matter shall be equal to the repulsion of the matter in the same small part on the same particle. A body in this state I call saturated with electric fluid: if the body contains more than this quantity of electric fluid, I call it overcharged: if less, I call it undercharged. This is the hypothesis; I now proceed to examine the consequences which will flow from it.

LEMMA I.

Let $E A e$ (TAB. XX. fig. 1.) represent a cone continued infinitely; let A be the vertex, and $B b$ and $D d$ planes parallel to the base; and let the cone be filled with uniform matter, whose particles repel each other with a force inversely as the n power of the distance. If n is greater than 3, the force with which a particle

at

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at A is repelled by $EBbe$ or all that part of the cone beyond Bb is as $\frac{1}{AB^{n-3}}$.

For supposing AB to flow, the fluxion of $EBbe$ is proportional to $-AB \times AB^2$, and the fluxion of its repulsion on A is proportional to $\frac{-AB}{AB^{n-2}}$; the fluent of which is $\frac{1}{n-3 \times AB^{n-3}}$; which when AB is infinite is equal to nothing; consequently the repulsion of $EBbe$ is proportional to $\frac{1}{n-3 \times AB^{n-3}}$ or to $\frac{1}{AB^{n-3}}$.

COROLLARY.

If AB is infinitely small, $\frac{1}{AB^{n-3}}$ is infinitely great; therefore the repulsion of that part of the cone between A and Bb , on A, is infinitely greater than the repulsion of all that beyond it.

LEMMA II.

By the same method of reasoning it appears, that if n is equal to 3, the repulsion of the matter between Bb and Dd on a particle at A, is proportional to the logarithm of $\frac{AD}{AB}$; consequently, the repulsion of that part is infinitely small in respect of that between A and Bb , and also infinitely small in respect of that beyond Dd .

LEMMA III.

In like manner, if n is less than 3, the repulsion of the part between A and Bb on A is proportional to AB^{3-n} : consequently the repulsion of the matter between A and Bb on A , is infinitely small in respect of that beyond it.

COROLLARY.

It is easy to see from these three lemmata, that, if the electric attraction and repulsion had been supposed to be inversely, as some higher power of the distance than the cube; a particle could not have been sensibly affected by the repulsion of any fluid, except what was placed close to it. If the repulsion was inversely, as the cube of the distance, a particle could not be sensibly affected by the repulsion of any finite quantity of fluid, except what was close to it. But as the repulsion is supposed to be inversely as some power of the distance less than the cube, a particle may be sensibly affected by the repulsion of a finite quantity of fluid, placed at any finite distance from it.

DEFINITION.

If the electric fluid in any body, is by any means confined in such manner that it cannot move from one part of the body to the other; I call it immovable: if it is able to move readily from one part to another, I call it moveable.

PROPOSITION I.

A body overcharged with electric fluid attracts or repels a particle of matter or fluid, and is attracted or repelled by it, with exactly the same force as it would, if the matter in it, together with so much of the fluid as is sufficient to saturate it, was taken away, or as if the body consisted only of the redundant fluid in it. In like manner an undercharged body attracts or repels with the same force, as if it consisted only of the redundant matter; the electric fluid, together with so much of the matter as is sufficient to saturate it, being taken away.

This is evident from the definition of saturation.

PROP. II.

Two over or undercharged bodies attract or repel each other with just the same force that they would, if each body consisted only of the redundant fluid in it, if overcharged, or of the redundant matter in it, if undercharged.

For, let the two bodies be called A and B; by the last proposition the redundant substance in B impels each particle of fluid and matter in A, and consequently impels the whole body A, with the same force that the whole body B impels it: for the same reason the redundant substance in A impels the redundant substance in B, with the same force that
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the whole body A impels it. It is shewn therefore, that the whole body B impels the whole body A, with the same force that the redundant substance in B impels the whole body A, or with which the whole body A impels the redundant substance in B; and that the whole body A impels the redundant substance in B, with the same force that the redundant substance in A impels the redundant substance in B; therefore the whole body B impels the whole body A, with the same force with which the redundant substance in A impels the redundant substance in B, or with which the redundant substance in B impels the redundant substance in A.

C O R O L L A R Y.

Let the matter in all the rest of space, except in two given bodies, be saturated with immoveable fluid; and let the fluid in those two bodies be also immoveable. Then, if one of the bodies is saturated, and the other either over or undercharged, they will not at all attract or repel each other.

If the bodies are both overcharged, they will repel each other.

If they are both undercharged, they will also repel each other.

If one is overcharged and the other undercharged, they will attract each other.

N. B. In this corollary, when I call a body overcharged, I would be understood to mean, that it is overcharged in all parts, or at least no where under-

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undercharged: in like manner, when I call it undercharged, I mean that it is undercharged in all parts, or at least no where overcharged.

P R O P. III.

If all the bodies in the universe are saturated with electric fluid, it is plain that no part of the fluid can have any tendency to move.

P R O P. IV.

If the quantity of electric fluid in the universe is exactly sufficient to saturate the matter therein, but unequally dispersed, so that some bodies are overcharged and others undercharged; then, if the electric fluid is not confined, it will immediately move till all the bodies in the universe are saturated.

For, supposing that any body is overcharged, and the bodies near it are not, a particle at the surface of that body will be repelled from it by the redundant fluid within; consequently some fluid will run out of that body; but if the body is undercharged, a particle at its surface will be attracted towards the body by the redundant matter within, so that some fluid will run into the body.

N. B. In Prob. IV. Case III. there will be shewn an exception to this proposition; there may perhaps be some other exceptions to it: but I think

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think there can be no doubt, but what this proposition must hold good in general.

LEMMA IV.

Let BDE, bde , and $\beta\delta\epsilon$ (fig. 2.) be concentric spherical surfaces, whose center is C: if the space * Bb is filled with uniform matter, whose particles repel with a force inversely, as the square of the distance, a particle placed any where within the space Cb, as at P, will be repelled with as much force in one direction as another, or it will not be impelled in any direction. This is demonstrated in Newt. Princip. liber I. prop. lxx. It follows also from his demonstration, that if the repulsion is inversely, as some higher power of the distance than the square, the particle P will be impelled towards the center; and if the repulsion is inversely as some lower power than the square, it will be impelled from the center.

LEMMA V.

If the repulsion is inversely as the square of the distance, a particle placed any where without the sphere BDE, is repelled by that sphere, and also by the space Bb, with the same force that it would if all the matter therein was collected in the center of the

* By the space Bb or B β , I mean the space comprehended between the spherical surfaces BDE and bde , or between BDE and $\beta\delta\epsilon$: by the space Cb or C β , I mean the spheres bde or $\beta\delta\epsilon$.

sphere;

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sphere ; provided the density of the matter therein is every where the same at the same distance from the center. This is easily deduced from prop. 71. of the same book, and has been demonstrated by other authors.

P R O P. V.

PROBLEM I. Let the sphere BDE be filled with uniform solid matter, overcharged with electric fluid: let the fluid therein be moveable, but unable to escape from it: let the fluid in the rest of infinite space be moveable, and sufficient to saturate the matter therein; and let the matter in the whole of infinite space, or at least in the space $B\beta$, whose dimensions will be given below, be uniform and solid; and let the law of the electric attraction and repulsion be inversely as the square of the distance: it is required to determine in what manner the fluid will be disposed both within and without the globe.

Take the space Bb such, that the interstices between the particles of matter therein shall be just sufficient to hold a quantity of electric fluid, whose particles are pressed close together, so as to touch each other, equal to the whole redundant fluid in the globe, besides the quantity requisite to saturate the matter in Bb ; and take the space $B\beta$ such, that the matter therein shall be just able to saturate the redundant fluid in the globe: then, in all parts of the space Bb , the fluid will be pressed close together, so

that its particles shall touch each other; the space $B\beta$ will be intirely deprived of fluid; and in the space Cb , and all the rest of infinite space, the matter will be exactly saturated.

For, if the fluid is disposed in the above-mentioned manner, a particle of fluid placed anywhere within the space Cb will not be impelled in any direction by the fluid in Bb , or the matter in $B\beta$, and will therefore have no tendency to move: a particle placed anywhere without the sphere $\beta\delta\epsilon$ will be attracted with just as much force by the matter in $B\beta$, as it is repelled by the redundant fluid in Bb , and will therefore have no tendency to move: a particle placed anywhere within the space Bb , will indeed be repelled towards the surface, by all the redundant fluid in that space which is placed nearer the center than itself; but as the fluid in that space is already pressed as close together as possible, it will not have any tendency to move; and in the space $B\beta$ there is no fluid to move, so that no part of the fluid can have any tendency to move.

Moreover, it seems impossible for the fluid to be at rest, if it is disposed in any other form; for if the density of the fluid is not everywhere the same at the same distance from the center, but is greater near b than near d , a particle placed anywhere between those two points will move from b towards d ; but if the density is everywhere the same at the same distance from the center, and the fluid in Bb is not pressed close together, the space Cb will be overcharged, and consequently a particle at b will be repelled from the center, and cannot be at rest: in like manner, if there is any fluid in $B\beta$, it cannot be

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be at rest: and, by the same kind of reasoning, it might be shewn, that, if the fluid is not spread uniformly within the space Cb , and without the sphere $\beta\delta\epsilon$, it cannot be at rest.

COROLLARY I.

If the globe BDE is undercharged, every thing else being the same as before, there will be a space Bb , in which the matter will be intirely deprived of fluid, and a space $B\beta$, in which the fluid will be pressed close together; the matter in Bb being equal to the whole redundant matter in the globe, and the redundant fluid in $B\beta$, being just sufficient to saturate the matter in Bb : and in all the rest of space the matter will be exactly saturated. The demonstration is exactly similar to the foregoing.

COROL. II.

The fluid in the globe BDE will be disposed in exactly the same manner, whether the fluid without is immoveable, and disposed in such manner, that the matter shall be everywhere saturated, or whether it is disposed as above described; and the fluid without the globe will be disposed in just the same manner, whether the fluid within is disposed uniformly, or whether it is disposed as above described.

P R O P. VI.

PROB. 2. To determine in what manner the fluid will be disposed in the globe BDE, supposing every thing as in the last problem, except that the fluid on the outside of the globe is immovable, and disposed in such manner as everywhere to saturate the matter, and that the electric attraction and repulsion is inversely, as some other power of the distance than the square.

I am not able to answer this problem accurately; but I think we may be certain of the following circumstances.

CASE I. Let the repulsion be inversely as some power of the distance between the square and the cube, and let the globe be overcharged.

It is certain that the density of the fluid will be everywhere the same, at the same distance from the center. Therefore, first, There can be no space as Cb , within which the matter will be everywhere saturated; for a particle at b is impelled towards the center, by the redundant fluid in Bb , and will therefore move towards the center, unless Cb is sufficiently overcharged to prevent it. Secondly, The fluid close to the surface of the sphere will be pressed close together; for otherwise a particle so near to it, that the quantity of fluid between it and the surface should be very small, would move towards it; as the repulsion of the small quantity of fluid between
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it and the surface, would be unable to balance the repulsion of the fluid on the other side. Whence, I think, we may conclude, that the density of the fluid will increase gradually from the center to the surface, where the particles will be pressed close together: whether the matter exactly at the center will be overcharged, or only saturated, I cannot tell.

C O R O L L A R Y.

For the same reason, if the globe be undercharged, I think we may conclude, that the density of the fluid will diminish gradually from the center to the surface, where the matter will be entirely deprived of fluid.

CASE 2. Let the repulsion be inversely as some power of the distance less than the square; and let the globe be overcharged.

There will be a space Bb , in which the particles of the fluid will be everywhere pressed close together; and the quantity of redundant fluid in that space will be greater than the quantity of redundant fluid in the whole globe BDE ; so that the space Cb , taken all together, will be undercharged: but I cannot tell in what manner the fluid will be disposed in that space.

For it is certain, that the density of the fluid will be everywhere the same, at the same distance from the center. Therefore, let b be any point where the fluid is not pressed close together, then will a particle at b be impelled towards the surface, by the
redundant

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redundant fluid in the space Bb ; therefore, unless the space Cb is undercharged, the particle will move towards the surface.

C O R O L L A R Y.

For the same reason, if the globe is undercharged, there will be a space Bb , in which the matter will be intirely deprived of fluid, the quantity of matter therein being more than the whole redundant matter in the globe; and, consequently, the space Cb , taken all together, will be overcharged.

L E M M A VI.

Let the whole space comprehended between two parallel planes, infinitely extended each way, be filled with uniform matter, the repulsion of whose particles is inversely as the square of the distance; the plate of matter formed thereby will repel a particle of matter with exactly the same force, at whatever distance from it, it be placed.

For, suppose that there are two such plates, of equal thickness, placed parallel to each other, let A (fig. 3.) be any point not placed in or between the two plates: let BCD represent any part of the nearest plate: draw the lines AB , AC , and AD , cutting the furthest plate in b , c , and d ; for it is plain, that if they cut one plate, they must, if produced, cut the other: the triangle BCD is to the triangle bcd , as AB^2 to Ab^2 ; therefore a particle of matter at A will be repelled with the same force by the matter in the triangle BCD , as by that in bcd . Whence it appears, that a particle at A will
be

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be repelled with as much force by the nearest plate, as by the more distant; and consequently, will be impelled with the same force by either plate, at whatever distance from it it be placed.

C O R O L L A R Y.

If the repulsion of the particles is inversely as some higher power of the distance than the square, the plate will repel a particle with more force, if its distance be small than if it be great; and if the repulsion is inversely as some lower power than the square, it will repel a particle with less force, if its distance be small than if it be great.

P R O P. VII.

PROB. 3. In fig. 4. let the parallel lines Aa , Bb , &c. represent parallel planes infinitely extended each way: let the spaces * AD and EH be filled with uniform solid matter: let the electric fluid in each of those spaces be moveable and unable to escape: and let all the rest of the matter in the universe be saturated with immoveable fluid; and let the electric attraction and repulsion be inversely as the square of the distance. It is required to determine in what manner the fluid will be disposed in the spaces AD and EH , according as one or both of them are over or undercharged.

* By the space AD or AB , &c. I mean the space comprehended between the planes Aa and Dd , or between Aa and Bb .

Let

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Let AD be that space which contains the greatest quantity of redundant fluid, if both spaces are overcharged, or which contains the least redundant matter, if both are undercharged; or, if one is overcharged, and the other undercharged, let AD be the overcharged one. Then, first, There will be two spaces, AB and GH , which will either be intirely deprived of fluid, or in which the particles will be pressed close together; namely, if the whole quantity of fluid in AD and EH together, is less than sufficient to saturate the matter therein, they will be intirely deprived of fluid; the quantity of redundant matter in each being half the whole redundant matter in AD and EH together: but if the fluid in AD and EH together is more than sufficient to saturate the matter, the fluid in AB and GH will be pressed close together; the quantity of redundant fluid in each being half the whole redundant fluid in both spaces. 2dly, In the space CD the fluid will be pressed close together; the quantity of fluid therein being such, as to leave just enough fluid in BC to saturate the matter therein. 3dly, The space EF will be intirely deprived of fluid; the quantity of matter therein being such, that the fluid in FG shall be just sufficient to saturate the matter therein: consequently, the redundant fluid in CD will be just sufficient to saturate the redundant matter in EF ; for as AB and GH together contain the whole redundant fluid or matter in both spaces, the spaces BD and EG together contain their natural quantity of fluid; and therefore, as BC and FG each contain their natural quantity of fluid, the spaces CD and EF together contain their
natural

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natural quantity of fluid. And, 4thly, The spaces BC and FG will be saturated in all parts.

For, first, If the fluid is disposed in this manner, no particle of it can have any tendency to move: for a particle placed anywhere in the spaces BC and FG, is attracted with just as much force by EF, as it is repelled by CD; and it is repelled or attracted with just as much force by AB, as it is in a contrary direction by GH, and, consequently, has no tendency to move. A particle placed anywhere in the space CD, or in the spaces AB and GH, if they are overcharged, is indeed repelled with more force towards the planes D*d*, A*a*, and H*b*, than it is in the contrary direction; but as the fluid in those spaces is already as much compressed as possible, the particle will have no tendency to move.

2dly, It seems impossible that the fluid should be at rest, if it is disposed in any other manner: but as this part of the demonstration is exactly similar to the latter part of that of Problem the first, I shall omit it.

C O R O L. I.

If the two spaces AD and EH are both overcharged, the redundant fluid in CD is half the difference of the redundant fluid in those spaces: for half the difference of the redundant fluid in those spaces, added to the quantity in AB, which is half the sum, is equal to the whole quantity in AD. For a like reason, if AD and EH are both undercharged, the redundant matter in EF is half the difference of the redundant matter in those spaces; and if AD is

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overcharged, and EH undercharged, the redundant fluid in CD exceeds half the redundant fluid in AD , by a quantity sufficient to saturate half the redundant matter in EH .

C O R O L. II.

It was before said, that the fluid in the spaces AB and GH (when there is any fluid in them) is repelled against the planes Aa and Hb ; and, consequently, would run out through those planes, if there was any opening for it to do so. The force with which the fluid presses against the planes Aa and Hb , is that with which the redundant fluid in AB is repelled by that in GH ; that is, with which half the redundant fluid in both spaces is repelled by an equal quantity of fluid. Therefore, the pressure against Aa and Hb depends only on the quantity of redundant fluid in both spaces together, and not at all on the thickness or distance of those spaces, or on the proportion in which the fluid is divided between the two spaces. If there is no fluid in AB and GH , a particle placed on the outside of the spaces AD and EH , contiguous to the planes Aa or Hb , is attracted towards those planes by all the matter in AB and GH , *id est*, by all the redundant matter in both spaces; and, consequently, endeavours to insinuate itself into the space AD or EH ; and the force with which it does so, depends only on the quantity of redundant matter in both spaces together. The fluid in CD also presses against the plane Dd , and the force with which it does so, is that with
which

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which the redundant fluid in CD is attracted by the matter in EF .

COROL. III.

If AD is overcharged, and EH undercharged, and the redundant fluid in AD is exactly sufficient to saturate the redundant matter in EH , all the redundant fluid in AD will be collected in the space CD , where it will be pressed close together: the space EF will be intirely deprived of fluid, the quantity of matter therein being just sufficient to saturate the redundant fluid in CD , and the spaces AC and FH will be everywhere saturated. Moreover, if an opening is made in the planes Aa or Hb , the fluid within the spaces AD or EH will have no tendency to run out thereat, nor will the fluid on the outside have any tendency to run in at it: a particle of fluid too placed anywhere on the outside of both spaces, as at P , will not be at all attracted or repelled by those spaces, any more than if they were both saturated; but a particle placed anywhere between those spaces, as at S , will be repelled from d towards e ; and if a communication was made between the two spaces, by the canal de , the fluid would run out of AD into EH , till they were both saturated.

PRO P. VIII.

PROB. 4. To determine in what manner the fluid will be disposed in the space AD , supposing that all the rest of the universe is saturated with immoveable fluid, and that the electric attraction and repulsion is inversely as some other power of the distance than the square.

I am not able to answer this Problem accurately, except when the repulsion is inversely as the simple or some lower power of the distance; but I think we may be certain of the following circumstances.

CASE I. Let the repulsion be inversely as some power of the distance between the square and the cube, and let AD be overcharged.

First, It is certain that the density of the fluid must be everywhere the same, at the same distance from the planes Aa and Dd . 2dly, There can be no space as BC , of any sensible breadth, in which the matter will not be overcharged. And, 3dly, The fluid close to the planes Aa and Dd will be pressed close together. Whence, I think, we may conclude, that the density of the fluid will increase gradually from the middle of the space to the outside, where it will be pressed close together. Whether the matter exactly in the middle will be overcharged, or only saturated, I cannot tell.

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CASE 2. Let the repulsion be inversely as some power of the distance between the square and the simple power, and let AD be overcharged.

There will be two spaces AB and DC , in which the fluid will be pressed close together, and the quantity of redundant fluid in each of those spaces will be more than half the redundant fluid in AD ; so that the space BC , taken all together, will be undercharged; but I cannot tell in what manner the fluid will be disposed in that space. The demonstrations of these two cases are exactly similar to those of the two cases of Prob. 2.

CASE 3. If the repulsion is inversely as the simple or some lower power of the distance, and AD is overcharged, all the fluid will be collected in the spaces AB and CD , and BC will be entirely deprived of fluid. If AD contains just fluid enough to saturate it, and the repulsion is inversely as the distance, the fluid will remain in equilibrio, in whatever manner it is disposed; provided its density is everywhere the same, at the same distance from the planes Aa and Dd : but if the repulsion is inversely as some less power than the simple one, the fluid will be in equilibrio, whether it is either spread uniformly, or whether it is all collected in that plane which is in the middle between Aa and Dd , or whether it is all collected in the spaces AB and CD ; but not, I believe, if it is disposed in any other manner.

The demonstration depends upon this circumstance; namely, that if the repulsion is inversely as the distance, two spaces AB and CD , repel a particle

ticle, placed either between them, or on the outside of them, with the same force as if all the matter of those spaces was collected in the middle plane between them.

It is needless mentioning the three cases in which AD is undercharged, as the reader will easily supply the place.

Though the four foregoing problems do not immediately tend to explain the phenomena of electricity, I chose to insert them; partly because they seem worth engaging our attention in themselves; and partly because they serve, in some measure, to confirm the truth of some of the following propositions, in which I am obliged to make use of a less accurate kind of reasoning.

In the following propositions, I shall always suppose the bodies I speak of to consist of solid matter, confined to the same spot, so as not to be able to alter its shape or situation by the attraction or repulsion of other bodies on it: I shall also suppose the electric fluid in these bodies to be moveable, but unable to escape, unless when otherwise expressed. As for the matter in all the rest of the universe, I shall suppose it to be saturated with immoveable fluid. I shall also suppose the electric attraction and repulsion to be inversely as any power of the distance less than the cube, except when otherwise expressed.

By a canal, I mean a slender thread of matter, of such kind that the electric fluid shall be able to move readily along it, but shall not be able to escape from it, except at the ends, where it communicates with other bodies. Thus, when I say that two bodies
com-

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communicate with each other by a canal, I mean that the fluid shall be able to pass readily from one body to the other by that canal.

P R O P. IX.

If any body at a distance from any over or under-charged body be overcharged, the fluid within it will be lodged in greater quantity near the surface of the body than near the center. For, if you suppose it to be spread uniformly all over the body, a particle of fluid in it, near the surface, will be repelled towards the surface, by a greater quantity of fluid than that by which it is repelled from it; consequently, the fluid will flow towards the surface, and make it denser there: moreover, the particles of fluid close to the surface will be pressed close together; for otherwise, a particle placed so near it, that the quantity of redundant fluid between it and the surface should be very small, would move towards it; as the small quantity of redundant fluid between it and the surface would be unable to balance the repulsion of that on the other side.

From the four foregoing problems it seems likely, that if the electric attraction or repulsion is inversely as the square of the distance, almost all the redundant fluid in the body will be lodged close to the surface, and there pressed close together, and the rest of the body will be saturated. If the repulsion is inversely as some power of the distance between

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the square and the cube, it is likely that all parts of the body will be overcharged : and if it is inverfely as fome lefs power than the fquare, it is likely that all parts of the body, except thofe near the furface, will be undercharged.

C O R O L L A R Y.

For the fame reason, if the body is undercharged, the deficiency of fluid will be greater near the furface than near the center, and the matter near the furface will be intirely deprived of fluid. It is likely too, if the repulfion is inverfely as fome higher power of the diftance than the fquare, that all parts of the body will be undercharged : if it is inverfely as the fquare, that all parts, except near the furface, will be faturated : and if it is inverfely as fome lefs power than the fquare, that all parts, except near the furface, will be overcharged.

P R O P. X.

Let the bodies A and D (fig. 5.) communicate with each other, by the canal EF ; and let one of them, as D, be overcharged ; the other body A will be fo alfo.

For as the fluid in the canal is repelled by the redundant fluid in D, it is plain, that unlefs A was overcharged, fo as to balance that repulfion, the fluid would run out of D into A.

In like manner, if one is undercharged, the other muft be fo too.

P R O P.

P R O P. XI.

Let the body A (fig. 6.) be either saturated or over or undercharged ; and let the fluid within it be in equilibrio. Let now the body B, placed near it, be rendered overcharged, the fluid within it being supposed immoveable, and disposed in such manner, that no part of it shall be undercharged ; the fluid in A will no longer be in equilibrio, but will be repelled from B : therefore, the fluid will flow from those parts of A which are nearest to B, to those which are more distant from it ; and, consequently, the part adjacent to MN (that part of the surface of A which is turned towards B) will be made to contain less electric fluid than it did before, and that adjacent to the opposite surface RS will contain more than before.

It must be observed, that when a sufficient quantity of fluid has flowed from MN towards RS, the repulsion which the fluid in the part adjacent to MN exerts on the rest of the fluid in A, will be so much weakened, and the repulsion of that in the part near RS will be so much increased, as to compensate the repulsion of B, which will prevent any more fluid flowing from MN to RS.

The reason why I suppose the fluid in B to be immoveable is, that otherwise a question might arise, whether the attraction or repulsion of the body A might not cause such an alteration in the disposition of the fluid in B, as to cause some parts of it to be

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undercharged; which might make it doubtful, whether B did on the whole repel the fluid in A. It is evident, however, that this proposition would hold good, though some parts of B were undercharged, provided it did on the whole repel the fluid in A.

C O R O L L A R Y.

If B had been made undercharged, instead of overcharged, it is plain that some fluid would have flowed from the further part RS to the nearer part MN, instead of from MN to RS.

P R O P. XII.

Let us now suppose that the body A communicates by the canal EF, with another body D, placed on the contrary side of it from B, as in fig. 5; and let these two bodies be either saturated, or over or undercharged; and let the fluid within them be in equilibrio. Let now the body B be overcharged: it is plain that some fluid will be driven from the nearer part MN to the further part RS, as in the former proposition; and also some fluid will be driven from RS, through the canal, to the body D; so that the quantity of fluid in D will be increased thereby, and the quantity in A, taking the whole body together, will be diminished; the quantity in the part near MN will also be diminished; but whether the quantity in the part near RS will be diminished or not, does not appear for certain; but I should imagine it would be not much altered.

C O R O L -

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COROLLARY.

In like manner, if B is made undercharged, some fluid will flow from D to A, and also from that part of A near RS, to the part near MN.

PROP. XIII.

Suppose now that the bodies A and D communicate by the bent canal $M P N n p m$ (fig. 7.) instead of the straight one EF: let the bodies be either saturated or over or undercharged as before; and let the fluid be at rest; then if the body B is made overcharged, some fluid will still run out of A into D; provided the repulsion of B on the fluid in the canal is not too great.

The repulsion of B on the fluid in the canal, will at first drive some fluid out of the leg $M P p m$ into A, and out of $N P p n$ into D, till the quantity of fluid in that part of the canal which is nearest to B is so much diminished, and its repulsion on the rest of the fluid in the canal is so much diminished also as to compensate the repulsion of B: but as the leg $N P p n$ is longer than the other, the repulsion of B on the fluid in it will be greater; consequently some fluid will run out of A into D, on the same principle that water is drawn out of a vessel through a syphon: but if the repulsion of B on the fluid in the canal is so great, as to drive all the fluid out of the space $G P H p G$, so that the fluid in the leg $M G p m$ does not

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join

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join to that in $NH \rho n$; then it is plain that no fluid can run out of A into D ; any more than water will run out of a vessel through a syphon, if the height of the bend of the syphon above the water in the vessel, is greater than that to which water will rise in vacuo.

COROLLARY.

If B is made undercharged, some fluid will run out of D into A ; and that though the attraction of B on the fluid in the canal is ever so great.

P R O P. XIV.

Let ABC (fig. 8.) be a body overcharged with immoveable fluid, uniformly spread; let the bodies near ABC on the outside be saturated with immoveable fluid; and let D be a body inclosed within ABC , and communicating by the canal DG with other distant bodies saturated with fluid; and let the fluid in D and the canal and those bodies be moveable; then will the body D be rendered undercharged.

For let us first suppose that D and the canal are saturated, and that D is nearer to B than to the opposite part of the body, C ; then will all the fluid in the canal be repelled from C by the redundant fluid in ABC ; but if D is nearer to C than to B , take the point F , such that a particle placed there would be repelled from C with as much force as one at D is repelled towards C ; the fluid in DF , taking the
whole

whole together, will be repelled with as much force one way as the other; and the fluid in FG is all of it repelled from C: therefore in both cases the fluid in the canal, taking the whole together, is repelled from C; consequently some fluid will run out of D and the canal, till the attraction of the unsaturated matter therein is sufficient to balance the repulsion of the redundant fluid in ABC.

P R O P. XV.

If we now suppose that the fluid on the outside of ABC is moveable; the matter adjacent to ABC on the outside, will become undercharged. I see no reason however to think that that will prevent the body D from being undercharged; but I cannot say exactly what effect it will have, except when ABC is spherical and the repulsion is inversely as the square of the distance; in this case it appears by Prob. I. that the fluid in the part DB of the canal will be repelled from C, with just as much force as in the last proposition; but the fluid in the part BG will not be repelled at all: consequently D will be undercharged, but not so much as in the last proposition.

C O R O L L A R Y.

If ABC is now supposed to be undercharged, it is certain that D will be overcharged, provided the matter near ABC on the outside is saturated with immoveable

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moveable fluid; and there is great reason to think that it will be so, though the fluid in that matter is moveable.

P R O P. XVI.

Let $A E F B$ (fig. 9.) be a long cylindric body, and D an undercharged body; and let the quantity of fluid in $A E F B$ be such, that the part near $E F$ shall be saturated. It appears from what has been said before, that the part near $A B$ will be overcharged; and moreover there will be a certain space, as $A a b B$, adjoining to the plane $A B$, in which the fluid will be pressed close together; and the fluid in that space will press against the plane $A B$, and will endeavour to escape from it; and by Prop. II. the two bodies will attract each other: now I say that the force with which the fluid presses against the plane $A B$, is very nearly the same with which the two bodies attract each other in the direction $E A$; provided that no part of $A E F B$ is undercharged.

Suppose so much of the fluid in each part of the cylinder as is sufficient to saturate the matter in that part, to become solid; the remainder, or the redundant fluid remaining fluid as before. In this case the pressure against the plane $A B$ must be exactly equal to that with which the two bodies attract each other, in the direction $E A$: for the force with which D attracts that part of the fluid which we supposed to become solid, is exactly equal to that, with which it
repels

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repels the matter in the cylinder; and the redundant fluid in $EabF$ is at liberty to move, if it had any tendency to do so, without moving the cylinder; so that the only thing which has any tendency to impel the cylinder in the direction EA is the pressure of the redundant fluid in $AabB$ against AB ; and as the part near EF is saturated, there is no redundant fluid to press against the plane EF , and thereby to counteract the pressure against AB . Suppose now all the electric fluid in the cylinder to become fluid; the force with which the two bodies attract each other will remain exactly the same; and the only alteration in the pressure against AB , will be, that that part of the fluid in $AabB$, which we at first supposed solid and unable to press against the plane, will now be at liberty to press against it; but as the density of the fluid when its particles are pressed close together may be supposed many times greater than when it is no denser than sufficient to saturate the matter in the cylinder, and consequently the quantity of redundant fluid in $AabB$ many times greater than that which is required to saturate the matter therein, it follows that the pressure against AB will be very little more than on the first supposition.

N. B. If any part of the cylinder is undercharged, the pressure against AB is greater than the force with which the bodies attract. If the electric repulsion is inversely as the square or some higher power of the distance, it seems very unlikely that any part of the cylinder should be undercharged; but if the repulsion is inversely as some lower power than the square, it

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is not improbable but some part of the cylinder may be undercharged.

LEMMA VII.

Let AB (fig. 10.) represent an infinitely thin flat circular plate, seen edgewise, so as to appear to the eye as a straight line; let C be the center of the circle; and let DC passing through C, be perpendicular to the plane of the plate; and let the plate be of uniform thickness, and consist of uniform matter, whose particles repel with a force inversely as the n power of the distance; n being greater than one, and less than three: the repulsion of the plate on a particle at D is proportional to $\frac{DC}{DC^{n-1}} - \frac{DC}{DA^{n-1}}$; provided the thickness of the plate and size of the particle D is given.

For if CA is supposed to flow, the corresponding fluxion of the quantity of matter in the plate, is proportional to $CA \times C\dot{A}$; and the corresponding fluxion of the repulsion of the plate on the particle D, in the direction DC, is proportional to $\frac{CA \times C\dot{A}}{DA^n} \times \frac{DC}{DA}$, = $\frac{D\dot{A} \times DC}{DA^n}$; for $D\dot{A}$ is to $C\dot{A} :: CA : DA$; the variable part of the fluent of which is $\frac{-DC}{n-1 \times DA^{n-1}}$; whence the repulsion of the plate on the particle D is proportional to $\frac{DC}{n-1 \times DC^{n-1}} - \frac{DC}{n-1 \times DA^{n-1}}$, or to $\frac{DC}{DC^{n-1}} - \frac{DC}{DA^{n-1}}$.

COROL-

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COROLLARY.

If DC^{n-1} is very small in respect of CA^{n-1} , the particle D is repelled with very nearly the same force as if the diameter of the plate was infinite.

LEMMA VIII.

Let L and l represent the two legs of a right angled triangle, and b the hypotenuse; if the shorter leg l is so much less than the other, that l^{n-1} is very small in respect of L^{n-1} , $b^{3-n} - L^{3-n}$ will be very small in respect of l^{3-n} .

For $b^{3-n} = \frac{L^2 + l^2}{2}^{\frac{3-n}{2}}$, $= L^{3-n} \times \frac{1 + \frac{l^2}{L^2}}{2}^{\frac{3-n}{2}}$, $= L^{3-n} \times 1 + \frac{3-n \times l^2}{2L^2} - \frac{3-n \times n-1 \times l^4}{8L^4}$, &c. therefore
 $b^{3-n} - L^{3-n} = \frac{3-n \times l^2}{2L^{n-1}} - \frac{3-n \times n-1 \times l^4}{8L^{n+1}}$, &c. $= \frac{l^{3-n} \times 3-n \times l^{n-1}}{2L^{n-1}} - \frac{l^{3-n} \times 3-n \times n-1 \times l^{n+1}}{8L^{n+1}}$, &c. which
 is very small in respect of l^{3-n} ; as l^{n-1} is by the supposition very small in respect of L^{n-1} .

LEMMA IX.

Let DC now represent the axis of a cylindric or prismatic column of uniform matter; and let the diameter of the column be so small, that the repulsion of the plate AB on it shall not be sensibly different from what it would be, if all the matter

in it was collected in the axis: the force with which the plate repells the column, is proportional to $DC^{3-n} + AC^{3-n} - DA^{3-n}$; supposing the thickness of the plate and base of the column to be given.

For, if DC is supposed to flow, the corresponding fluxion of the repulsion is proportional to $\frac{D\dot{C}}{DC^{n-2}} - \frac{DC \times D\dot{C}}{DA^{n-1}} = \frac{D\dot{C}}{DC^{n-2}} - \frac{D\dot{A}}{DA^{n-2}}$; the fluent of which, $\frac{AC^{3-n} + DC^{3-n} - DA^{3-n}}{3-n}$, vanishes when DC vanishes.

COROLL. I.

If the length of the column is so great that AC^{n-1} , is very small in respect of DC^{n-1} , the repulsion of the plate on it is very nearly the same as if the column was infinitely continued.

For by Lemma 8, $AC^{3-n} + DC^{3-n} - DA^{3-n}$ differs very little in this case from AC^{3-n} ; and if DC is infinite, it is exactly equal to it.

COROLL. II

If AC^{n-1} is very small in respect of DC^{n-1} ; and the point E be taken in DC such that EC^{n-1} , shall be very small in respect of AC^{n-1} , the repulsion of the plate on the small part of the column EC, is, to its repulsion on the whole column DC, very nearly as EC^{3-n} to AC^{3-n} .

LEMMA

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LEMMA X.

If we now suppose all the matter of the plate to be collected in the circumference of the circle, so as to form an infinitely slender uniform ring, its repulsion on the column DC will be less than when the matter is spread uniformly all over the plate, in the ratio of $\frac{3-n \times AC^2}{2} \times \frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}$ to $DC^{3-n} + AC^{3-n} - DA^{3-n}$.

For it was before said, that if the matter of the plate be spread uniformly, its repulsion on the column will be proportional to $DC^{3-n} + AC^{3-n} - DA^{3-n}$, or may be expressed thereby; let now AC, the semi-diameter of the plate, be increased by the infinitely small quantity AĈ; the quantity of matter in the plate will be increased by a quantity, which is to the whole, as $2 AĈ$ to AC; and the repulsion of the plate on the column, will be increased by $3-n \times AĈ \times AC^{2-n} - AĈ \times \frac{AC}{DA} \times 3-n \times DA^{2-n}$, $= 3-n \times AĈ \times AC \times \frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}$: therefore if a quantity of matter, which is to the whole quantity in the plate, as $2 AĈ$ to AC be collected in the circumference, its repulsion on the column DC, will be to that of the whole plate, as $3-n \times AĈ \times AC \times \frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}$, to $DC^{3-n} + AC^{3-n} - DA^{3-n}$; and consequently the repulsion of the plate when all the matter is collected in its circumference, is to its repulsion

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pulsion when the matter is spread uniformly, as $\frac{3-n \times AC^2}{2} \times \frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}$, to $DC^{3-n} + AC^{3-n} - DA^{3-n}$.

COROLL. I.

If the length of the column is so great, that AC^{n-1} is very small in respect of DC^{n-1} , the repulsion of the plate, when all the matter is collected in the circumference, is to its repulsion when the matter is spread uniformly, very nearly as $\frac{3-n \times AC^{3-n}}{2}$ to AC^{3-n} , or as $3-n$ to 2.

COROLL. II.

If EC^{n-1} is very small in respect of AC^{n-1} , the repulsion of the plate on the short column EC , when all the matter in the plate is collected in its circumference, is to its repulsion when the matter is spread uniformly, very nearly as $\frac{3-n \times n-1 \times EC^2}{4AC^{n-1}}$ to EC^{3-n} , or as $3-n \times n-1 \times EC^{n-1}$ to $4AC^{n-1}$; and is therefore very small in comparison of what it is when the matter is spread uniformly.

For by the same kind of process as was used in Lemma 8, it appears, that if EC^2 is very small in

respect of AC^2 , $AC^2 \times \frac{1}{AC^{n-1}} - \frac{1}{EA^{n-1}}$ differs very

little.

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little from $\frac{n-1 \times EC^2}{2EA^{n-1}}$, or from $\frac{n-1 \times EC^2}{2AC^{n-1}}$; and if EC^{n-1} is very small in respect of AC^{n-1} , EC^2 is *a fortiori* very small in respect of AC^2 .

COROLL. III.

Suppose now that the matter of the plate is denser near the circumference than near the middle, and that the density at and near the middle is to the mean density, or the density which it would everywhere be of if the matter was spread uniformly, as δ to one; the repulsion of the plate on EC will be less than if the matter was spread uniformly, in a ratio approaching much nearer to that of δ to one, than to that of equality.

COROLL. IV.

Let every thing be as in the last corollary, and let π be taken to one, as the force with which the plate actually repels the column DC (DC^{n-1} being very great in respect of AC^{n-1}) is to the force with which it would repel it, if the matter was spread uniformly; the repulsion of the plate on EC will be to its repulsion on DC, in a ratio between that of $EC^{3-n} \times \delta$ to $AC^{3-n} \times \pi$, and that of EC^{3-n} to $AC^{3-n} \times \pi$, but will approach much nearer to the former ratio than to the latter.

LEMMA

LEMMA XI.

In the line DC produced, take CF equal to CA: if all the matter of the plate AB is collected in the circumference, its repulsion on the column CD, infinitely continued, is equal to the repulsion of the same quantity of matter collected in the point F, on the same column.

For the repulsion of the plate on the column in the direction CD, is the same, whether the matter of it be collected in the whole circumference, or in the point A. Suppose it therefore to be collected in A; and let an equal quantity of matter be collected in F; take FG constantly equal to AD; and let AD and FG flow: the fluxion of CD is to the fluxion of FG, as AD to CD; and the repulsion of A on the point D, in the direction CD, is to the repulsion of F on G, as CD to AD; and therefore the fluxion of the repulsion of A on the column CD, in the direction CD, is equal to the fluxion of the repulsion of F on CG; and when AD equals AC, the repulsion of both A and F on their respective columns vanishes; and therefore the repulsion of A on the whole column CD equals that of F on CG; and when CD and CG are both infinitely extended, they may be looked upon as the same column.

P R O P. XVII.

Let two similar bodies, of different sizes, and consisting of different sorts of matter, be both overcharged,

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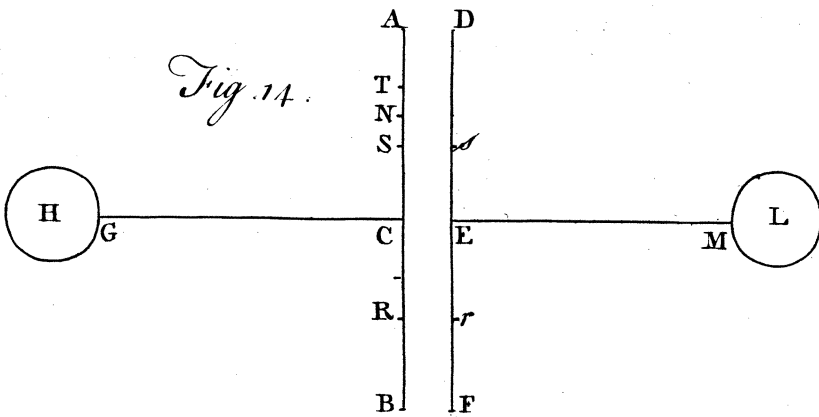
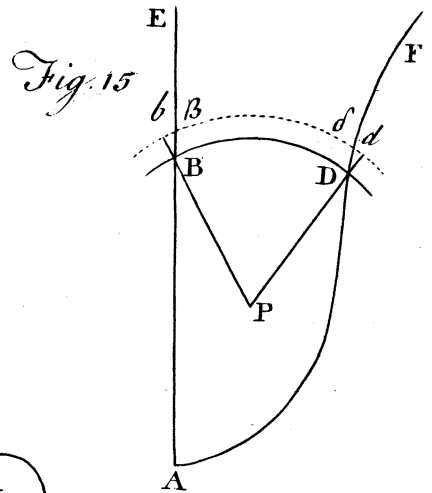
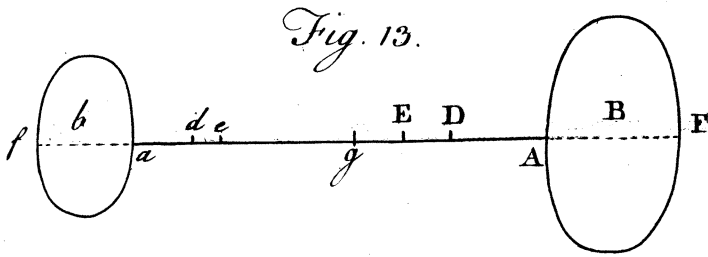
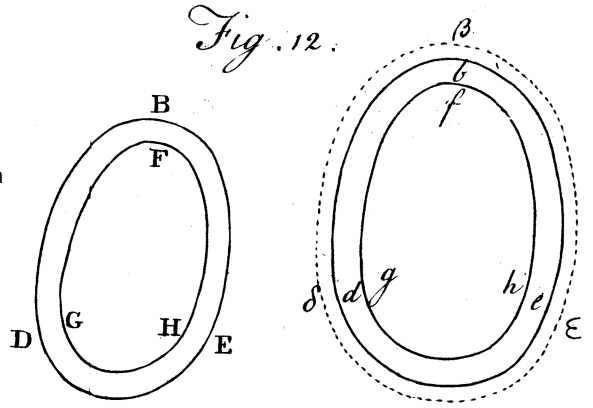
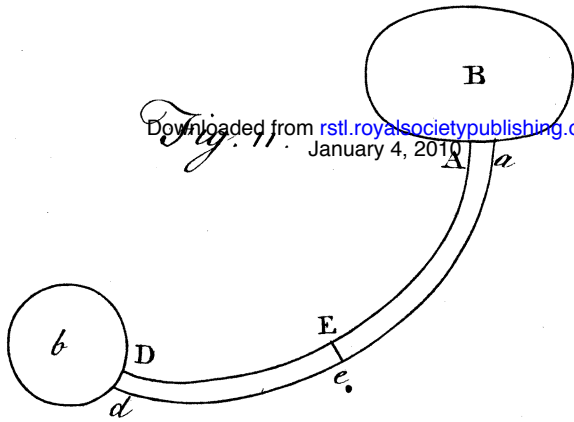
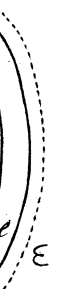
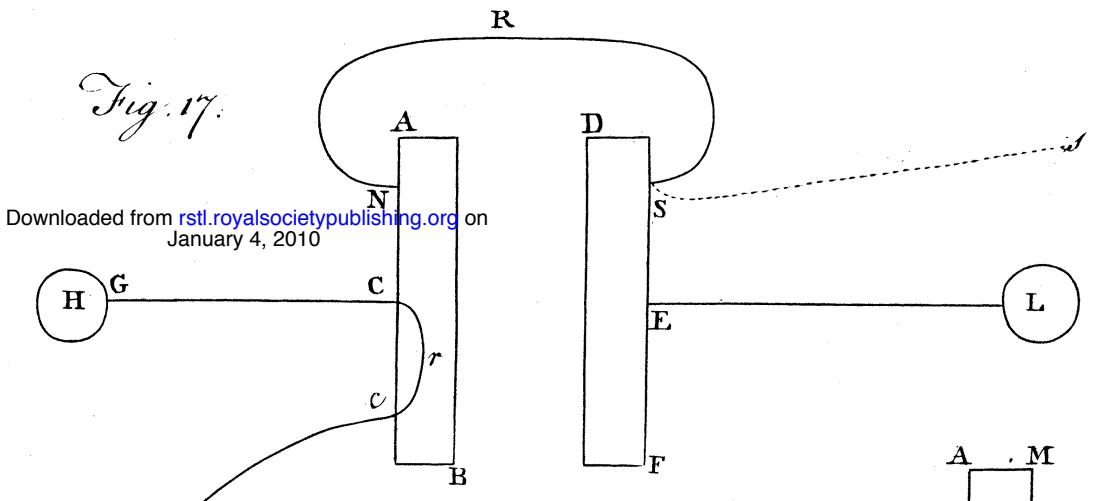
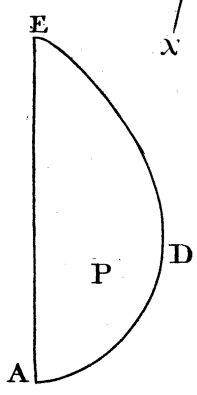


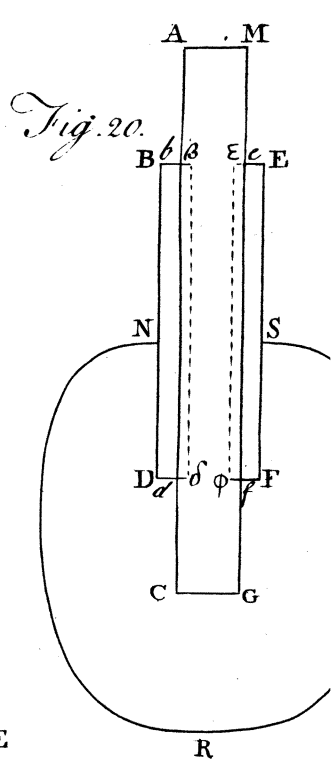
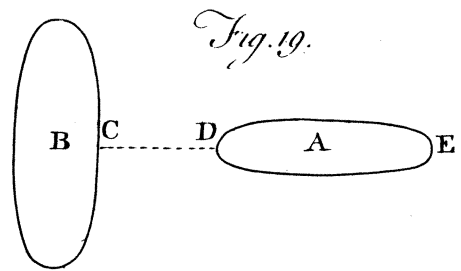
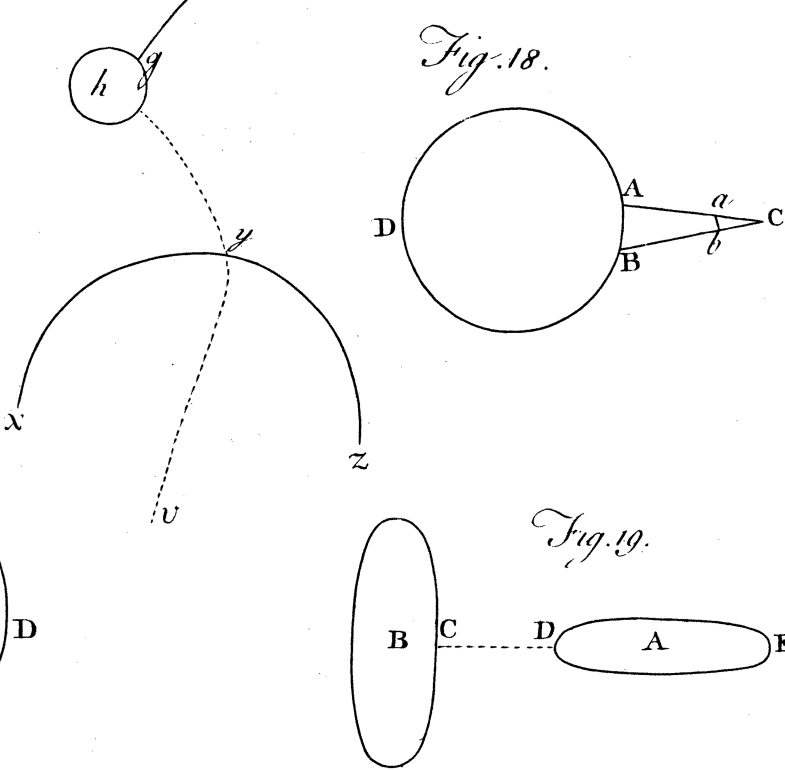
Fig. 16.



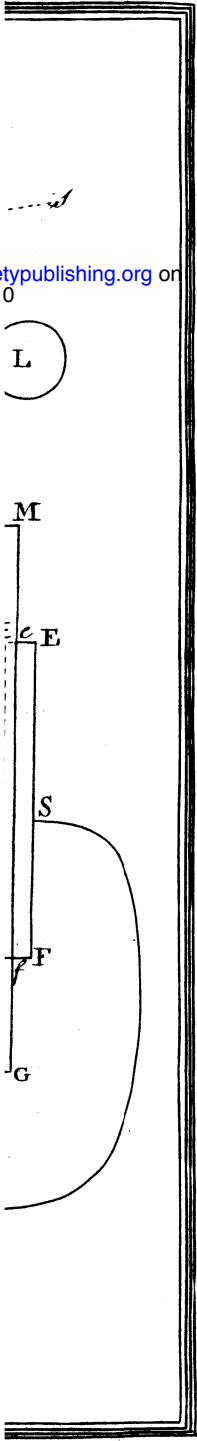
16.



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charged, or both undercharged, but in different degrees; and let the redundance or deficiency of fluid in each be very small in respect of the whole quantity of fluid in them: it is impossible for the fluid to be disposed accurately in a similar manner in both of them*; as it has been shewn that there will be a space, close to the surface, which will either be as full of fluid as it can hold, or will be intirely deprived of fluid; but it will be disposed as nearly in a similar manner in both, as is possible. To explain this, let BDE and *bde* (fig. 12) be the two similar bodies; and let the space comprehended between the surfaces BDE and FGH (or the space BF as I shall call it for shortness) be that part of BDE, which is either as full of fluid as it can hold, or intirely deprived of it: draw the surface *fgb*, such that the space *bf*, shall be to the space BF, as the quantity of redundant or deficient fluid in *bde*, to that in BDE, and that the thickness of the space *bf* shall every where bear the same proportion to the corresponding thickness of BF: then will the space *bf* be either as full of fluid as it can hold, or

* By the fluid being disposed in a similar manner in both bodies, I mean that the quantity of redundant or deficient fluid in any small part of one body, is to that in the corresponding small part of the other, as the whole quantity of redundant or deficient fluid in one body, to that in the other. By the quantity of deficient fluid in a body, I mean the quantity of fluid wanting to saturate it. Notwithstanding the impropriety of this expression, I must beg leave to make use of it, as it will frequently save a great deal of circumlocution.

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intirely deprived of it; and the fluid within the space fgb will be disposed very nearly similarly to that in the space FGH .

For it is plain, that if the fluid could be disposed accurately in a similar manner in both bodies, the fluid would be in equilibrio in one body, if it was in the other: therefore draw the surface $\beta\delta\epsilon$, such that the thickness of the space βf shall be every where to the corresponding thickness of BF , as the diameter of bde to the diameter of BDE ; and let the redundant fluid or matter in bf be spread uniformly over the space βf ; then if the fluid in the space fgb is disposed exactly similarly to that in FGH , it will be in equilibrio; as the fluid will then be disposed exactly similarly in the spaces $\beta\delta\epsilon$ and BDE : but as by the supposition, the thickness of the space βf is very small in respect of the diameter of bde , the fluid or matter in the space bf will exert very nearly the same force on the rest of the fluid, whether it is spread over the space βf , or whether it is collected in bf .

P R O P. XVIII.

Let two bodies, B and b , be connected to each other by a canal of any kind, and be either over or undercharged: it is plain that the quantity of redundant or deficient fluid in B , would bear exactly the same proportion to that in b , whatever sort of matter B consisted of, if it was possible for the redundant or deficient fluid in
I any

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any body, to be disposed accurately in the same manner, whatever sort of matter it consisted of. For suppose B to consist of any sort of matter; and let the fluid in the canal and two bodies be in equilibrio: let now B be made to consist of some other sort of matter, which requires a different quantity of fluid to saturate it; but let the quantity and disposition of the redundant or deficient fluid in it remain the same as before: it is plain that the fluid will still be in equilibrio; as the attraction or repulsion of any body depends only on the quantity and disposition of the redundant and deficient fluid in it. Therefore, by the preceding proposition, the quantity of redundant or deficient fluid in B , will actually bear very nearly the same proportion to that in b , whatever sort of matter B consists of; provided the quantity of redundant or deficient fluid in it is very small in respect of the whole.

P R O P. XIX.

Let two bodies B and b (fig. 11.) be connected together by a very slender canal $ADda$, either straight or crooked: let the canal be everywhere of the same breadth and thickness; so that all sections of this canal made by planes perpendicular to the direction of the canal in that part, shall be equal and similar: let the canal be composed of uniform matter; and let the electric fluid therein be supposed incompressible, and of such density as exactly to saturate the matter

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therein ; and let it, nevertheless, be able to move readily along the canal ; and let each particle of fluid in the canal be attracted and repelled by the matter and fluid in the canal and in the bodies B and *b*, just in the same manner that it would be if it was not incompressible * ; and let the bodies B and *b* be either over or undercharged. I say that the force with which the whole quantity of fluid in the canal is impelled from A towards D, in the direction of the axis of the canal, by the united attractions and repulsions of the two bodies, must be nothing ; as otherwise the fluid in the canal could not be at rest : observing that by the force with which the whole quantity of fluid is impelled in the direction of the axis of the canal, I mean the sum of the forces, with which the fluid in each part of the canal is impelled in the direction of the axis of the canal in that place, from A towards D ; and observing also, that an impulse in the contrary direction from D towards A must be looked upon as negative.

For as the canal is exactly saturated with fluid, the fluid therein is attracted or repelled only by the redundant matter or fluid in the two bodies. Suppose now that the fluid in any section of the canal, as E *e*,

* This supposition of the fluid in the canal being incompressible, is not mentioned as a thing which can ever take place in nature, but is merely imaginary ; the reason for making of which will be given hereafter.

other, or at so great an one, that the repulsion of either body on the fluid in the canal shall not be sensibly less than if they were at an infinite distance: then, if the electric attraction and repulsion is inversely as the n power of the distance, n being greater than one, and less than three, the quantity of redundant fluid in the two bodies will be to each other, as the $n - 1$ power of their corresponding diameters AF and af .

For if the quantity of redundant fluid in the two bodies is in this proportion, the repulsion of one body on the fluid in the canal, will be equal to that of the other body on it in the contrary direction; and, consequently, the fluid will have no tendency to flow from one body to the other, as may thus be proved. Take the points D and E very near to each other; and take da to DA , and ea to EA , as af to AF ; the repulsion of the body B on a particle at D , will be to the repulsion of b on a particle at d , as $\frac{1}{AF}$ to $\frac{1}{af}$; for, as the fluid is disposed similarly in both bodies, the quantity of fluid in any small part of B , is to the quantity in the corresponding part of b , as AF^{n-1} to af^{n-1} ; and, consequently, the repulsion of that small part of B , on D , is to the repulsion of the corresponding part of b , on d , as $\frac{AF^{n-1}}{AF^n}$, or $\frac{1}{AF}$, to $\frac{1}{af}$. But the quantity of fluid in the small part DE of the canal, is to that in de , as DE to de , or as AF to af ; therefore the repulsion of

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of B on the fluid in DE, is equal to that of b on the fluid in de : therefore, taking ag to Aa , as af to AF , the repulsion of b on the fluid in ag , is equal to that of B on the fluid in Aa ; but the repulsion of b on ag may be considered as the same as its repulsion on Aa ; for, by the supposition, the repulsion of B on Aa may be considered as the same as if it was continued infinitely; and therefore, the repulsion of b on ag may be considered as the same as if it was continued infinitely.

N. B. If n was not greater than one, it would be impossible for the length of Aa to be so great, that the repulsion of B on it might be considered as the same as if it was continued infinitely; which was my reason for requiring n to be greater than one.

C O R O L L A R Y .

By just the same method of reasoning it appears, that if the bodies are undercharged, the quantity of deficient fluid in b will be to that in B, as af^{n-1} to AF^{n-1} .

P R O P . XXI.

Let a thin flat plate be connected to any other body, as in the preceding proposition, by a canal of incompressible fluid, perpendicular to the plane of the plate; and let that body be overcharged, the quantity of redundant fluid in the plate will bear very nearly the same
3 proportion

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proportion to that in the other body, whatever the thickness of the plate may be, provided its thickness is very small in proportion to its breadth, or smallest diameter.

For there can be no doubt, but what, under that restriction, the fluid will be disposed very nearly in the same manner in the plate, whatever its thickness may be; and therefore its repulsion on the fluid in the canal will be very nearly the same, whatever its thickness may be.

P R O P. XXII.

Let AB and DF (fig. 14.) represent two equal and parallel circular plates, whose centers are C and E ; let the plates be placed so, that a right line joining their centers shall be perpendicular to the plates; let the thickness of the plates be very small, in respect of their distance CE ; let the plate AB communicate with the body H , and the plate DF with the body L , by the canals CG and EM of incompressible fluid, such as are described in Prop. XIX; let these canals meet their respective plates in their centers C and E , and be perpendicular to the plane of the plates; and let their length be so great, that the repulsion of the plates on the fluid in them may be considered as the same, as if they were continued infinitely; let the body H be overcharged, and let L be saturated. It is plain, from Prop. XII. that DF will be undercharged, and AB will be more overcharged

charged than it would otherwise be. Suppose, now, that the redundant fluid in AB is disposed in the same manner as the deficient fluid is in DF ; let P be to one as the force with which the plate AB would repel the fluid in CE , if the canal ME was continued to C , is to the force with which it would repel the fluid in CM ; and let the force with which AB repels the fluid in CG , be to the force with which it would repel it, if the redundant fluid in it was spread uniformly, as π to 1; and let the force with which the body H repels the fluid in CG , be the same with which a quantity of redundant fluid, which we will call B , spread uniformly over AB , would repel it in the contrary direction. Then will the redundant fluid in AB be equal to $\frac{B}{2P\pi - P^2\pi}$, and therefore, if P is very small, will be very nearly equal to $\frac{B}{2P\pi}$; and the deficient fluid in DF will be to the redundant fluid in AB , as $1 - P$ to one, and therefore, if P is very small, will be very nearly equal to the redundant fluid in AB .

For it is plain, that the force with which AB repels the fluid in EM , must be equal to that with which DF attracts it; for otherwise, some fluid would run out of DF into L , or out of L into DF : for the same reason, the excess of the repulsion of AB on the fluid in CG , above the attraction of FD thereon, must be equal to the force with which a
quantity

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quantity of redundant fluid equal to B , spread uniformly over AB , would repel it, or it must be equal to that with which a quantity equal to $\frac{B}{\pi}$, spread in the manner in which the redundant fluid is actually spread in AB , would repel it. By the supposition, the force with which AB repels the fluid in EM , is to the force with which it would repel the fluid in CM , supposing EM to be continued to C , as $1 - P$ to one; but the force with which any quantity of fluid in AB would repel the fluid in CM , is the same with which an equal quantity similarly disposed in DF , would repel the fluid in EM ; therefore, the force with which the redundant fluid in AB repels the fluid in EM , is to that with which an equal quantity similarly disposed in DF , would repel it, as $1 - P$ to one: therefore, if the redundant fluid in AB be called A , the deficient fluid in DF must be $A \times 1 - P$: for the same reason, the force with which DF attracts the fluid in CG , is to that with which AB repels it, as $A \times 1 - P \times 1 - P$, or $A \times \overline{1 - P^2}$, to A ; therefore, the excess of the force with which AB repels CG above that with which DF attracts it, is equal to that with which a quantity of redundant fluid equal to $A - A \times \overline{1 - P^2}$, or $A \times 2P - P^2$, spread over AB , in the manner in which the redundant fluid therein is actually spread, would repel it: therefore, $A \times 2P - P^2$ must be equal to $\frac{B}{\pi}$, or A must be equal to $\frac{B}{2P\pi - P^2\pi}$.

C O R O L.

COROL. I.

If the density of the redundant fluid near the middle of the plate AB , is less than the mean density, or the density which it would everywhere be of, if it was spread uniformly, in the ratio of δ to one; and if the distance of the two plates is so small, that EC^{n-1} is very small in respect of AC^{n-1} , and that EC^{3-n} is very small in respect of AC^{3-n} , the quantity of redundant fluid in AB will be greater than $\frac{B}{2} \times \frac{\overline{AC}}{EC}^{3-n}$, and less than $\frac{B}{2\delta} \times \frac{\overline{AC}}{EC}^{3-n}$, but will approach much nearer to the latter value than the former. For, in this case, $P\pi$ is, by Lemma X. Corol. IV. less than $\frac{\overline{EC}}{AC}^{3-n}$, and greater than $\frac{\overline{EC}}{AC}^{3-n} \times \delta$, but approaches much nearer to the latter value than the former; and if EC^{3-n} is very small in respect of AC^{3-n} , P is very small.

REMARKS.

If DF was not undercharged, it is certain that AB would be considerably more overcharged near the circumference of the circle than near the center; for if the fluid was spread uniformly, a particle placed anywhere at a distance from the center, as at N , would be repelled with considerably more force towards the circumference than it would towards the

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center. If the plates are very near together, and, consequently, DF nearly as much undercharged as AB is overcharged, AB will still be more overcharged near the circumference than near the center, but the difference will not be near so great as in the former case: for, let NR be many times greater than CE, and NS less than CE; and take Er and Es equal to CR and CS, there can be no doubt, I think, but that the deficient fluid in DF will be lodged nearly in the same manner as the redundant fluid in AB; and therefore, the repulsion of the redundant fluid at R, on a particle at N, will be very nearly balanced by the attraction of the redundant matter at r, for R is not much nearer to N than r is; but the repulsion of S will not be near balanced by that of s; for the distance of S from N is much less than that of s. Let now a small circle, whose diameter is ST, be drawn round the center N, on the plane of the plate; as the density of the fluid is greater at T than at S, the repulsion of the redundant fluid within the small circle tends to impel the point N towards C; but as there is a much greater quantity of fluid between N and B, than between N and A, the repulsion of the fluid without the small circle tends to balance that; but the effect of the fluid within the small circle is not much less than it would be, if DF was not undercharged; whereas much the greater part of the effect of that part of the plate on the outside of the circle, is taken off by the effect of the corresponding part of DF: consequently, the difference of density between T and S will not be near so great, as if DF was not undercharged. Hence I should imagine, that if the two plates are very

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very near together, the density of the redundant fluid near the center will not be much less than the mean density, or δ will not be much less than one; moreover, the less the distance of the plates, the nearer will δ approach to one.

C O R O L. II.

Let now the body H consist of a circular plate, of the same size as A B, placed so, that the canal C G shall pass through its center, and be perpendicular to its plane; by the supposition, the force with which H repels the fluid in the canal C G, is the same with which a quantity of fluid, equal to B, spread uniformly over A B, would repel it in the contrary direction: therefore, if the fluid in the plate H was spread uniformly, the quantity of redundant fluid therein would be B, and if it was all collected in the circumference, would be $\frac{2B}{3-n}$; and therefore the real quantity will be greater than B, and less than $\frac{2B}{3-n}$.

C O R O L. III.

Therefore, if we suppose δ to be equal to one, the quantity of redundant fluid in A B will exceed that in the plate H, in a greater ratio than that of $\left(\frac{AC}{CE}\right)^{3-n} \times \frac{3-n}{4}$ to one, and less than that of $\left(\frac{AC}{CE}\right)^{3-n} \times \frac{1}{2}$ to one; and from the preceding remarks it appears, that the real quantity of redundant fluid in A B can hardly

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hardly be much greater than it would if δ was equal to one.

COROL. IV.

Hence, if the electric attraction and repulsion is inversely as the square of the distance, the redundant fluid in AB , supposing δ to be equal to one, will exceed that in the plate H , in a greater ratio than that of AC to $4CE$, and less than that of AC to $2CE$.

COROL. V.

Let now the body H consist of a globe, whose diameter equals AB ; the globe being situated in such a manner, that the canal CG , if continued, would pass through its center; and let the electric attraction and repulsion be inversely as the square of the distance, the quantity of redundant fluid in the globe will be $2B$: for the fluid will be spread uniformly over the surface of the globe, and its repulsion on the canal will be the same as if it was all collected in the center of the sphere, and will therefore be the same with which an equal quantity, disposed in the circumference of AB , would repel it in the contrary direction, or with which half that quantity, or B , would repel it, if spread uniformly over the plate.

COROL.

COROL. VI.

Therefore, if δ was equal to one, the redundant fluid in AB would exceed that in the globe, in the ratio of AC to $4CE$; and therefore, it will in reality exceed that in the globe, in a rather greater ratio than that of AC to $4CE$; but if the plates are very near together, it will approach very near thereto, and the nearer the plates are, the nearer it will approach thereto.

COROL. VII.

Whether the electric repulsion is inverfely as the square of the distance or not, if the body H is as much undercharged, as it was before overcharged, AB will be as much undercharged as it was before overcharged, and DF as much overcharged as it was before undercharged.

COROL. VIII.

If the size and distance of the plates be altered, the quantity of redundant or deficient fluid in the body H remaining the same, it appears, by comparing this proposition with the 20th and 21st propositions, that the quantity of redundant and deficient fluid in AB will be as $AC^{n-1} \times \left[\frac{AC}{EC} \right]^{3-n}$, or as $\frac{AC^2}{EC^{3-n}}$, supposing the value of δ to remain the same.

PROP.

P R O P. XXIII.

Let $A E$ (fig. 15.) be a cylindric canal, infinitely continued beyond E ; and let $A F$ be a bent canal, meeting the other at A , and infinitely continued beyond F : let the section of this canal, in all parts of it, be equal to that of the cylindric canal, and let both canals be filled with uniform fluid of the same density: the force with which a particle of fluid P , placed anywhere at pleasure, repels the whole quantity of fluid in $A F$, in the direction of the canal, is the same with which it repels the fluid in the canal $A E$, in the direction $A E$.

On the center P , draw two circular arches $B D$ and $b d$, infinitely near to each other, cutting $A E$ in B and β , and $A F$ in D and δ , and draw the radii $P b$ and $P d$. As $P B = P D$, the force with which P repels a particle at B , in the direction $B \beta$, is to that with which it repels an equal particle at D , in the direction $D \delta$, as $\frac{B b}{B \beta}$ to $\frac{D d}{D \delta}$, or as $\frac{1}{B \beta}$ to $\frac{1}{D \delta}$; and therefore, the force with which it repels the whole fluid in $B \beta$, in the direction $B \beta$, is the same with which it repels the whole fluid in $D \delta$, in the direction $D \delta$, that is in the direction of the canal; and therefore, the force with which it repels the whole fluid in $A E$, in the direction $A E$, is the same with which it repels the whole fluid in $A F$, in the direction of the canal.

C O R O L L A R Y.

If the bent canal ADF, instead of being infinitely continued, meets the cylindric canal in E, as in fig. 16. the repulsion of P on the fluid in the bent canal ADE, in the direction of the canal, will still be equal to its repulsion on that in the cylindric canal AE, in the direction AE.

P R O P. XXIV.

If two bodies, for instance the plate AB, and the body H, of Prop. XXII. communicate with each other, by a canal filled with incompressible fluid, and are either over or undercharged, the quantity of redundant fluid in them will bear the same proportion to each other, whether the canal by which they communicate is straight or crooked, or into whatever part of the bodies the canal is inserted, or in whatever manner the two bodies are situated in respect of each other; provided that their distance is infinite, or so great that the repulsion of each body on the fluid in the canal shall not be sensibly less than if it was infinite.

Let the parallelograms AB and DF (fig. 17.) represent the two plates, and H and L the bodies communicating with them: let now H be removed to *b*; and let it communicate with AB, by the bent canal *gc*; the quantity of fluid in the plates and
bodies

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bodies remaining the same as before ; and let us, for the sake of ease in the demonstration, suppose the canal gc to be every where of the same thickness as the canal GC ; though the proposition will evidently hold good equally, whether it is or not : the fluid will still be in equilibrio. For let us first suppose the canal gc to be continued through the substance of the plate AB , to C , along the line crC ; the part crC being of the same thickness as the rest of the canal, and the fluid in it of the same density : by the preceding proposition, the repulsion or attraction of each particle of fluid or matter in the plates AB and DF , on the fluid in the whole canal $Crcg$, in the direction of that canal, is equal to its repulsion or attraction on the fluid in the canal CG , in the direction CG ; and therefore the whole repulsion or attraction of the two plates on the canal $Crcg$, is equal to their repulsion or attraction on CG : but as the fluid in the plate AB is in equilibrio, each particle of fluid in the part Crc of the canal, is impelled by the plates, with as much force in one direction as the other ; and consequently the plates impel the fluid in the canal cg , with as much force as they do that in the whole canal $Crcg$, that is, with the same force that they impel the fluid in CG . In like manner the body b impels the fluid in cg , with the same force that H does the fluid in CG ; and consequently b impels the fluid in cg , one way in the direction of the canal, with the same force that the two plates impel it the contrary way ; and therefore the fluid in cg has no tendency to flow from one body to the other.

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COROLLARY.

By the same method of reasoning, with the help of the corollary to the 23d proposition, it appears, that if AB and H each communicate with a third body, by canals of incompressible fluid, and a communication is made between AB and H by another canal of incompressible fluid, the fluid will have no tendency to flow from one to the other through this canal; supposing that the fluid was in equilibrio before this communication was made. In like manner if AB and H communicate with each other, or each communicate with a third body, by canals of real fluid, instead of the imaginary canals of incompressible fluid used in these propositions, and a communication is also made between them by a canal of incompressible fluid, the fluid can have no tendency to flow from one to the other. The truth of the latter part of this corollary will appear by supposing an imaginary canal of incompressible fluid to be continued through the whole length of the real one.

P R O P. XXV.

Let now a communication be made between the two plates AB and DF, by the canal NRS of incompressible fluid, of any length; and let the body H and the plate AB be overcharged. It is plain that the fluid will flow through that canal from AB to DF. Now the whole force with which the fluid in the canal is impelled

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along it, by the joint action of the two plates, is the same with which the whole quantity of fluid in the canal CG or cg is impelled by them; supposing the canal $NR S$ to be every where of the same breadth and thickness as CG or cg .

For suppose that the canal $NR S$, instead of communicating with the plate DF , is bent back just before it touches it, and continued infinitely along the line Ss ; the force with which the two plates impel the fluid in Ss , is the same with which they impel that in EL , supposing Ss to be of the same breadth and thickness as EL ; and is therefore nothing; therefore the force with which they impel the fluid in $NR S$, is the same with which they impel that in $NR Ss$; which is the same with which they impel that in CG .

P R O P. XXVI.

Let now xyz be a body of an infinite size, containing just fluid enough to saturate it; and let a communication be made between b and xyz , by the canal by of incompressible fluid, of the same breadth and thickness as gc or GC ; the fluid will flow through it from b to xyz ; and the force with which the fluid in that canal is impelled along it, is equal to that with which the fluid in $NR S$ is impelled by the two plates.

If

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If the canal by is of so great a length, that the repulsion of b thereon is the same as if it was continued infinitely, then the thing is evident: but if it is not, let the canal by , instead of communicating with xyz , so that the fluid can flow out of the canal into xyz , be continued infinitely through its substance, along the line yv : now it must be observed that a small part of the body xyz , namely, that which is turned towards b , will by the action of b upon it, be rendered undercharged; but all the rest of the body will be saturated; for the fluid driven out of the undercharged part will not make the remainder, which is supposed to be of an infinite size, sensibly overcharged: now the force with which the fluid in the infinite canal byv , is impelled by the body b and the undercharged part of xyz , is the same with which the fluid in gc is impelled by them; but as the fluid in all parts of xyz is in equilibrio, a particle in any part of yv cannot be impelled in any direction; and therefore the fluid in by is impelled with as much force as that in byv ; and therefore the fluid in by is impelled with as much force as that in gc ; and is therefore impelled with as much force as the fluid in $NR S$ is impelled by the two plates.

It perhaps may be asked, whether this method of demonstration would not equally tend to prove that the fluid in by was impelled with the same force as that in $NR S$, though xyz did not contain just fluid enough to saturate it. I answer not; for this demonstration depends on the canal yv being continued, within the body xyz , to an infinite distance beyond any over or undercharged part; which could

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not be if xyz contained either more or less fluid than that.

PROP. XXVII.

Let two bodies B and b (fig. 13.) be joined by a cylindric or prismatic canal Aa , filled with real fluid; and not by an imaginary canal of incompressible fluid as in the 20th proposition; and let the fluid therein be in equilibrio: the force with which the whole or any given part of the fluid in the canal, is impelled in the direction of its axis, by the united repulsions and attractions of the redundant fluid or matter in the two bodies and the canal, must be nothing; or the force with which it is impelled one way in the direction of the axis of the canal, must be equal to that with which it is impelled the other way.

For as the canal is supposed cylindric or prismatic, no particle of fluid therein can be prevented from moving in the direction of the axis of it, by the sides of the canal; and therefore the force with which each particle is impelled either way in the direction of the axis, by the united attractions and repulsions of the two bodies and the canal, must be nothing, otherwise it could not be at rest; and therefore the force with which the whole, or any given part of the fluid in the canal, is impelled in the direction of the axis, must be nothing.

COROL. I.

If the fluid in the canal is disposed in such manner, that the repulsion or attraction of the redundant fluid or matter in it, on the whole or any given part of the fluid in the canal, has no tendency to impel it either way in the direction of the axis; then the force with which that whole or given part is impelled by the two bodies must be nothing; or the force with which it is impelled one way in the direction of the axis, by the body B, must be equal to that with which it is impelled in the contrary direction by the other body; but not if the fluid in the canal is disposed in a different manner.

COROL. II.

If the bodies, and consequently the canal, is overcharged; then, in whatever manner the fluid in the canal is disposed, the force with which the whole quantity of redundant fluid in the canal is repelled by the body B in the direction A *a*, must be equal to that with which it is repelled by *b* in the contrary direction. For the force with which the redundant fluid is impelled in the direction A *a* by its own repulsion, is nothing; for the repulsion of the particles of any body on each other have no tendency to make the whole body move in any direction.

R E M A R K S.

When I first thought of the 20th and 22d propositions, I imagined that when two bodies were connected by a cylindric canal of real fluid, the repulsion of one body on the whole quantity of fluid in the canal, in one direction, would be equal to that of the other body on it in the contrary direction, in whatever manner the fluid was disposed in the canal; and that therefore those propositions would have held good very nearly, though the bodies were joined by cylindric canals of real fluid; provided the bodies were so little over or undercharged, that the quantity of redundant or deficient fluid in the canal should be very small in respect of the quantity required to saturate it; and consequently that the fluid therein should be very nearly of the same density in all parts. But from the foregoing proposition it appears that I was mistaken, and that the repulsion of one body on the fluid in the canal is not equal to that of the other body on it, unless the fluid in the canal is disposed in a particular manner: besides that, when two bodies are both joined by a real canal, the attraction or repulsion of the redundant matter or fluid in the canal, has some tendency to alter the disposition of the fluid in the two bodies; and in the 22d proposition, the canal CG exerts also some attraction or repulsion on the canal EM : on all which accounts the demonstration of those propositions is defective, when the bodies are joined by real canals. I have good reason however to think, that those propositions actually hold good very nearly when the bodies
are

are joined by real canals; and that, whether the canals are straight or crooked, or in whatever direction the bodies are situated in respect of each other: though I am by no means able to prove that they do: I therefore chose still to retain those propositions, but to demonstrate them on this ideal supposition, in which they are certainly true, in hopes that some more skilful mathematician may be able to shew whether they really hold good or not.

What principally makes me think that this is the case, is that as far as I can judge from some experiments I have made, the quantity of fluid in different bodies agrees very well with those propositions, on a supposition that the electric repulsion is inversely as the square of the distance. It should also seem from those experiments, that the quantity of redundant or deficient fluid in two bodies, bore very nearly the same proportion to each other, whatever is the shape of the canal by which they are joined, or in whatever direction they are situated in respect of each other.

Though the above propositions should be found not to hold good, when the bodies are joined by real canals, still it is evident, that in the 22d proposition, if the plates AB and DF are very near together, the quantity of redundant fluid in the plate AB will be many times greater than that in the body H, supposing H to consist of a circular plate of the same size as AB, and DF will be near as much undercharged as AB is overcharged.

Sir Isaac Newton supposes that air consists of particles which repel each other with a force invertely as the distance: but it appears plainly from the foregoing pages, that if the repulsion of the particles was

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in this ratio; and extended indefinitely to all distances, they would compose a fluid extremely different from common air. If the repulsion of the particles was inversely as the distance, but extended only to a given very small distance from their centers, they would compose a fluid of the same kind as air, in respect of elasticity, except that its density would not be in proportion to its compression: if the distance to which the repulsion extends, though very small, is yet many times greater than the distance of the particles from each other, it might be shewn, that the density of the fluid would be nearly as the square root of the compression. If the repulsion of the particles extended indefinitely, and was inversely as some higher power of the distance than the cube, the density of the fluid would be as some power of the compression less than $\frac{3}{2}$. The only law of repulsion, I can think of, which will agree with experiment, is one which seems not very likely; namely, that the particles repel each other with a force inversely as the distance; but that, whether the density of the fluid is great or small, the repulsion extends only to the nearest particles: or, what comes to the same thing, that the distance to which the repulsion extends, is very small, and also is not fixed, but varies in proportion to the distance of the particles.

P A R T

P A R T II.

*Containing a comparison of the foregoing theory with
experiment.*

§ 1. It appears from experiment, that some bodies suffer the electric fluid to pass with great readiness between their pores; while others will not suffer it to do so without great difficulty; and some hardly suffer it to do so at all. The first sort of bodies are called conductors, the others non-conductors. What this difference in bodies is owing to I do not pretend to explain.

It is evident that the electric fluid in non-conductors may be considered as moveable, or answers to the definition given of that term in p. 588. As to the fluid contained in non-conducting substances, though it does not absolutely answer to the definition of immoveable, as it is not absolutely confined from moving, but only does so with great difficulty; yet it may in most cases be looked upon as such without sensible error.

Air does in some measure permit the electric fluid to pass through it; though, if it is dry, it lets it pass but very slowly, and not without difficulty; it is therefore to be called a non-conductor.

It appears that conductors would readily suffer the fluid to run in and out of them, were it not for the air which surrounds them: for if the end of a conductor is inserted into a vacuum, the fluid runs in and out of it with perfect readiness; but

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when it is furrounded on all sides by the air, as no fluid can run out of it without running into the air, the fluid will not do so without difficulty.

If any body is furrounded on all sides by the air, or other non-conducting substances, it is said to be insulated: if on the other hand it any where communicates with any conducting body, it is said to be not insulated. When I say that a body communicates with the ground, or any other body, I would be understood to mean that it does so by some conducting substance.

Though the terms positively and negatively electrified are much used, yet the precise sense in which they are to be understood, seems not well ascertained; namely, whether they are to be understood in the same sense in which I have used the words over or undercharged, or whether, when any number of bodies, insulated and communicating with each other by conducting substances, are electrified by means of excited glass, they are all to be called positively electrified (supposing, according to the usual opinion, that excited glass contains more than its natural quantity of electricity); even though some of them, by the approach of a stronger electrified body, are made undercharged. I shall use the words in the latter sense; but as it will be proper to ascertain the sense in which I shall use them more accurately, I shall give the following definition.

In order to judge whether any body, as A, is positively or negatively electrified: suppose another body B, of a given shape and size, to be placed at an infinite distance from it, and from any other
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over or undercharged body ; and let B contain the same quantity of electric fluid, as if it communicated with A by a canal of incompressible fluid : then, if B is overcharged, I call A positively electrified ; and if it is undercharged, I call A negatively electrified ; and the greater the degree in which B is over or undercharged, the greater is the degree in which A is positively or negatively electrified.

It appears from the corollary to the 24th proposition, that if several bodies are insulated, and connected together by conducting substances, and one of these bodies is positively or negatively electrified, all the other bodies must be electrified in the same degree : for supposing a given body B to be placed at an infinite distance from any over or undercharged body, and to contain the same quantity of fluid as if it communicated with one of those bodies by a canal of incompressible fluid, all the rest of those bodies must by that corollary contain the same quantity of fluid as if they communicated with B by canals of incompressible fluid : but yet it is possible that some of those bodies may be overcharged, and others undercharged : for suppose the bodies to be positively electrified, and let an overcharged body D be brought near one of them, that body will become undercharged, provided D is sufficiently overcharged ; and yet by the definition it will still be positively electrified in the same degree as before.

Moreover, if several bodies are insulated and connected together by conducting substances, and one of these bodies is electrified by excited glass, there can be no doubt, I think, but what they

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will all be positively electrified; for if there is no other over or undercharged body placed near any of these bodies, the thing is evident; and though some of these bodies may, by the approach of a sufficiently overcharged body, be rendered undercharged; yet I do not see how it is possible to prevent a body placed at an infinite distance, and communicating with them by a canal of incompressible fluid, from being overcharged.

In like manner if one of these bodies is electrified by excited sealing wax, they will all be negatively electrified.

It is impossible for any body communicating with the ground to be either positively or negatively electrified: for the earth, taking the whole together, contains just fluid enough to saturate it, and consists in general of conducting substances; and consequently though it is possible for small parts of the surface of the earth to be rendered over or undercharged, by the approach of electrified clouds or other causes; yet the bulk of the earth, and especially the interior parts, must be saturated with electricity. Therefore assume any part of the earth which is itself saturated, and is at a great distance from any over or undercharged part; any body communicating with the ground, contains as much electricity as if it communicated with this part by a canal of incompressible fluid, and therefore is not at all electrified.

If any body A, insulated and saturated with electricity, is placed at a great distance from any over or undercharged body, it is plain that it cannot be electrified; but if an overcharged body is brought

brought near it, it will be positively electrified; for supposing A to communicate with any body B, at an infinite distance, by a canal of incompressible fluid, it is plain that unless B is overcharged, the fluid in the canal could not be in equilibrio, but would run from A to B. For the same reason a body insulated and saturated with fluid, will be negatively electrified if placed near an undercharged body.

§ 2. The phenomena of the attraction and repulsion of electrified bodies seem to agree exactly with the theory; as will appear by considering the following cases.

CASE I. Let two bodies, A and B, both conductors of electricity, and both placed at a great distance from any other electrified bodies, be brought near each other. Let A be insulated, and contain just fluid enough to saturate it; and let B be positively electrified. They will attract each other; for as B is positively electrified, and at a great distance from any overcharged body, it will be overcharged; therefore, on approaching A and B to each other, some fluid will be driven from that part of A which is nearest to B to the further part: but when the fluid in A was spread uniformly, the repulsion of B on the fluid in A was equal to its attraction on the matter therein; therefore, when some fluid is removed from those parts where the repulsion of B is strongest to those where it is weaker, B will repel the fluid in A with less force than it attracts the matter; and consequently the bodies will attract each other.

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CASE II. If we now suppose that the fluid is at liberty to escape from out of A, if it has any disposition to do so, the quantity of fluid in it before the approach of B being still sufficient to saturate it; that is, if A is not insulated and not electrified, B being still positively electrified, they will attract with more force than before: for in this case, not only some fluid will be driven from that part of A which is nearest to B to the opposite part, but also some fluid will be driven out of A.

It must be observed, that if the repulsion of B on a particle at E, (fig. 19.) the farthest part of A, is very small in respect of its repulsion on an equal particle placed at D, the nearest part of A, the two bodies will attract with very nearly the same force, whether A is insulated or not; but if the repulsion of B, on a particle at E, is very near as great as on one at D, they will attract with very little force if A is insulated. For instance, let a small overcharged ball be brought near one end of a long conductor not electrified; they will attract with very near the same force, whether the conductor be insulated or not; but if the conductor be overcharged, and brought near a small un-electrified ball, they will not attract with near so much force, if the ball is insulated, as if it is not.

CASE III. If we now suppose that A is negatively electrified, and not insulated, it is plain that they will attract with more force than in the last case;

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case; as A will be still more undercharged in this case, than in the last.

N. B. In these three cases, we have not as yet taken notice of the effect which the body A will have in altering the quantity and disposition of the fluid in B; but in reality this will make the bodies attract each other with more force than they would otherwise do; for in each of these cases the body A attracts the fluid in B; which will cause some fluid to flow from the farther parts of B to the nearer, and will also cause some fluid to flow into it, if it is not insulated, and will consequently cause B to act upon A with more force than it would otherwise do.

CASE IV. Let us now suppose that B is negatively electrified; and let A be insulated, and contain just fluid enough to saturate it; they will attract each other; for B will be undercharged; it will therefore attract the fluid in A, and will cause some fluid to flow from the farthest part of A, where it is attracted with less force, to the nearer part, where it is attracted with more force; so that B will attract the fluid in A with more force than it repels the matter.

CASE V, and VI. If A is now supposed to be not insulated and not electrified, B being still negatively electrified, it is plain that they will attract with more force than in the last case: and if A is positively electrified, they will attract with still more force.

In these three last cases also, the effect which A has in altering the quantity and disposition of the fluid

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fluid in B, tends to increase the force with which the two bodies attract.

CASE VII. It is plain that a non-conducting body saturated with fluid, is not at all attracted or repelled by an over or undercharged body, until, by the action of the electrified body on it, it has either acquired some additional fluid from the air, or had some driven out of it, or till some fluid is driven from one part of the body to the other.

CASE VIII. Let us now suppose that the two bodies A and B are both positively electrified in the same degree. It is plain, that were it not for the action of one body on the other, they would both be overcharged, and would repel each other. But it may perhaps be said, that one of them as A may, by the action of the other on it, be either rendered undercharged on the whole, or at least may be rendered undercharged in that part nearest to B; and that the attraction of this undercharged part on a particle of the fluid in B, may be greater than the repulsion of the more distant overcharged part; so that on the whole the body A may attract a particle of fluid in B. If so, it must be affirmed that the body B repels the fluid in A; for otherwise, that part of A which is nearest to B could not be rendered undercharged. Therefore, to obviate this objection, let the bodies be joined by the straight canal DC of incompressible fluid (fig. 19.). The body B will repel the fluid in all parts of this canal; for as A is supposed to attract the fluid in B, B will not only be more overcharged than it would otherwise be, but it will also be more
over-

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overcharged in that part nearest to A than in the opposite part. Moreover, as the near undercharged part of A is supposed to attract a particle of fluid in B with more force than the more distant overcharged part repels it; it must, *a fortiori*, attract a particle in the canal with more force than the other repels it; therefore the body A must attract the fluid in the canal; and consequently some fluid must flow from B to A, which is impossible; for as A and B are both electrified in the same degree, they contain the same quantity of fluid as if they both communicated with a third body at an infinite distance, by canals of incompressible fluid; and therefore, by the corollary to Prop. 24, if a communication is made between them by a canal of incompressible fluid, the fluid would have no disposition to flow from one to the other.

CASE IX. But if one of the bodies as A is positively electrified in a less degree than B, then it is possible for the bodies to attract each other; for in this case the force with which B repels the fluid in A may be so great, as to make the body A either intirely undercharged, or at least to make the nearest part of it so much undercharged, that A shall on the whole attract a particle of fluid in B.

It may be worth remarking with regard to this case, that when two bodies, both electrified positively but unequally, attract each other, you may by removing them to a greater distance from each other, cause them to repel; for as the stronger electrified body repels the fluid in the weaker with less force when removed to a greater distance, it will not be

able to drive so much fluid out of it, or from the nearer to the further part, as when placed at a less distance.

CASE X, and XI. By the same reasoning it appears, that if the two bodies are both negatively electrified in the same degree, they must repel each other: but if they are both negatively electrified in different degrees, it is possible for them to attract each other.

All these cases are exactly conformable to experiment.

CASE XII. Let two cork balls be suspended by conducting threads from the same positively electrified body, in such manner that if they did not repel, they would hang close together: they will both be equally electrified, and will repel each other: let now an overcharged body, more strongly electrified than them, be brought under them; they will become less overcharged, and will separate less than before: on bringing the body still nearer, they will become not at all overcharged, and will not separate at all: and on bringing the body still nearer, they will become undercharged, and will separate again.

CASE XIII. Let all the air of a room be overcharged, and let two cork balls be suspended close to each other by conducting threads communicating with the wall. By Prop. 15, it is highly probable that the balls will be undercharged; and therefore they should repel each other.

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These two last cases are experiments of Mr. Canton's, and are described in *Philos. Transf.* 1753, p. 350, where are other experiments of the same kind, all readily explicable by the foregoing theory.

I have now considered all the principal or fundamental cases of electric attractions and repulsions which I can think of; all of which appear to agree perfectly with the theory.

§ 3. On the cases in which bodies receive electricity from or part with it to the air.

L E M M A I.

Let the body A (fig. 6.) either stand near some over or undercharged body, or at a distance from any. It seems highly probable, that if any part of its surface, as MN, is overcharged, the fluid will endeavour to run out through that part, provided the air adjacent thereto is not overcharged.

For let G be any point in that surface, and P a point within the body, extremely near to it; it is plain that a particle of fluid at P, must be repelled with as much force in one direction as another (otherwise it could not be at rest) unless all the fluid between P and G is pressed close together, in which case it may be repelled with more force towards G than it is in the contrary direction: now a particle at G is repelled in the direction PG, *i. e.* from P to G, by all the redundant fluid between P and G; and a particle at P is repelled by the same fluid in the contrary direction; so that as the particle at P is repelled with not less force in the direction PG than in the

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contrary, I do not see how a particle at *G* can help being repelled with more force in that direction than the contrary, unless the air on the outside of the surface *M N* was more overcharged than the space between *P* and *G*.

In like manner, if any part of the surface is undercharged, the fluid will have a tendency to run in at that part from the air.

The truth of this is somewhat confirmed by the third problem; as in all the cases of that problem, the fluid was shewn to have a tendency to run out of the spaces *A D* and *E H*, at any surface which was overcharged, and to run in at any which was undercharged.

C O R O L. I.

If any body at a distance from other over or undercharged bodies, be positively electrified, the fluid will gradually run out of it from all parts of its surface into the adjoining air; as it is plain that all parts of the surface of that body will be overcharged: and if the body is negatively electrified, the fluid will gradually run into it at all parts of its surface from the adjoining air.

C O R O L. II.

Let the body *A* (fig. 6.) insulated and containing just fluid enough to saturate it, be brought near the overcharged body *B*; that part of the surface of *A* which is turned towards *B* will by Prop. 11. be rendered

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dered undercharged, and will therefore imbibe electricity from the air; and at the opposite surface R S, the fluid will run out of the body into the air.

C O R O L. III.

If we now suppose that A is not insulated, but communicates with the ground, and consequently that it contained just fluid enough to saturate it before the approach of B, it is plain that the surface M N will be more undercharged than before; and therefore the fluid will run in there with more force than before; but it can hardly have any disposition to run out at the opposite surface R S; for if the canal by which A communicates with the ground is placed opposite to B, as in figure 5, then the fluid will run out through that canal till it has no longer any tendency to run out at R S; and by the remarks at the end of Prop. 27, it seems probable, that the fluid in A will be nearly in the same quantity, and disposed nearly in the same manner, into whatever part of A the canal is inserted by which it communicates with the ground.

C O R O L. IV.

If B is undercharged the case will be reversed; that is, it will run out where it before run in, and will run in where it before run out.

As far as I can judge, these corollaries seem conformable to experiment: thus far is certain, that bodies at a distance from other electrified bodies receive

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ceive electricity from the air, if negatively electrified, and part with some to it if positively electrified: and a body not electrified and not insulated receives electricity from the air if brought near an overcharged body, and loses some when brought near an undercharged body: and a body insulated and containing its natural quantity of fluid, in some cases, receives, and in others loses electricity, when brought near an over or undercharged body.

§ 4. The well-known effects of points in causing a quick discharge of electricity seem to agree very well with this theory.

It appears from the 20th proposition, that if two similar bodies of different sizes are placed at a very great distance from each other, and connected by a slender canal, and overcharged, the force with which a particle of fluid placed close to corresponding parts of their surface is repelled from them, is inversely as the corresponding diameters of the bodies. If the distance of the two bodies is small, there is not so much difference in the force with which the particle is repelled by the two bodies; but still, if the diameters of the two bodies are very different, the particle will be repelled with much more force from the smaller body than from the larger. It is true indeed that a particle placed at a certain distance from the smaller body, will be repelled with less force than if it be placed at the same distance from the greater body; but this distance is, I believe, in most cases pretty considerable; if the bodies are spherical, and the repulsion inversely as the square of the distance, a particle placed at any distance from the surface of the
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smaller body less than a mean proportional between the radii of the two bodies, will be repelled from it with more force than if it be placed at the same distance from the larger body.

I think therefore that we may be well assured that if two similar bodies are connected together by a slender canal, and are overcharged, the fluid must escape faster from the smaller body than from an equal surface of the larger; but as the surface of the larger body is greatest, I do not know which body ought to lose most electricity in the same time; and indeed it seems impossible to determine positively from this theory which should, as it depends in great measure on the manner in which the air opposes the entrance of the electric fluid into it. Perhaps in some degrees of electrification the smaller body may lose most, and in others the larger.

Let now ACB (fig. 18.) be a conical point standing on any body DAB , C being the vertex of the cone; and let DAB be overcharged: I imagine that a particle of fluid placed close to the surface of the cone anywhere between b and C , must be repelled with at least as much, if not more, force than it would, if the part $AabB$ of the cone was taken away, and the part aCb connected to DAB by a slender canal; and consequently, from what has been said before, it seems reasonable to suppose that the waste of electricity from the end of the cone must be very great in proportion to its surface; though it does not appear from this reasoning whether the waste of electricity from the whole cone should be greater or less than from a cylinder of the same base and altitude.

All which has been here said relating to the flowing out of electricity from overcharged bodies, hold^s equally true with regard to the flowing in of electricity into undercharged bodies.

But a circumstance which I believe contributes as much as any thing to the quick discharge of electricity from points, is the swift current of air caused by them, and taken notice of by Mr. Wilson and Dr. Priestly (*vide* Priestly, p. 117 and 591); and which is produced in this manner.

If a globular body ABD is overcharged, the air close to it, all round its surface, is rendered overcharged, by the electric fluid, which flows into it from the body; it will therefore be repelled by the body; but as the air all round the body is repelled with the same force, it is in equilibrio, and has no tendency to fly off from it. If now the conical point ACB be made to stand out from the globe, as the fluid will escape much faster in proportion to the surface from the end of the point than from the rest of the body, the air close to it will be much more overcharged than that close to the rest of the body; it will therefore be repelled with much more force; and consequently a current of air will flow along the sides of the cone, from B towards C ; by which means there is a continual supply of fresh air, not much overcharged, brought in contact with the point; whereas otherwise the air adjoining to it would be so much overcharged, that the electricity would have but little disposition to flow from the point into it.

The same current of air is produced in a less degree, without the help of the point, if the body, instead of being globular, is oblong or flat, or has
knobs

knobs on it, or is otherwise formed in such manner as to make the electricity escape faster from some parts of it than the rest.

In like manner, if the body *ABD* be undercharged, the air adjoining to it will also be undercharged, and will therefore be repelled by it; but as the air close to the end of the point will be more undercharged than that close to the rest of the body, it will be repelled with much more force; which will cause exactly the same current of air, flowing the same way, as if the body was overcharged; and consequently the velocity with which the electric fluid flows into the body, will be very much increased. I believe indeed that it may be laid down as a constant rule, that the faster the electric fluid escapes from any body when overcharged, the faster will it run into that body when undercharged.

Points are not the only bodies which cause a quick discharge of electricity; in particular, it escapes very fast from the ends of long slender cylinders; and a swift current of air is caused to flow from the middle of the cylinder towards the end: this will easily appear by considering that the redundant fluid is collected in much greater quantity near the ends of the cylinders than near the middle. The same thing may be said, but I believe in a less degree, of the edges of thin plates.

What has been just said concerning the current of air, serves to explain the reason of the revolving motion of Dr. Hamilton's and Mr. Kinnersley's bent pointed wires, vide *Phil. Trans.* vol. *LI*, p. 905, and vol. *LIII*, p. 86; also Priestly, p. 429: for the same repulsion which impels the air from the thick part of the

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wire towards the point, tends to impel the wire in the contrary direction.

It is well known, that if a body B is positively electrified, and another body A, communicating with the ground, be then brought near it, the electric fluid will escape faster from B, at that part of it which is turned towards A, than before. This is plainly conformable to theory; for as A is thereby rendered undercharged, B will in its turn be made more overcharged, in that part of it which is turned towards A, than it was before. But it is also well known that the fluid will escape faster from B, if A be pointed, than if it be blunt; though B will be less overcharged in this case than in the other; for the broader the surface of A, which is turned towards B, the more effect will it have in increasing the overcharge of B. The cause of this phenomenon is as follows:

If A is pointed, and the pointed end turned towards B, the air close to the point will be very much undercharged, and therefore will be strongly repelled by A, and attracted by B, which will cause a swift current of air to flow from it towards B; by which means a constant supply of undercharged air will be brought in contact with B, which will accelerate the discharge of electricity from it in a very great degree: and moreover, the more pointed A is, the swifter will be this current. If, on the other hand, that end of A which is turned towards B, is so blunt, that the electricity is not disposed to run into A faster than it is to run out of B, the air adjoining to B may be as much overcharged as that adjoining to A is undercharged; and therefore may by the joint repulsion
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of B and attraction of A, be impelled from B to A, with as much or more force than the air adjoining to A is impelled in the contrary direction; so that what little current of air there is may flow in the contrary direction.

It is easy applying what has been here said to the case in which B is negatively electrified.

§ 5. In the paper of Mr. Canton's, quoted in the second section, and in a paper of Dr. Franklin's (Phil. Trans. 1755, p. 300, and Franklin's letters p. 155.) are some remarkable experiments, shewing that when an overcharged body is brought near another body, some fluid is driven to the further end of this body, and also some driven out of it, if it is not insulated. The experiments are all strictly conformable to the 11th, 12th, and 13th propositions: but it is needless to point out the agreement, as the explanation given by the authors does it sufficiently.

§ 6. On the Leyden vial.

The shock produced by the Leyden vial seems owing only to the great quantity of redundant fluid collected on its positive side, and the great deficiency on its negative side; so that if a conductor was prepared of so great a size, as to be able to receive as much additional fluid by the same degree of electrification as the positive side of a Leyden vial, and was positively electrified in the same degree as the vial, I do not doubt but what as great a shock would be produced by making a communication between this conductor and the ground, as between the two surfaces of the

Leyden vial, supposing both communications to be made by canals of the same length and same kind.

It appears plainly from the experiments which have been made on this subject, that the electric fluid is not able to pass through the glass; but yet it seems as if it was able to penetrate without much difficulty to a certain small depth, perhaps I might say an imperceptible depth within the glass; as Dr. Franklin's analysis of the Leyden vial shews that its electricity is contained chiefly in the glass itself, and that the coating is not greatly over or undercharged.

It is well known that glass is not the only substance which can be charged in the manner of the Leyden vial; but that the same effect may be produced by any other body, which will not suffer the electricity to pass through it.

* Hence the phenomena of the vial seem easily explicable by means of the 22d proposition. For let $ACGM$, fig. 20, represent a flat plate of glass or any other substance which will not suffer the electric fluid to pass through it, seen edgewise; and let $BbdD$, and $EefF$, or Bd and Ef , as I shall call them for shortness, be two plates of conducting matter of the same size, placed in contact with the glass opposite to each other; and let Bd be positively electrified; and let Ef communicate with the ground; and let the fluid be supposed either

* The following explication is strictly applicable only to that sort of Leyden vial, which consists of a flat plate of glass or other matter. It is evident, however, that the result must be nearly of the same kind, though the glass is made into the shape of a bottle as usual, or into any other form: but I propose to consider those sort of Leyden vials more particularly in a future paper.

able to enter a little way into the glass, but not to pass through it, or unable to enter it at all; and if it is able to enter a little way into it, let $b\beta\delta d$, or $b\delta$, as I shall call it, represent that part of the glass into which the fluid can enter from the plate Bd , and $e\phi$, that which the fluid from $E\phi$ can enter. By the abovementioned proposition, if be , the thickness of the glass, is very small in respect of bd , the diameter of the plates, the quantity of redundant fluid forced into the space Bd , or $B\delta$, (that is, into the plate Bd , if the fluid is unable to penetrate at all into the glass, or into the plate Bd , and the space $b\delta$ together, if the fluid is able to penetrate into the glass) will be many times greater than what would be forced into it by the same degree of electrification if it had been placed by itself; and the quantity of fluid driven out of $E\phi$, will be nearly equal to the redundant fluid in $B\delta$.

If a communication be now made between $B\delta$ and $E\phi$, by the canal NRS , the redundant fluid will run from $B\delta$ to $E\phi$; and if in its way it passes through the body of any animal, it will by the rapidity of its motion produce in it that sensation called a shock.

It appears from the 26th proposition, that if a body of any size was electrified in the same degree as the plate Bd , and a communication was made between that body and the ground, by a canal of the same length, breadth and thickness as NRS ; that then the fluid in that canal would be impelled with the same force as that in NRS , supposing the fluid in both canals to be incompressible; and consequently, as the quantity of fluid to be moved,
and

and the resistance to its motion is the same in both canals, the fluid should move with the same rapidity in both: and I see no reason to think that the case will be different, if the communication is made by canals of real fluid.

Therefore what was said in the beginning of this section, namely, that as great a shock would be produced by making a communication between the conductor and the ground, as between the two sides of the Leyden vial, by canals of the same length and same kind, seems a necessary consequence of this theory; as the quantity of fluid which passes through the canal is, by the supposition, the same in both; and there is the greatest reason to think, that the rapidity with which it passes will be nearly if not quite the same in both. I hope soon to be able to say whether this agrees with experiment as well as theory.

It may be worth observing, that the longer the canal NRS is, by which the communication is made, the less will be the rapidity with which the fluid moves along it; for the longer the canal is, the greater is the resistance to the motion of the fluid in it; whereas the force with which the whole quantity of fluid in it is impelled, is the same whatever be the length of the canal. Accordingly, it is found in melting small wires, by directing a shock through them, that the longer the wire the greater charge it requires to melt it.

As the fluid in $B\delta$, is attracted with great force by the redundant matter in $E\phi$, it is plain that if the fluid is able to penetrate at all into the glass, great part of the redundant fluid will be lodged in $b\delta$,

$b\delta$, and in like manner there will be a great deficiency of fluid in $e\phi$. But in order to form some estimate of the proportion of the redundant fluid, which will be lodged in $b\delta$, let the communication between Ef and the ground be taken away, as well as that by which Bd is electrified; and let so much fluid be taken from $B\delta$, as to make the redundant fluid therein equal to the deficient fluid in $E\phi$. If we suppose that all the redundant fluid is collected in $b\delta$, and all the deficient in $e\phi$, so as to leave Bd and Ef saturated; then, if the electric repulsion is inversely as the square of the distance, a particle of fluid placed anywhere in the plane bd , except near the extremities b and d , will be attracted with very near as much force by the redundant matter in $e\phi$, as it is repelled by the redundant fluid in $b\delta$; but if the repulsion is inversely, as some higher power than the square, it will be repelled with much more force by $b\delta$, than it is attracted by $e\phi$, provided the depth $b\beta$ is very small in respect of the thickness of the glass; and if the repulsion is inversely, as some lower power than the square, it will be attracted with much more force by $e\phi$, than it is repelled by $b\delta$. Hence it follows, that if the depth to which the fluid can penetrate is very small in respect of the thickness of the glass, but yet is such that the quantity of fluid naturally contained in $b\delta$, or $e\phi$, is considerably more than the redundant fluid in $B\delta$; then, if the repulsion is inversely as the square of the distance, almost all the redundant fluid will be collected in $b\delta$, leaving the plate Bd not very much overcharged; and in like manner Ef will be

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not very much undercharged: if the repulsion is inverfely as some higher power than the fquare, Bd will be very much overcharged, and Ef very much undercharged: and if the repulsion is inverfely, as some lower power than the fquare, Bd will be very much undercharged, and Ef very much overcharged.

Suppofe, now, the plate Bd to be feparated from the plate of glafs, ftill keeping it parallel thereto, and oppofite to the fame part of it that it before was applied to; and let the repulfion of the particles be inverfely, as some higher power of the diftance than the fquare. When the plate is in contact with the glafs, the repulfion of the redundant fluid in that plate, on a particle in the plane bd , *id eft*, the inner furface of the plate, muft be equal to the excefs of the repulfion of the redundant fluid in $b\delta$ on it, above the attraction of $E\phi$ on it; therefore, when the plate Bd is removed ever fo fmall a diftance from the glafs, the repulfion of the redundant fluid in the plate, on a particle in the inner furface of that plate, will be greater than the excefs of the repulfion of $b\delta$ on it, above the attraction of $E\phi$; for the repulfion of $b\delta$ will be much more diminifhed by the removal, than the attraction of $E\phi$: confequently, fome fluid will fly from the plate to the glafs, in the form of fparks: fo that the plate will not be fo much overcharged when removed from the glafs, as it was when in contact with it. I fhould imagine, however, that it would ftill be confiderably overcharged.

If one part of the plate is feparated from the glafs before the reft, as muft neceffarily be the cafe, if it confifts of bending materials, I fhould guefs it would
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be at least as much, if not more, overcharged, when separated, as if it is separated all at once.

In like manner, it should seem that the plate *Ef* will be considerably undercharged, when separated from the glass, but not so much so as when in contact with it.

From the same kind of reasoning I conclude, that if the repulsion is inversely, as some lower power of the distance than the square, the plate *Bd* will be considerably undercharged, and *Ef* considerably overcharged, when separated from the glass, but not in so great a degree as when they are in contact with it.

§ 7. There is an experiment of Mr. Wilke and Æpinus, related by Dr. Priestly, p. 258. called by them, electrifying a plate of air: it consisted in placing two large boards of wood, covered with tin plates, parallel to each other, and at some inches asunder. If a communication was made between one of these and the ground, and the other was positively electrified, the former was undercharged; the boards strongly attracted each other; and, on making a communication between them, a shock was felt like that of the Leyden vial.

I am uncertain whether in this experiment the air contained between the two boards is very much overcharged on one side, and very much undercharged on the other, as is the case with the plate of glass in the Leyden vial; or whether the case is, that the redundant or deficient fluid is lodged only in the two boards, and that the air between them serves only to prevent the electricity from running from one

board to the other : but whichever of these is the case, the experiment is equally conformable to the theory.

It must be observed, that a particle of fluid placed between the two plates is drawn towards the under-charged plate, with a force exceeding that with which it would be repelled from the overcharged plate, if it was electrified with the same force, the other plate being taken away, nearly in the ratio of twice the quantity of redundant fluid actually contained in the plate, to that which it would contain, if electrified with the same force by itself ; so that, unless the plate is very weakly electrified, or their distance is very considerable, the fluid will be apt to fly from one to the other, in the form of sparks.

§ 8. Whenever any conducting body as A, communicating with the ground, is brought sufficiently near an overcharged body B, the electric fluid is apt to fly through the air from B to A, in the form of a spark : the way by which this is brought about seems to be this. The fluid placed anywhere between the two bodies, is repelled from B towards A, and will consequently move slowly through the air from one to the other : now it seems as if this motion increased the elasticity of the air, and made it rarer : this will enable the fluid to flow in a swifter current, which will still further increase the elasticity of the air, till at last it is so much rarified, as to form very little opposition to the motion of the electric fluid, upon which it flies in an uninterrupted mass from one body to the other.

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In the same manner may the electric fluid pass from one body to another, in the form of a spark, if the first body communicates with the ground, and the other body is negatively electrified, or in any other case in which one body is strongly disposed to part with its electricity to the air, and the other is strongly disposed to receive it.

In like manner, when the electric fluid is made to pass through water, in the form of a spark, as in Signor Beccaria's * and Mr. Lane's † experiments, I imagine that the water, by the rapid motion of the electric fluid through it, is turned into an elastic fluid, and so much rarified as to make very little opposition to its motion : and when stones are burst or thrown out from buildings struck by lightning, in all probability that effect is caused by the moisture in the stone, or some of the stone itself, being turned into an elastic fluid.

It appears plainly, from the sudden rising of the water in Mr. Kinnerley's electrical air thermometer ‡, that when the electric fluid passes through the air, in the form of a spark, the air in its passage is either very much rarified, or intirely displaced : and the bursting of the glass vessels, in Beccaria's and Lane's experiments, shews that the same thing happens with regard to the water, when the electric fluid passes through it in the form of a spark. Now, I see no means by which the displacing of the air or

* *Elettricismo artificiale e naturale*, p. 110. Priestly, p. 209.

† *Phil. Trans.* 1767, p. 451.

‡ *Phil. Trans.* 1763, p. 84. Priestly, p. 216.

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water can be brought about, but by supposing its elasticity to be increased, by the motion of the electric fluid through it, unless you suppose it to be actually pushed aside, by the force with which the electric fluid endeavours to issue from the overcharged body: but I can by no means think, that the force with which the fluid endeavours to issue, in the ordinary cases in which electric sparks are produced, is sufficient to overcome the pressure of the atmosphere, much less that it is sufficient to burst the glass vessels in Beccaria's and Lane's experiments.

The truth of this is confirmed by Prop. XVI. For, let an undercharged body be brought near to, and opposite to the end of a long cylindrical body communicating with the ground, by that proposition the pressure of the electric fluid against the base of the cylinder is scarcely greater than the force with which the two bodies attract each other, provided that no part of the cylinder is undercharged; which is very unlikely to be the case, if the electric repulsion is inversely as the square of the distance, as I have great reason to believe it is; and, consequently, if the spark was produced, by the air being pushed aside by the force with which the fluid endeavours to issue from the cylinder, no sparks should be produced, unless the electricity was so strong, that the force with which the bodies attracted each other was as great as the pressure of the atmosphere against the base of the cylinder: whereas it is well known, that a spark may be produced, when the force, with which the bodies attract, is very trifling in respect of that.

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One may frequently observe, in discharging a Leyden vial, that if the two knobs are approached together very slowly, a hissing noise will be perceived before the spark ; which shews, that the fluid begins to flow from one knob to the other, before it passes in the form of a spark ; and therefore serves to confirm the truth of the opinion, that the spark is brought about in the gradual manner here described.

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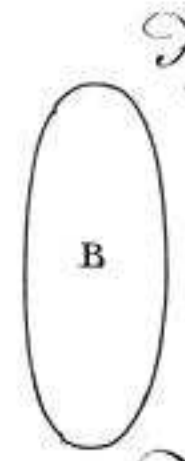
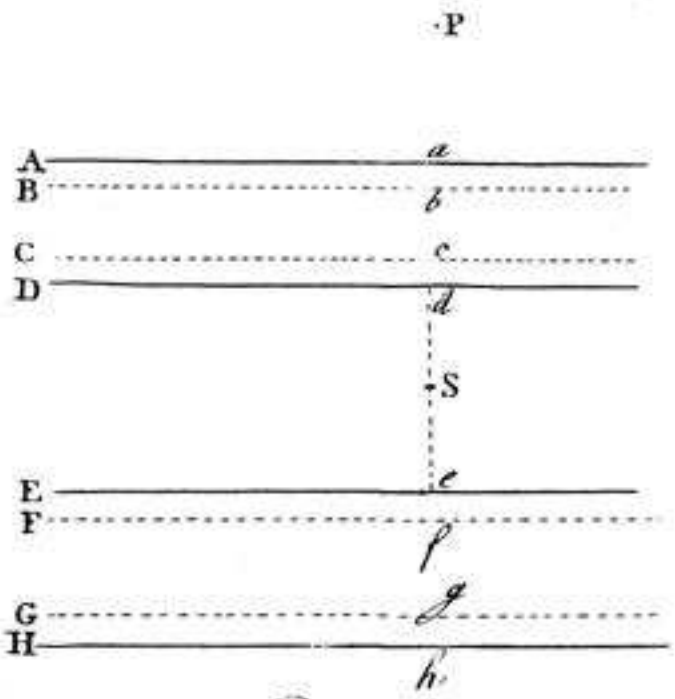
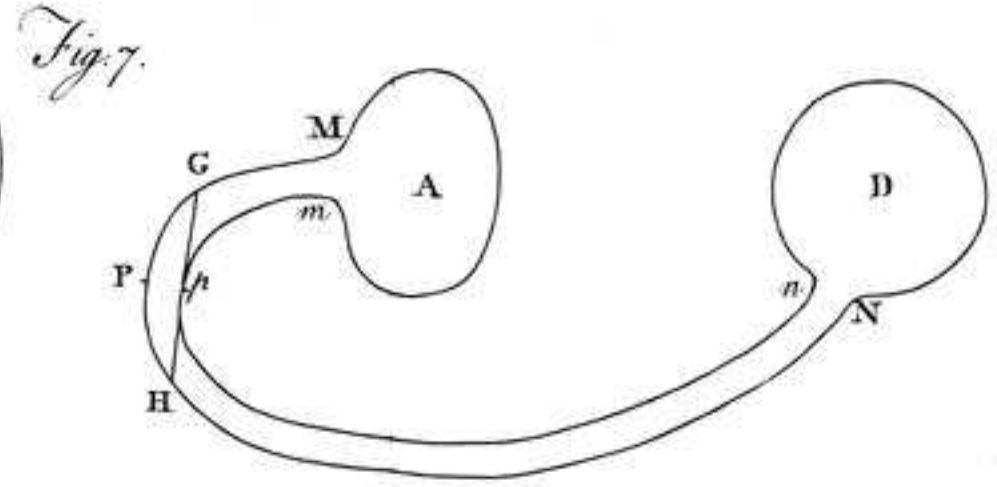
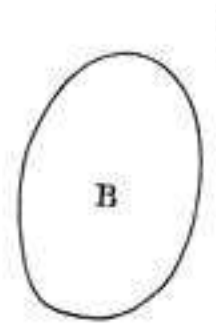
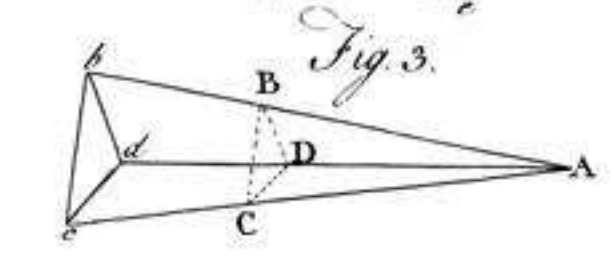
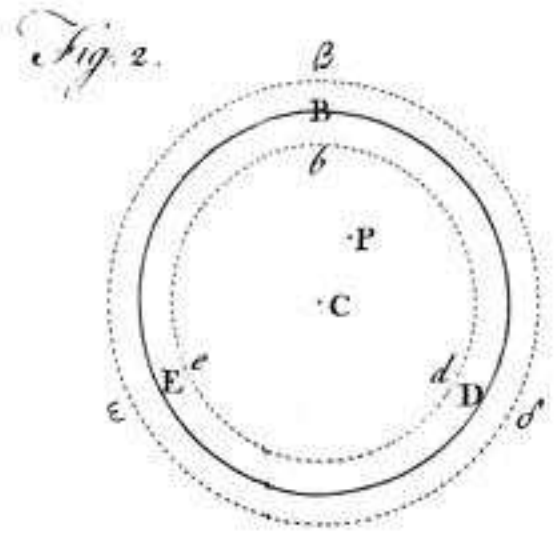
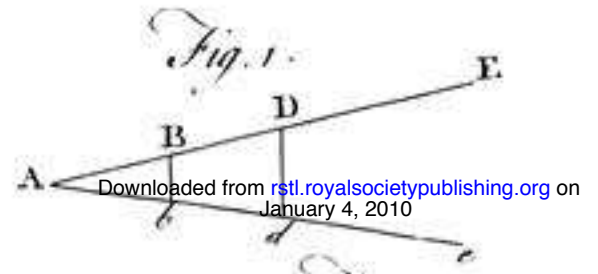


Fig. 5.

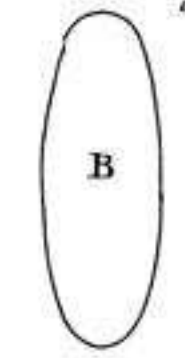
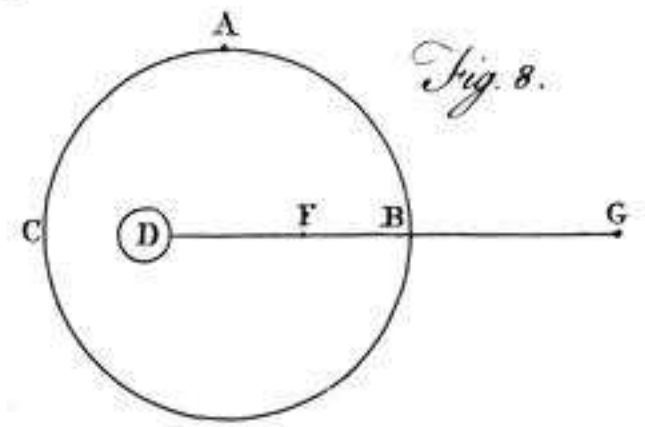
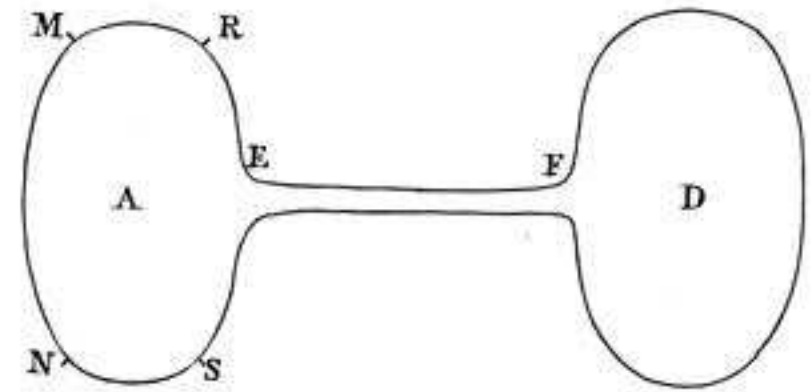
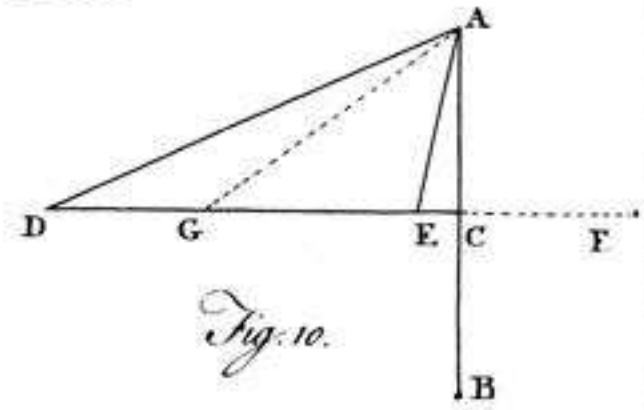
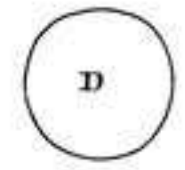
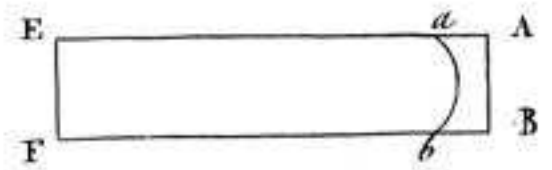
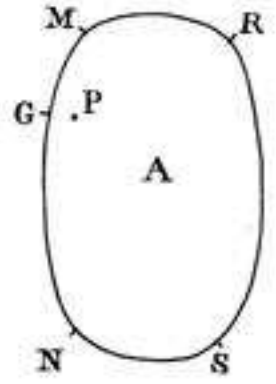


Fig. 6.



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