

Satellites that come to pass conformable to the calculus of *Cassini*, and differ days and hours from the calculus and predictions made upon the hypotheses of *Galilæi*: Besides that there should happen a great many which do not happen according to the system of *Cassini*. *E.g.* according to the hypothesis of *Galilæi*, the fourth of the Satellites should have more than 90 Eclipses in a year, of the duration of three or four hours; but according to the system of *Cassini*, the same Satellit will be three or four years without suffering any Eclipse. Which proceeds from nothing but the false situation of the Orbs supposed by *Galilæi*; as the great difference of the time of the Eclipses that happen depends from this, that neither *Galilæo* nor the other Astronomers do separate from the proper motion of the Satellites the appearances which do befall it by that of *Jupiter* about the Sun. And therefore 'tis, that they have taken for a simple and equal motion a motion compounded of an equal and unequal; whence they have slipped into an error about the Mean motions, which in progress of time hath so increased, that the Configurations drawn from their hypotheses for that time have almost no likeness at all with those that are observed.

These old hypotheses were therefore far off from serving to find the Longitudes, as their Authors intended them; since it was impossible for them nor only to observe the Eclipses of the Satellites for some years to the nearness of an hour, but even to make us know and distinguish at this time one Satellit from another, whereas by the System of Signor *Cassini* one may predict for many years to come the Eclipses of the Satellites with as much preciseness, as those of the Sun and Moon by the Astronomical Tables.

Methodus directâ & Geometrica, cujus ope investigantur Aphelia, Eccentricitates, Proportioneseque orbium Planetarum primariorum, absque supposita æqualitate anguli motûs, ad alterum Ellipseos focum, ab Astronomis hætenus usurpatâ. Auth. *Edmundo Halley Jun.* è Collegio Reginae Oxon.

Motus Terræ annuus per Eclipticam, opticam inæqualitatem inducit motibus cæterorum planetarum, Astronomis Copernicani nomine Parallaxeos orbis notissimam; quam quidem inæqualitatem, ex observationibus non multâ operâ datam; methodi sequentis basin firmissimam constituo; ubi præter observata nihil aliud supponitur, quàm quòd orbis Planetarum sint Ellipses, quòdque Sol in foco, omnium orbibus communi, sit constitutus, & denique, quòd tempora periodica singulorum

singulorum ita innotescant, ut non sentiatur error aliquis, saltem in duobus vel tribus revolutionibus: His concessis, motus Terræ, pro cæteris Planetis necessariò requisitus, primò aggregandus est.

Tab.I.
Fig.2.

Sit S Sol; ABCDE, orbis Terræ; P, Planeta Mars, (qui in hanc rem plurimis de causis longè præferendus est;) & primò observetur verum tempus & locus, quo Mars opponitur Soli; tunc enim Sol & Terra coincidunt in lineam rectam cum Marte; vel, (quod fere semper accidit) si habuerit latitudinem, cum puncto, ubi perpendicularis à Marte demissa in planum Eclipticæ incidit. Sic in Schemate, S, A, & P sunt in linea recta; deinde post 687 dies, Mars revertitur ad idem punctum P, ubi in priori observatione Soli opponebatur; Terra verò, cum non revertatur ad A, nisi post 730½ dies, in B, Solem respicit in linea SB, Martem verò in linea BP, & observatis longitudinibus Solis & Martis, omnes anguli Trianguli PBS dantur, & suppositâ PS 100000, in iisdem partibus invenitur longitudo lineæ SB; pari ratione post alteram Martis periodum, Terra existente in C invenitur linea SC, nec absimiliter lineæ SD, SE, SF; differentiæque observatorum locorum Solis, sunt anguli ad Solem ASB, BSC, CSD, DSE: Sic tandem ventum est ad hoc problema Geometricum: Datis tribus lineis, in uno Ellipseos foco coeuntibus, tam longitudine quàm positione, invenire longitudinem transversæ diametri, cum distantia focorum: Cujus resolutio extenditur etiam ad reliquos planetas, si, post Theoriam motus Terræ cognitam, scrutemur (secundum methodum præpositam à Reverendiss. Episcopo Sarisburiensi in Astronomia ejus Geometricâ lib. 2. part. 2. cap. 5.) tres distantias planetæ alicujus à Sole in positionibus suis. Quoniam verò Rev. Episcopus supponit planetam ita ferri in orbe suo, ut equalibus temporibus æquales angulos ad focum alterum Ellipseos absolvat, & ei calculum suum superstruit, non incongruum videtur, ostendere, quomodo id ipsum fieri possit absque ista suppositione, quam observatio nos rejiciendam monet.

Tab.I.
Fig.3.

Sit S, Sol; ALBK, orbis Terræ; P, Planeta, vel Punctum in plano Eclipticæ, ubi perpendicularis, à planeta demissa, incidit; AB linea Apfidum orbis Terræ: Observentur primò Planeta, in P, longitudo & latitudo, simulque Solis Longitudo à Terra in K; & post periodum ejusdem planetæ, Terra existente in L, observentur denuo positiones Planetæ Solisque, ut prius: Jam ex observatis longitudinibus Solis & Aphelii Terræ, anguli ASK, ASL dantur, & consequenter latera SK, SL: (Nam si angulus Anomalie coequata sit acutus, proportio est, ut differentia distantie mediæ & Co-sinus anguli in Eccentricitatem ducti, ad distantiam Apheliam, ita Perihelia distantia, ad distantiam Planetæ à Sole in datâ Anomaliâ: quod si angulus fuerit obtusus, primus terminus proporti-

onis

onis est summa duarum partium, quarum in priori analogia fuit differentia : Hujus Theorematis demonstrationem neminem Analytices modicè peritum latere posse arbitror, & idcirco ei supersedeo :) Jam in Triangulo KSL dantur latera KS, LS, & angulus KSL, queruntur Latus KL, & anguli SKL, SLK : Deinde in Triangulo KLP, dantur KL, KLP, differentia observatarum Longitudinum planetæ, & PKL differentia angulorum SKL ultimò inventi, & SKP Elongationis Planetæ à Sole in prima observatione, queritur LP : Tum in Triangulo LSP, latera LS, LP, & angulus PLS elongatio Planetæ à Sole in secunda observatione, dantur; latus SP & angulus LSP requiruntur, quibus inventis, ut SP ad LP, ita Tangens Latitudinis observatæ ex L, ad Tangentem Inclinationis sive Latitudinis ad Solem; & ut Co-sinus Inclinationis ad Radium, ita SP curtata distantia, ad veram distantiam planetæ à Sole : Sic tandem invenimus positionem & longitudinem desideratam. Jam restat ut ostendam, quomodo ex datis tribus distantiiis à Sole cum angulis interceptis, invenienda sit media distantia cum Eccentricitate Ellipseos.

Sit S Sol, & SA, SB, SC tres distantie in debita positione, ductisque AB, BC, sit AB distantia focorum Hyperbolæ, & SA-SB=EH transversa diameter; quibus positis, describatur linea ista Hyperbolica, cujus focus interior est punctum A, extremitas lineæ longioris SA: Pari modo sist B, C, foci alterius Hyperbolæ, cujus diameter SB-SC=KL; ex quibus describatur linea Hyperbolica focum habens interiorē in puncto B: Dico has duas Hyperbolas sic descriptas sese intersectare in puncto F, qui est alter Ellipseos quæsitæ focus, ductæque lineæ FA, FB, vel FC, SA+FA, SB+FB vel SC+FC æquabitur transverse diametro, & SF est distantia focorum: quibus positis descriptio Ellipseos facillima est. Cum verò hujus constructionis ratio non omnibus ita facile percipiatur, non abs re erit, illustrationem ejus aliquam afferre; Ideò dico, quòd ex notissima Ellipseos proprietate SB+FE=SA+FA, & transpositis æquationis partibus FB-FA=SA-SB, ita ut etiamsi FB & FA nos lateant, earum tamen differentia equalis sit SA-SB, hæc est, EH, cùmque sit ex natura Hyperbolæ, ut habeat quasvis duas lineas à suis focis ad quodvis punctum in sua curva constanter differentes quantitate transverse diametri; constat, punctum F esse alicubi in curva Hyperbolæ, cujus diameter transversa æquatur SA-SB, & Foci A, B: Pari modo demonstrari potest punctum F esse in Hyperbola cujus diameter est SB-SC, & foci B, C. Ergo necesse est, ut sit in intersectione duarum istarum Hyperbolarum, quæ, cùm sese intersectent in unico solum puncto, clarè ostendunt ubi sit Focus alter Ellipseos quæsitæ.

Jam ut id ipsum Analyticè expediatur, puta factum, sique $FE=1$, $SA-SB=FB-FA=b$, $AB=c$, $SB-SC=FC-FB=d$, $BC=f$, sitque Sinus anguli $ABC=s$, Co-sinus ejusdem $=s$.

Tab.I.
Fig.4.

Tum

Tum ut c ad b, ita 2a-b ad $\frac{2ab-bb}{c}$ & $\frac{2f-bb+c}{2c} = BD$ per 36. 3 Eucl.
 & ut f ad d, ita 2a+d ad $\frac{2ad+dd}{f}$ & $\frac{ff-2ad-dd}{2f} = BG$ per eandem, & ut mi-
 nuatur labor calculi sit $\frac{cc-bb}{2c} = g$ & $\frac{b}{c} = h$, similiter sit $\frac{ff-dd}{2f} = k$
 Per 12 & 1 lib. 2. Eclid. 3 & $\frac{d}{f} = 1$, tunc $DE = g+ha$, & $BG = k-la$; & quoniam in omni

Triangulo $\left. \begin{array}{l} \text{obtusangulo} \\ \text{acutangulo} \end{array} \right\} \text{quadratum basis aequatur} \left. \begin{array}{l} \text{summa} \\ \text{differentia} \end{array} \right\}$

summa quadratorum laterum, & dupli rectanguli laterum in Co-
 sinum anguli comprehensi ducti, erit $gg+2gha+hhaa+kk-2kla+llaa$
 $+2gks-2glsat+2khsa-2hlsaa$ aequalis quadrato DG: Sed DG aequalis
 est sinui Anguli DFG vel DBG in a, id est FB ducto, (est enim qua-
 drilaterum FBDG circulo, cujus diameter est FB, inscriptum;) ideo
 $SSaa=gg+2gha+hhaa+kk-2kla+llaa+2gks-2glsat+2khsa-2hlsaa$;
 quae aequatio facile resolvitur, cum non excedat quadraticam affectam,
 semperque componitur ex istis Quadratis & Rectangulis; signa tamen
 + & -- ob diversam trium linearum constitutionem multâ cautione
 sunt rectangulis adhibenda. Nostram equationem aptavimus figu-
 ra IV, sed in alio quovis casu non erit difficile attendenti, ex his
 quae superius tradidimus, similem constituere. Sic tandem absolvi
 propositum meum, & ostendi, quomodo ex tribus locis Heliocentricis
 Planetae, & distantibus à Sole observatis, describi possit Orbita istius
 Planetae; quod non nisi quinque talibus observationibus haëtenuſ effe-
 ctum vidimus.

