

*An Easie Demonstration of the Analogy of the
Logarithmick Tangents to the Meridian Line or
sum of the Secants: with various Methods
for computing the same to the utmost Exact-
ness, by E. Halley.*

IT is now near 100 Years since our Worthy Countryman Mr. *Edward Wright* published his *Correction of Errors in Navigation*, a Book well deserving the perusal of all such as design to use the Sea. Therein he considers the Course of a Ship on the Globe, steering obliquely to the Meridian; and having shewn, that the *Departure* from the Meridian, is in all cases less than the *Difference of Longitude*, in the ratio of *Radius* to the *secant* of the *Latitude*, he concludes, That the sum of the *Secants* of each point in the *Quadrant* being added successively, would exhibit a Line divided into Spaces, such as the intervals of the parallels of *Latitude* ought to be in a true Sea Chart, whereon the Meridians are made parallel Lines, and the *Rhombs* or *Oblique Courses* represented by right Lines. This is commonly known by the name of the *Meridian Line*, which though it generally be called *Mercators*, was yet undoubtedly Mr. *Wrights* Invention, (as he has made it appear in his Preface.) And the Table thereof is to be met with in most Books treating of Navigation, computed with sufficient exactness for the purpose.

It was first discovered by chance, and as far as I can learn, first published by Mr. *Henry Bond*, as an addition to *Norwoods Epitome of Navigation*, about 50 Years since, that the *Meridian Line* was Analogous to a Scale of *Logarithmick Tangents of half the Complements of the Latitudes*. The difficulty to prove the truth of this Proposition, seemed such to Mr. *Mercator*, the Author of *Logarithmotechnia*, that he proposed to wager a good sum of Money, against whoso would fairly undertake it, that he should not demonstrate either, that it was true or false: And about that time Mr. *John Collins*, holding a Correspondence with all the Eminent Mathematicians of the Age, did excite them to this Enquiry.

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The first that demonstrated the said *Analogy*, was the excellent Mr. *James Gregory* in his *Exercitationes Geometricæ*, published *Anno* 1668. which he did, not without a long train of Consequences and Complication of Proportions, whereby the evidence of the Demonstration is in a great measure lost, and the Reader wearied before he attain it. Nor with less work and apparatus hath the celebrated Dr. *Barrow* in his *Geometrical Lectures* (Lect. XI. App. I.) proved, that the Sum of all the *Secants* of any arch is analogous to the *Logarithm* of the ratio of *Radius + Sine* to *Rad. — Sine*, or which is all one, that the *Meridional parts* answering to any degree of Latitude, are as the *Logarithms* of the *rationes* of the *Versed Sines* of the distances from both the *Poles*. Since which the incomparable Dr. *Wallis* (on occasion of a paralogism committed by one Mr. *Norris* in this matter) has more fully and clearly handled this Argument, as may be seen in Num. 176. of these *Transactions*. But neither Dr. *Wallis* nor Dr. *Barrow* in their said Treatises have any where touched upon the aforesaid relation of the *Meridian-line* to the *Logarithmick Tangent*; nor hath any one, that I know of, yet discovered the Rule for computing independently the interval of the *Meridional parts* answering to any two given Latitudes.

Wherefore having attained, as I conceive, a very facile and natural demonstration of the said *Analogy*, and having found out the Rule for exhibiting the *difference of Meridional parts*, between any two parallels of Latitude, without finding both the Numbers whereof they are the difference: I hope I may be entituled to a share in the improvements of this useful part of Geometry. Desiring no other favour of some Mathematical Pretenders, than that they think fit to be so just, as neither to attribute my desire to please the Honourable the *Royal Society* in these Exercises, to any kind of *Vanity* or Love of Applause in me, (who too well know how very few these things oblige, and how small reward they procure) nor yet to complain *coram non iudice*, that I arrogate to my self the Inventions of others, and upon that pretext to depreciate what I do, unless at the same time, they can produce the Author I wrong, to prove their assertions. Such dissingenuity as I have always most carefully avoided, so I wish I had not too much experience of it in the very same persons

persons, who make it their business to detract from that little share of Reputation I have in these things. But to return to the matter in hand, Let us demonstrate the following Proposition.

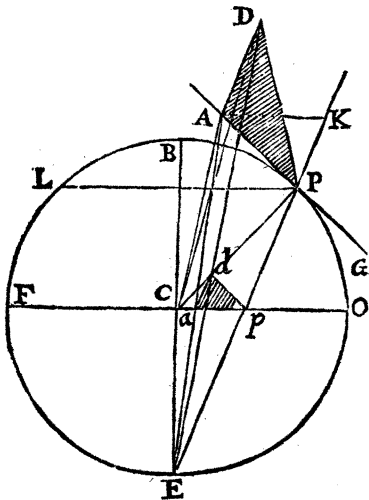
The Meridian Line is a Scale of Logarithmick Tangents of the half Complements of the Latitudes.

For this Demonstration, it is requisite to premise these four Lemmata.

Lemma. I. In the *Stereographick Projection* of the Sphere upon the plain of the Equinoctial, the distances from the Center, which in this case is the Pole, are laid down by the Tangents of half those distances, that is, of half the Complements of the Latitudes. This is evident from *Eucl. 3. 20.*

Lemma. II. In the *Stereographick Projection*, the Angles, under which the Circles intersect each other, are in all cases equal to the Spherical Angles they represent: Which is perhaps as valuable a property of this *Projection*, as that of all the Circles of the Sphere thereon appearing Circles: But this not being vulgarly known, must not be assumed without a *Demonstration.*

Let $EBPL$ be any great circle of the Sphere, E the Eye placed in its Circumference, C its Center, P any point thereof, and let FCO be supposed a plain erected at right Angles to the Circle $EBPL$, on which FCO we design the Sphere to be projected. Draw EP crossing the Plain FCO in p , and p shall be the point P projected.



To the point P draw the Tangent APG , and on any point thereof, as A , erect a perpendicular AD , at right angles to the plain $EBPL$, and draw the lines PD, AC, DC : and the angle APD shall be equal to the Spherical Angle contained

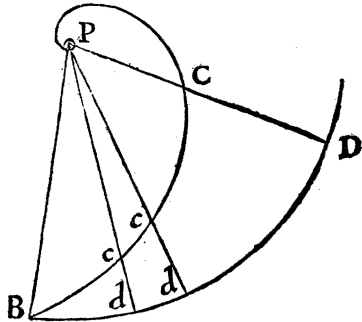
between the plains APC, DPC . Draw also AE, DE , intersecting

secting the plain FCO in the points a and d ; and joyn ad , pd : I say the Triangle adp is similar to the triangle ADP , and the angle apd equal to the angle APD . Draw PL , AK parallel to FO , and by reason of the parallels, ap will be to ad as AK to AD : But (by *Eucl.* 3. 32.) in the triangle AKP , the angle $AKP=LPE$ is also equal to $APK=EPG$ wherefore the sides AK , AP are equal, and 'twill be, as ap to ad so AP to AD . Whence the angles DAP , dap being right, the angle APD will be equal to the angle apd that is, the Spherical Angle is equal to that on the Projection, and that in all Cafes. *Which was to be proved.*

This *Lemma* I lately received from Mr. *Ab. de Moivre*, though I since understand from Dr. *Hook* that he long ago produced the same thing before the *Society*. However the demonstration and the rest of the discourse is my own.

Lemma III. On the *Globe*, the *Rhumb Lines* make equal angles with every *Meridian*, and by the foregoing *Lemma*, they must likewise make equal angles with the *Meridians* in the *Stereographick Projection* on the plain of the *Equator*: They are therefore, in that *Projection*, *Proportional Spirals* about the *Pole Point*.

Lemma IV. In the *Proportional Spiral* it is a known property that the angles BPC or the arches BD , are *Exponents* of the *rationes* of BP to PC : for if the arch BD be divided into innumerable equal parts, right lines drawn from them to the Center P , shall divide the Curve $BccC$ into an infinity of proportionals; and all the lines Pc shall be an infinity of proportionals between PB and PC whose number is equal to all the points d, d , in the arch BD : Whence, and by what I have delivered in Num. 216, it follows, that as BD to Bd , or as the angle BPC to the angle BPc , so is the *Logarithm* of the *ratio* of PB to PC , to the *Logarithm* of the *ratio* of PB to Pc .



From these *Lemmata* our Proposition is very clearly demonstrated: For by the first, PB, Pc, PC are the *Tangents* of half the *Complements* of the *Latitudes* in the *Stereographick Projection*: And by the last of them, the differences of Longitude, or angles at the Pole between them, are *Logarithms* of the *rationes* of those *Tangents* one to the other. But the *Nautical Meridian Line* is no other than a Table of the Longitudes, answering to each minute of Latitude, on the *Rhumb-line* making an angle of 45 degrees with the Meridian. Wherefore the Meridian Line is no other than a Scale of *Logarithmick Tangents* of the half *Complements* of the *Latitudes*. *Quod erat demonstrandum.*

Coroll. 1. Because that in every point of any *Rhumb Line*, the difference of Latitude is to the *Departure*, as the *Radius* to the *Tangent* of the angle that Rhumb makes with the Meridian; and those equal *Departures* are every where to the differences of Longitude, as the *Radius* to the *Secant* of the Latitude; it follows that the differences of Longitude are, on any Rhumb, *Logarithms* of the same *Tangents*, but of a differing *Species*; being proportioned to one another as are the *Tangents* of the angles made with the Meridian.

Coroll. 2. Hence any Scale of *Logarithm Tangents*, (as those of the *Vulgar Tables* made after *Brigg's* form; or those made to *Napiers*, or any other form whatsoever) is a Table of the differences of Longitude, to the several *Latitudes*, upon some determinate *Rhumb* or other: And therefore, as the *Tangent* of the angle of such *Rhumb*, to the *Tangent* of any other *Rhumb*: So the difference of the *Logarithms* of any two *Tangents*, to the difference of Longitude, on the proposed *Rhumb*, intercepted between the two *Latitudes*, of whose half *Complements* you took the *Logarithm Tangents*.

And since we have a very compleat Table of *Logarithm Tangents* of *Brigg's* form, published by *Vlacq*, Anno 1633, in his *Canon Magnus Triangulorum Logarithmicus*, computed to ten *Decimal places* of the *Logarithm*, and to every ten *Seconds* of the *Quadrant*, (which seems to be more than sufficient for the nicest *Calculator*) I thought fit to enquire the *Oblique angle*, with which that *Rhumb Line* crosses the *Meridian*, whereon the said *Canon* of *Vlacq* precisely answers to the differences of Longitude, putting *Unity* for one minute thereof,

thereof, as in the Common Meridian Line. Now the *momentary augment* or *fluxion* of the Tangent Line at 45 degrees, is exactly double to the *fluxion* of the arch of the Circle, (as may easily be proved) and the Tangent of 45, being equal to *Radius*, the *fluxion* also of the Logarithm Tangent will be double to that of the arch, if the Logarithm be of *Napeirs* form : But for *Brigg's* form it will be as the same doubled arch multiplied into, 0, 43429, &c. or divided by 2, 30258, &c. Yet this must be understood only of the addition of an indivisible arch, for it ceases to be true if the arch have any determinate magnitude.

Hence it appears, that if one minute be supposed Unity, the length of the arch of one minute being, 0,00290888208665721596154 &c. in parts of the Radius, the proportion will be as Unity to 2,908882 &c. so Radius to the Tangent of $71^{\circ} 1' 42''$ whose Logarithm is 10, 46372611720718325204 &c. and under that angle is the Meridian intersected by that Rumb Line, on which the *differences* of *Napeirs* Logarithm Tangents of the half Completments of the Latitudes are the true differences of Longitude, estimated in minutes and parts, taking the first Four Figures for Integers. But for *Vlacq's* Tables we must say.

As 2,302585 &c. to 2908882 &c. So Radius to 1,26331143874244569212, &c. which is the Tangent of $51^{\circ} 38' 9''$, and its Logarithm 10, 101510428507720941162 &c. wherefore in the Rhumb Line, which makes an angle of $51^{\circ} 38' 9''$ with the Meridian, *Vlacq's* Logarithm Tangents are the true differences of Longitude. And this compared with our second *Corollary* may suffice for the use of the Tables already computed.

But if a Table of Logarithm Tangents be made by extraction of the root of the Infiniteth power, whose Index is the length of the arch you put for Unity, (as for minutes the 0,002908882th &c. power) which we will call a ; such a Scale of Tangents, shall be the true Meridian Line or sum of all the Secants taken infinitely many. Here the Reader is desired to have recourse to my little Treatise of *Logarithms*, published in N^o 216. p. 58. that I may not need to repeat it. By what is there delivered, it will follow, that putting t for the excess or defect of any Tangent above or under the *Radius* or *Tangent* of 45 ; the Logarithm of the

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ratio of Radius to such Tangent will be.

$$\frac{1}{m} \text{ into } t - \frac{1}{2}tt + \frac{1}{4}t^3 - \frac{1}{8}t^5 + \frac{1}{16}t^7, \&c.$$

when the arch is greater than 45^{gr} , or

$$\frac{1}{m} \text{ into } t + \frac{1}{2}tt + \frac{1}{4}t^3 + \frac{1}{8}t^5 + \frac{1}{16}t^7, \&c.$$

when it is less than 45^{gr} . And by the same doctrine putting T for the Tangent of any *arch*, and t for the difference thereof from the Tangent of another arch, the Logarithm of their *ratio* will be

$$\frac{1}{m} \text{ into } \frac{t}{T} + \frac{tt}{2TT} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$$

when T is the greater Term, or

$$\frac{1}{m} \text{ into } \frac{t}{T} - \frac{tt}{2TT} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$$

when T is the lesser Term:

And if m be supposed ,0002908882, &c. = a , its reci-

procal $\frac{r}{a}$ will be, 3437;7467707849392526, &c. which multiplied into the aforesaid *Series*, shall give precisely the difference of Meridional parts, between the two Latitudes to whose half complements the assumed Tangents belong. Nor is it material from whether Pole you estimate the Complements, whether the elevated or depressed; the Tangents being to one another in the same *ratio* as their Complements, but inverted.

In the same Discourse I also shewed that the *Series* might be made to converge twice as swift, all the even Powers being omitted: and that putting τ for the sum of the two Tangents the same Logarithm would be.

$$\frac{2}{m} \text{ or } \frac{2r}{a} \text{ into } \frac{t}{\tau} + \frac{t^3}{3\tau^3} + \frac{t^5}{5\tau^5} + \frac{t^7}{7\tau^7} + \frac{t^9}{9\tau^9}, \&c.$$

but the *ratio* of τ to t , or of the sum of two Tangents to their difference, is the same as that of the *sine* of the sum of the arches, to the *sine* of their difference. Wherefore if S be put for the *sine* Complement of the Middle Latitude, and s for the *sine* of half the difference of Latitudes, the same *Series* will be

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \&c.$$

wherein as the differences of Latitude are smaller, fewer steps will suffice. And if the Equator be put for the Middle Latitude, and consequently $S = R$, and s to the *sine* of the Latitude, the Meridional parts reckoned from the Equator will be

$$\frac{s}{a} + \frac{s^3}{3ra} + \frac{s^5}{5r^3a} + \frac{s^7}{7r^5a}, \&c.$$

which is coincident with Dr. *Wallis's* solution in Numb. 176. And this same *Series*, being half the Logarithm of the ratio of $R + s$ to $R - s$, that is, of the *Versed-sines* of the distances from both Poles, does agree with what Dr. *Barrow* had shewn in his XI. *Lecture*.

The same ratio of τ to t may be expressed also by that of the Sum of the Co-sines of the two Latitudes, to the sine of their difference: As likewise by that of the Sine of the Sum of the two Latitudes, to the difference of their Co-sines: Or by that of the *Versed-sine* of the Sum of the Co-latitudes, to the difference of the sines of the Latitudes; Or as the same difference of the sines of the Latitudes, to the *Versed-sine* of the difference of the Latitudes; all which are in the same ratio of the Co-sine of the Middle Latitude, to the Sine of half the difference of the Latitudes. As it were easie to demonstrate, if the Reader were not supposed capable to do it himself, upon a bare inspection of a Scheme duly representing these Lines.

This variety of Expression of the same ratio I thought not fit to be omitted, because by help of the rationality of the Sine of 30^{th} , in all cases where the Sum or difference of the Latitudes is 30^{th} , 60^{th} , 90^{th} , 120^{th} or 150 degrees, some one of them will exhibit a simple series, wherein great part of the Labour will be saved: And besides I am willing to give the Reader his choice which of these equipollent methods to make use of; but for his exercise I shall leave the prosecution of them, and the *compendia* arising therefrom, to his own industry. Contenting my self to consider only the former, which for all uses seems the most convenient, whether we design to make the whole Meridian Line, or any part thereof, *viz.*

$$\frac{2r}{a} \text{ into } \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}, \text{ \&c.}$$

Wherein a is the length of any Arch which you design shall be the Integer or Unity in your Meridional Parts; (whether it be a Minute, League or Degree, or any other,) S the Co-fine of the Middle Latitude, and s the Sine of half the difference of Latitudes; But the Secants being the Reciprocals of the Co-fines, $\frac{s}{S}$ will be equal to $\frac{f}{rr}$ putting f for the Secant of the Middle Latitude; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2fs}{ar}$ This multiplied by $\frac{s^3}{3SS}$ that is by $\frac{f^3ss}{3rrrr}$, will give the second step: and that again by $\frac{3f^5ss}{5rrrr}$, the third step; and so forward till you have completed as many Places as you desire. But the squares of the Sines being in the same ratio with the *Verfed-fines* of the double Arches, we may instead of $\frac{s^3}{3SS}$ assume for our Multiplicator $\frac{v}{3V}$, or the *Verfed-fine* of the difference of the Latitudes divided by thrice the *Verfed-fine* of the sum of the Co-latitudes, &c. which is the utmost *Compendium* I can think of for this purpose, and the same *series* will become.

$$\frac{2sr}{aS} \text{ into } 1 + \frac{v}{3V} + \frac{v^2}{5V^2} + \frac{v^3}{7V^3} + \frac{v^4}{9V^4}$$

Hereby we are enabled to estimate the defect of the method of making the Meridian line by the continual addition of the Secants of æquidifferent Arches, which as the differences of those Arches are smaller, does still nearer and nearer approach the Truth. If we assume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common difference of a rank of Arches: It will be in all cases, As the Arch of one Minute, to its Chord: So the Secant of the Middle Latitude, to the first step of our *series*. This by reason of the near equality between a and $2s$, which are to one another in the ratio of Unity to $1-0,0000000352566457713$, &c. will not differ from the
Secant

Secant f but in the ninth Figure ; being less than it in that proportion. The next step being $\div \frac{2f^3s^3}{3ar^5}$ will be equal to the Cube of the Secant of the middle Latitude multiplied into

$$\frac{2sss}{3arr} = 0,0000000705132908715; \text{ which therefore unless}$$

the Secant exceed *ten times Radius*, can never amount to 1 in the fifth place. These two steps suffice to make the Meridian Line or Logarithm Tangent to far more places than any Tables of Natural Secants yet extant, are computed to; but if the third step be required it will be found to be

$$\div f^5 \text{ into } \frac{2s^5}{5ar^4} = 0,000000000000000089498; \text{ By all}$$

which it appears that Mr. *Wright's* Table does no where exceed the true Meridian Parts by fully half a Minute; which small difference arises by his having added continually the Secants of $1', 2', 3', \&c.$ instead of $0\frac{1}{2}', 1\frac{1}{2}', 2\frac{1}{2}', 3\frac{1}{2}', \&c.$ But as it is, it is abundantly sufficient for *Nautical Uses*. That in Sr. *Jonas Moor's New Systeme of the Mathematicks* is much nearer the Truth, but the difference from *Wright's* is scarce sensible, till you exceed those Latitudes where Navigation ceases to be practicable, the one exceeding the Truth by about half a Minute, the other being a very small matter deficient therefrom.

For an example easie to be imitated by who so pleases, I have added the true Meridional Parts to the first and last Minutes of the Quadrant; not so much that there is any occasion for such accuracy, as to shew that I have obtained, and laid down herein, the full *Doctrine* of these *spiral Rhumbs* which are of so great concern in the Art of *Navigation*.

The first Minute is, 1.00000001410265862178

The Second, 2,00000005641063806707

The Last, or $89^\circ.59'$ is 30374,9634311414228643

and not 32348,5279 as Mr. *Wright* has it, by the addition of the *Secants* of every whole Minute: Nor 30249,8 as Mr. *Oughtred's* Rule makes it, by adding the *Secants* of every other half Minute. Nor 30364,3 as Sir *Jonas Moor* had concluded it by I know not what method, tho' in the rest of his Table he follow *Oughtred*. And.

And this may suffice to shew how to derive the true Meridian Line from the Sines, Tangents or Secants supposed ready made; but we are not destitute of a Method for deducing the same independently, from the Arch it self. If the Latitude from the Equator be estimated by the length of its Arch A ; Radius being Unity, and the Arch put for an Integer be a , as before; the Meridional Parts answering to that Latitude will be

$$\frac{1}{2} \text{ into } A + \frac{1}{6} A^3 + \frac{1}{120} A^5 + \frac{1}{5040} A^7 \text{ or } \frac{61}{5040} A^7 + \frac{171}{113400} A^9 \text{ or } \frac{1111}{113400} A^9, \&c.$$

which converges much swifter than any of the former Series, and besides has the advantage of A encreasing in Arithmetical progression, which would be of great ease, if any should undertake *de novo* to make the *Logarithm Tangents*, or the Meridian Line to many more places than now we have them. The *Logarithm Tangent* to the Arch of $45 + \frac{1}{2} A$ being no other than the aforesaid Series $A + \frac{1}{6} A^3 + \frac{1}{120} A^5$, &c. in *Napiers* form, or the same multiplied into 0,43429, &c. for *Briggs's*.

But because all these Series towards the latter end of the Quadrant do converge exceeding slowly, so as to render this Method almost useless, or at least very tedious. It will be convenient to apply some other Arts, by assuming the Secants of some intermediate Latitudes; and you may for s or the Sine of a the Arch of half the difference of Latitudes, substitute $a - \frac{1}{2} a^3 + \frac{1}{120} a^5 - \frac{1}{5040} a^7 + \frac{1}{113400} a^9$, &c. according to Mr. *Newton's* Rule for giving the Sine from the Arch; And if a be no more than a degree, a very few steps will suffice for all the accuracy that can be desired.

And if a be commensurable to a , that is, if it be a certain number of those Arches which you make your Integer, then will $\frac{a}{a}$ be that number: which if we call n . the parts of the Meridional Line will be found to be.

$$\frac{f^n}{r} \text{ into } \left\{ \begin{array}{l} 1 + \frac{aa}{3r^4} + \frac{f^4 a^4}{5r^8} + \frac{f^6 a^6}{7r^{12}}, \&c. \\ - \frac{aa}{6rr} - \frac{ff a^4}{6r^6} - \frac{f^4 a^6}{6r^{10}}, \&c. \\ + \frac{r a^4}{120r^4} + \frac{13f^2 a^6}{360r^8}, \&c. \\ - \frac{1 a^6}{5040r^6}, \&c. \end{array} \right.$$

In this the first two steps are generally sufficient for Nautical uses, especially when neither of the Latitudes exceed 60 degrees, and the difference of Latitudes doth not pass 30 degrees.

But I am sensible I have already said too much for the Learned, tho' too little for the Learner; to such I can recommend no better Treatise, than that of Dr. *Wallis* in Numb. 176. wherein he has with his usual brevity, and that perspicuity peculiar to himself, handled this Subject from the first Principles, which here for the most part we suppose known.

I need not shew how, by regressive work, to find the Latitudes from the Meridional Parts, the Method being sufficiently obvious. I shall only conclude with the proposal of a Problem which remains to make this Doctrine compleat, and that is this.

A ship sails from a given Latitude, and having run a certain number of Leagues, has altered her Longitude by a given angle. It is required to find the Course she steered. The solution hereof would be very acceptable, if not to the publick, at least to the Author of this Tract, being likely to open some further Light into the Mysteries of Geometry.

To Conclude, I shall only add, that Unity being Radius, the *Cosine* of the Arch *A*, according to the same Rules of Mr. *Newton*, will be

$1 - \frac{1}{2} A^2 + \frac{1}{24} A^4 - \frac{1}{720} A^6 + \frac{1}{40320} A^8 - \frac{1}{362880} A^{10} \&c.$
from which and the former Series exhibiting the *Sine* by the Arch, by division it is easie to conclude, that the *Natural Tangent* to the Arch *A* is

$A + \frac{1}{3} A^3 + \frac{1}{15} A^5 + \frac{1}{315} A^7 + \frac{1}{1575} A^9 \&c.$
and the *Natural Secant* to the same Arch

$1 + \frac{1}{2} A^2 + \frac{1}{24} A^4 + \frac{1}{720} A^6 + \frac{1}{3024} A^8 \&c.$
and from the Arithmetick of Infinites, the Number of these Secants being the Arch A , it follows, that the sum Total of all the Infinite Secants on that Arch is

$A + \frac{1}{2} A^3 + \frac{1}{24} A^5 + \frac{1}{720} A^7 + \frac{1}{3024} A^9 \&c.$
the which, by what foregoes, is the *Logarithm Tangent*, of *Napeirs* form, for the Arch of $45^{\text{th}} + \frac{1}{2} A$, as before.

And Collecting the Infinite Sum of all the *Natural Tangents* on the said Arch A , there will arise

$\frac{1}{2} AA + \frac{1}{12} A^4 + \frac{1}{24} A^6 + \frac{1}{1512} A^8 + \frac{1}{14175} A^{10} \&c.$
which will be found to be the *Logarithm* of the Secant of the same Arch A .

Accounts of Books.

L. Catoptrica & Dioptrica Elementa, Auctore Davide Gregorio, D. M. Astronomiæ Professore Saviliano Oxonia, & Soc. Reg. Socio, 8°. è Theatro Oxon. 1695.

IN this Treatise the Learned Author demonstrates the Principal Laws of Reflection and Refraction, without restraining himself to any Sect of Philosophers; as also the properties of plain and spherical Surfaces in reflecting and refracting of Rays, and by the way shews how it comes that spherical Surfaces produce the same effects with those of certain Spheroids and Conoids, *viz.* because they have the same degree of Curvature. In the Catoptricks he determines the place of the Image, when the Object and the Eye are not in the same axis of the reflecting Sphere: an inconvenience that Dioptrical Machines are not subject to.

Then he proceeds to determine the situation and bigness of the Images of sensibly big Objects, with the quan-