Wikibooks

Bibliografische Information
Detaillierte Daten zu dieser Publikation sind bei Wikibooks zu erhalten:
HTTP://DE.WIKIBOOKS.ORG/
Namen von Programmen und Produkten sowie sonstige Angaben sind häufig geschützt. Da es auch freie Bezeichnungen gibt, wird das Symbol ® nicht verwendet.
Erstellt am 11. März 2011.

Inhaltsverzeichnis

UNKNOWN TEMPLATE "Print version notice"

Note: current version of this book can be found at http://en.wikibooks.org/wiki/Trigonometry

Remember to click "refresh" to view this version.

1 Contents

Algebra

2 Prerequisites And Basics

To be able to study Trigonometry successfully, it is recommended that students complete GEOMETRY[1] and ALGEBRA[2] prior to digging in to the course material. Students should also be familiar with the arithmetic of the real number system. It is helpful to have a graphing calculator and graph paper on hand to be able to follow along as well. If one is not available, software available on sites such as GRAPHCALC[3] or GEOGEBRA[4] may be helpful. Geometric constructions proposed in the text can be drawn using GEOPS[5], free software for performing geometric constructions in the manner of the Ancient Greeks.

Next Page: IN SIMPLE TERMS[6]

Home: TRIGONOMETRY[7]

CATEGORY:TRIGONOMETRY[8]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/GEOMETRY
2 HTTP://DE.WIKIBOOKS.ORG/WIKI/ALGEBRA
3 HTTP://WWW.GRAPHCALC.COM/
4 HTTP://WWW.GEOGEBRA.ORG/CMS/
5 HTTP://WWW.CPAN.ORG/AUTHORS/ID/P/PR/PRBRENAN/GEOPS-{}1.2.PL
6 Kapitel 5 auf Seite 15
7 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY
8 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

3 Trigonometry

the trigonometric tables help you to find the sine cosine and tangent of any theta

4 Introduction

Trigonometry is the study of the properties of triangles.

- **"Tri"** is Ancient Greek word for three,
- **"gon"** means side,
- **"metry"** measurement

Together they make "measuring three sides".

If you know some facts about a triangle, such as the lengths of its sides, then using trigonometry you can find out other facts about it. If you know the lengths of sides then you can find what the angles are. If you know the length of one side and of two of the angles, then you can work out what the remaining angle is, and also what the lengths of the other two sides are.

As a consequence the Ancient Greeks were able to use trigonometry to calculate the distance from the Earth to the Moon.

4.1 Starting Learning Trigonometry

One of the first things we learn in trigonometry is how to calculate what the third angle in a triangle is when given the other two angles. For example, in a triangle if two of the angles are $60°$ then the third angle must be $60°$ too.

However, if we're instead told two lengths, say 7cm and 10cm, and if we're not told anything about the angles, then the third length has a range of possibilities. It could be 5cm. It could be 15cm.

To calculate lengths we start out by examining a right-angled triangle, i.e. one where one of the angles is 90° .

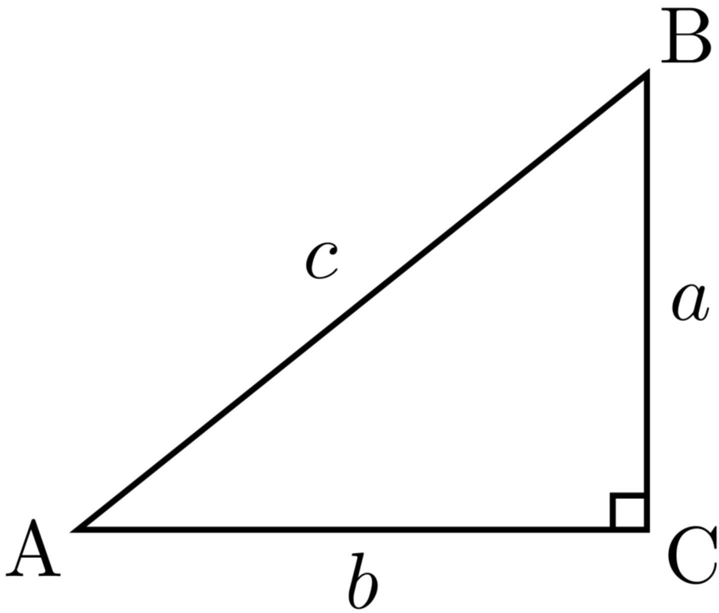

Abb. 1: A right-angle triangle

We'll be using right-angled triangles a lot.

In a right-angled triangle we *can* work out the length of the longest side if given the lengths of the two shorter sides. In a right-angled triangle the lengths of the sides have a particularly nice relationship to each other. The formula for this relationship is written:

$$a^2 + b^2 = c^2$$

How you use this formula to calculate the third side when you know two of the sides in a right-angled triangle is explained in the page on the 'PYTHAGORAS' THEOREM[1]'

What if the triangle isn't a right-angled triangle, though? What if it doesn't have a right angle?

More complex triangles can be built by joining right-angled triangles together. What we learn about right-angled triangles enables us to work with other kinds of triangles too. Trigonometry and trigonometric functions can also be used with more complex shapes such as squares, hexagons, circles and ellipses. Ultimately the most important mathematical tools we have for measuring the universe are based on the study of the mathematics of triangles.

Trigonometry is a fundamental step in your mathematical education. From the seemingly simple shape, the right triangle, we gain tools and insight that help us in practical and theoretical endeavors. The subtle mathematical relationships between the right triangle, the circle, the sine wave, and the exponential curve can only be fully understood with a firm foundation in trigonometry.

UNKNOWN TEMPLATE "Trig/NAV "

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/..%2FPYTHAGOREAN%20THEOREM

4.2 Applications

Abb. 2: Triangles used in architecture. This is part of a roof.

Abb. 3: Triangles and trigonometry are important in engineering.

Trigonometry is a branch of mathematics concerned with the lengths of the sides and size of the angles of triangles. Because many physical questions can be framed in terms of triangles, trigonometry has found wide use in the physical sciences and engineering.

Trigonometry is needed in surveying and architecture since relationships between lengths and angles in triangles are used directly.

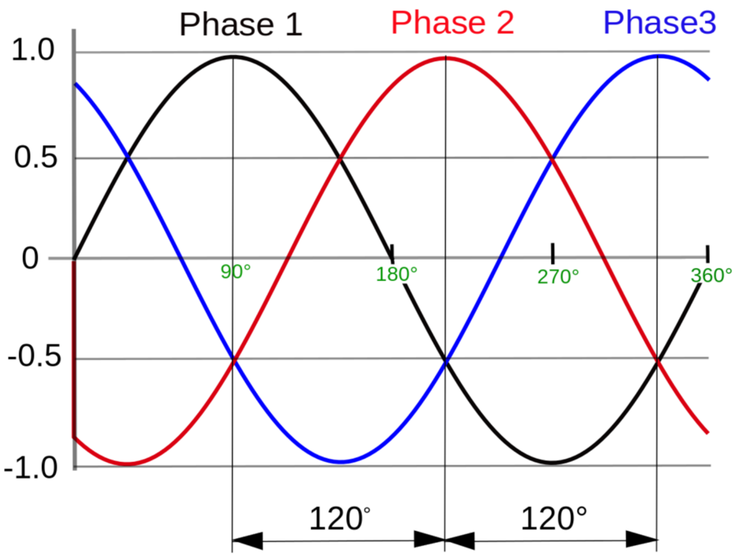

Abb. 4: 3 phase AC waveform. Trigonometry functions are used in Electrical Engineering

Trigonometry is also used in electrical engineering. The functions that relate angles and side lengths in right triangles turn out to also be useful in expressing how AC electric current varies with time.

UNKNOWN TEMPLATE "Trig/NAV "

UNKNOWN TEMPLATE "BookCat"

5 In simple terms

1. REDIRECT TRIGONOMETRY/FOR ENTHUSIASTS/TRIGONOMETRY DONE RIGOROUSLY[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FFOR%20ENTHUSIASTS%2FTRIGONOMETRY%20DONE%20RIGOROUSLY

6 Radian and Degree Measure

1. REDIRECT TRIGONOMETRY/RADIANS[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FRADIANS

7 The Unit Circle

1. REDIRECT TRIGONOMETRY/THE UNIT CIRCLE[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FTHE%20UNIT%20CIRCLE

8 Trigonometric Angular Functions

1. REDIRECT TRIGONOMETRY/GEOMETRIC DEFINITIONS OF TRIG FUNCTIONS[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FGEOMETRIC%20DEFINITIONS%20OF%20TRIG%20FUNCTIONS

9 Right Angle Trigonometry

1. REDIRECT TRIGONOMETRY/THALES THEOREM[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FTHALES%20THEOREM

10 Graphs of Sine and Cosine Functions

The graph of the sine function looks like this:

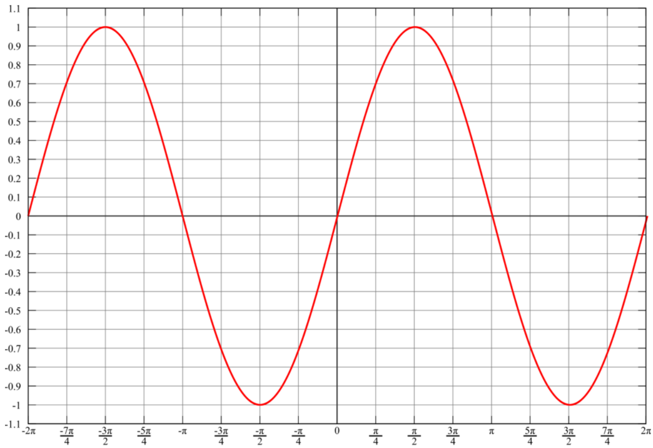

Abb. 5

Careful analysis of this graph will show that the graph corresponds to the unit circle. X is essentially the degree measure (in radians), while Y is the value of the sine function.

The graph of the cosine function looks like this:

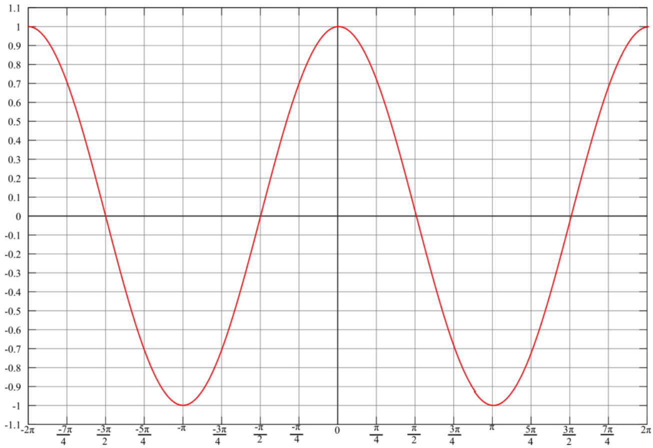

Abb. 6

As with the sine function, analysis of the cosine function will show that the graph corresponds to the unit circle. One of the most important differences between the sine and cosine functions is that sine is an odd function (i.e. $\sin(-\theta) = -\sin\theta$) while cosine is an even function (i.e. $\cos(-\theta) = \cos\theta$).

Sine and cosine are periodic functions; that is, the above is repeated for preceding and following intervals with length 2π.

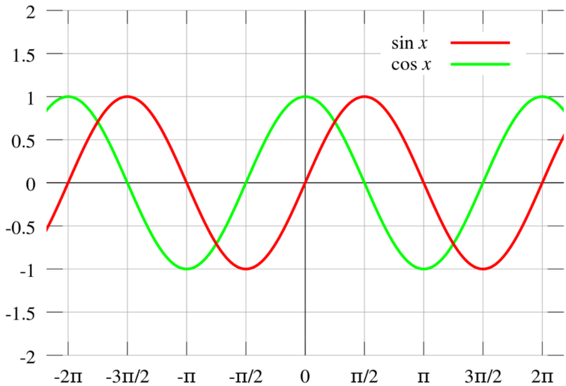

Abb. 7

10.1 Exercises

UNKNOWN TEMPLATE ËxerciseRobox"

For what value of θ does $\sin\theta = \cos\theta$? (Choose a value between 0 and 90° or equivalently in radians between 0 and $\frac{\pi}{2}$).

- Convince yourself you are right by drawing the triangle and using **SOH-CAH-TOA**.

UNKNOWN TEMPLATE "Robox/Close"

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

CATEGORY:TRIGONOMETRY[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

11 Graphs of Other Trigonometric Functions

A graph of $\tan(x)$. $\tan(x)$ is defined as $\frac{\sin(x)}{\cos(x)}$.

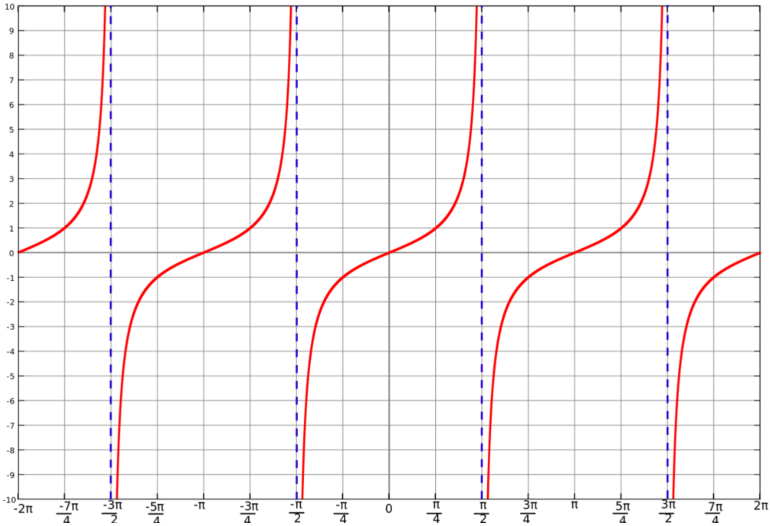

Abb. 8

A graph of $\csc(x)$. $\csc(x)$ is defined as $\frac{1}{\sin(x)}$.

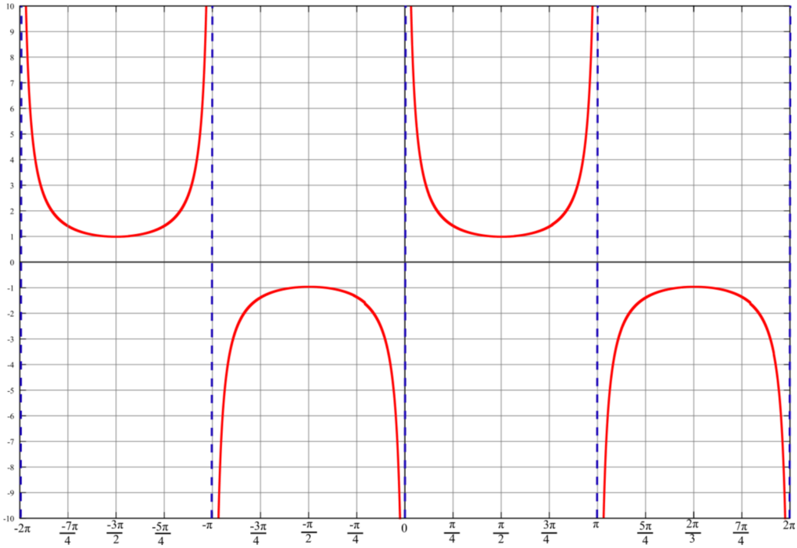

Abb. 9

A graph of $\sec(x)$. $\sec(x)$ is defined as $\frac{1}{\cos(x)}$.

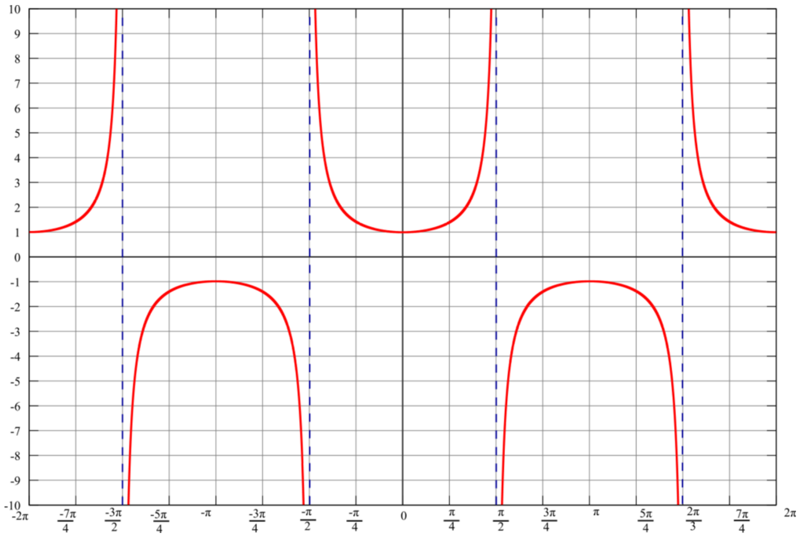

Abb. 10

A graph of $\cot(x)$. $\cot(x)$ is defined as $\frac{1}{\tan(x)}$ or $\frac{\cos(x)}{\sin(x)}$.

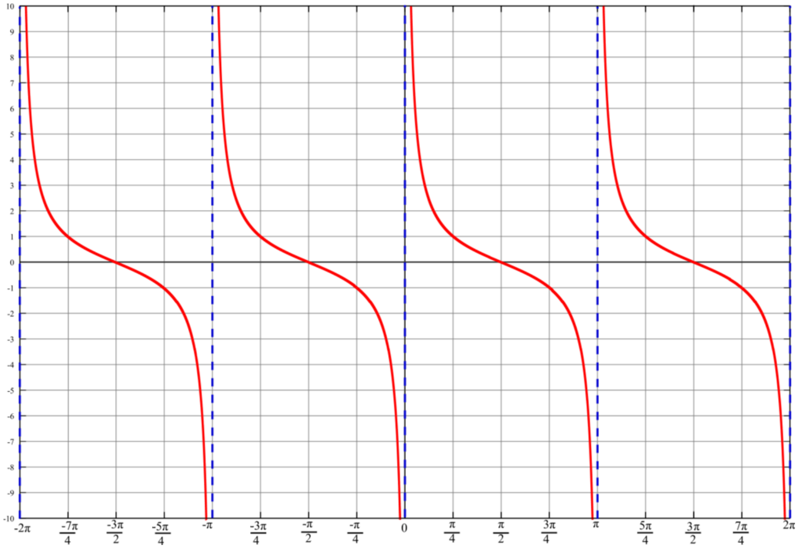

Abb. 11

Note that tan(x), sec(x), and csc(x) are unbounded, positive or negative. While tan(x) (and cot(x)) can take any value, sec(x) and csc(x) can never lie between -1 and 1.

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

12 Inverse Trigonometric Functions

12.1 The Basic Idea

In the equation

$$y = \sin x$$

if we are given x we can work out y.

If we are given y, can we work out x?

Look at the graph of $\sin x$ and you will see that for $y > 1$ or $y < -1$ there aren't any answers at all, whilst for other values of y there are infinitely many answers.

If

$$y = \sin x$$

we write the inverse of sin as follows

$$x = \sin^{-1} y$$

Notice the strange notation. It's just a convention, and does not really fit in well with the convention of $\sin^2 x$ meaning $(\sin x)^2$. However, we are stuck with this notation. It is so widely used and so familiar in mathematical work that in practice it does not cause confusion. On many calculators the alternative notation arcsin is used.

A calculator will only give one answer (or error) for $\sin^{-1} y$. You can get some of the other answers by adding or subtracting multiples of 360^o. In a later section we spell out the conventions as to which answer is given by $\sin^{-1} y$.

12.2 arcsin

A common notation used for the inverse functions is the "arcfunction" notation, prefixing the function name by "arc" or sometimes just "a".

$$\mathrm{Sin}^{-1} x = \arcsin x = \mathrm{asin}\, x$$

,

$$\mathrm{Cos}^{-1} x = \arccos x = \mathrm{acos}\, x$$

, and

$$\mathrm{Tan}^{-1} x = \arctan x = \mathrm{atan}\, x$$

.

The arcfunctions might perhaps be so named because of the relationship between radian measure of angles and arclength--the arcfunctions yield arc lengths on a unit circle.

12.3 The Inverse Functions, Domain and Range

The inverse of sine or cosine for some values has multiple answers and some values does not exist at all. We'd like it if the inverse were a function, but according to the mathematical definition of a function it is not:

- A **function** is something that given one value always gives back a unique value as its 'answer'.

$\sin x$ is a function because given any x it gives back some value. There is not really a function that is an inverse for $\sin x$ because, for example, $\sin 20^\circ$ and $\sin 160^\circ$ have the same value. The inverse 'function' does not know whether to go back to 20^o or 160^o.

To deal with this we need some more mathematical language. We have mathematical terminology for what a function operates on and where its values end up. A function like $\sqrt{x}$ needs to be accompanied by some agreement as to what values it can operate on and where they end up. By convention $\sqrt{25} = 5$, though $x = -5$ is also a valid solution to $x^2 = 25$. So how do we describe this kind of thing?

- The **domain** of a function is the set of values which it is defined on.

UNKNOWN TEMPLATE ËxampleRobox"

The function $\frac{1}{x}$ is defined on all values of x except for 0. Its domain is the real numbers excluding zero.

UNKNOWN TEMPLATE "Robox/Close"

UNKNOWN TEMPLATE ËxampleRobox"

The factorial function, operates on the positive whole numbers. As a function f - $f(1) = 1, f(2) = 2, f(3) = 6, f(4) = 24\cdots$. The factorial function is usually written by writing an exclamation point after a number. So, 3 factorial is usually written as 3! and it is $3 \times 2 \times 1 = 6$. 4 factorial is written as 4! and it is $4 \times 3 \times 2 \times 1 = 24$.

The domain of the factorial function is the positive whole numbers.

UNKNOWN TEMPLATE "Robox/Close"

- The **range** of a function is the set of values which it can take. The range will depend on the domain too.

UNKNOWN TEMPLATE ËxampleRobox"

Consider the function

$$f(x) = x^2$$

If we use ordinary numbers for x like 37.2 or -1001.56 we always find $f(x)$ is a positive number (or zero). The range of $f(x)$ is the numbers greater than or equal to zero.

UNKNOWN TEMPLATE "Robox/Close"

UNKNOWN TEMPLATE ËxampleRobox"

Consider the function

$$f(x) = 2x$$

If we choose the domain of $f(x)$ to be the numbers greater than or equal to 1, then the range of $f(x)$ is the numbers greater than or equal to two.

UNKNOWN TEMPLATE "Robox/Close"

12.4 More Notation

- The **integers** are whole numbers like 1,2,3,4 and also include 0, -1, -2,-3... A symbol we use to indicate the integers is $\mathbb{Z}$. The statement $x \in \mathbb{Z}$ means exactly the same thing as x is an integer.

- The **reals** include all the integers, and also fractions, and also other numbers like pi, and square root of 2. The reals 'fill in the gaps' between the integers. You'll get a better definition of the reals in a later book. A symbol we use to indicate the reals is $\mathbb{R}$. The statement $x \in \mathbb{R}$ means exactly the same thing as x is a real number.

- A range of numbers in the reals can often be written using **interval notation**. Here is an example for numbers greater than or equal to zero and less than one: $[0,1)$. The square and round brackets have special meaning. The square bracket means the number is included. A round bracket means the number is excluded.

UNKNOWN TEMPLATE ËxampleRobox"

- The notation $(1.3,100.7]$ means all numbers between 1.3 and 100.7, including 100.7 but excluding 1.3.
- The notation $[-59.1,12.5]$ means all numbers between -59.1 and 12.5, including -59.1 and including 12.5.
- The notation $(0,71.2)$ means all numbers between 0 and 71.2, excluding 0 and excluding 71.2.

UNKNOWN TEMPLATE "Robox/Close"

- Function **composition** is applying one function after another. If we have two functions f and g, we write the composite function, applied to x as: $f \circ g(x)$. This means first apply g to x, then apply f to that result. We can regard $f \circ g$ as a new function in its own right.

- An **inverse** to a function f is written f^{-1}. If f is the function that adds 12 to a number, then f^{-1} is the function that subtracts 12 from a number.

12.5 Inverses to Trig functions

Some textbooks 'solve' the problem of inverses for trig functions by defining new functions $\operatorname{Sin} x$, $\operatorname{Cos} x$, and $\operatorname{Tan} x$ (all with initial capitals) to equal the original functions but with restricted domain.

With suitably restricted domain the functions $\operatorname{Sin} x$, $\operatorname{Cos} x$, and $\operatorname{Tan} x$ (all with initial capitals) do have inverses which are functions too.

The restrictions to allow the inverses to be functions are standard. Here (and for the rest of this page) we are using radians to measure angles rather than degrees:

$$\operatorname{Sin} x$$

has domain

$$\left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right]$$

;

$$\operatorname{Cos} x$$

has domain

$$[0, \pi]$$

; and

$$\operatorname{Tan} x$$

has domain

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

For each function, the restricted domain includes first-quadrant angles as well as an adjacent quadrant.

12.6 Inverses, Really?

It is important to note that because of these restricted domains, the inverse trigonometric functions do not necessarily behave as one might expect an inverse function to behave. While

$$\mathrm{Sin}^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = \mathrm{Sin}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

(following the expected

$$\mathrm{Sin}^{-1}(\sin x) = x$$

),

$$\mathrm{Sin}^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \mathrm{Sin}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

.

For the inverse trigonometric functions, $f^{-1} \circ f(x) = x$ only when x is in the range of the inverse function.

The other direction, however: $f \circ f^{-1}(x) = x$ for all x to which we can apply the inverse function.

12.7 The Inverse Relations

’For completeness’, here are definitions of the inverse trigonometric *relations* based on the inverse trigonometric *functions*. This section is really mostly about using mathematical notation to express how adding multiples of 360^o to an angle gives us another solution for the inverse. Because we are working in radians we’re adding multiples of two pi. Some more notation being used here:

- Curly braces like so {} mean the ’set of’, i.e. the set of whatever is inside them.
- The symbol ∪ is read as **union**. The union of two sets is everything that is in either of them.

- $\sin^{-1} x = \left\{\mathrm{Sin}^{-1} x + 2\pi n, n \in \mathbb{Z}\right\} \cup \left\{\pi - \mathrm{Sin}^{-1} x + 2\pi n, n \in \mathbb{Z}\right\}$ (the sine function has period 2π, but within any given period may have two solutions and $\sin x = \sin(\pi - x)$)
- $\cos^{-1} x = \left\{\pm\mathrm{Cos}^{-1} x + 2\pi n, n \in \mathbb{Z}\right\}$ (the cosine function has period 2π, but within any given period may have two solutions and cosine is even--$\cos x = \cos(-x)$)
- $\tan^{-1} x = \left\{\mathrm{Tan}^{-1} x + \pi n, n \in \mathbb{Z}\right\}$ (the tangent function has period π and is one-to-one within any given period)

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

13 Applications and Models

13.1 Simple harmonic motion

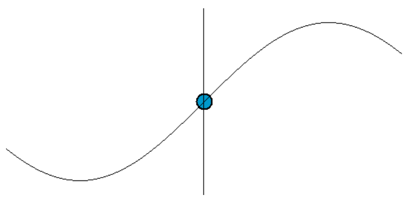

Abb. 12: Simple harmonic motion. Notice that the position of the dot matches that of the sine wave.

Simple harmonic motion (SHM) is the motion of an object which can be modeled by the following function:

$$x = A\sin\left(\omega t + \phi\right)$$

or

$$x = c_1 \cos(\omega t) + c_2 \sin(\omega t)$$

where $c_1 = A\sin\phi$ and $c_2 = A\cos\phi$.

In the above functions, A is the amplitude of the motion, ω is the angular velocity, and ϕ is the phase.

The velocity of an object in SHM is

$$v = A\omega\cos\left(\omega t + \phi\right)$$

The acceleration is

$$a = -A\omega^2\sin\left(\omega t + \phi\right) = -\omega^2 x$$

An alternative definition of harmonic motion is motion such that

$$a = -\omega^2 x$$

13.1.1 Springs and Hooke's Law

An application of this is the motion of a weight hanging on a spring. The motion of a spring can be modeled approximately by HOOKE'S LAW[1]:

$F = -kx$

where F is the force the spring exerts, x is the position of the end of the spring, and k is a constant characterizing the spring (the stronger the spring, the higher the constant).

1 HTTP://DE.WIKIPEDIA.ORG/WIKI/HOOKE%27S%20LAW

Calculus-based derivation

From NEWTON'S LAWS[2] we know that $F = ma$ where m is the mass of the weight, and a is its acceleration. Substituting this into Hooke's Law, we get

$ma = -kx$

Dividing through by m:

$$a = -\frac{k}{m}x$$

The calculus definition of acceleration gives us

$$x'' = -\frac{k}{m}x$$

$$x'' + \frac{k}{m}x = 0$$

Thus we have a second-order differential equation. Solving it gives us

$$x = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

(2)

2 HTTP://DE.WIKIBOOKS.ORG/WIKI/GENERAL%20MECHANICS%2FFUNDAMENTAL%20PRINCIPLES%20OF%20DYNAMICS

with an independent variable t for time.

We can change this equation into a simpler form. By lettting c_1 and c_2 be the legs of a right triangle, with angle ϕ adjacent to c_2, we get

$$\sin\phi = \frac{c_1}{\sqrt{c_1^2 + c_2^2}}$$

$$\cos\phi = \frac{c_2}{\sqrt{c_1^2 + c_2^2}}$$

and

$$c_1 = \sqrt{c_1^2 + c_2^2}\sin\phi$$

$$c_2 = \sqrt{c_1^2 + c_2^2}\cos\phi$$

Substituting into (2), we get

$$x = \sqrt{c_1^2 + c_2^2}\sin\phi\cos\left(\sqrt{\frac{k}{m}}t\right) + \sqrt{c_1^2 + c_2^2}\cos\phi\sin\left(\sqrt{\frac{k}{m}}t\right)$$

Using a trigonometric identity, we get:

$$x = \sqrt{c_1^2 + c_2^2}\left[\sin\left(\phi + \sqrt{\frac{k}{m}}t\right) + \sin\left(\phi - \sqrt{\frac{k}{m}}t\right)\right] + \sqrt{c_1^2 + c_2^2}\left[\sin\left(\sqrt{\frac{k}{m}}t + \right.\right.$$

$$x = \sqrt{c_1^2 + c_2^2} \sin\left(\sqrt{\frac{k}{m}} t + \phi\right)$$

(3)

Let $A = \sqrt{c_1^2 + c_2^2}$ and $\omega^2 = \frac{k}{m}$. Substituting this into (3) gives

$$x = A \sin\left(\omega t + \phi\right)$$

*Next Page: ../*USING FUNDAMENTAL IDENTITIES/[3]

Previous Page: ../INVERSE TRIGONOMETRIC FUNCTIONS/[4]

Home: TRIGONOMETRY[5]

UNKNOWN TEMPLATE "BookCat"

3 HTTP://DE.WIKIBOOKS.ORG/WIKI/..%2FUSING%20FUNDAMENTAL%20IDENTITIES%2F

4 HTTP://DE.WIKIBOOKS.ORG/WIKI/..%2FINVERSE%20TRIGONOMETRIC%20FUNCTIONS%2F

5 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY

14 Analytic Trigonometry

15 Using Fundamental Identities

1. REDIRECT TRIGONOMETRY/REMEMBERING TRIG FORMULAE[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FREMEMBERING%20TRIG%20FORMULAE

16 Verifying Trigonometric Identities

To verify an identity means to make sure that the equation is true by setting both sides equal to one another.

There is no set method that can be applied to verifying identities; there are, however, a few different ways to start based on the identity which is to be verified.

16.1 Introduction

Trigonometric identities are used in both course texts and in real life applications to abbreviate trigonometric expressions. It is important to remember that merely verifying an identity or altering an expression is not an end in itself, but rather that identities are used to simplify expressions according to the task at hand. Trigonometric expressions can always be reduced to sines and cosines, which can be more manageable than their inverse counterparts.

16.2 To verify an identity:

1) Always try to reduce the larger side first.

2) Sometimes getting all trigonometric functions on one side can help.

3) Remember to use and manipulate already existing identities. The Pythagorean identities are usually the most useful in simplifying.

4) Remember to factor if needed.

5) Whenever you have a squared trigonometric function such as Sin^2(t), always go to your Pythagorean identities, which deal with squared functions.

6) Sometimes doing the reverse of the normal steps helps. For example, adding 1 in unique forms (such as $cos(t)/cos(t)$ can help simplify expressions by matching denominators and simplifying numerators.

Easy example $1/cot(t) = sin(t)/cos(t)$

```
1/cot(t)= tan(t) ,so tan(t) = sin(t)/cos(t)
```

tan(t) is the same as sint(t)/cos(t) and therefore can be rewritten as $sin(t)/cos(t) = sin(t)/cos(t)$

Identity verified

UNKNOWN TEMPLATE "Trigonometry:Navigation"

CATEGORY:TRIGONOMETRY[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

17 Solving Trigonometric Equations

Trigonometric equations are equations including trigonometric functions. If they have only such functions and constants, then the solution involves finding an unknown which is an argument to a trigonometric function.

17.1 Basic trigonometric equations

17.1.1 sin x = n

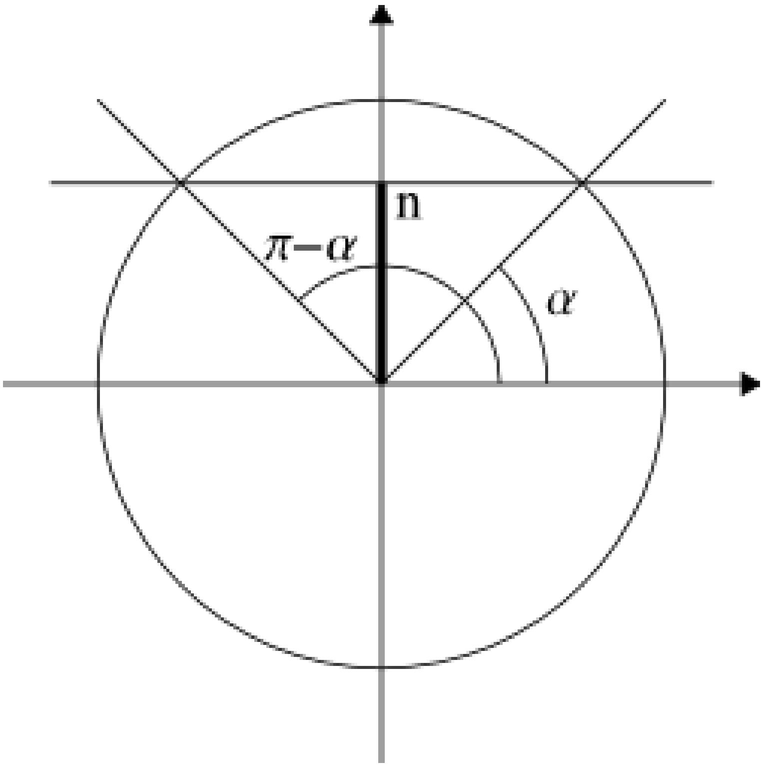

Abb. 13

n	$\sin x = n$
	$x = \alpha + 2k\pi$
$\lvert n\rvert < 1$	$x = \pi - \alpha + 2k\pi$
	$\alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$
$n = -1$	$x = -\frac{\pi}{2} + 2k\pi$
$n = 0$	$x = k\pi$
$n = 1$	$x = \frac{\pi}{2} + 2k\pi$
$\lvert n\rvert > 1$	$x \in \varnothing$

The equation $\sin x = n$ has solutions only when n is within the interval [-1; 1]. If n is within this interval, then we need to find an α such that:

$$\alpha = \sin^{-1} n$$

The solutions are then:

$$x = \alpha + 2k\pi$$

$$x = \pi - \alpha + 2k\pi$$

Where k is an integer.

In the cases when n equals 1, 0 or -1 these solutions have simpler forms which are summarized in the table on the right.

For example, to solve:

$$\sin \frac{x}{2} = \frac{\sqrt{3}}{2}$$

First find α:

$$\alpha = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

Then substitute in the formulae above:

$$\frac{x}{2} = \frac{\pi}{3} + 2k\pi$$

$$\frac{x}{2} = \pi - \frac{\pi}{3} + 2k\pi$$

Solving these linear equations for x gives the final answer:

$$x = \frac{2\pi}{3}(1 + 6k)$$

$$x = \frac{4\pi}{3}(1 + 3k)$$

Where k is an integer.

17.1.2 cos x = n

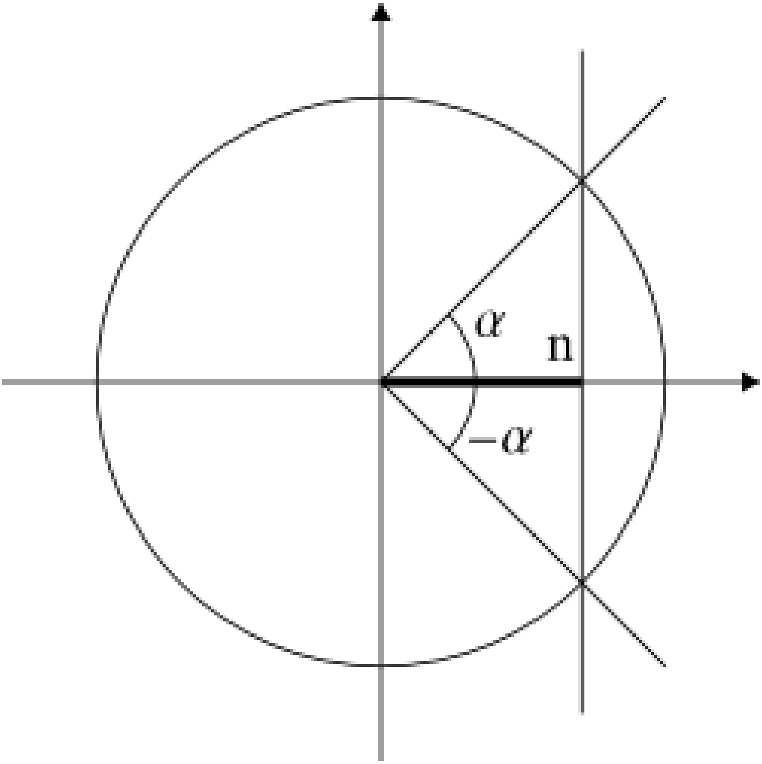

Abb. 14

n	$\cos x = n$
$\lvert n\rvert < 1$	$x = \pm\alpha + 2k\pi$ $\alpha \in [0;\pi]$
$n = -1$	$x = \pi + 2k\pi$
$n = 0$	$x = \frac{\pi}{2} + k\pi$
$n = 1$	$x = 2k\pi$
$\lvert n\rvert > 1$	$x \in \varnothing$

Like the sine equation, an equation of the form $\cos x = n$ only has solutions when n is in the interval [-1; 1]. To solve such an equation we first find the angle α such that:

$$\alpha = \cos^{-1} n$$

Then the solutions for x are:

$$x = \pm\alpha + 2k\pi$$

Where k is an integer.

Simpler cases with n equal to 1, 0 or -1 are summarized in the table on the right.

17.1.3 tan x = n

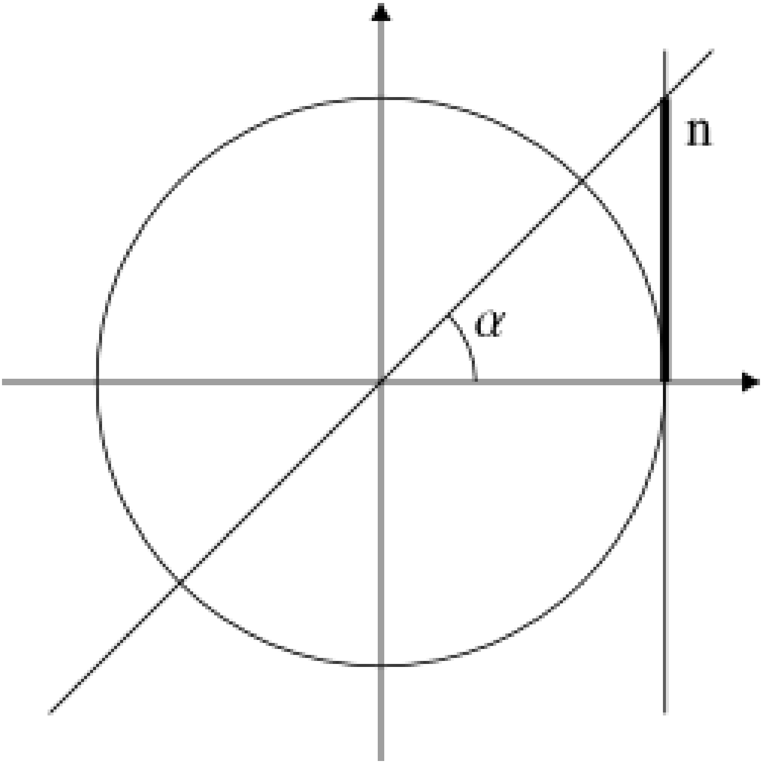

Abb. 15

n	$\tan x = n$
General case	$x = \alpha + k\pi$ $\alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$
$n = -1$	$x = -\frac{\pi}{4} + k\pi$
$n = 0$	$x = k\pi$
$n = 1$	$x = \frac{\pi}{4} + k\pi$

An equation of the form $\tan x = n$ has solutions for any real n. To find them we must first find an angle α such that:

$$\alpha = \tan^{-1} n$$

After finding α, the solutions for x are:

$$x = \alpha + k\pi$$

When n equals 1, 0 or -1 the solutions have simpler forms which are shown in the table on the right.

17.1.4 cot x = n

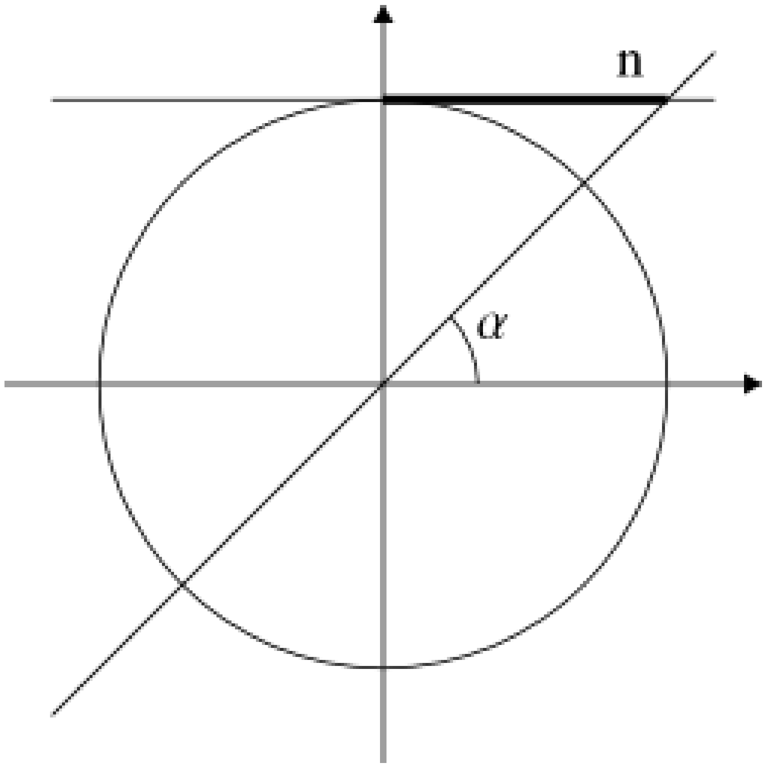

Abb. 16

n	$\cot x = n$
General case	$x = \alpha + k\pi$ $\alpha \in [0;\pi]$
$n = -1$	$x = -\frac{3\pi}{4} + k\pi$
$n = 0$	$x = \frac{\pi}{2} + k\pi$
$n = 1$	$x = \frac{\pi}{4} + k\pi$

The equation $\cot x = n$ has solutions for any real n. To find them we must first find an angle α such that:

$$\alpha = \cot^{-1} n$$

After finding α, the solutions for x are:

$$x = \alpha + k\pi$$

When n equals 1, 0 or -1 the solutions have simpler forms which are shown in the table on the right.

17.1.5 csc x = n and sec x = n

The trigonometric equations csc x = n and sec x = n can be solved by transforming them to other basic equations:

$$\csc x = n \Leftrightarrow \frac{1}{\sin x} = n \Leftrightarrow \sin x = \frac{1}{n}$$

$$\sec x = n \Leftrightarrow \frac{1}{\cos x} = n \Leftrightarrow \cos x = \frac{1}{n}$$

17.2 Further examples

Generally, to solve trigonometric equations we must first transform them to a basic trigonometric equation using the TRIGONOMETRIC IDENTITIES[1]. This sections lists some common examples.

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/..%2FTRIGONOMETRIC_IDENTITIES_REFERENCE

17.2.1 $a \sin x + b \cos x = c$

To solve this equation we will use the identity:

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \alpha)$$

$$\alpha = \begin{cases} \tan^{-1}(b/a), & \text{if } a > 0 \\ \pi + \tan^{-1}(b/a), & \text{if } a < 0 \end{cases}$$

The equation becomes:

$$\sqrt{a^2 + b^2} \sin(x + \alpha) = c$$

$$\sin(x + \alpha) = \frac{c}{\sqrt{a^2 + b^2}}$$

This equation is of the form $\sin x = n$ and can be solved with the formulae given above.

For example we will solve:

$$\sin 3x - \sqrt{3} \cos 3x = -\sqrt{3}$$

In this case we have:

$$a = 1, b = -\sqrt{3}$$

$$\sqrt{a^2 + b^2} = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$$

$$\alpha = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$$

Apply the identity:

$$2\sin\left(3x - \frac{\pi}{3}\right) = -\sqrt{3}$$

$$\sin\left(3x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

So using the formulae for $\sin x = n$ the solutions to the equation are:

$$3x - \frac{\pi}{3} = -\frac{\pi}{3} + 2k\pi \Leftrightarrow x = \frac{2k\pi}{3}$$

$$3x - \frac{\pi}{3} = \pi + \frac{\pi}{3} + 2k\pi \Leftrightarrow x = \frac{\pi}{9}(6k+5)$$

Where k is an integer.

UNKNOWN TEMPLATE "Trigonometry/Navigation"

PT:MATEMÁTICA ELEMENTAR/TRIGONOMETRIA/EQUAÇÕES E INEQUAÇÕES ENVOLVENDO FUNÇÕES TRIGONOMÉTRICAS[2]

2 HTTP://PT.WIKIBOOKS.ORG/WIKI/MATEM%E1TICA%20ELEMENTAR%2FTRIGONOMETRIA%2FEQUA%E7%F5ES%20E%20INEQUA%E7%F5ES%20ENVOLVENDO%20FUN%E7%F5ES%20TRIGONOM%E9TRICAS

18 Sum and Difference Formulas

1. REDIRECT TRIGONOMETRY/ADDITION FORMULA FOR SINES[1]

Multiple-Angle and Product-to-sum Formulas

1. REDIRECT TRIGONOMETRY/DOUBLE-ANGLE FORMULAS[2]

---

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FADDITION%20FORMULA%20FOR%20SINES
2 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FDOUBLE-{}ANGLE%20FORMULAS

19 Additional Topics in Trigonometry

20 Law of Sines

UNKNOWN TEMPLATE "warning"

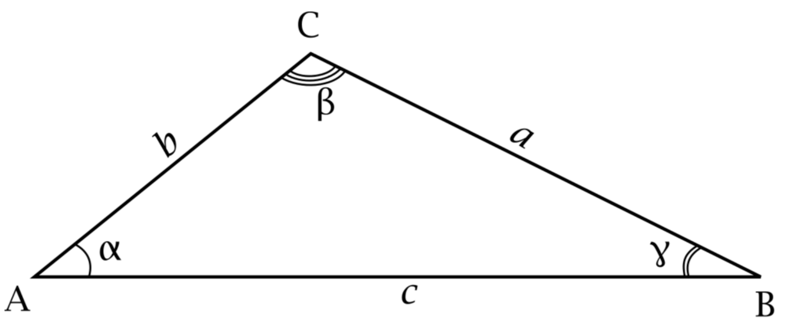

Abb. 17

For any triangle with vertices A, B, and C, corresponding angles A, B, and C, and corresponding opposite side lengths *a*, *b*, and *c*, the Law of Sines states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Each of these expressions is also equal to the diameter of the triangle's CIRCUMCIRCLE[1] (the circle that passes through the points A, B, and C). The law can also be written in terms of the reciprocals:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

20.1 Proof

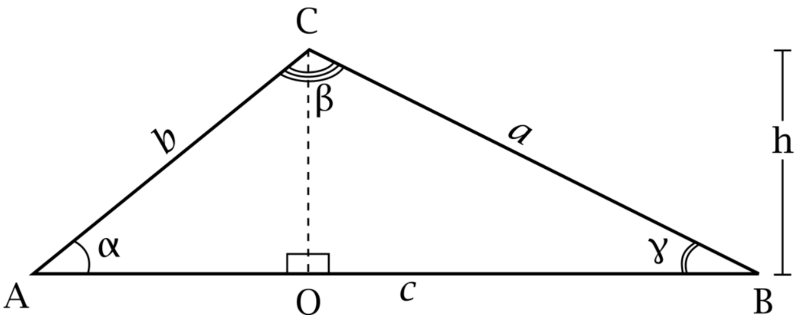

Abb. 18

Dropping a perpendicular *OC* from vertex C to intersect AB (or AB extended) at O splits this triangle into two right-angled triangles AOC and BOC. We can calculate the length *h* of the altitude OC in two different ways:

- Using the triangle AOC gives

$$h = b \sin A;$$

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FCIRCLES%20AND%20TRIANGLES%2FTHE%20CIRCUMCIRCLE

- and using the triangle BOC gives

$$h = a \sin B.$$

- Eliminate h from these two equations:

$$a \sin B = b \sin A.$$

- Rearrange to obtain

$$\frac{a}{\sin A} = \frac{b}{\sin B}.$$

By using the other two perpendiculars the full law of sines can be proved. **QED.**

20.2 Application

This formula can be used to find the other two sides of a triangle when one side and the three angles are known. (If two angles are known, the third is easily found since the sum of the angles is 180º.) See SOLVING TRIANGLES GIVEN ASA[2]. It can also be used to find an angle when two sides and the angle opposite one side are known.

2 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FSOLVING%20TRIANGLES%20GIVEN%20ASA

20.3 Area of a triangle

The area of a triangle may be found in various ways. If all three sides are known, use Heron's theorem.

If two sides and the included angle are known, consider the second diagram above. Let the sides b and c, and the angle between them α be known. From triangle ACO, the altitude h = CO is $b\sin(\alpha)$ so the area is

UNKNOWN TEMPLATE "frac"

$ab\sin(\alpha)$.

If two angles and the included side are known, again consider the second diagram above. Let the side c and the angles α and γ be known. Let AO =x. Then

$$\frac{x}{h} = cot(\alpha); \frac{c-x}{h} = cot(\gamma); \text{adding these, } \frac{c}{h} = cot(\alpha)+cot(\gamma)$$

Thus

$$h = \frac{c}{cot(\alpha)+cot(\gamma)} \text{ so area } = \frac{c^2}{2(cot(\alpha)+cot(\gamma))}$$

.

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

CATEGORY:TRIGONOMETRY[3]

3 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

pt:Matemática elementar/Trigonometria/Lei dos senos e dos cossenos[4]

4 http://pt.wikibooks.org/wiki/Matem%E1tica%20elementar%2FTrigonometria%2FLei%20dos%20senos%20e%20dos%20cossenos

21 Law of Cosines

21.1 Law of Cosines

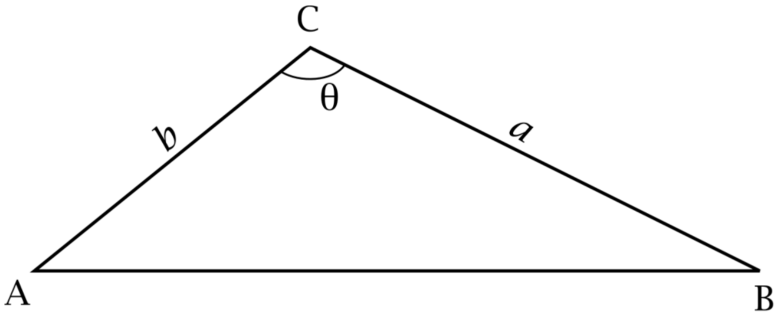

Abb. 19

The Pythagorean theorem is a special case of the more general theorem relating the lengths of sides in any triangle, the law of cosines:[1]

$$a^2 + b^2 - 2ab\cos\theta = c^2,$$

where is the angle between sides *a* and *b*.

1
UNKNOWN TEMPLATE "cite book "

21.1.1 Does the formula make sense?

This formula had better agree with the Pythagorean Theorem when $\theta = 90^\circ$.

So try it...

When $\theta = 90^\circ$, $\cos\theta = \cos 90^\circ = 0$

The $-2ab\cos\theta = 0$ and the formula reduces to the usual Pythagorean theorem.

21.2 Permutations

For any triangle with angles A, B, and C and corresponding opposite side lengths *a*, *b*, and *c*, the Law of Cosines states that

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A,$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B,$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C.$$

21.2.1 Proof

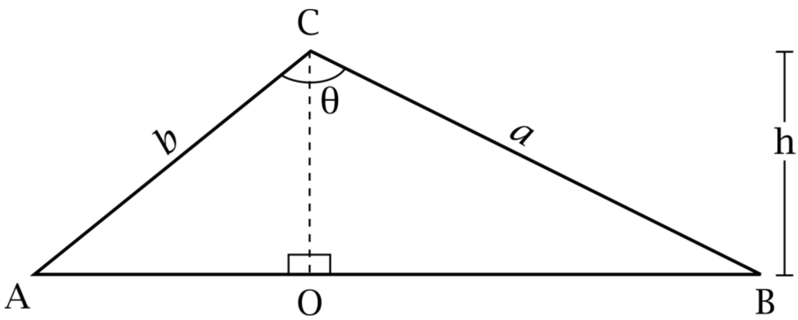

Abb. 20

Dropping a perpendicular OC from vertex C to intersect AB (or AB extended) at O splits this triangle into two right-angled triangles AOC and BOC, with altitude h from side c.

First we will find the lengths of the other two sides of triangle AOC in terms of known quantities, using triangle BOC.

$h = a \sin B$

Side c is split into two segments, with total length c.

OB has length $a \cos B$
AO has length $c - a \cos B$

Now we can use the Pythagorean Theorem to find b, since $b^2 = AO^2 + h^2$.

$$\begin{aligned} b^2 &= (c - a\cos B)^2 + a^2 \sin^2 B \\ &= c^2 - 2ac\cos B + a^2\cos^2 B + a^2 \sin^2 B \\ &= a^2 + c^2 - 2ac\cos B \end{aligned}$$

The corresponding expressions for a and c can be proved similarly.

The formula can be rearranged:

$cos(C) = \frac{a^2+b^2-c^2}{2ab}$

and similarly for cos(A) and cos(B).

21.3 Applications

This formula can be used to find the third side of a triangle if the other two sides and the angle between them are known. The rearranged formula can be used to find the angles of a triangle if all three sides are known. See SOLVING TRIANGLES GIVEN SAS[2].

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

CATEGORY:TRIGONOMETRY[3]

2 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FSOLVING%20TRIANGLES%20GIVEN%20SAS

3 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

22 Solving Triangles

1. REDIRECT TRIGONOMETRY/SOLVING TRIANGLES GIVEN ASA[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FSOLVING%20TRIANGLES%20GIVEN%20ASA

23 Vectors in the Plane

In practice, one of the most useful applications of trigonometry is in calculations related to vectors, which are frequently used in PHYSICS[1]. A vector is a quantity which has both magnitude (such as three or eight) and direction (such as north or 30 degrees south of east). It is represented in diagrams by an arrow, often pointing from the origin to a specific point.

A plane vector $\vec{A}$ can be expressed in two ways -- as the sum of a horizontal vector of magnitude A_x and a vertical vector of magnitude A_y, or in terms of its angle θ and magnitude $\left|\vec{A}\right|$ (or simply A). These two methods are called "rectangular" and "polar" respectively.

23.1 Rectangular to Polar conversion

For simplicity, assume $\vec{A}$ is in the first quadrant and has x-component A_x and y-component A_y (which will necessarily be positive). Given these components, we want to find the angle θ and the magnitude A.

If we draw all three of these vectors, they form a right triangle. It is easy to see that $\tan\theta = \frac{A_y}{A_x}$, or $\theta = \arctan\frac{A_y}{A_x}$ (A vector with an angle of zero is defined to be pointing directly to the right.) Furthermore, by the Pythagorean Theorem, ${A_x}^2 + {A_y}^2 = A^2$, or $A = \sqrt{{A_x}^2 + {A_y}^2}$.

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/PHYSICS

23.2 Polar to Rectangular conversion

This is essentially the same problem as above, but in reverse. Here, θ and A are known and we want to calculate the values of A_x and A_y.

Using the same triangle as above, we can see that $\cos\theta = \frac{A_x}{A}$, or $A_x = A\cos\theta$. Also, $\sin\theta = \frac{A_y}{A}$, or $A_y = A\sin\theta$.

23.3 Review of conversions

- $\theta = \arctan\frac{A_y}{A_x}$
- $\left|\vec{A}\right| = A = \sqrt{{A_x}^2 + {A_y}^2}$
- $A_x = A\cos\theta$
- $A_y = A\sin\theta$

UNKNOWN TEMPLATE "Trigonometry:Navigation"

CATEGORY:TRIGONOMETRY[2]

2 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

24 Vectors and Dot Products

Consider the vectors **U** and **V** (with respective magnitudes |**U**| and |**V**|). If those vectors enclose an angle θ then the dot product of those vectors can be written as:

$$\mathbf{U} \cdot \mathbf{V} = |\mathbf{U}||\mathbf{V}| \cos(\theta)$$

If the vectors can be written as:

$$\mathbf{U} = (U_x, U_y, U_z)$$

$$\mathbf{V} = (V_x, V_y, V_z)$$

then the *dot product* is given by:

$$\mathbf{U} \cdot \mathbf{V} = U_x V_x + U_y V_y + U_z V_z$$

For example,

$$(1,2,3) \cdot (2,2,2) = 1(2) + 2(2) + 3(2) = 12.$$

and

$$(0,5,0)\cdot(4,0,0) = 0.$$

We can interpret the last case by noting that the product is zero because the angle between the two vectors is 90 degrees.

Since

$$|\mathbf{U}| = \sqrt{U_x^2 + U_y^2 + U_z^2}$$

and

$$|\mathbf{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

this means that

$$\cos(\theta) = \frac{U_x V_x + U_y V_y + U_z V_z}{\sqrt{U_x^2 + U_y^2 + U_z^2}\sqrt{V_x^2 + V_y^2 + V_z^2}}$$

UNKNOWN TEMPLATE "Trigonometry:Navigation"

UNKNOWN TEMPLATE "BookCat"

25 Trigonometric Form of the Complex Number

$z = a + bi = r\left(\cos\phi + i\sin\phi\right)$

where

- i is the IMAGINARY NUMBER[1] $\left(i = \sqrt{-1}\right)$
- the modulus $r = \mathrm{mod}(z) = |z| = \sqrt{a^2 + b^2}$
- the argument $\phi = \arg(z)$ is the angle formed by the complex number on a polar graph with one real axis and one imaginary axis. This can be found using the RIGHT ANGLE TRIGONOMETRY[2] for the trigonometric functions.

This is sometimes abbreviated as $r\left(\cos\phi + i\sin\phi\right) = r\,\mathrm{cis}\,\phi$ and it is also the case that $r\,\mathrm{cis}\,\phi = re^{i\phi}$ (provided that ϕ is in radians). The latter identity is called Euler's formula.

Euler's formula can be used to prove DeMoivre's formula: $(\cos\phi + i\sin\phi)^n = \cos(n\phi) + i\sin(n\phi)$. This formula is valid for all values of n, real or complex.

UNKNOWN TEMPLATE "Trigonometry:Navigation"

CATEGORY:TRIGONOMETRY[3]

1 HTTP://DE.WIKIPEDIA.ORG/WIKI/IMAGINARY%20NUMBER
2 HTTP://DE.WIKIBOOKS.ORG/WIKI/ALGEBRA%2FCOMPLEX%20NUMBERS%23SOME_EXAMPLES
3 HTTP://DE.WIKIBOOKS.ORG/WIKI/CATEGORY%3ATRIGONOMETRY

26 Trigonometry References

27 Trigonometric Unit Circle and Graph Reference

The Unit Circle

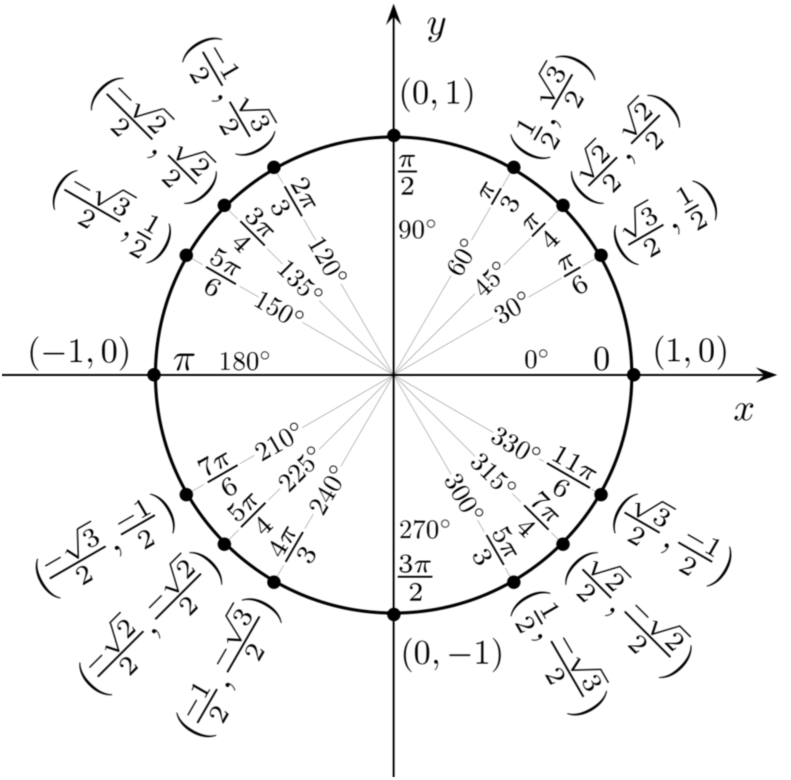

Abb. 21

The unit circle is a commonly used tool in trigonometry because it helps the user to remember the special angles and their trigonometric functions. The unit circle is a circle drawn with its center at the origin of a graph(0,0), and with a radius of 1. All angles are measured starting from the x-axis in quadrant one and may go around the unit circle any number of degrees. Points on the outside of the circle that are in line with the terminal (ending) sides of the angles are very useful to know, as they give the trigonometric function of the angle through their coordinants. The format is (cos, sin).

Note that in trigonometry, an angle can be of any size, positive or negative. An angle larger than 360º means that you have gone round the circle more than once.

UNKNOWN TEMPLATE "Trig/NAV"

UNKNOWN TEMPLATE "BookCat"

28 Trigonometric Formula Reference

UNKNOWN TEMPLATE ËxerciseRobox"

Cover the right hand side of each formula, and use the information about remembering formulas from the previous page to get the right hand side.

UNKNOWN TEMPLATE "Robox/Close"

28.1 Principal Trig Relationships

The following identities give relationships between the trigonometric functions.

1. $\sin x = \cos\left(\frac{\pi}{2} - x\right)$
2. $\cos x = \sin\left(\frac{\pi}{2} - x\right)$
3. $\tan x = \dfrac{\sin x}{\cos x}$
4. $\csc x = \dfrac{1}{\sin x}$
5. $\sec x = \dfrac{1}{\cos x}$

28.1.1 Pythagoras related

1. $\sin^2\theta + \cos^2\theta = 1$

2. $\tan^2\theta + 1 = \sec^2\theta$

UNKNOWN TEMPLATE ËxampleRobox"

One formula is missing.

By dividing the $\sin^2\theta + \cos^2\theta = 1$ by

$$\sin^2\theta$$

or by

$$\cos^2\theta$$

we can get two other formulas.

The missing formula is obtained by dividing through by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

The missing formula is:

$$1 + \cot^2\theta = \csc^2\theta$$

UNKNOWN TEMPLATE "Robox/Close"

28.1.2 Periodicity

Four trigonometric functions are 2π periodic:

1. $\sin\theta = \sin(\theta + 2\pi)$
2. $\cos\theta = \cos(\theta + 2\pi)$

3. $\csc\theta = \csc(\theta + 2\pi)$
4. $\sec\theta = \sec(\theta + 2\pi)$

Two trigonometric functions are π periodic:

1. $\tan\theta = \tan(\theta + \pi)$
2. $\cot\theta = \cot(\theta + \pi)$

28.1.3 Angle Sums

Formulas involving sums of angles are as follows:

1. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
4. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

28.1.4 Multiple Angle Formulas

Substituting $\beta = \alpha$ gives the **double angle formulae**

1. $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$
2. $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

Substituting $\sin^2\alpha + \cos^2\alpha = 1$ gives

1. $\cos(2\alpha) = 2\cos^2\alpha - 1$
2. $\cos(2\alpha) = 1 - 2\sin^2\alpha$

These can be obtained by putting $\beta = 2\theta, \alpha = \theta$ in the addition formula.

1. $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$
2. $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$
3. $\tan(3\theta) = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

This can also be obtained from the angle sums formula.

1. $2\sin(A)\cos(B) = \sin(A+B) + \sin(A-B)$

28.2 Trigonometric functions of some closely related angles

This list may duplicate some of the periodicity formulas above, but all the formulas are given for the sake of completeness. Angles are expressed in degrees rather than radians. Similar relations for cot, sec and cosec follow immediately from the definitions of these functions; just replace sin by cosec, cos by sec and tan by cot (and vice versa).

28.2.1 sin(x)

1. $\sin(-x) = -\sin(x)$
2. $\sin(90^\circ - x) = \cos(x)$
3. $\sin(90^\circ + x) = \cos(x)$
4. $\sin(180^\circ - x) = \sin(x)$
5. $\sin(180^\circ + x) = -\sin(x)$
6. $\sin(270^\circ - x) = -\cos(x)$
7. $\sin(270^\circ + x) = -\cos(x)$
8. $\sin(360^\circ - x) = -\sin(x)$
9. $\sin(360^\circ + x) = \sin(x)$

28.2.2 cos(x)

1. $\cos(-x) = \cos(x)$
2. $\cos(90^\circ - x) = \sin(x)$
3. $\cos(90^\circ + x) = -\sin(x)$
4. $\cos(180^\circ - x) = -\cos(x)$

5. $\cos(180^\circ + x) = -\cos(x)$
6. $\cos(270^\circ - x) = -\sin(x)$
7. $\cos(270^\circ + x) = \sin(x)$
8. $\cos(360^\circ - x) = \cos(x)$
9. $\cos(360^\circ + x) = \cos(x)$

28.2.3 tan(x)

1. $\tan(-x) = -\tan(x)$
2. $\tan(90^\circ - x) = \cot(x)$
3. $\tan(90^\circ + x) = -\cot(x)$
4. $\tan(180^\circ - x) = -\tan(x)$
5. $\tan(180^\circ + x) = \tan(x)$
6. $\tan(270^\circ - x) = \cot(x)$
7. $\tan(270^\circ + x) = -\cot(x)$
8. $\tan(360^\circ - x) = -\tan(x)$
9. $\tan(360^\circ + x) = \tan(x)$

UNKNOWN TEMPLATE ËxerciseRobox"

Rewrite the above formulas using radians.

UNKNOWN TEMPLATE "Robox/Close"

UNKNOWN TEMPLATE "Trig/NAV"

29 Trigonometric Identities Reference

1. REDIRECT TRIGONOMETRY/FOR ENTHUSIASTS/LESS-USED TRIG IDENTITIES[1]

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FFOR%20ENTHUSIASTS%2FLESS-{}USED%20TRIG%20IDENTITIES

30 Natural Trigonometric Functions of Primary Angles

1. REDIRECT TRIGONOMETRY/SELECTED ANGLES REFERENCE[1]

UNKNOWN TEMPLATE "BookCat"

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/TRIGONOMETRY%2FSELECTED%20ANGLES%20REFERENCE

31 Autoren

Edits	User
1	ADAM MAJEWSKI[1]
5	ADRIGNOLA[2]
2	ALBMONT[3]
4	ALSOCAL[4]
2	AZ1568[5]
1	BANANACLAW[6]
8	CARANDOL[7]
1	DBROADWELL[8]
1	DERBETH[9]

1 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:ADAM_MAJEWSKI
2 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:ADRIGNOLA
3 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:ALBMONT
4 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:ALSOCAL
5 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:AZ1568
6 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:BANANACLAW
7 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:CARANDOL
8 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:DBROADWELL
9 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:DERBETH

7	DOUGLAS W. MITCHELL[10]
1	EL SLAMERON[11]
15	EVIL SALTINE[12]
1	GHE[13]
1	GOTSOMEIDEAS[14]
3	HERBYTHYME[15]
7	ILG[16]
1	JAK86[17]
5	JAMES.SPUDEMAN[18]
47	JAMESCROOK[19]
18	JGUK[20]
2	JOMEGAT[21]
3	KEYTOTIME[22]

10 http://en.wikibooks.org/w/index.php?title=User:Douglas_W._Mitchell
11 http://en.wikibooks.org/w/index.php?title=User:El_Slameron
12 http://en.wikibooks.org/w/index.php?title=User:Evil_saltine
13 http://en.wikibooks.org/w/index.php?title=User:GHe
14 http://en.wikibooks.org/w/index.php?title=User:Gotsomeideas
15 http://en.wikibooks.org/w/index.php?title=User:Herbythyme
16 http://en.wikibooks.org/w/index.php?title=User:Ilg
17 http://en.wikibooks.org/w/index.php?title=User:Jak86
18 http://en.wikibooks.org/w/index.php?title=User:James.Spudeman
19 http://en.wikibooks.org/w/index.php?title=User:JamesCrook
20 http://en.wikibooks.org/w/index.php?title=User:Jguk
21 http://en.wikibooks.org/w/index.php?title=User:Jomegat
22 http://en.wikibooks.org/w/index.php?title=User:Keytotime

2 Kriak[23]
8 Lmov[24]
1 Markjoseph125[25]
1 MatrixFrog[26]
1 Miforbes[27]
4 Nguyenthephuc[28]
4 Philiprbrenan[29]
7 Programmermatt[30]
1 QuiteUnusual[31]
1 R3m0t[32]
3 Recent Runes[33]
2 Robert Horning[34]

23 http://en.wikibooks.org/w/index.php?title=User:Kriak
24 http://en.wikibooks.org/w/index.php?title=User:Lmov
25 http://en.wikibooks.org/w/index.php?title=User:Markjoseph125
26 http://en.wikibooks.org/w/index.php?title=User:MatrixFrog
27 http://en.wikibooks.org/w/index.php?title=User:Miforbes
28 http://en.wikibooks.org/w/index.php?title=User:Nguyenthephuc
29 http://en.wikibooks.org/w/index.php?title=User:Philiprbrenan
30 http://en.wikibooks.org/w/index.php?title=User:Programmermatt
31 http://en.wikibooks.org/w/index.php?title=User:QuiteUnusual
32 http://en.wikibooks.org/w/index.php?title=User:R3m0t
33 http://en.wikibooks.org/w/index.php?title=User:Recent_Runes
34 http://en.wikibooks.org/w/index.php?title=User:Robert_Horning

9	RobinH[35]
31	Robinson0120[36]
2	RockRichard[37]
1	Satish annigeri[38]
1	Sigma 7[39]
1	Srujankumar[40]
1	SuperJ587[41]
15	Swift[42]
1	TakuyaMurata[43]
2	Taxman[44]
15	The Grimm Ripper[45]
6	Thenub314[46]

35 http://en.wikibooks.org/w/index.php?title=User:RobinH
36 http://en.wikibooks.org/w/index.php?title=User:Robinson0120
37 http://en.wikibooks.org/w/index.php?title=User:RockRichard
38 http://en.wikibooks.org/w/index.php?title=User:Satish_annigeri
39 http://en.wikibooks.org/w/index.php?title=User:Sigma_7
40 http://en.wikibooks.org/w/index.php?title=User:Srujankumar
41 http://en.wikibooks.org/w/index.php?title=User:SuperJ587
42 http://en.wikibooks.org/w/index.php?title=User:Swift
43 http://en.wikibooks.org/w/index.php?title=User:TakuyaMurata
44 http://en.wikibooks.org/w/index.php?title=User:Taxman
45 http://en.wikibooks.org/w/index.php?title=User:The_Grimm_Ripper
46 http://en.wikibooks.org/w/index.php?title=User:Thenub314

1 UNCLE G[47]
1 VAN DER HOORN[48]
3 WEYRICK[49]
2 WHITEKNIGHT[50]
1 WIKIBOOKS IS COMMUNISM[51]
19 WISDEN[52]
1 XEROL[53]
2 XOCHIMEC[54]

47 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:UNCLE_G
48 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:VAN_DER_HOORN
49 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:WEYRICK
50 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:WHITEKNIGHT
51 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:WIKIBOOKS_IS_COMMUNISM
52 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:WISDEN
53 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:XEROL
54 HTTP://EN.WIKIBOOKS.ORG/W/INDEX.PHP?TITLE=USER:XOCHIMEC

32 Bildnachweis

In der nachfolgenden Tabelle sind alle Bilder mit ihren Autoren und Lizenen aufgelistet.

Für die Namen der Lizenzen wurden folgende Abkürzungen verwendet:

- GFDL: Gnu Free Documentation License. Der Text dieser Lizenz ist in einem Kapitel diese Buches vollständig angegeben.
- cc-by-sa-3.0: Creative Commons Attribution ShareAlike 3.0 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by-sa/3.0/ nachgelesen werden.
- cc-by-sa-2.5: Creative Commons Attribution ShareAlike 2.5 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by-sa/2.5/ nachgelesen werden.
- cc-by-sa-2.0: Creative Commons Attribution ShareAlike 2.0 License. Der Text der englischen Version dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by-sa/2.0/ nachgelesen werden. Mit dieser Abkürzung sind jedoch auch die Versionen dieser Lizenz für andere Sprachen bezeichnet. Den an diesen Details interessierten Leser verweisen wir auf die Onlineversion dieses Buches.

- cc-by-sa-1.0: Creative Commons Attribution ShareAlike 1.0 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by-sa/1.0/ nachgelesen werden.
- cc-by-2.0: Creative Commons Attribution 2.0 License. Der Text der englischen Version dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by/2.0/ nachgelesen werden. Mit dieser Abkürzung sind jedoch auch die Versionen dieser Lizenz für andere Sprachen bezeichnet. Den an diesen Details interessierten Leser verweisen wir auf die Onlineversion dieses Buches.
- cc-by-2.0: Creative Commons Attribution 2.0 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by/2.0/deed.en nachgelesen werden.
- cc-by-2.5: Creative Commons Attribution 2.5 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by/2.5/deed.en nachgelesen werden. Mit dieser Abkürzung sind jedoch auch die Versionen dieser Lizenz für andere Sprachen bezeichnet. Den an diesen Details interessierten Leser verweisen wir auf die Onlineversion dieses Buches.
- cc-by-3.0: Creative Commons Attribution 3.0 License. Der Text dieser Lizenz kann auf der Webseite http://creativecommons.org/licenses/by/3.0/deed.en nachgelesen werden. Mit dieser Abkürzung sind jedoch auch die Versionen dieser Lizenz für andere Sprachen bezeichnet. Den an diesen Details interessierten Leser verweisen wir auf die Onlineversion dieses Buches.
- GPL: GNU General Public License Version 2. Der Text dieser Lizenz kann auf der Webseite

Bild	Autor	Lizenz
1		GFDL
2		
3	Vermont Timber Works Inc.	cc-by-sa-3.0
4	USER:J JMESSERLY[1] modification of original svg by USER:SIRIUSA[2]	PD
5	Self: Commons user KEYTOTIME[3]	PD
6	Self: Commons user KEYTOTIME[4]	PD
7	QUALC1[5]	GFDL
8		PD
9		PD
10		PD
11		PD
12		PD
13		PD
14	Original uploader was LMOV[6] at EN.WIKIBOOKS[7]	PD
15		PD
16	Original uploader was LMOV[8] at EN.WIKIBOOKS[9]	PD
17	JAMESCROOK[10]	PD
18	JAMESCROOK[11]	PD

1 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AJ%20JMESSERLY
2 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3ASIRIUSA
3 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AKEYTOTIME
4 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AKEYTOTIME
5 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AQUALC1
6 HTTP://DE.WIKIBOOKS.ORG/WIKI/%3AWIKIBOOKS%3AEN%3AUSER%3ALMOV
7 HTTP://EN.WIKIBOOKS.ORG
8 HTTP://DE.WIKIBOOKS.ORG/WIKI/%3AWIKIBOOKS%3AEN%3AUSER%3ALMOV
9 HTTP://EN.WIKIBOOKS.ORG
10 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AJAMESCROOK
11 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AJAMESCROOK

19	JAMESCROOK[12]	PD
20	JAMESCROOK[13]	PD
21		GFDL

12 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AJAMESCROOK
13 HTTP://DE.WIKIBOOKS.ORG/WIKI/USER%3AJAMESCROOK