

rectangled Cone whose Altitude is equal to AR . And its Curve Surface will be equal to half the Surface of a Globe whose Radius is AR . So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius AR .

The Description of the Rular and Wheel, *Fig. 2.* is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig. 5.* where AB is a brass Rular; w the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. s represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixt Rular. p is a Stud for convenient holding the Rular in its Motion.

Note, *Most of these Properties of this Curve by the Name of la Traçtrice, are to be found in a Memoire of M. Bome among those of the Royal Academy of Sciences for the Year 1712. but not publish'd till 1715: Whereas this Paper of Mr. Perks was produced before the Royal Society in May 1714, as appears by their Journal.*

VI. *An Account of a Book entituled Methodus Incrementorum, Auctore Brook Taylor, LL.D. & R. S. Secr. By the Author.*

WHEN I apply'd my self to consider throughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir *Isaac Newton* our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Fluxions

ions of Quantities are proportional to the nascent Increments of those Quantities: in order to understand that Method thoroughly, I found it necessary to consider well the Properties of Increments in general. And from those Properties I saw it would be easy to draw a perfect Knowledge of the Method of Fluxions: for if in any case the Increments are supposed to vanish and to become equal to nothing, their Proportions become immediately the same with the Proportions of the Fluxions. In this Method I consider Quantities, as formed by a continual Addition of parts of a finite Magnitude, and those parts I call the Increments of the Quantities they belong to, because that by the Addition of them the Quantities are increased. These parts being consider'd as formed in the same manner by a continual Addition of other parts; thence follows the Consideration of second Increments; and so on to third, fourth, and other Increments of a higher kind. For Example, if x stands for any Number in the Series 0. 1. 4. 10. 20. 35, &c. in which the Numbers are formed by a continual Addition of the Numbers in the Series 1. 3. 6. 10. 15, &c. then the Numbers in the latter Series are call'd the Increments of the Numbers in the foregoing Series; thus, for Example, if to the third Number (4) in the first Series, I add the corresponding third Number (6) in the second Series, I shall produce the next, that is the fourth Number (10) in the first Series, and so the rest. Any Number in the first Series being call'd x , the corresponding Number (which is its Increment) in the second Series I express by x . And these Numbers x being form'd in the same manner by the Numbers in the Series 1. 2. 3. 4. 5, &c. I call these last Numbers x , they being the first Increments of the Numbers x , and the second Increments of the Numbers x ; and so on. Hence having given any Series of Numbers that are call'd by a general Character x , their Increments are found by taking their

their Differences; thus in the present Example, the first Increments x in the Series 1. 3. 6. 10. 15, &c. are found by taking the Differences of the Numbers x in the Series 1. 4. 10. 20. 35, &c. and the second Increments x in the Series 1. 2. 3. 4. 5, &c. are found in the like manner, by taking the Differences of the Numbers x , and so of the third and other Increments. This Method consists of two parts; One is concerned in shewing how to find the Relations of the Increments of several variable Quantities, having given the Relations of the Quantities themselves; and the Other is concerned in finding the Relations of the Integral Quantities themselves freed from the consideration of their Increments, having given the Relations of the Increments: either simply, or they being any how compounded with their Integral Quantities. In the Method of Fluxions Quantities are not consider'd with their parts, but with the Velocities of the Motions they are supposed to be formed by; or to speak more accurately, they are consider'd with the Quantities of the Motions by which they are supposed to be generated; for the Fluxions are proportional to the Velocities, only when the moving Quantities, which produce the flowing Quantities consider'd, are equal. These Quantities of Motion, or Velocities when the moving Quantities are equal, are what *Sir Isaac Newton* calls Fluxions. As in the Method of Increments there are second, third, and other Increments; so in the Method of Fluxions there are second, third, and other Fluxions; the Fluxions themselves being consider'd as Quantities that are formed by Motion, the Quantity of which Motion is their Fluxions. As the Method of Increments consists of two Parts; one being concern'd in finding the Increments from the Integrals given, and the other in finding the Integrals having given the Increments; so does the Method of Fluxions consist of two Parts; the one shewing how to find the Fluxions, having the

the Fluents given; and the other shewing how to find the Fluents freed from Fluxions, having given the Relations of the Fluxions, whether compounded with their Fluents or otherwise. The Principles of this Method may all be drawn directly as a Corollary from the Principles of the Method of Increments. For Sir *Isaac Newton* having demonstrated (*Phil. Nat. Princ. Math. Sect. 1.* and in the Beginning of his Treatise *De Quadratura Curvarum*) that the Fluxions of Quantities are proportional to their nascent or evanescent Increments, if in any Proposition relating to Increments, you make the Increments to vanish and to become equal to nothing, and for their Proportion put the Fluxions, you will have a Proposition that will be true in the Method of Fluxions. This is but a Corollary to Sir *Isaac Newton's* Demonstration of the Fluxions being proportional to the nascent Increments. For this reason, to make the Method of Fluxions to be understood more throughly, I thought it proper to treat of these two Methods together, and I have handled them promiscuously as if they were but one Method. Some people, because that the Fluxions are proportional to the nascent Increments of Quantities, have thought that by the Method of Fluxions Sir *Isaac Newton* has introduced into Mathematicks the Consideration of infinitely little Quantities; as if there were any such thing as a real Quantity infinitely little. But in this they are mistaken, for Sir *Isaac* does only consider the first or last Ratio's of Quantities, when they begin to be, or when they vanish, not after they are become something, or just before they vanish; but in the very moment when they do so. In this case Quantities are not consider'd as infinitely little; but they are really nothing at the time that Sir *Isaac* takes the Proportions of their Fluxions; and the Truth of this Method is demonstrated from the Principles of the Method of Increments, in the same manner as the Ancients demonstrated

ted their Conclusions in the Method of Exhaustions, by a *Deductio ad Absurdum*.

Having premised thus much in general concerning the two Methods here treated of, to come to a particular Description of this Book; In the Preface I give a short Description of the Method of Increments, and an Account of Sir *Isaac Newton's* Notion of the Fluxions which I have already spoke of. The Book consists of two Parts, and contains 118 Pages in *Quarto*; the Propositions being number'd throughout from the Beginning. In the first Part I explain the Principles of both Methods: and in the second Part I shew the Usefulness of them in some particular Examples.

After having explain'd the Notation I make use of in the Introduction, in the first Proposition I explain the direct Method both of Increments and of Fluxions. The second Proposition shews how to transform an Equation wherein Integrals and their Increments, or wherein Fluents and their Fluxions are concern'd; so as in the Room of the Integrals or Fluents, to substitute their Compliments to a given Quantity with their Increments or their Fluxions, they increasing in a contrary Sense to the Quantities in the first Supposition. In the third Proposition I shew how to transform a fluxional Equation, so as to change the Characters of the Fluents, making that Quantity to flow uniformly which in the first Supposition flow'd unequally, having second, third and other Fluxions, and making that Quantity which in the first Supposition flow'd uniformly, now to flow unequally, so as to have second and third Fluxions, &c. This Proposition is of great use in the inverse Method, when you would invert the Expression of the Relation of the flowing Quantities; for Example, if in the Supposition z flows uniformly and x variably, by the inverse Method of Fluxions you will find x express'd by the Powers of z : but if you would find z

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express'd by the Powers of x , you must then transform the Equation by this Proposition. Sir *Isaac Newton* and Mr. *de Moivre* do this by the Reversion of Serieses; but I take this to be the more proper and more genuine Method of doing it directly. In the fourth and fifth Propositions are explain'd the Method of judging of the Nature and Number of the Conditions that may accompany an Incremental or a Fluxional Equation. This is a Circumstance that I don't find to have been explain'd by any one before, and the Propositions are something intricate; wherefore it will not be improper to explain this Matter a little more at large. Suppose then that z and x are two variable Quantities, and supposing z to increase uniformly by the continual Addition of its constant Increment z , (according to the Notation I make use of in this Book) suppose $x + z = x$. Then if it be propos'd as a Problem to find the Value of x , express'd by the Powers of z , and quite freed from the Increments; by the fourth Proposition there may be three Conditions added to this Problem. The Demonstration of this is taken from the Formation of the Integrals by a continual Addition of their Increments. Suppose that all the Values of $z, x, x, x, x, \&c.$ were set down in order in so many Columns, and that at the Head of the Table, the corresponding Values of $z, x, x, x, x, \&c.$ were express'd by the Symbols $a, c, c, c, c, \&c.$ Then by means of the given Equation $x + z = x$, or $x = x - z$, will $c = c - a$, whence the second corresponding Values of z, x, x, x, x will be $a + z, c + c, c + c, c + c - a (= c + c)$ and $c + c - a - z$ (by the Eq.) Whence again the third Values are $a + 2z, c + 2c + c, c + 2c + c - a (= c + 2c + c)$ $c + 2c - 2a + c - z$, and $c + 2c + c - a - 2z$, and so you may proceed to find the fourth, fifth, and all the

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the other Values of $z, x, \dot{x}, \ddot{x}, \ddot{\dot{x}}$. But by what is already set down, it is evident that all their Values will be express'd by a and z , and the three Symbols c, \dot{c}, \ddot{c} : and consequently all the Values of the rest of the Increments of x , viz. $\dot{x}, \ddot{x}, \ddot{\dot{x}}$, &c. will be express'd by the same Symbols. Whence it follows that to determine the Values of the Symbols c, \dot{c}, \ddot{c} , there may be taken three Conditions relating to the Values of $x, \dot{x}, \ddot{x}, \ddot{\dot{x}}, \ddot{\dot{\dot{x}}}$, &c. promiscuously, as the fourth Proposition directs. The same Rule holds good in the Method of Fluxions. For Example, having given the Equation $a z^2 + x^2 \dot{z} = 0$, if it be propos'd to describe the Curve that it belongs to; by the fourth Proposition it may also be required as a Condition, that the Curve shall pass through two given Points; that it shall touch two given Lines; that it shall pass thro' a given Point, and when it cuts a given Line shall have a given Degree of Curvature; or that it shall have any other two Circumstances that depend upon the Values of the third, fourth or other Fluxions. These Conditions that attend Incremental or Fluxional Equations, I don't know to have been sufficiently taken notice of by any Body: but they ought well to be attended to in the Inverse Methods; the Solutions of particular Problems being never perfect, unless there be provision made for the satisfying of them, by the indetermin'd Coefficients in the Equation that contains the Solution of the Problem. Examples of this may be seen in Prop. 17 and 18, where I give the Solution of the Problems concerning the *Isoperimeter*, and the *Catenaria*.

The sixth Proposition contains the general Explanation of the Inverse Method both of Fluxions and of Increments, which consists in the Solution of this Problem. Having given the Relations of the Increments, or of the Fluxions of several Quantities, whether they be considered

red with their proper Integrals or with their proper Fluents or not ; to find the Relations of the Integrals or of the Fluents, freed from their Increments or from their Fluxions. The Direction I have given for finding the Solution in finite Terms is but tentative. And I must confess I know of no other Method that is general for all Cases. For I can find no certain Rule to judge in general, whether any proposed Equation, involving Increments or involving Fluxions, can be resolved in finite Terms. For this Reason we are obliged to seek the general Solution in infinite Serieses ; which when they break off, or when they can any way be reduced to finite Terms, they then contain the Solutions which we always hope for. The Method of finding these Serieses is explain'd in the eighth Proposition, and that is by means of a Series that is demonstrated in the seventh Proposition. And this I take to be the only genuine and general Solution of the inverse Methods. For in this Solution you always have those indetermined Coefficients, which are necessary to adapt the Equation that is found to the Conditions of the Problem proposed. For want of this Circumstance all other Methods are imperfect ; and particularly Sir *Isaac Newton's* Method of finding Serieses by a Rular and Parallelograms labours under this Difficulty, because it brings no new Coefficients into the resulting Equation, which may afterwards be determined by the Conditions of the Problem. However because this Method is very ingenious and very elegant, I thought it proper to explain it in the following (*viz.* the 9th) Prop. The 10th, 11th, and 12th Propositions conclude the first Part, and in them I treat of the manner of finding the Integral or the Fluent, having given the Expression of a particular Increment, or of a particular Fluxion of it ; without being involved with the Integrals, or with the Fluents, or with any other Increments, or with any other Fluxions of

it. This is a particular Case of the Inverse Method, but for its great usefulness I thought it deserved particularly to be taken notice of. This Problem is treated of in general in the 10th Proposition. The Method of solving it in finite Terms is only tentative; and when that does not succeed, recourse must necessarily be had to the Solution by a Series in the 8th Proposition. In the 11th and 12th Propositions I have shew'd how Serieses may be conveniently found, in some particular Cases when Fluxions are proposed.

In the second Part I have endeavour'd to shew the Usefulness of these Methods in the Solution of several Problems; The 13th Proposition is much the same with *Sir Isaac Newton's Methodus Differentialis*, when the Ordinates are at equal Distances: and in an Example at the End of this Proposition I have shew'd how easily *Sir Isaac Newton's* Series for expressing the Dignity of a Binomial may be found by this Incremental Method. The 14th Proposition shews in some measure how this Method may be of use in summing up of Arithmetical Serieses. In the 15th Proposition I shew by some Examples how the Proportions of the Fluxions are to be found in Geometrical Figures; from whence immediately flows the Method of finding the Radiuses of their inosculating Circles, the Invention of the Points of contrary Flexure, and the Solution of other Problems of the like nature. In the 16th Proposition I shew how the Method of Fluxions is to be applied to the Quadrature of all sorts of Curves. In the following Proposition I give a general Solution of the Problem of the *Isoperimeter*, which has been treated of by the two famous Mathematical Brothers the *Bernullii's*. In the 18th Proposition I give the Solution of the Problem about the *Catenaria*, not only when the Chain is of a given Thickness every where, but in general, when its Thickness alters according to any given Law. In the fol-

following Proposition I shew the *Fornix* or Arch which supports its own Weight to be the same with the *Catena-ria*. In the two next Propositions I shew how to find the Figures of pliable Surfaces which are charged with the Weight of a Fluid. In the 22^d and 23^d Propositions I treat of the Motion of a Musical String, and give the Solution of this Problem: To find the Number of Vibrations that a String will make in a certain time, having given its Length, its Weight, and the Weight that stretches it. This Problem I take to be entirely new, and in the Solution of it (in the last part of Prop. 23.) there is a remarkable Instance of the Usefulness of the Method of first and last Ratio's. The 24th Proposition gives the Invention of the Center of Oscillation of all Bodies; and in the 25th Proposition I have given the Investigation of the Center of Percussion. It is known that this Problem is solved by the same *Calculus* as the foregoing; wherefore it is generally thought that these two Centers are the same. But that is a Mistake, because the Center of Oscillation can be but one Point; but the Center of Percussion may be any where in a certain Line, which this Proposition shews how to find. There is an Error in this Proposition, which I was not sensible of till after the Book was publish'd, wherefore I take this Opportunity of correcting of it. It does not affect the Reasoning by which I find the Distance of the Center of Percussion from the Axis of Rotation; but it is this, that I supposed the Center of Percussion to be in the Plane passing thro' the Center of Gravity, and perpendicular to the Axis of Rotation: which is a Mistake. It is corrected by the following Proposition.

P R O P.

P R O P. P R O B,

To find the Distance of the Center of Percussion from the Plane passing thro' the Center of Gravity and perpendicular to the Axis of Rotation.

S O L U T I O N.

Let the sixth Figure be supposed in the Plane passing thro' the Axis of Rotation, and in which the Center of Percussion is sought.

Let AB be the Axis of Rotation, AGC be the Intersection of this Figure with the Plane passing thro' the Center of Gravity, and perpendicular to the Axis of Rotation, G be the Point whereon a Line, rais'd perpendicular to this Figure, will pass thro' the Center of Gravity; BE be a Line parallel to AG wherein is the Center of Percussion. Then to find the Distance AB , let p stand for an Element of the Body proposed standing perpendicularly on any point D . Draw DC perpendicular to AGC , and AB will be equal to the Summ of all the Quantities $p \times GC \times CD$ taken with their proper Signs, divided by the Body it self multiplied into the Distance AG .

Having thus found the Distance AB , suppose the Plane of the Figure in *Prop. 25.* to cut the present Figure at right Angles in the Line BE , and the Center of Percussion will be rightly determined by that Proposition.

The 26th Proposition shews how to determine the Density of the Air at any Distance from the Center of the Earth supposing the Density always to be proportional to the compressing Force, and that the Power of Gravitation is reciprocally as the Distances from the Center of the Earth.

The last Proposition shews how to find the Refraction of a Ray of Light in its passage thro' the Atmosphere, upon the Supposition that Light is a Body, and that the Refraction of it is caused by the Attraction of the Bodies the Rays approach to. In this Proposition there is a remarkable Instance of the Usefulness of the Method of Increments in finding the Coefficients of a Series, which according to the Values of a certain Symbol, as n , expresses both all the Fluents, and all the Fluxions of a certain Quantity.

II. Ludovici Ferdinandi Marfilii *Dissertatio de Generatione Fungorum.* Rom. 1714. 4to.

THIS Author tells us that he gave his youthful Inclinations to the Study of the Mathematicks and Observations of Nature, under the Tuition of the celebrated *Malpigi*, and *Lelius Triumphettus* Botanick Professor of *Bononia*: and amongst the various Productions of Nature, his chief Delight was in the Contemplation of the sudden Growth and various sorts of *Mushrooms* which both the Earth and Trees brought forth. Of the first Kind he observed the greatest Number to arise in Camps, produced from the Horses Dung, and are commonly called *Prataioli*.

In the Years 1699 and 1700, being then in *Croatia* and *Transylvania*, in the Armies there, he made a large Volume of Designs of *Fungi*, which he sent to *Triumphetti* to put in order, who added a great Number to them of such as he found about *Bononia*; yet after all the most diligent Search, he could never find them to produce any Seed either in their Gills or other Parts.

The Origin and Generation of *Mushrooms* he says is not easy to demonstrate, since both the Antients and Moderns dis-

disagree very much about it. The late Botanists seem to be of three different Sentiments concerning their Produce. Mr. Ray, Dr. Sherrard, Mr. Doody, Boccone and Mentzelius having observed some *Mushrooms* to have had Seed, were of opinion others might have the same Original. *Clusius* and *John Baptista Porta* had in some also observed their Seed: Others, viz. *Sharrock* and the accurate *Malpigi*, who could not find any Seed in them, altho' with the Assistance of Microscopes, did suppose they might be produced by Pieces of themselves, carried by the Winds from place to place, as other Plants are by slips and Offsets.

The third Opinion, which he says most agree in, is that they arise from Putrefaction, or a Mixture of certain Salts, Sulphur and Earth impregnated with the Dung of Beasts.

The *Fungus seminifer Campaniformis Mentzelii*, &c. being the Mushroom which first gave the occasion of the Opinion of their having Seed, this nice Author has accurately figured and observed, and supposes with others that these seedlike Bodies may be the *Ovaria* of some Insects; and the rather because they are so very large in proportion to the smallness of the *Mushroom*: and that they had often been sowed by Dr. *Amadoes* a curious Botanist, without any Success towards raising them. From whence he concludes these Bodies ought to have another Denomination than Seed; neither is he of the Opinion that they are produced by parts of themselves.

In his Division of *Mushrooms* he first treats of the *Truffles* and their Increase, Situation and Soyl, Colour, Taste and Consistence. He next proceeds to soft *Mushrooms*, such as he observed in his own Garden; which having in the Spring been meliorated with Horse-dung, about the middle of *June* there sprung up divers of that sort which the *Italians* call *Prataiuoli*, amongst a Bed of *Lettice*. These continued till near the midst of *August* before they went off.

off. Of these and some other Kinds he accurately figures the first Shootings and Fibres.

His next Tribe are such as grow from Wood, but yet are themselves soft. Of these he observes three Kinds; the first a large one in his Window, out of a piece of Firwood which it had often rained on; with two smaller sorts from some rotten Boards in his Garden. All these he figures both in their natural and divided States, as also Microscopically

Treating of hard woody *Mushrooms* (of which he also gives you some accurate Figures) he observes they rarely appear on the Trees, in *Germany* and *Croatia*, before they are twenty or thirty Years old; but most commonly when forty or fifty: and the Original of them he attributes chiefly to the Rottenness of the Wood, and says they generally break out in the Spring, when the Leaves begin to shoot. And that usually they grow below the middle of the Trees, and are cause of so much Decay in them, that they often die in three or four Years.

It may not here be amiss to subjoin what Dr. *Lancisus* communicates to our Author, concerning the *Lapis Fungarius*, viz. that altho' this *Mushroom-producer* has the Name of a Stone, it ought not to be reckoned of that Genus, it being really no other than a Mass or *Congeries* of Roots, Seeds and Juices coagulated with Earth into, as it were, a stony Substance. Upon which pouring Water and setting it in a warm Place; it loosens its hardened Substance; and by mollifying its Fibres and moistning its concrete Juices, out of the Cliffs and Chinks thereof the *Mushrooms* spring, as they do in other places from simple Dung and loose Earth. And it is also farther to be noted, that when this stony Mass has thus yielded these its Offspring, the Remains grows light, porous and decay'd, its nutritive Juices being then exhausted.