

A Correction of the Theory of the Motion of the Satellite of Saturn, by that Ingenious Astronomer Mr. EDMUND HALLY.

S I R,

I here send you an *Astronomical account* of the most remote of all the *Planets* of our *Vortex*, and withal of the most inconsiderable, I mean of the *Satellite* of *Saturn*, discovered in the year 1655 by *Monsieur Christian Hugen* of *Zulichem*, who in that accurate *Treatise* of his *Systema Saturnium*, from pag. 25 to 34 gives us the *Theory* of its *Motion* as well as the shortness of the interval of time between his *Observations* would admit; and since him I know none that have gone about to improve the said *Theory*.

The late *Conjunctions* of *Jupiter* and *Saturn* giving me frequent occasions of viewing them both, with a *Telescope* that I have of about 24 foot, and pretty good of that length, I easily remarked this *Satellite* of *Saturn*, and having found it, in a convenient position to determine its place, I perceived that *Hugen's* numbers were considerably run out, and about 15 *degrees* in twenty years too swift; this made me resolve more nicely to enquire into its *period*; and accordingly I waited till I had gotten a competent number of *Observations*, the most considerable whereof are these.

1682. *November* 13^o 13^h 00' PM. the *Satellite* appeared on the *North* side of *Saturn*, and a *perpendicular* let fall from it on the *transvers* *diameter* of the *Ring*, fell upon the middle of the dark space of the following *Anse*; and the same night 19^h 00' it had past the *Conjunction*, and the *perpendicular* fell exactly on the *Western* edge of the *Globe* of *Saturn*, as in *Tab. 2. Figure 8*. The *Northern* *Latitude*, and retrograde motion, made it evident that the *Satellite* was then in *Perigæo*.

Again

Again November 21^o. 16^h. 15'. this *Satellite* of *Saturn* was on his *South* side, the *perpendicular* on the line of the *Anse* fell on the middle of the dark space of the *Western Anse*, and the same night 19^h 00' the *perpendicular* fell precisely on the *Center* of *Saturn*, and the distance therefrom was somewhat less than one *diameter* of the *Ring* (as in *Fig. 9.*) by this it was evident that the *Satellite* was in *Apogæo*.

I observed it in *Apogæo* again on the 24th of *January* 1683, at 8^h. 00' P.M. the *perpendicular* on the line of the *Anse* fell exactly on the *Western* limb of the *Globe* of *Saturn*, and at 9^h 30' P.M. the said *perpendicular* fell within the *Globe* more than half way to the *Center*, and the distance from the line of the *Anse* towards the *South* seemed much about one *diameter* of the *Ring*. *Fig. 10.*

Lastly, *February* 9^o. 1683. 8^h 10' P.M. it was again in *Apogæo*, and I could by no means discern towards which side it inclin'd most, nor whether the transvers *diameter* of the *Ring*, or the distance of the *Satellite* therefrom were the greater; so that at that time it was precisely *Apogæon*. *Fig. 11.*

To compare with these, I chose two out of those of *Monsieur Hugen*, which seem'd the most to be confid'd in; the first made 1659, *March* 14^o *ft. n.* 12^h 00'. at the *Hague*; when the *Satellite* appear'd about one *diameter* of the *Ring* under *Saturn*, but it was gone so farr to the *Westward*, that he concluded, that about four hours before, or 7^h. 40'. at *London*. it had been in *Perigæo*. *Fig. 12.*

Again *March* 22^o. 1659. 10^h 45'. the *Satellite* was a whole *diameter* above the line of the *Anse*, and the *perpendicular* thereon fell nearly upon the extremity of the *Eastern Anse*. See *Fig. 6.*

I could wish that we had some intermediate *Observations*, but there are none extant that I can hear of; so I proceed to the search of this *Satellit*'s period.

By the first of my *Observations* it appears that the *Satellite* was in *Perigæo* 1682 *November* 13^o. 17^h 00'. *circiter*, at which time *Saturn* was 3^s. 21^o. 39' from the first *Star* of *Aries*, in the *Ecliptick*, but the *Earth* reduced to *Saturns Equinoctial*, and the *Satellite* was 9^s. 23^o. 46'. a 1^a * γ . And *March* 4^o 1659. 7^h. 40'. *Saturns* place in the *Ecliptick*, was 6^s. 0^o. 41', but the *Earth* reduced and consequently

consequently the *Satellite* in $11^{\circ}.28'.18''$ à *prima Stella Arietis*. The interval of time is 8655 daies, 9 hours, 20 minutes; in which the *Satellite* had made a certain number of *Revolutions* to the fixt Stars, and besides $9^{\circ}.25'.28''$, or 295 degrees, 28 min. whose *Complement* to a Circle $64^{\circ}.32'$, is 2 daies, 20^h, 36' motion of the *Satellite*, according to *Hugens*. So that 8658 daies, 5^h, 56' or 12467876 minutes of time, is the time of some number of intire *Revolutions*; and dividing that interval by 15 daies 22 hours, 39 minutes, or 22959 min. (the *Period* of *Hugens*) the *Quotient* 543 shewes the number of *Revolutions*; and again dividing 12467876 min. by 543, the 22961 $1\frac{1}{2}$ min. or 15 daies, 22^h. 41'. 6'' appears to be the true time of this *Satellit's* *Period*. Hence the *diurnal motion* will be $22^{\circ}.34'.38''.18'''$, and the *Annual* besides 22 *Revolutions* $10^{\circ}.20'.43'$. Having made *Tables* to this *Period*, I found that in the *Apogæon* Observation of *Hugens* the *Satellite* was above 3 degrees faster than by my *calculus*, and that in the three other Observations of my own being likewise in the superior part it was about $2\frac{1}{2}$ degrees slower than by the same *Calculation*. Now 'tis evident that these differences must arise from some *Eccentricity* in the *Orbite* of this *Satellite*, and that in *March* 1659, the *Apocronion* (as I may call it,) was somewhere in the *Oriental Semicircle*, and that in *November* 1682 it was in the *Western Semicircle*, and supposing the *Apocronion* fixt, it must necessarily be between $9^{\circ}.23'.46''$ and $11^{\circ}.28'.18''$. a $1^{\text{a}} * \gamma$ that being the common part between those two *Semicircles*: and because the difference was greater in *Hugen's* Observation than in *Mine*, 'twill follow that the *Linea Apfæum*, or *Apocronion*, should be nearer to $9^{\circ}.23'.46''$ than to $11^{\circ}.28'.18''$. I will suppose $10^{\circ}.22'.00'$ a *prima Stella Arietis*, (which happens to be also the place of *Saturn Equinox*.) and the greatest *equation* about $2\frac{1}{2}$ degrees. Upon the score of this *inequality* the *mean motion* of the *Satellite* will be found about $2^{\circ}.45'$ slower in $23\frac{1}{2}$ years, or 7 min. in a year, whence I state the *Annual motion* $10^{\circ}.20'.36''$ above 22 *Revolutions*, and the correct *Epocha* for the last day of *December* 1682 at Noon in the *Meridian* of *London* $9^{\circ}.10'.15''$. a $1^{\text{a}} * \gamma$, from which *Elements* I compose the following *Table*.

*Tabula Motus Medii Satellitis Saturnii,
ab Hugenio inventi, a prima * r.*

Ann. Christ. Civit.	Epocha			Annus	Mot. Med.			Diebus	Mot. Med.			H.	Mot. Med.			Mot. Med.		
	s.	o.	l.		s.	o.	l.		s.	o.	l.		M.	o.	l.	M.	o.	l.
1641	8.	29.	17.	1	10.	20.	36	1	0.	22.	35	1	0.	56	31	29.	10	
1661	10.	14.	10.	2	9.	11.	12	2	1.	15.	9	2	1.	53	32	30.	6	
1681	11.	29.	3.	3	8.	1.	48	3	2.	7.	44	3	2.	49	33	31.	3	
1682	10.	19.	39.	4	7.	14.	59	4	3.	0.	18	4	3.	46	34	31.	59	
1683	9.	10.	15.	5	6.	5.	35	5	3.	22.	53	5	4.	42	35	32.	55	
1684	8.	00.	51.	6	4.	26.	11	6	4.	15.	28	6	5.	39	36	33.	52	
1685	7.	14.	2.	7	3.	16.	47	7	5.	8.	2	7	6.	35	37	34.	48	
Mens.	Mot. Med.			8	2.	26.	57	8	6.	0.	37	8	7.	32	38	35.	45	
Ann.	Com.			9	1.	20.	23	9	5.	23.	12	9	8.	28	39	36.	41	
Jan.	0.	0.	0.	10	0.	11.	9	10	7.	15.	46	10	9.	24	40	37.	38	
Feb.	11.	9.	53	11	11.	1.	45	11	8.	8.	21	11	10.	21	41	38.	34	
Mart.	8.	12.	2.	12	10.	14.	56	12	9.	0.	55	12	11.	17	42	39.	31	
April.	7.	21.	56	13	9.	5.	32	13	9.	23.	30	13	12.	14	43	40.	27	
Maii	6.	9.	14.	14	7.	26.	8	14	10.	16.	5	14	13.	10	44	41.	24	
Junii	5.	19.	7.	15	6.	16.	44	15	11.	8.	39	15	14.	7	45	42.	20	
Julii	4.	6.	26.	16	5.	29.	54	16	0.	1.	14	16	15.	3	46	43.	17	
Aug.	3.	16.	19	17	4.	20.	30	17	0.	23.	48	17	16.	0	47	44.	13	
Sept.	2.	26.	12.	18	3.	11.	6	18	1.	16.	23	18	16.	58	48	45.	10	
Octob.	1.	13.	31.	19	2.	1.	42	19	2.	8.	58	19	17.	52	49	46.	6	
Novem.	0.	23.	24.	20	1.	14.	53	20	3.	1.	32	20	18.	49	50	47.	3	
Decem.	11.	10.	43.	21				21	3.	24.	7	21	19.	45	51	47.	59	
				22				22	4.	16.	42	22	20.	42	52	48.	56	
				23				23	5.	9.	16	23	21.	38	53	49.	52	
				24				24	6.	1.	51	24	22.	35	54	50.	49	
				25				25	6.	24.	25	25	23.	31	55	51.	45	
				26				26	7.	17.	00	26	24.	27	56	52.	42	
				27				27	8.	9.	35	27	25.	24	57	53.	38	
				28				28	9.	2.	0	28	26.	20	58	54.	35	
				29				29	9.	24.	44	29	27.	17	59	55.	31	
				30				30	10.	17.	18	30	28.	13	60	56.	27	
				31				31	11.	9.	53							
				32				32	0.	2.	28							

*In Anno Biffextili post Fe-
bruarium adde unum di-
em, motumq; ei competen-
tem,*

I here suppose the *Linea Apfiduum* fixt, as having no arguments from Observation to prove the contrary, tho' it be very probable that as the *Apogeon* of our *Moon* has a motion about the *Earth* in about 9 years, so that if this *Satellite* ought to have about *Saturn*, but with a much longer *Period*, which future Observation may discover.

The distance of this *Satellite* from the *Center* of *Saturn* seems to be much about 4 *Diameters* of the *Ring*, or 9 of the *Globe*, and the plane wherein it moves very little or no thing differing from that of the *Ring*, that is to say, intersecting the *Orb* of *Saturn* $4^{\circ}.22^{\circ}$. and $10^{\circ}.22^{\circ}$. γ , with an *Angle* of $23\frac{1}{2}$ degrees, so as to be nearly *Parallel* to the *Earths Equator*; whence the *Latitude* of the *Apogeon* Semicircle from $4^{\circ}.22^{\circ}$. to $10^{\circ}.22^{\circ}$ of *Saturns Longitude*, will be *Northern*, and of the other Semicircles *Southern*; and the contrary in the other half of *Saturns Longitude*, to wit, from $10^{\circ}.22^{\circ}$. to $4^{\circ}.22^{\circ}$ of his distance from the first Star of γ .

It follows now to shew how by the help of this *Table* to compute the place of this *Satellite*, to any time required.

First we must have the true *Longitude* of *Saturn* from the *Earth*, and nuabred from the first Star of γ , (or rather the place of the *Earth* viewed from *Saturn* together with its *Latitude* from the *Orb* of *Saturn*, but that being never fully $\frac{1}{4}$ of a *degree* we neglect it as a *nicety*) and therefrom *subtract* $10^{\circ}.22^{\circ}$. there remains the distance of *Saturn* from this *Equinoctial* point, with which distance as with the *Longitude* of the *Sun*, take out the *Right Ascension* and *Declination* thereto ($23\frac{1}{2}$ degrees being the *obliquity* common to both) and to the *Right Ascension* adding $10^{\circ}.22^{\circ}$. the sum shall be the *Longitude* of the *Satellit's Apogeon*. Then say, As *Radius* to *sine* of the *Declination*, so 8 to the greatest *Latitude* in *Apogeo*, or *Perigeo* in the parts of the *semidiameter* of the *Ring*.

Next collect the *middle motion* of the *Satellite*, and from it *subtract* $10^{\circ}.22^{\circ}$; the *remainder* shall be the *mean Anomaly*, with which in the *Table* of the *Moons primary Equation*; take out the *Equation*: answering thereto, and the half thereof *added* or *substracted* to or from the *middle motion*, according to the *Title*, gives the *true motion* of the *Satellite*, from which *subtract* the *Apogeon*, and if the *remainder* be more than 6 *Signs*, the *Satellite* is *Occidental*; if less *Oriental*, and as *Radius* to *Sine* of the *remainder*, so 8 to the *Semidiameters* of the *Ring*; or 18, to the *Semidiameters* of the *Globe*; that the *Satellite* is to the *Eastward* or *Westward* of the *center* of *Saturn*, according to the *afore-going Precept*.

Lastly, As *Radius*, to *Co-sine* of the said *remainder*, so is the greatest *Latitude* from the *line* of the *Anse*, to the *Latitude* sought.

arise-

Here Note, that I purposely neglect the *inequality* of the distance, arising from the *Eccentricity*, as being too small to be any way observable.

Lastly to clear all difficulties that may arise to them that are but little versed in this sort of *Calculation*, I have added *Two Examples* of the work, that where the *Precept* may seem obscure it may be thereby illustrated.

Anno 1657 Maii 19th. n. *Hugens* Observed the *Satellite* very near to *Saturn* on the *Western* side, and very little above the *line* of the *Ansa*. I suppose this about 10^h. p. m. Anno 1658 Martii 11^o. 10^h fl. n. he Observed it again, and said of it, *difficile conspiciebatur, quippe propinquius admodum Saturno, Orientem spectabat, eratq; Anserum lineâ aliquanto inferior & quasi sub Saturno transiturus*. Let us now Calculate to these two times.

1657 Maii 9 ^o .	9 ^h . 40 ^l .	Londini
Saturni Locus	13. 28. 57 ^l	
h a 1 ^a * γ	5 ^s . 0. 32	
Equinoct. sub.	10. 22. 00	
h ab Equinoct.	6. 8. 32	
Ascen. Recta.---	6. 7. 50	
Apogæon.	4. 29. 50	
Declin. Aust.	3. 23	

1658. Martii 1 ^o	10 ^h
Saturni Locus	≈ 16 ^o . 25 ^l
h a 1 ^a * γ	5 ^s 17 ^o . 59
Equinoctium	10 22. 00
h ab Equin.	6. 25. 59
Ascen. Recta.	6. 24. 5
Apogæon	5. 16. 5.
Declin. Aust.	10. 4

	Med. Mot. Satel.
1641.	8 ^s . 29 ^o . 17 ^l
16.	5. 29. 54
Maii	6. 9. 14
9.	6. 23. 12.
9 ^h . 40 ^l	9. 5

	Med. Mot. Satel.
1641.	8 ^s . 29 ^o . 17 ^l
17.	4. 20. 30 ^½
Martii.	8. 12. 2 ^½
1 ^d .	0 22. 34 ^½
10 ^h	0. 24 ^½

Long. Med. Satel.	4. 10. 42
Apocron.	10. 22. 00
Anomalial	5. 18. 42
Equatio sub.	31
Long. Ver. Satel.	4. 10. 11
Apogæon	4. 29. 50
Residuum	11. 10. 21
h. e. ante Apogæon	19. 39
ergo 2 ⁷ / ₁₀ . Semid. Annali ad occasum	
& 2 ² / ₁₀ . ad Boream.	

Long. Med. Satel.	11. 3. 49
Apocron.	10. 22. 00
Anomalial	0. 11. 49
Equatio sub.	0. 30
Long. Ver. Satel.	11. 3. 19
Apogæon	5. 1. 5
Residuum	5. 17. 14
h. e. ante Perigæon	12. 46
hinc provenit 1 ⁷ / ₁₀ . Semid. Ann. ad ortum,	
& 1 ² / ₁₀ . ad Austrum sive infra.	

In each agreeing exactly with the Description and Figure of
 Monsieur *Hugens*. M 2

I here call the *Plane* of this *Satellit's Orb*, which hitherto I suppose the same with that of the *Ring, Saturns Equinoctial*, not that any discovery hath been able to prove that the *Axis* of that *Globe* is at *right Angles* thereto, but because it has pleased Mr. *Hugens* to call it so, and likewise because it is so nearly *Parallel* to our *Globes Equinoctial*; Nevertheless to speak my Opinion, I believe that the *Axis* is inclined, and that not a little, to the *Plane* of the *Ring*; for as the reflection of the *Suns* light from the *Ring* is a great convenience to that *Hemisphere* of *Saturn*, which beholds its illuminate side; so the other *Hemisphere* is very much incommoded by the shadow of the *Ring*, which for many *Months*, and in some *Parallels* for several *Years*, occasions a continual Night by the interception of the *Suns* beams, which is a consequence that demonstratively follows the position of the *Ring* in the *plane* of *Saturns Equator*. Now this great inconvenience would be in some measure relieved by the oblique position of the *Axis*, for then the *Parallels* of *Latitude* intersecting the *plane* of the *Ring*, many and in most cases all of them, might for some time in every *Diurnal revolution* of the *Globe* free themselves from this *Eclipse*, which otherwise were sufficient to render this *Globe* of *Saturn* unfit for any settled habitation; but this is but conjecture.

The other Two *Satellites* of *Saturn* discovered by Signor *Cassini* at *Paris Anno 1672* and *1673*, I must confess I could never yet see. I have been told that they disappear for about $\frac{2}{3}$ of *Saturns revolution*, and were only to be seen when the *Ansa* were very small, it being supposed that the light which proceeds from the *Ansa* when considerably opened might hide these *Satellites*. In the year *1685* when the *Ansa* will be quite vanished, will be a proper time to look after them, that so we may bring their Motion to Rule, and know where to find them, for want of which knowledge 'tis likely they are at present not to be found.

Obser.

Fig. 1.

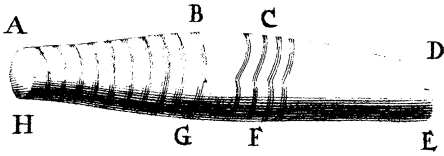


Fig. 2.

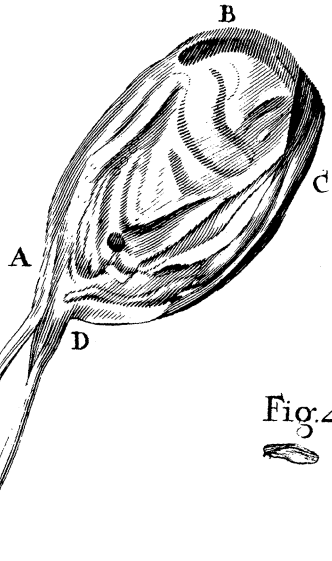


Fig. 3.

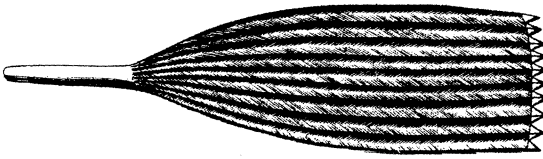


Fig. 4.



Fig. 5.

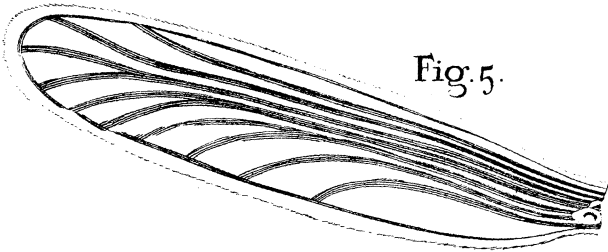


Fig. 6.



Fig. 7.

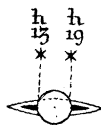
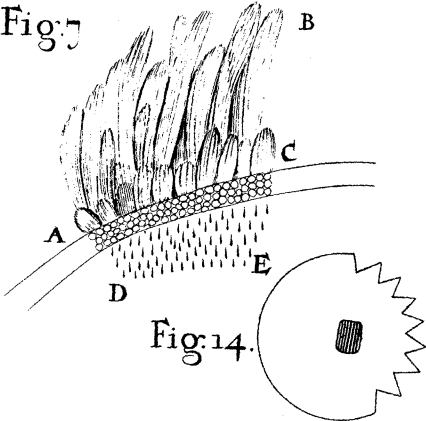


Fig. 8.

Fig. 9.

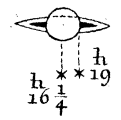


Fig. 10.

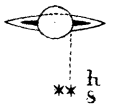


Fig. 11.

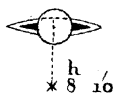


Fig. 12.

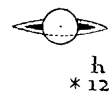


Fig. 13.



Fig. 13.