V. *De* Figuris *quas* Fluida *rotata induere possunt*, *Problemata duo*; *cum conjectura de* Stellis *quæ aliquando prodeunt vel deficiunt*; *& de* Annulo Saturni. *Authore* Petro Ludovico De Maupertuis, *Regiæ Societatis* Londinensis, *& Academiæ Scientiarum* Parisiensis *Socio.*

PROBLEMA I.

INVENIRE Figuram Sphæroidis fluidi circa axem rotantis, posito quod fluidi partes versus centrum attrahantur secundum aliquam distantiæ a centro dignitatem.

SOLUTIO. Fig. 1.

Sit P Q axis revolutionis, & P A Q B sectio Sphæroidis per axem; jam cum partes fluidi inter se quiescant, columnarum unaquæque C D idem habebit pondus versus C; considerando ergo e columnis unam C D quæ efficit cum C P datum angulum cujus sinus $= h$ pro radio $= 1$, & quæ ex infinitis cylindrulis G g componitur; cylindruli cujusque pondus versus C quæro.

Gravitas absoluta in A cum sit data & $= p$, pro habenda gravitate in G, erit $p . p' :: CA^n . CG^n$; unde habebitur gravitas in G seu $p' = \dfrac{p . CG^n}{CA^n}$.

Sed cum propter revolutionis motum pars quævis fluidi repellitur vi centrifuga secundum G H; & cum in in mobilibus quæ contemporaneas circulationes absolvunt vires centrifugæ sint ut circulorum descriptorum radij ; si vis centrifuga in A sit data & $= f$, pro habenda vi centrifuga in G, erit $f . f' :: CA . LG =$ (ob $LG . CG :: h . 1$) hCG; unde habebitur vis centrifuga in G seu $f' = \frac{fh . CG}{CA}$: Sed vis hæc cum secundum G H agat decomponenda est in duas vires K H & G K ex quibus una tantum G K partem aliquam vis secundum G C tollit. Habebitur ergo vis illa G K dicendo $GH . GK$ vel $1 . h :: \frac{fh . CG}{CA}$. $f' = \frac{fhh . CG}{CA} =$ vi cylindrulum Gg versus D trahenti. Vis ergo cylindrulum Gg versus C trahens erit tantum $\frac{p . CG^n}{CA^n} - \frac{fhh . CG}{CA}$; & pondus cylindruli versus C, erit $\left(\frac{p . CG^n}{CA^n} - \frac{fhh . CG}{CA}\right) Gg$. Jam columnæ C G ex cylindrulis istis conflatæ pondus erit $\left(\frac{p . CG^n}{CA^n} - \frac{fhh . CG}{CA}\right) Gg$ quod cum Gg sit Elementum ipsius C G, dabit pro pondere columnæ C G, $\frac{p . CG^{n+1}}{n+1 . CA^n} - \frac{fhhCG^2}{2CA}$; & pro pondere totius columnæ C D, $\frac{p . CD^{n+1}}{n+1 . CA^n} - \frac{fhh . CD^2}{CA}$, quod efficere debet pondus constans A.

Si ergo vocentur $CA = a$, $CD = r$, habebitur $\frac{pr^{n+1}}{n+1 . a^n} - \frac{fhhrr}{2a} = A$. Et cum æquatio hæc, quæcunque ſit h, ſemper obtineat, jam ſi h pro indeterminata ſumatur, æquatio præcedens relationem dabit inter radium quemvis C D & ſinum anguli quem cum axe P Q facit.

Nunc determinanda eſt quantitas conſtans A. Ut æquatio præcedens, ſit ad ſectionem ſphæroidis illius cujus ſemi axis C A $= a$, oportet, quando angulus D C P eſt rectus, vel quando $h = 1$, ſit $r = a$; tunc ergo habetur $\frac{p\, a^{n+1}}{n+1 \,.\, a^n} - \frac{f\, a\, a}{2\, a} = A$, vel A $=$ $\left(\frac{2p - nf - f}{2 \,.\, n+1}\right) a$.

Et ſic æquatio correcta, erit $\frac{p\, r^{n+1}}{n+1 \,.\, a^n} - \frac{f\, h\, h\, r\, r}{2\, a}$ $= \left(\frac{2p - nf - f}{2 \,.\, n+1}\right) a$ vel $2\, p\, r^{n+1} - (n+1)$ $f\, h\, h\, a^{n-1}\, r\, r = (2p - nf - f)\, a^{n+1}$.

Æquatio hæc, omnium ſphæroidum ſectiones determinat quæcunque ſit dignitas diſtantiæ, ſecundum quam fit attractio; unâ tantum excepta hypotheſi in qua attractio foret in ratione ſimplicis diſtantiæ a centro inverſa.

In hoc caſu recurrendum erit ad $\left(\frac{n \,.\, CG^n}{CA^n} - \frac{f\, h\, h \,.\, CG}{CA}\right) G\, g$ quod tunc fit $\left(\frac{p \,.\, CA}{CG} - \frac{f\, h\, h \,.\, CG}{CA}\right)$ $G\, g$, cujus fluens non niſi per Logarithmos habetur, & prodit $p \,.\, CA \log. CG - \frac{f\, h\, h \,.\, CG^2}{2\, CA} = A$; vel pro pondere totius columnæ $p\, a \log. r - \frac{f\, h\, h\, r\, r}{2\, a} = A$.

Ut

Ut corrigatur hæc æquatio, oportet ut quando $h = 1$, ſit $r = a$; tunc ergo habetur $p\, a \log. a - \frac{f\, a}{2} = A$; & æquatio correcta, eſt $p\, a \log. r - \frac{f\, h\, h\, r\, r}{2\, a} = p\, a \log. a - \frac{f\, a}{2}$; vel $2\, p\, a \log\left(\frac{r}{a}\right) = \frac{f\, h\, h\, r\, r}{a} - f\, a$; vel tranſeundo ad numeros & ſumendo $c =$ numero cujus $\log. = 1$, habetur $r = a\, c^{\left(\frac{f\, h\, h\, r\, r}{2\, p\, a\, a} - \frac{f}{2\, p}\right)}$.

Patet meridianos ſphæroidum ſemper prodire curvas algebraicas excepta tantum iſta hac hypotheſi.

Si harum omnium curvarum deſideretur aequatio more ſolito per coordinatas rectangulas, facile habetur. Nam faciendo $C\,E = x$, & $D\,E = y$, erit $r\, r = x\, x + y\, y$, & $h\, r = y$. Exterminando ergo h & r ex æquatione generali, invenietur $2\, p\, (x\, x + y\, y)^{\frac{n+1}{2}} - (n + 1)\, f\, a^{n-1}\, y\, y = (2\, p - n\, f - f)\, a^{n+1}$.

Et in caſu $n = -1$; $x\, x + y\, y = a\, a\, c^{\left(\frac{f\, y\, y}{p\, a\, a} - \frac{f}{p}\right)}$.

Sed prima noſtra ratio definiendi curvas per radios & angulos æque, & forſan hic magis commoda eſt quam illa quæ definit curvas per coordinatas.

Quamvis h, ut variabilis tractatur, tamen non ultra certos limites variat, & hi limites ſunt 0 & 1; noſtra itaque æquatio radialis non definit niſi partem curvæ cujus amplitudo eſt angulus rectus; ſed cum curvæ iſtæ ex quatuor arcubus ſimilibus & æqualibus conſtent,

conſtent, dantur curvæ meridianorum integræ per æquationem noſtram.

Jam facile determinatur ratio inter ambos Sectionis axes in quavis Hypotheſi.

Cum æquatio generalis ſit $2\,p\,r^{n+1} - (n+1)\,f\,h\,h\,a^{n-1}\,r\,r = (2\,p - n\,f - f)\,a^{n+1}$; ut inveniatur r quando $h = o$, habetur $2\,p\,r^{n+1} = (2\,p - n\,f - f)\,a^{n+1}$. Ex quo elicitur $CA \,.\, CP :: (2\,p)^{\frac{1}{n+1}} \,.\, (2\,p - n\,f - f)^{\frac{1}{n+1}}$.

Et in Hypotheſi gravitatis ſimplici diſtantiæ reciproce proportionalis, habetur Log. $\left(\frac{r}{a}\right) = -\frac{f}{2\,p}$.

Ex quo elicitur Log. CA — Log. $CP = \frac{f}{2\,p}$.

Patet quod n exiſtente numero affirmativo, integro, ſeu fracto, hoc eſt in omnibus hypotheſibus gravitatis directe proportionalis alicui diſtantiæ dignitati, diameter æquatoris axe revolutionis major ſemper erit. Sed ſi ſit n numerus aliquis negativus, hoc eſt, ſi gravitas proportionalis ſit inverſe alicui dignitati diſtantiæ, habebitur $CA \,.\, CP :: (2\,p)^{\frac{1}{-n+1}} \,.\, (2\,p + n\,f - f)^{\frac{1}{-n+1}}$; nunc ſi $n < 1$, ſit $k = 1 - n$; & habebitur $CA \,.\, CP :: (2\,p)^{\frac{1}{k}} \,.\, (2\,p - k\,f)^{\frac{1}{k}}$; & ſi $n > 1$, ſit $n - 1 = k$; & habebitur $CA \,.\, CP :: (2\,p)^{\frac{1}{-k}} \,.\, (2\,p + k\,f)^{\frac{1}{-k}}$, vel $CA \,.\, CP :: (2\,p + k\,f)^{\frac{1}{k}} \,.\, (2\,p)^{\frac{1}{k}}$. Inſuper invenimus quod n exiſtente $= -1$, habe-

habetur Log. C A — Log. C P $= \frac{f}{2p}$. Ex quibus patet nullam esse hypothesin in qua diameter æquatoris non superet meridiani diametrum.

Sphæroidum figura, ut satis apparet, a ratione, quam habet vis centrifuga ad gravitatem, dependet. Nunc, qualis esse possit in quibusdam hypothesibus ista ratio, videamus, & quæ inde figura sphæroidibus eveniet.

Si gravitas uniformis supponatur, erit $n = 0$ & habebitur C A . C P :: $2p \,.\, 2p - f$. Itaque in terra ubi vis centrifuga sub æquatore 289^{am} gravitatis partem æquat, si quæratur ratio quam habet diameter æquatoris ad axem in hypothesi gravitatis uniformis (ponendo 289 pro p, & 1 pro f) habebitur C A . C P :: 578. 577.

Posset vis centrifuga æquari gravitati, quod obtineret si terræ revolutio diurna 17 vicibus celerior redderetur; & tunc haberetur C A . C P :: 2 . 1. Sed si revolutio magis ac magis cita fieret, partes successive dissiparentur donec tandem terra ad atomum unicam redigeretur. Ex quo patet quod in hac hypothesi gravitatis uniformis, terra circa polos nunquam potest esse depressior quam si diameter æquatoris sit duplo major axe revolutionis. In hoc casu terra constaret ex duobus paraboloidibus sicut invenit D. Huygens *in tractatu de causa gravitatis* pro hac hypothesi particulari quam solam examinavit.

Si gravitas distantiæ a centro proportionalis statuatur, erit $n = 1$, & habebitur C A . C P :: $\surd p \,.\, \surd(p - f)$. Si igitur vis centrifuga, gravitati fieret æqualis, diameter æquatoris, axe revolutionis fieret infinite major. Hoc est, sphærois planum tantum circulare foret.

Et

Et cum in hac hypothesi vis centrifuga ad gravitatem omnes possit habere rationes à ratione nullâ, usque ad æqualitatis rationem, patet æquatoris diametrum ad axem revolutionis omnes has rationes habere posse ; & sphæroidem quæ in hac hypothesi, semper est Ellypsois, posse esse omnes Ellypsoides à sphæra usque ad circulum. Sed in hac etiam hypothesi, vis centrifuga ultra crescere nequit.

Si gravitas quadrato distantiæ reciproce proportionalis ponatur, erit $n = - 2$; & habebitur $CA . CP :: 2p + f . 2p$. Ex quo liquet in hac hypothesi vim centrifugam semper crescere posse, vel quod eodem redit, motum revolutionis citiorem semper fieri posse, nec tamen sphæroidis partes dissiparentur.

SCHOLION.

Cæterum, ex his omnibus hypothesibus nullam quasi in natura revera datam hic usurpo : siquidem interiores corporum partes non gravitant versus centrum aliquod unicum juxta proportionem quamvis distantiarum ab hoc centro in corporibus posito. Attractio partium ex forma corporis dependet, ut & vicissim forma dependet ex attractione. Idcirco omnes hæ determinationes, sunt magis mathematicæ quam physicæ. Unde fit, quod D. Newton indeterminatione axis terræ & diametri æquatoris rationem invenerit diversam ab Huygeniana & a nostris, nempe eam quæ est inter 229 & 230. Summus vir solutionem mere geometricam per hypotheses neglexit, ut naturæ magis consentaneam daret.

PROB-

PROBLEMA II.

Posito quod materia fluens circa axem extra fluentum sumtum, attrahatur versus centrum in hoc axe positum vi alicui distantiæ a centro dignitati proportionali; dum interea propter fluenti partium attractionem mutuam, fit altera attractio versus aliud centrum intra fluentum sumtum, quæ in quavis sectione fluenti revolutionis perpendiculariter per centrum exterius facta, sit alicui distantiæ a centro interiori dignitati proportionalis: invenire figuram quam fluentum induet.

SOLUTIO. (*Fig.* 2.)

Sit ADPad QA sectio fluenti gyrantis circa axem $A\lambda$ per planum revolutioni rectum quod transit per centrum γ facta. Sit γ centrum virium centripetarum extra fluentum sumtum; & C centrum versus quod partes fluenti attrahuntur in sectione sumtum.

Ut fluidi partes in æquilibrio maneant, oportet pondus cujusque columnæ CD tum a gravitate versus γ, tum versus C, tum a vi centrifuga ortum, idem ubique maneat.

Sit ergo gravitas in A versus γ, data & $= \pi$, gravitas in A versus C, data & $= p$, & vis centrifuga in A, etiam data & $= f$ Sit A C $= a$, C $\gamma = b$, $cg = r$; sinus ang. DCP $= h$ pro radio $= 1$; erit GL $= hr$, & è γ demissa perpendiculari γ R in radium C D productum, erit CR $= hb$, & γ G $=$ (per 12^{am} Elem. lib. 2.) $\sqrt{(bb + 2bhr + rr)}$.

 Jam

Jam cum ſit gravitas in A verſus $\gamma = \overset{\prime}{\pi}$; dicendo $\pi . \overset{\prime}{\pi} :: (a+b)^{m} (bb + 2bhr + rr)^{\frac{1}{2}m}$ habebitur gravitas in G ſeu $\overset{\prime}{\pi} = \dfrac{\pi (bb + 2bhr + rr)^{\frac{1}{2}m}}{(a+b)^{m}}$.

Et ut verſus C derivetur, dicatur $\overset{\prime}{\pi} . \overset{\prime\prime}{\pi} :: G\gamma . GR$, vel $\dfrac{\pi (bb + 2bhr + rr)^{\frac{1}{2}m}}{(a+b)^{m}} . \overset{\prime\prime}{\pi} :: (bb + 2bhr + rr)^{\frac{1}{2}} . bh + r$; unde habetur vis ab attractione verſus γ, derivata verſus C, ſeu

$$\overset{\prime\prime}{\pi} = \frac{\pi (bh + r) (bb + 2bhr + rr)^{\frac{m-1}{2}}}{(a+b)^{m}}$$

Habetur inſuper (cum gravitas in A verſus C, ſit $= p$) gravitas in G verſus C $= \dfrac{pr^{n}}{a^{n}}$; Gravitas ergo tota verſus C ex gravitatibus ambabus verſus γ & C orta habebitur $= \dfrac{\pi . (bh + r)(bb + 2bhr + rr)^{\frac{m-1}{2}}}{(a+b)^{m}} + \dfrac{pr^{n}}{a^{n}}$;

Nunc cum ſit vis centrifuga in A, $= f$; dicendo $f . \overset{\prime}{f} :: a + b . b + hr$ habetur vis centrifuga in G ſeu $\overset{\prime}{f} = \dfrac{f(b + hr)}{a + b}$; & ut pars iſtius vis quæ verſus D trahit inveniatur; fiat $\overset{\prime}{f} . \overset{\prime\prime}{f} :: GH . GK$, vel $\dfrac{f(b + hr)}{a + b} . \overset{\prime\prime}{f} :: 1 . h$; unde habetur vis gravitati verſus C oppoſita ſeu $\overset{\prime\prime}{f} = \dfrac{fh(b + hr)}{a + b}$. Vis

Vis ergo versus C ex omnibus his viribus resultans, erit $\frac{\pi(bh+r)(bb+2bhr+rr)^{\frac{m-1}{2}}}{(a+b)^m}+\frac{pr^n}{a^n}-\frac{fb(b+hr)}{a+b}$.

Concipiendo ergo ut in primo problemate colummam C D, ex infinitis cylindrulis $\dot{r}$ compositam, habebitur $F\left(\frac{\pi(bh+r)(bb+2bhr+rr)^{\frac{m-1}{2}}}{(a+b)^m}+\frac{pr^n}{a^n}-\frac{fb(b+hr)}{a+b}\right)\dot{r}$, quod æquari debet alicui constanti ponderi. Erit ergo $\frac{\pi(bb+2bhr+rr)^{\frac{m+1}{2}}}{m+1.(a+b)^m}+\frac{pr^{n+1}}{(n+1).a^n}-\frac{fbhr}{a+b}-\frac{fbhrr}{2.(a+b)}=A$.

Ut corrigatur hæc æquatio, oportet quando $h=1$, esse $r=a$; tunc ergo habetur $\frac{\pi(a+b)}{m+1}+\frac{pa}{n+1}-\frac{fab}{a+b}-\frac{faa}{2(a+b)}=A$. Et æquatio corecta, erit $\frac{\pi(bb+2bhr+rr)^{\frac{m+1}{2}}}{(m+1).(a+b)^m}+\frac{pr^{n+1}}{(n+1)a^n}-\frac{fbhr}{a+b}-\frac{fbhrr}{2(a+b)}=\frac{\pi(a+b)}{m+1}+\frac{pa}{n+1}-\frac{fab}{a+b}-\frac{faa}{2(a+b)}$.

Vel scribendo c pro $a+b$, & q pro $(m+1) \times (n+1)$ $2(n+1)\pi a^n (bb+2bhr+rr)^{\frac{m+1}{2}} + 2(m+1)pc^m r^{n+1} - 2qfa^n bc^{m-1}hr - qfa^n c^{m-1}hhrr = 2(n+1)\pi a^n c^{m+1} + 2(m+1)pa^{n+1}c^m - 2qfa^{n+1}bc^{m-1} - qfa^{n+2}c^{m-1}$.

Patet, in omnibus hypothesibus, sectionem fluenti esse curvam algebraicam, exceptis tantum hypothesibus attractionis versus γ vel versus C in ratione simplicis distantiæ inversâ; nam si sit tantum $m=-1$, habebitur pro sectione fluenti

$$\frac{\pi(a+b)}{2}\,\mathrm{L}\,(bb+2bhr+rr) + \frac{pr^{n+1}}{n+1.\,a^n} - \frac{fbhr}{a+b} - \frac{fhhrr}{2(a+b)} = \frac{\pi(a+b)}{2}\,\mathrm{L}\,(a+b)^2 + \frac{pa}{n+1} - \frac{fab}{a+b} - \frac{faa}{2(a+b)}.\ \text{vel}$$

$$\frac{\pi c}{2}\,\mathrm{L}\left(\frac{bb+2bhr+rr}{cc}\right) = -\frac{pr^{n+1}}{(n+1)a^n} + \frac{fbhr}{c} + \frac{fhhrr}{2c} + \frac{pa}{n+1} - \frac{fab}{c} - \frac{faa}{2c}.$$

Et si tantum $n=-1$, habebitur

$$\frac{\pi(bb+2bhr+rr)^{\frac{m+1}{2}}}{m+1.\,(a+b)^m} + pa\,\mathrm{L}r - \frac{fbhr}{a+b} - \frac{fhhrr}{2(a+b)} = \frac{\pi(a+b)}{m+1} + pa\,\mathrm{L}a - \frac{fab}{a+b} - \frac{faa}{2(a+b)}.\ \text{vel}\ pa\,\mathrm{L}\left(\frac{r}{a}\right) = -$$

$\pi(bb$

$$\frac{\pi(bb+2bhr+rr)^{\frac{m+1}{2}}}{(m+1).c^m}+\frac{fbhr}{c}+\frac{fbhrr}{2c}+$$
$$\frac{\pi c}{m+1}-\frac{fab}{c}-\frac{faa}{2c}.$$

Sed si sint simul $m=-1$, & $n=-1$, habebitur

$$\frac{\pi(a+b)}{2}\,\mathrm{L}\,(bb+2bhr+rr)+pa\,\mathrm{L}\,r-$$
$$\frac{fbhr}{a+b}-\frac{fbhrr}{2(a+b)}=\frac{\pi(a+b)}{2}\,\mathrm{L}\,(a+b)^2+$$
$$pa\,\mathrm{L}\,a-\frac{fab}{a+b}-\frac{faa}{2(a+b)}.\ \text{vel}$$

$$\frac{\pi c}{2}\,\mathrm{L}\left(\frac{bb+2bhr+rr}{cc}\right)+pa\,\mathrm{L}\left(\frac{r}{a}\right)=$$
$$\frac{fbhr}{c}+\frac{fbhrr}{2c}-\frac{fab}{c}-\frac{faa}{2c}.$$

Si desideretur æquatio sectionis fluenti per coordinatas rectangulas; faciendo $CE=x$ & $DE=y$ habebuntur duæ æquationes $rr=xx+yy$ & $hr=y$, quarum ope exterminabuntur r & h ex æquationibus supra inventis; & habebitur pro casu generali,

$$2(n+1)\pi a^n(bb+2by+yy+xx)^{\frac{m+1}{2}}+$$
$$2(m+1)pc^m(xx+yy)^{\frac{n+1}{n}}-2qfa^nbc^{m-1}y-$$
$$qfa^nc^{m-1}yy=2(n+1)\pi a^nc^{m+1}+2(m+$$
$$1)pa^{n+1}c^m-2qfa^{n+1}bc^{m-1}-qfa^{n+2}c^{m-1}.$$

Et eodem modo in casibus $m=-1$, $n=-1$, reperientur æquationes per coordinatas rectangulas.

Ut curvam P A Q invenimus, ita quoque invenietur curva P a Q mutatis mutandis. Nam tunc si sit gravitas in a versus γ data & $= \pi$, gravitas in a versus C $= p$, vis centrifuga in $a = f$; C $a = a$, C$\gamma = b$, C$g = r$, $g\,l = hr$, & $\gamma\, g = \sqrt{(bb - 2bhr + rr)}$ invenietur gravitas in g versus C, ab attractione versus γ orta, $\overset{\prime\prime}{\pi} = \frac{\pi\,(bh - r)\,(bb - 2bhr + rr)^{\frac{m-1}{2}}}{(b-a)^m}$.

Habetur insuper gravitas in g versus C, $\overset{\prime}{p} = \frac{p\,r^n}{a^n}$.

Sic etiam vis centrifugæ pars in g quæ trahit versus C invenietur $\overset{\prime\prime}{f} = \frac{f\,h\,(b - hr)}{b-a}$.

Sed hæ posteriores vires nunc primæ opponuntur. Habebitur ergo $\int\left(\frac{\pi(-bh+r)\,(bb-2bhr+rr)^{\frac{m-1}{2}}}{(b-a)^m} + \frac{p\,r^n}{a^n} + \frac{f\,h\,(b-hr)}{b-a}\right)\dot{r} = A$. Unde deducitur

$$\frac{\pi\,(bb - 2bhr + rr)^{\frac{m+1}{2}}}{(m+1)\,(b-a)^m} + \frac{p\,r^{n+1}}{(m+1)\,a^n} + \frac{fbhr}{b-a} - \frac{f\,h\,h\,r\,r}{2\,(b-a)} = \frac{\pi\,(b-a)}{m+1} + \frac{p\,a}{n+1} + \frac{f\,a\,b}{b-a} - \frac{f\,a\,a}{2\,(b-a)}.$$

Et in casibus $m = -1$, $n = -1$, invenientur ut supra æquationes sectionum, debitis tantum signis mutatis.

Et per has æquationes radiales invenientur æquationes ad coordinatas ut factum est pro curva P A Q.

Et cum pondus columnæ tam in superiori quam in inferiori curva debeat idem esse, habebitur æquatio inter pondus A in curva superiori, & pondus A in infe-

inferiori, ex qua determinabitur C a pro determinata C A, & ſic ſectio fluenti integra determinabitur.

Quæcunque ſit hypotheſis gravitatis, ſemper pro dato angulo D C P, radius C D obtineri poteſt datæ longitudinis, & ſic figura fluenti vel craſſior vel tenuior fiet, & quidem modis infinitis; ponendo in æquatione pro b & r valores determinatos. Sic fieri poteſt ut puncta P & Q coeant, ſcribendo o pro b & r; & tunc ſectio fluenti ex duabus ovalibus figuris in C junctis conſtabit. Nam infinitæ rationes inter π, p, & f quæ ad id efficiendum conveniunt, obtinebuntur.

Si ex.grat.ultimum hoc deſideretur, nempe ut P & Q coeant in C, habebitur $2\,(n+1)\,\pi\, b^{m+1} = 2\,(n+1)\,\pi\, c^{m+1} + 2\,(m+1)\,p\,a\,c^{m} - 2\,q\,f\,a\,b\,c^{m-1} - q\,f\,a\,a\,c^{m-1}$. Unde eliciuntur infinitæ rationes inter π, p, & f.

Si ponatur gravitas tum verſus γ, tum verſus C ſimplici diſtantiæ a centro proportionalis; ſectio fluenti erit coniſectio. Et ſi tunc deſideretur ut puncta P, Q & C coeant, figura ex duabus Ellypſibus in C junctis, conſtabit.

Nunc ſi diſtantia C γ evaneſcat, vel duo centra coeant; erit $b = o$ & $e = a$; & fluentum fiet ſphærois.

Si inſuper ponatur $m = n$, & $\pi = o$, æquatio generalis ſectionis fluenti fiet $2\,p\,r^{n+1} - (n+1)\,f\,a^{n-1}\,b\,b\,r\,r = (2\,p - nf - f)\,a^{n+1}$. Vel in caſu $n = -1$, $2\,p\,a\,\mathrm{L}\left(\frac{r}{a}\right) = \frac{f\,b\,b\,r\,r}{a} - f\,a$.
ut invenimus in primo problemate quod eſt iſtius caſus tantum ſpecialis.

SCHOLION.

Hæc confideratio formarum quas pro diverfa gravitatis ad vim centrifugam ratione, fluida induere poffunt, me induxit ut cogitarem tales planetarum formas forfitan in cœlis reperiri, cum ad hoc celeriori tantum circa axem motu, vel minori materiæ denfitate opus fit. Etenim quamvis pauci quos novimus planetæ fatis ad fphæroidicam formam accedant, cur non alij aliarum formarum fupra dictarum admitterentur vel circa alios foles, vel etiam circa noftrum? Hi planetæ lentiformes, vel propter diftantiam, a nobis nunquam confpicerentur, vel quia in plano Eclipticæ verfarentur, aut in plano parum ad Eclipticam inclinato, cui plano illorum axis revolutionis effet rectus, aut fere rectus; nam in hoc fitu è terra confpici nequirent.

Cur etiam talis formarum varietas inter fixas, locum non haberet? præfertim cum illas circa axem gyrari, folis inftar noftri, fit admodum verifimile. Forfitan fixæ lentiformes in cœlis dantur. Forfitan planetis admodum excentricis vel cometis cinguntur, qui cum in plano æquatoris fixæ non verfentur, quando ad perihelium accedunt, directionem axis ftellæ turbant; & tunc quæ nobis propter fitum non apparebat, apparet ftella, vel quæ apparebat non apparet. Et fic ratio redderetur cur quædam ftellæ per vices accendi & extingui videntur.

Sed fi in quovis fyftemate cometa aliquis caudam trahens, fertur in viciniam alicujus potentis planetæ, quid eventurum? Materia quæ à corpore cometæ effluit, circa planetam trahetur; & cometa novam mate-

materiam effundente, vel sufficiente materiæ jam effusæ copia, orietur fluxus circa planetam continuus: & quamvis columnæ fluenti vel cylindrica, vel conica, vel quælibet alia forma primùm fuerit, vis ejus centrifuga cum gravitatibus tum a planeta tum a materia fluenti ortis, semper eam latiorem & tenuiorem reddet; & columna hæc curvata ad aliquam e formis supra definitis in Probl. 2° accedet. Et sic omnium naturæ phænomenorum maxime stupendi, Saturni annuli ratio redderetur.

Interea dum cometæ cauda talem planetæ annulum daret, corpus ipsum cometæ forsan etiam traheretur si in distantia debita esset, & novus planetæ satelles fieret. Sic forsan plures cometæ satellitibus & annulo Saturnum ditarunt: nam annulum Saturni unius cometæ effluvio tribuendum non videtur, cum umbram in Saturni discum projiciat dum materia tamen caudarum cometarum adeo sit rara ut trans illam lucentes stellæ videri queant. Annulus ergo Saturni ex plurium cometarum caudis constare videtur, & quarum materia propter attractionem Saturni densior facta est.

Patet planetam satellites, nec tamen annulum, acquirere posse; nam non omnes cometæ caudam habent: Et si cometa caudâ carens trahatur, planetæ satellitem sine annulo dabit.

Summus Newton statuit vapores cometarum in planetas spargi: imò etiam hanc communicationem necessariam duxit, ut quidquid liquoris consumitur, reparetur. Viri illustrissimi D.D. Halley & Whiston, cometas & cometarum caudas planetis infestas mutationes, ut polorum variationem, diluvia, & incendia inferre posse crediderunt; sed cometæ benigniores effectus producere possunt, & etiam planetis aliquando res miras & utiles dare.

VI. *An Extract of a Letter from* Oliver St. John, *Esq; F. R. S. dated from* Florence, November *the* 30*th*, 1731, N. S. *Communicated by* R. Graham, *F. R. S.*

WHEN I consider how many are charged overlaid in the Bills of Mortality, I wonder that the *Arcutio's*, universally used here, are not used in *England*. I here send you the Design of one, drawn in Perspective, with the Dimensions, which are larger than usual.

The ARCUCCIO. *Vide Fig.* 3.

a, The Place where the Child lies.

b, The Head-board.

c, The Hollows for the Nurses Breasts.

d, A Bar of Wood to lean on when she suckles the Child.

e, A small Iron Arch to support the said Bar.

The Length 3 Feet, 2 Inches and a half.

Every Nurse in *Florence* is obliged to lay the Child in it, under Pain of Excommunication. The *Arcutio*, with the Child in it, may be safely laid entirely under the Bed-cloaths in the Winter, without Danger of smothering.

VII. *An*

Fig. 1.

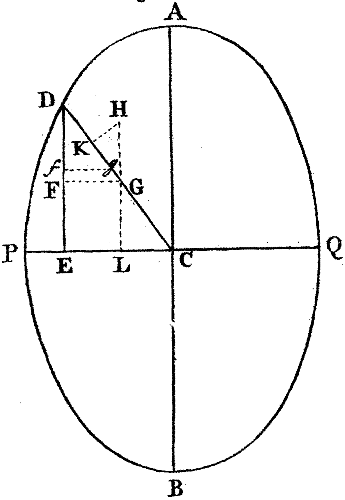


Fig. 3.

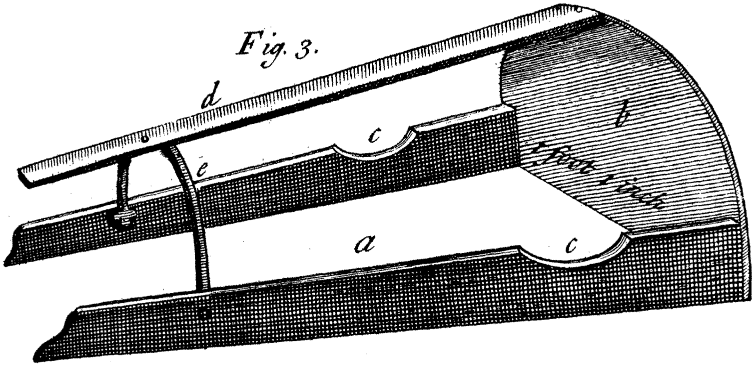


Fig. 2.

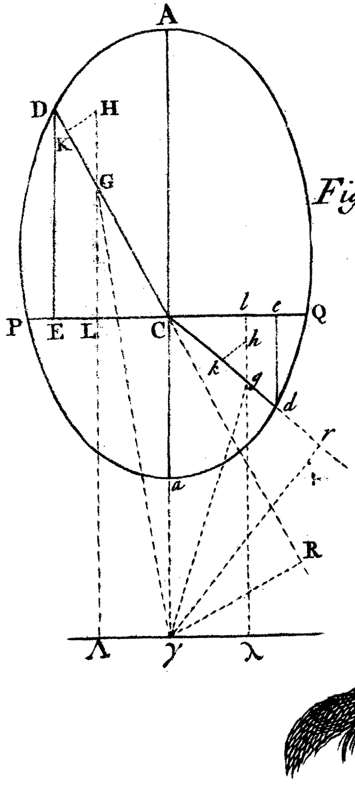
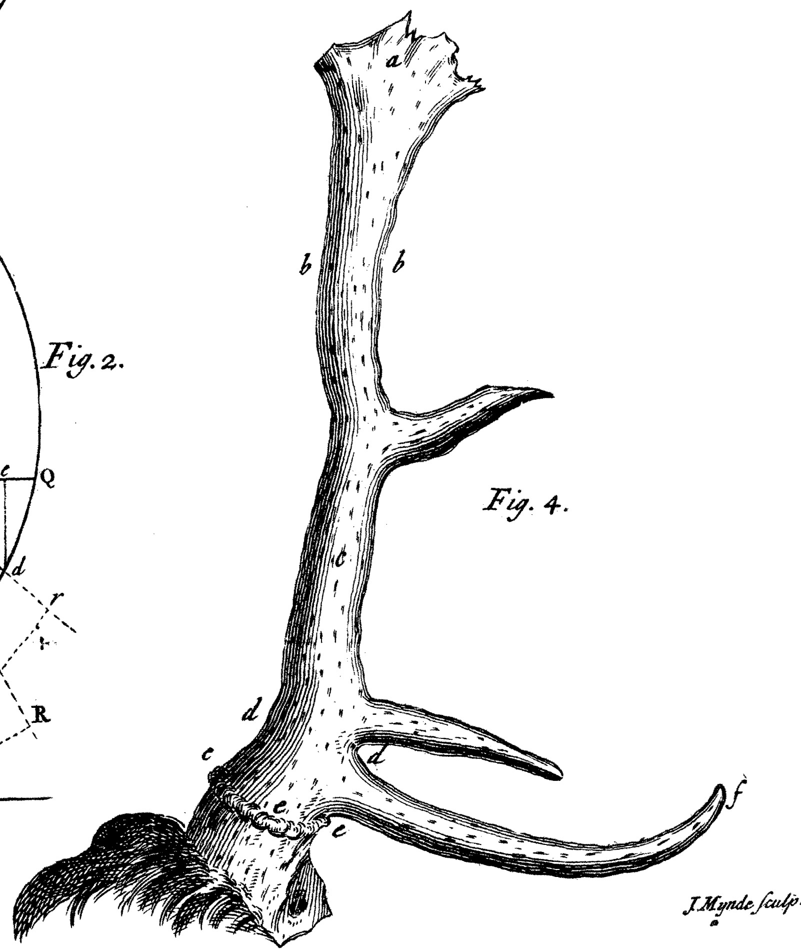


Fig. 4.



J. Mynde sculp.