HOMEWORK

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II.7.1. $|S_3| = 6 \Rightarrow 1/6,\ 2/6,\ 3/6,\ 6/6$
$Since\ S_3\ is\ not\ cyclic,\ we\ may\ or\ may\ not\ find\ a\ subgroup\ whose\ order\ is\ 2\ or\ 3.$
$S_3 = \{(1),\ (12),\ (13),\ (23),\ (123),\ (132)\}$
$H_1 =<(1)>= \{(1)\}$
$H_2 =<(12)>= \left\{(1),\ (12),\ (12)^2,\ ...\right\} = \{(1),\ (12)\}$
$H_3 =<(13)>= \left\{(1),\ (13),\ (13)^2,\ ...\right\} = \{(1),\ (13)\}$
$H_4 =<(23)>= \left\{(1),\ (23),\ (23)^2,\ ...\right\} = \{(1),\ (23)\}$
$A_3 =<(123)>= \left\{(1),\ (123),\ (123)^2,\ ...\right\} = \{(1),\ (123),\ (132)\}$
$A_3 =<(132)>= \left\{(1),\ (132),\ (132)^2,\ ...\right\} = \{(1),\ (132),\ (123)\}$
$These\ are\ all\ subgroups\ of\ S_3.$
$''Definition\ 7.1.\ A\ subgroup\ N\ of\ a\ group\ G\ is\ normal\ if\ \forall g \in G,\ \forall n \in N,\ gng^{-1} \in N''$
$S_3\ is\ normal\ subgroup\ of\ S_3,\ because\ \forall n,\ g \in S_3,\ gng^{-1} \in S_3$
$H_1\ is\ normal\ subgroup\ of\ S_3,\ because\ \forall g \in S_3,\ \forall n \in H_1,\ gng^{-1} \in H_1$
$H_2\ is\ not\ normal\ subgroup\ of\ S_3,\ because\ (13) \in S_3,\ (12) \in H_2,\ (13)(12)(13)^{-1} = (23) \notin H_2$
$H_3\ is\ not\ normal\ subgroup\ of\ S_3,\ because\ (12) \in S_3,\ (13) \in H_3,\ (12)(13)(12)^{-1} = (23) \notin H_3$
$H_4\ is\ not\ normal\ subgroup\ of\ S_3,\ because\ (12) \in S_3,\ (23) \in H_4,\ (12)(23)(12)^{-1} = (13) \notin H_4$
$Since\ A_3 \leq S_3\ and\ (S_3 : A_3) = 2,\ then\ A_3\ is\ a\ normal\ subgroup\ of\ S_3$