### ( 277 )

## I. A Differtation concerning the Figure of the Earth. Part the Second. By the Reverend J. T. Defaguliers, LL. D. F. R. S.

**S** INCE my \* Paper concerning the Figure of the Earth was read before the Royal Society, I met with a Differtation of Monf. Mairan (in the Memoirs of the Royal Academy of Paris, for the Year 1720.) wherein the learned and ingenious Author has taken a great deal of Pains to reconcile the Obfervations made on Pendulums, (found to be shorter at the Æquator than at Paris, when they fwing Seconds) with the oblong fpheroidical Figure of the Earth, deduced from Monf. Callini's Measures. And tho' upon a strict Examination of his Conjectures, and what he gives for Demonstrations. I do not find Reafon to alter my Opinion concerning the oblate or flatted Spheroid, which Sir I (aac Newton has shewn to be the Figure of the Earth ; yet fince it might be thought by fome, who have read Monf. Mairan's Treatife, and afterwards may read mine, that I have not confider'd all the Circumftances that He has done, and that I have not been exact enough in the Mathematical Part of my Differtation, because I have drawn fome Conclusions from fuppoing the Figure of the Earth fpherical, when I should have supposed it an oblong Spheroid; I beg Leave to fhew here, wherein I think Monf. Mairan is miltaken, and to give those additional Proofs of my Affertions, which I promis'd the Society when I gave in my laft Paper.

\* Vid. Philof. Tranf. Nº. 386, 387,

VOL. XXXIII.

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## (278)

#### First then I begin with the Conjectures.

Monf. *Mairan* fays, that it is as reafonable to fuppofe the Earth (if it was once fluid) to have been an oblong Spheroid at first, as a Sphere; and that, in fuch a Cafe, the Centrifugal Force of the feveral Parts of the Earth, arising from its Revolution about its Axis, which might convert a Sphere into an oblate Spheroid, wou'd only change an oblong Spheroid into one lefs oblong.

If the Earth was at first a Fluid, (fupposed homogeneous, and of any given Form,) and left to those Laws, which we find to obtain at present; it must put on a spherical Figure, for the same Reason that Drops of Mercury, of Water, and other Fluids, put on such a Figure. And to suppose any Change made in that Figure from the Pressure of an external Fluid, filling up all Space, is contrary to what has been demonstrated by Sir Isaac Newton in his Principia Lib. 2. Prop. 19. where he shews, That if any Portion of a Fluid be compress'd by the same or any other homogeneous Fluid, that Portion will not have its Figure alter'd by that Pressure.

And indeed we fee, that in the Receiver of the Air-Pump, Lumps of Butter, coagulated Oil, or Honey, Drops of Quickfilver or Water, &c. have the fame Figure, whether the Preffure of the Air acts upon them, or be taken off by exhaufting the Receiver.

That a fluid Substance, of any Figure, will by the Gravity of its Parts become spherical, is plain by the following

#### DEMONSTRATION. Fig. I,

Let ABCDE be a Portion of an homogeneous Fluid, whofe Parts tend towards one another, and whofe Figure is not fpherical. If in fuch a Fluid we fuppofe a Syphon as ACE (or which is the fame thing, if all the Fluid fhou'd be frozen, except the Canal ACE) whofe Legs AC and CE are unequal, and meet at C, the Center of the Fluid, towards which there is the greatest Tendency; the Fluid will run out at A in the Leg AC, till it be come down as far as g in the Leg CE, fuppoling  $C_g$ equal to AC. But if the Leg AC be lengthened as far as c, then the Fluid will only come down as far as e in the Leg CE, and at the fame time rife up to a in the Leg Ca, Ca being equal to Ce.

If fuch another Canal or Syphon be fuppos'd at BCD. the Fluid in it will come down from D to d, and rife from B to b. And fince fuch Syphons may be fuppos'd all over the Fluid ABDE; that Fluid, by the mutual Tendency of its Parts towards one another, must be reduc'd to the fpherical Figure abde. Which was to be demonstrated.

Now, without confidering the Unreafonablenefs of the Supposition, let us imagine the Earth to have been an oblong Spheroid at first, and then to have a diurnal Revolution given to it, which fhou'd by Degrees fhorten its Axis, to bring it to what Meffieurs Caffini and Mairan suppose it at present to be. If in such a Cafe the Earth be fuppos'd fluid enough to change its Figure. by the Revolution about its Axis, why fhould it ftop when the Æquatorial Diameter comes to want just one 96th Part of the Length of the Axis? fince two Powers act upon it to fhorten its Axis, viz. Gravity, and the Centrifugal Force; the first of which has already been fhewn capable to reduce it to a Sphere, and the Centrifugal Force is acknowledged by Monf. Mairan to be (as Sir Ifaac Newton has prov'd it) at the Æquator equal to is Part of the Gravity there. Certainly the Alteration of Figure wou'd not have ftopp'd, before the R r 2 Earth

Earth came to be a Sphere; nay, and it must have rifen at the Æquator; and how much, I have already shewn in my former Paper.

Again, if we fuppole the Earth of an heterogeneous Fluid, before the diurnal Revolution, the heaviest Parts wou'd go towards the Center, and the lighter towards the Surface; and that Way the Terraqueous Globe wou'd also become a Sphere. Then if, when the Central Parts are fix'd, and the fuperficial Strata are full fluid, the Earth receives a diurnal Motion; it will rife at the Æquatorial Parts, and that to a greater Height than what I have thewn in my former Paper, where I fuppos'd the Earth of uniform Matter. And that fomething like this must be the Cafe, appears from what Sir Ifaac Newton has faid upon this Subject. For after having thewn, from fuppoling the Earth of uniform Matter, that the Centrifugal Force of all its Parts wou'd bring it to be 17's English Miles higher at the Æquator than at the Poles, and after having given a Table of the proportionable Decreafe of the Length of the Degrees of a Meridian of the Earth, going from the Poles to the Æquator, in fuch a Figure of the Earth, with the Lengths that Pendulums must have to fwing Seconds in feveral Latitudes; from a Comparison of the Lengths of Pendulums (observ'd by different Persons to be shorter towards the Æquator, than in greater Latitudes (when they fwing Seconds) he fhews that the Earth muft be 31,7 Miles higher at the Æquator than at the Poles; and therefore that it must be denser towards the Central than the Superficial Parts to produce a flatted Spheroid, where the Æquatorial Diameter must exceed the Axis fo much more; that is, be longer fomething more than Hart.

Laftly, let us fuppole the Earth, at its first Creation, to have been made of Land and Water, the first as folid, and and the laft as fluid as it is now, but of Monf. Caffini's Figure, and examine the Confequence. Since in that Figure the Axis is  $\frac{1}{5\sigma}$  Part longer than the Æquatorial Diameter; the Gravity will be fo much greater at the Æquator than at the Poles, that the Waters will all flow to the Æquatorial, and leave the Polar Regions; which will happen ftill more by the Centrifugal Force, which the Earth in its diurnal Motion will give to the Fluid; and therefore the Sea wou'd be  $43\frac{2}{3}\frac{2}{3}\frac{2}{3}$  Miles (reckoning 5000 Feet to a Mile) higher at the Æquator than at the Poles, which muft overflow all the Torrid Zone, and leave the Polar Regions dry.

I am very well aware, that it may be objected by fuch as have read Monf. Mairan's Differtation, and have not read Sir Isaac Newton's Principia, or have not read that Book with due Attention \_\_ " That I have not ar-" gued fairly in drawing Confequences from a greater " Gravity at the Æquator than at the Poles, in an ob-" long Spheroid ; becaufe Monf. Mairan has fhewn. " that, in fuch a Figure of the Earth, the Gravity is greater " at the Poles than at the Æquator; and that I fhou'd " have drawn my Confequences from these Principles." To which I answer, that his Demonstrations about Gravity are built upon wrong Suppositions, as I shall shew by Neverthelefs, fuppofing that Gravity was and by. greater towards the Poles than towards the Æquator, in the Proportion that he affigns, namely of the Ray of Curvature drawn into the Perpendicular to the Curve. terminated at the Axis; let us confider what will follow from his Principles.

\* Let us then suppose the Earth at first in a fluid State; A A the Axis, dÆ the Æquatorial Diameter, 06 a Ray of Curvature, dn another, ac and dC two Lines

\* Fig. II.

of Tendency or Perpendiculars to the Curve, intercepted by the Axis at c and C; and dC, A C, two Tubes or Canals of the Fluid, gravitating towards, and communicating at C. I fay that, according to Monf. Mairan's Principles of Gravity, the Earth cannot preferve its oblong fpheroidical Figure. For fince the Gravity at a : Is to the Gravity at  $d:: As dn \times dC$ : to  $ab \times ac$ , it will follow (from the Nature of the Ellipse) that the Gravity at A : will be to the Gravity at  $d :: A \otimes A C^{4}$ : to  $dC^*$ : and therefore the Forces, with which the Columns of Fluid A C and d C tend towards C, will be as their Maffes drawn into the Forces driving towards C. that is, as  $AC \times AC^4$  to  $dC \times dC^4$ . Now by the Principles of Hydroftaticks, it is evident that the Fluid. in the Canal A C, will caufe the Fluid in the Canal d C to run out at d as long as  $AC \times AC^{4}$  is greater than  $dC \times dC^4$ : And if the Canal Cd be continued quite to  $\delta$ , the Surface of the Fluid in AC will fink to  $\alpha$ , whilft the Surface of the Fluid in d Crifes up to  $\delta$ , in which Cafe as  $\alpha C = C \beta$ , the Point A will come to  $\alpha$ , and the Point d to  $\beta$ , and the Curve A d being chang'd into  $\alpha$   $\delta$ , the oblong Spheroid will be chang'd into a Sphere, the only Figure confiftent with the *Æquili*brium of the fluid Parts, according to Monf. Mairan's own Principles; becaufe then you will have  $AC^4 =$  $d C^{4}$ , and  $A C \times A C^{4} = d C \times d C^{4}$ . If we make ufe of Sir Ifaac Newton's Principles in this Reafoning, we shall also shew, that an oblong, spheroidical, fluid Earth will be chang'd into a Sphere; but not fo faft as it does by Monf. Mairan's Laws; for, according to Sir Ilaac Newton, the Gravity at A: Is to the Gravity at  $d:: As \sqrt[4]{Cd}: \sqrt[4]{AC}$ . Q. E. D.

NB. Here

## (283)

NB. Here we have suppos'd no diurnal Revolution, for as soon as that begins, the Centrifugal Force will raise the Æquatorial Parts, and change the Sphere into a flatted Spheroid, as has been before shewn, and is allow'd by Mons. Mairan.

Now if we fuppole the fame Figure of the Earth, but the Land (at its first Creation) as firm as it is now; it will in that Cafe follow from Monf. *Mairan*'s Principles, that the Sea must rife and overflow all the Æquatorial Regions, tho' the Earth had no diurnal Revolution; and much more fo, when the Centrifugal Force, arifing from the diurnal Motion, helps to carry the Water the fame Way.

#### DEMONSTRATION.

Let  $P \not a P \not E$  \* reprefent the Plane of a Meridian, PPthe Axis of the Earth (fuppos'd an oblong Spheroid)  $a \not E$  the Diameter of the Æquator,  $de a \not a$  Part of the Surface of the Earth,  $a \not A$  and  $e \not B$  two Perpendiculars to the Surface of the Earth (which are here two Rays of Curvature) fc the Surface of the Sea, and f deg, ba a ctwo Cylinders of Sea-Water of equal Bafes and equal Heights.

Since Gravity acts on the two equal Columns of Water  $b \ a \ c \ a$ ,  $f \ d \ e \ g$  in the reciprocal Ratio of the Ray of Curvature (at the refpective Places of the Columns) drawn into that Part of it which Monf. *Mairan* calls the Line of Tendency, (that is, in the Ratio of  $e \ B \times e \ Z$  to  $a \ A \times a \ C$ ) the Weight of  $f \ e$ : will be to the Weight of  $b \ a$ :: As  $a \ A \times a \ C$ : to  $e \ B \times e \ Z$ . Therefore if there be a Communication between the fluid Co-

\* Fig. III.

lumns fe and ba, there cannot be an *Æquilibrium*, till the Quantity of Matter in fe, becomes to the Quan. tity of Matter in b a, reciprocally as the Gravity at the Place *e* is to the Gravity at *e*; and in that Cafe the Height g e will be reduc'd to ke, if ke:ce::e Bx  $e Z : a A \times a C$ . And confequently the Surface of the Sea will go thro' the Points ik bc, where bc under the Æquator is higher than ik towards the Poles. Q. E. D.

NB. That the Centrifugal Force will fill add to the Height of the Sea at bc, is plain from what we have faid before. And if we apply these Principles to determine the different Lengths of Pendulums, swinging Seconds at Paris and at the Æquator; from the Gravity at Paris, compar'd to the Gravity at the Æquator (in this Supposition of the Action of Gravity and Figure of the Earth) a Pendulum must be shorter at the Æquator by more than 10 Lines, without considering the Centrifugal Force; and if the Centrifugal Force be taken into Consideration, the Pendulums must be shortened near a whole Inch. But this being about five Times more than agrees with Observation ; what proves too much, proves nothing at all.

Having thus thewn, that Monf. Mairan's Account of the Action of Gravity, on feveral Places upon the Earth's Surface, can be of no Service for reconciling the Experiments made on Pendulums, with the Figure of the Earth deduc'd from Monf. Callini's Measures: I proceed to fhew that his Demonstrations are founded upon wrong Principles. And first, in Relation to Gravity.

This Gentleman has follow'd Sir Ifaac Newton, in faying, that Gravity increases in a duplicate reciprocal Pro-

Proportion of the diminish'd Distance from the Center of the Force, and fo vice versa; but he has follow'd Sir Ilaac Newton no farther than ferv'd his prefent Purpofe: otherwife he wou'd have known. - That in refpect to a Central Body (as a Planet) towards which others are (attracted or) impell'd by Gravity, this Law obtains only as Bodies attracted, are remov'd from the Surface of the Planet, to greater Distances from the Center compar'd with that Diftance; or as from greater Diftances they approach nearer to the Planet. - That the greatest Action of Gravity is at the Surface of the Planet.—That afterwards in advancing towards the Center. the Force of Gravity, on the Body attracted, continually grows lefs, decreasing directly as the Diftance; and that this holds true in a Spheroid as well as a Sphere. - That on different Parts of the Surface of the Earth (in the Condition it is now) the Gravity on Bodies is reciprocally as their Diftance from the Center of the Earth. -That though at a confiderable Diftance we look upon the Earth, or any Planet, or even the Sun, as a Point (in the Center of the Forces tending towards it) endued with an absolute Force, proportional to its Quantity of Matter; yet when we come fo near the Body as to confider the Space it takes up, we are to take notice. that the whole Attraction or Gravity of the Body, is made up of the Sum of the Attraction of all its Parts properly combin'd; and therefore, that when a Corpufcle. or Body attracted, comes to be within the Planet, or Body attracting, the Matter above it draws it back in fuch a Manner, that it leaves it only a Force to go on to. wards the Center, which is directly as the Diffance, as we have already faid; just as if a Body concentric to the Planet (whether fpherical or fpheroidical) had its Surface just where the Corpuscle is, and all the exterior Cruft or Shell was annihilated.

VOL.XXXIII.

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I do not doubt but Monf. *Mairan* will be of this Opinion, when he has carefully and impartially examin'd the 12th and 13th *Sections* of the First Book of Sir *Ifaac Newton's Principia*, and the 18th, 19th, and 20th *Prop.* of the Third Book. And if he will be at the Pains to compare the 38th and 39th *Proposition* of the Third Book with the 66th of the First, he will find that the Precession of the *A*-quinoxes is owing to the broad spheroidical Figure of the Earth; and that if it had Monf. *Cassini's* Figure, the *A*-quinoctial Points wou'd move *in Consequentia* faster than they do now *in Antecedentia*.

Further, Monf. Mairan demonstrates, that in an oblong Spheroid, the Diminution of Gravity, by the Centrifugal Force, encreases faster in going from the Poles to the Æquator, than it wou'd do in a Sphere, and faster in a Sphere than it wou'd do in a broad Spheroid; and therefore wou'd shew, " That notwithstanding the Sur-" face of the Earth is nearer to the Center in Monf. Cas-" fini's Figure than in Sir Isac Newton's, yet the " Centrifugal Force will diminish the Gravity fo fast in " going from Paris to the Æquator, that the shortening " of Pendulums, to make them swing Seconds at the " Æquator, may very well be accounted for that Way." Now let us examine into this Matter, to see whether the Cause is adequate to the Effect.

If the Diftance from the Surface of the Earth at the Pole to the Center be 96, and the Diftance of the Surface at the Æquator be 95, the Diftance of the Surface at *Paris*, in the Latitude of 48°50', will be 95,562, &. by the Property of the Ellipfe. Now fince the Force of Gravity, in different Places on the Earth's Surface, is reciprocally as the Diftance from the Center, and the Lengths of Pendulums, that perform their Vibrations in the fame Time, are directly as the Force of Gravity; therefore therefore the Length of Pendulums at Paris, will be to their Length at the Equator, as 95 to 95,562, Gc. that is, as 440,555, Sc. to 443,165, Sc. and confequently they must be lengthen'd 2,61 Sc. Lines. But as from Monf. Mairan's Principles, the Diminution of Gravity by the Centrifugal Force, is greater at the Æquator than at Paris, hardly at Part of the whole Gravity at the Æquator, the Pendulums must be shortened in that Proportion; fo that then the Length of a Second-Pendulum, will be 440,555 + 2,61 - I Lines. But as that Quantity is greater than 440,555, Gc. therefore the Pendulums upon the Whole must be lengthen'd : Nay, though we fhou'd allow a fhortening of two Lines; fince by Obfervation Pendulums are found to be about two Lines fhorter at the Æquator, the oblong spheroidical Figure of the Earth cannot be confiftent with the Experiments on Pendulums.

I beg Leave to fet down Monf. *Mairan*'s aforefaid Demonstration here; that we may fee whether he has affum'd true Principles.

#### PROPOSITION V.

"\*XI. The Centrifugal Force at any Degree of Latitude, taken upon the oblong Spheroid, between "the Æquator and the Pole, is lefs in Comparifon "to the Centrifugal Force at the Æquator, than it "wou'd be at the fame Degree of Latitude taken upon "a Sphere; or, which is the fame thing, the Centri-"fugal Force encreafes more, going from the Poles towards the Æquator, upon an oblong Spheroid, "than upon a perfect Sphere; and confequently Gra-"vity diminisconter, and a Pendulum must be more

<sup>\*</sup> See Monf. Mairan's Differention, Article XI. &c.

" Shortened under the Æquator, in the Hypothesis of "the oblong Spheroid, than in that of a perfect "Sphere.

Having defcrib'd an oval Curve of any Kind, as
for Example, the Ellipfe \* ADBE abovementioned,
and infcrib'd the Circle DHE, whofe Radius is DC
= half the fhorter Axis DE; upon AD take any
Point as R, between the Æquator and the Pole, and
from that Point to the Evoluta OTX draw the Ray
of Curvature RT, which gives the Line of Tendency
R P (Art. IV.) Draw likewife from the common Center C, to the Circumference of the Circle DH, a Radius CV, parallel to PR, and meeting the Circle at V;
then from the Points R, V, draw the Lines R N, VZ,

" It must be observ'd, *First*, That as the Ellipse AD " represents a Meridian of the oblong Spheroid, the " Circle DH represents a Meridian of a Sphere in the " fame Plane.

" Secondly, That the Point V, on the Circular Meridian, answers to the fame Degree of Latitude as the Point R, upon the elliptical Meridian; because the Lines PR, CV, being parallel to each other, and perpendicular, the one to the Ellipse and the other to the Circle (by Construction) the touching Planes, or Horizons of the Points R, V, will also be parallel.

" Thirdly, Whence it follows that the Diminution of the Centrifugal Force (acting against Gravity) on account of its Obliquity to the Horizon (Art. X.) of

Fig. IV.

" the fame Degree of Latitude on the Elliptical and on " the Circular Meridian, is the fame in both Cafes, and " in the fame Ratio as the abfolute Centrifugal Forces " represented by the Perpendiculars R N, V Z, (Art. IX.) " Therefore to know whether the Centrifugal Force " (whether absolute or relative) of the Point R, upon the " oblong Spheroid ADBE, be less or greater in refrect " to the Centrifugal Force under the common Æquator " DE, than the Centrifugal Force (whether abfolute or " relative) of the correspondent PointVupon the Sphere: " nothing more is requir'd than to fee which is the " longeft of the two Perpendiculars, namely, R N in " the oblong Spheroid, or VZ in the Sphere; fince " these two Lines express the Radii of the Circles of " Revolution, and confequently the abfolute Quantity " of the Centrifugal Forces.

" 4thly and lastly, That the Ratio of the Centrifugal " Forces of two correspondent Points upon the oblong " Spheroid A DBE, and the infcrib'd Sphere DHE, to the " Centrifugal Force of their Æquators is the fame, fuppo-" fing the Sphere of any other Bigness; and that it has " been determin'd here of the Diameter DE, only to " render the Demonstration eafier, by giving the fame " Confequent to the Antecedents R N and VZ. For " if about the Center C and with the Radius Cd, the " Circle *dhe* be defcrib'd equal (for Example) to a " Meridian of a Sphere of the fame Solidity as the ob-" long Spheroid ADBE; and the Radius CV be pro-" duc'd till it meet the Circle db at the Point u, and " uz be let fall perpendicular to the common Axis of • Revolution, and parallel to VZ: It is plain, that " we fhall always have V Z: D C:: uz: dC, or "  $\frac{VZ}{DC} = \frac{uz}{dC}$ , and confequently  $\frac{RN}{DC}$  will have the " fame

- ( 290 )
- " fame Ratio to  $\frac{VZ}{DC}$  as to  $\frac{uz}{dC}$ .

" Therefore, in order to demonstrate that the Centri-"fugal Force of a Point, taken in any Latitude upon "the oblong Spheroid, is lefs when compar'd to the Cen-"trifugal Force of the like Point, taken upon a Sphere "in refpect to the Centrifugal Force at the Æquator; "there is nothing more requir'd than to fhew that R N " $\prec VZ$ , because by that means we shall have  $\frac{R N}{D C}$ " $\prec \frac{VZ}{DC}$ .

" This being obferv'd; from the Point R, draw the " Line R I, parallel to the Axis A B, and meeting the " Circle DH at K, and the Diameter D E of the Æqua-" tor at the Point I. From the Point K having let fall " the Perpendicular KL = R N, upon the Axis A B, " and drawn KC to the Center C; the Queftion will be " brought to this, viz. To know whether the Point V " coincides with the Point K; or whether it is above it " towards D, or below towards H.

" But CK = CV = CD > PR(Art. VIII.) there-"fore CK and PR being both between the Parallels "AC, RI, the greateft CK is more inclin'd to them "than the leaft PR, and the Angle KCA is lefs than the Angle RPA = VCA. And fince thefe two "Angles have each of them one of their Sides coinciding with the Line AC, namely, the Side A P of the Angle "R PA, and the Side AC of the Angle KCA, it follows that the Side VC of the Angle VCA = RPA > KCA, will go above CK between CK and CD, and meet the Line R I at the Point G, between K and "J, and the Circle D H at the Point V, which confe-" quently " quently will be above RI, between K and D. There-" fore CV = CG + GV is = PR + GV, and con-" fequently VZ, which meets RI at the Point F, is " = ZF + FV = RN + FV; and therefore RN " = VZ - FV. Therefore RN  $\leq VZ$ .

"And because the fame Thing may be demonstrated in respect of any other Point, taken between the *Æ*quator and the Pole; and that Gravity, and confequently the Length of a Pendulum diminishes, as the Centrifugal Force encreases. Therefore  $\mathcal{Oc. Q.E.D.}$ 

#### COROLLARY.

"XII. From what has been demonstrated, and from "Prop. 3. Art. VIII. it follows, that the Perpendicular which is drawn from any Point of an oval Meridian to the Axis, will be fo much shorter, in Comparison to the Perpendicular drawn from the correspondent Point of an inferib'd circular Meridian, as the Latitude is greater; and confequently (by Art. XI. Num. 3.) the Centrifugal Force will be fo much the lefs, and Gravity fo much the greater, upon the oblong Spheroid, in respect to the Centrifugal Force, and the Gravity under the Æquator.

"For as the Line R P does always decreafe, as the Point R is taken nearer to the Pole A, it is evident, that the Angle V CK will continually encreafe, in refpect to the Angles V C A, K C A, as it is their Difference, and confequently that the Perpendicular V Z will be fo much greater than the Perpendicular K L = R N.

I pass over the Demonstration of the latter Part of his Proposition above-mentioned, which he deduces justly from his Construction, if what he fays (Num. 2.) be right; because in such a Case it cannot be call'd in Queftion; ftion, and proceed to an Observation that he makes afterwards, viz, " We must take care to observe in the " foregoing Propositions and Corollaries, that the " Comparison is always made between two similar " Points of Latitude, taken upon the two Spheroids, " or upon one of the Spheroids and the Sphere, " between the Aguator and the Poles, in respect to " the Centrifugal Force upon the Æquator of any " one of these Spheroids, or of the Sphere. For if " we only compar'd absolutely the Centrifugal Force " of a Point of the Æquator of the one, to the Cen-" trifugal Force of a correspondent Point of the Æ. " quator of the other, it is plain that it wou'd be " greater upon a flatted Spheroid than upon a Sphere. " or than upon an oblong Spheroid of the fame Soli-" dity, in the Ratio of the great Axis of the gene." " rating Ellipse of the flatted Spheroid, to the Di-" ameter of the Sphere, or to the shorter Axis of the " generating Ellipse of the oblong Spheroid. And in " all Likelihood, this must be the Reason that has " made others, who have treated of this Subject, to " imagine the very contrary of what I have demon-" ftrated.

As Monf. Mairan confiders the Earth at reft, in the Conftruction for his Demonstration above quoted, and afterwards observes what Effect the Centrifugal Force will have upon Bodies on its Surface, to diminish the Gravity, with which they endeavour to defcend in their Line of Tendency R P: He fhou'd not only have taken notice (as he has done) that the whole Centrifugal Force **N R** is not to be fubltracted from the Gravity at **R**, as the whole Centrifugal Force CD is to be fubstracted from the whole Gravity at D, becaufe of the Obliquity of R N to P R; but he fhould have obferv'd alfo, that the Obliquity of the Plane of the Parallel N R, in which the

the Centrifugal Force acts, must alter the Line of Tendency R P, and change the Direction R P into R W, fomewhere between the Point P and the Center C; for if there be a heavy Body as a Plummet, hanging by a Thread in the Line SR, or SP, the Line of Tendency which has been fuppos'd perpendicular to the Curve A R D, without taking in the Effect of the Centrifugal Force; as foon as the Spheroid revolves about its Axis. the Body which wou'd fall in the Line SR, acted upon only by one Force, namely, that of Gravity, will now be acted upon by another Force, at the fame Time pufhing it in the Line Ss (which is the fame as Rr) and confequently will move in the Line Sr, diagonal of the Parallelogram sSRr; or, which is all one, a Body plac'd at R will have its Line of Tendency in RW, as I have already thewn in my first Differtation on this Subject; only I did not fuppole the Earth a Spheroid before the diurnal Motion, and therefore made use of the Line ZV inftead of the Line NR; fothat it may be objected that the Angle r SR will not be fo great in a Spheroid as in a Sphere, becaufe the Centrifugal Force which acts with the fame Obliquity (fince NRP = ZVC) is as much lefs in the Spheroid as NR is lefs than ZV: But I was aware of that, and therefore made the Angle **R** Sr only of 5 Minutes, when it really appears to be of almost fix Minutes, when the Earth is fuppos'd fpherical: and therefore, without coming to give the exact Quantity of the faid Angle, one may eafily perceive, that Monf. Caffini's Difference of the Axis and Æquatorial Diameter will produce a Figure, in which the Angle R Sr, will not be lefs than of 5 Minutes.

Such an Obliquity, caus'd in the Direction of Gravity, will render the oblong fpheroidical Figure of the Earth impoffible, becaufe then Fluids wou'd not have the Lines of their Gravity perpendicular to the Horizons of the VOL.XXXIII. T t Places Places where they are, (supposing the Horizons of Places to be Planes touching the Curve of the Earth in those Places) and Plumb Lines wou'd be so far out of the Perpendicular to Lines of Level, as to make an Angle easy to be observed, as I have shewn in my former Paper.

But if the fame Caufe be fuppos'd to act upon the Sea to make it level, as makes heavy Bodies to fall (which certainly muft) then indeed Lines of Level will be perpendicular to Plumb Lines, and the Level of the Sea, taken always for the Horizon of a Place, will not be a Plane touching the Earth, but cutting it towards the Poles, and confequently the Water will be carried towards the Æquator, as was before fhewn.

Befides, the Difference of the Action of the Centrifugal Force wou'd not be fo great between correspondent Points of the fame Latitude in the Spheroid and in the Sphere; for when the Line of Tendency R P is by the Centrifugal Force chang'd into R W, the Point R upon the Spheroid does no longer correspond in Latitude with the Point V upon the Sphere, but must be taken nearer to V, fo that the Line R W may become parallel to V C, and R W A = VC A.

If it be alledged here, that Monf. *Mairan* fuppofes the Earth in Motion, and takes in the Effect of the Centrifugal Force, when he makes the Line of Tendency to be R P; I anfwer, that if he had confider'd the Earth as revolving upon its Axis, he wou'd not have made V C. the Line of Tendency of a fpherical Earth in Motion, fince it is the Line of Tendency of fuch an Earth at reft.

In Monf. *Mairan*'s Obfervation above-mentioned, he fays, " that we are not to compare the Centrifugal Force at the Æquator of an oblong Spheroid, with the Centrifugal Force at the Æquator of a Sphere, or at the Æquator *"* Æquator of a flatted Spheroid of the fame Solidity ; *"* allowing that then it wou'd be greater in the Sphere, *"* and ftill greater in the flatted Spheroid : but only the *"* Centrifugal Forces in feveral Latitudes upon the fame *"* Figure." —But I beg Leave to differ from him for the following Reafons.

First, Because the Force of Gravity is not the fame at the Æquator of the flatted Spheroid, as it is at the Æquator of the Sphere, or as it is at the Æquator of the oblong Spheroid.

Secondly, Becaufe it is not the fame in different Latitudes, in either of the Spheroids. (See Sir Ifaac Newton Lib. 3. Prop. 19 and 20.) And Monf. Mairan's Way of arguing will only ferve, in Cafe the Gravity fhou'd be the fame in all the Points of the Surface of the Earth in his Figure, and also in the two other Figures.

For Example, let the uniform Gravity be call'd g; and *Firft*, let the Centrifugal Force at the Æquator of the flatted Spheroid be call'd c + 2; and the Centrifugal Force in any Latitude, as for Example, the Latitude of *Paris* (as it is diminished on Account of a shorter Co-fine of Latitude, and likewise on Account of its Obliquity to the Line of Tendency.) be call'd c + 2 - l; the Difference of the Diminution of Gravity at *Paris*, and at the Æquator will be g-c+2 - g-c+2-l = l.

Secondly, Let the Centrifugal Force at the Aquator of the Sphere be call'd c + 1, and the Centrifugal Force at the Latitude of *Paris* be call'd c + 1 - 1l + m; the Difference of the Diminution of Gravity at T t 2 *Paris*,

# (296) Paris and at the Æquator in a fpherical Earth, will be $\overline{g-c+1} - \overline{g-c+1} - \overline{l+m} = l+m.$

Thirdly, Let the Centrifugal Force at the Æquator of the oblong Spheroid be call'd c, and the Centrifugal Force at Paris be call'd c - l + m + n; the Difference of the Diminution of Gravity at Paris, and at the Æquator, in an oblong fpheroidical Earth, will be  $g - c - \frac{1}{g - c - l + m + n} = l + m + n$ .

Now, if Gravity shou'd in every Cafe be equal to g, it is evident, that the shortening of Pendulums, at the Æquator, wou'd be greater in the oblong Spheroid, than in the Sphere, or in the flatted Spheroid; because as the Lengths of Pendulums diminish with the Gravity, those Lengths will be at *Paris* and at the Æquator, when compar'd, as g - c + 2 - l to g - c + 2 in the flatted Spheroid; as g - c + 1 - l + m to g - c + 1in the Sphere, and as g - c - l + m + n to g - c + 1in the Sphere, and confequently, from what M. Mairan has demonstrated this Ratio of g - c - l + m + nto g - c, being greater than either of the others, the Pendulums must be shortened in the oblong Spheroid.

But as the Force of Gravity is lefs at the Æquator of the flatted Spheroid, than at the Æquator of the Sphere, or of the oblong Spheroid of the fame Solidity: let us express its Quantity in the three Cafes by g - s, g; and g + s, and we fhall then find the Lengths of the Pendulums, at the Æquator of the three Solids, as g - s - c + 2, g - c + 1, and g + s - c; confequently the Lengths of Pendulums Pendulums will be greatest at the Æquator of the oblong Spheroid, because g + s - c is the greatest Quantity.

Laftly, To compare the Lengths of Pendulums at the Auguator of the oblong Spheroid, thus found, with their Lengths at the Latitude of Paris upon the faid Spheroid — Let us express the Excess of Gravity at the Æquator, whereby it is greater than at Paris (becaufe in this Figure, Paris is farther from the Center of the Earth, than the Æquator, by so Part) by the Letter s. and the Excels of the Centrifugal Force at the Æquator, above that Part of it which acts directly against Gravity at Paris, by l + m + n, the Gravity at Paris by  $g_{1}$ , and the Centrifugal Force at the Æquator by c; then g + s - c will fill represent the diminish'd Gravity. and answer to the Length of Pendulums at the Æguator, whilf g = c = l + m + n or g = c + l + m + n re. prefents the diminish'd Gravity, and confequently the Length of Pendulums at *Paris*. If s be equal to l + m+ n, Pendulums will be as long at the Æquator as at **Paris**; and if s be greater than l + m + n, Pendulums will be longer at the Æquator. But making all poffible Allowance, in Favour of Monf. Mairan's Hy pothefis, no Calculation will bring l + m + n to be

greater than, or ever equal to s. Therefore Monf. Mairan's Demonstrations, above-mentioned, are of no Force to prove the Earth to be an oblong Spheroid.

And now, I think, I have answer'd all that relates to the Figure of the Earth in Mons. Mairan's Disfertation; in shewing, That his Conjectures can neither be supported by those Physical Principles which Sir Isaac Newton has Mathematically deduc'd from unguestionsd

questioned Observations and Experiments accurately made: nor even by those Principles which He (M. Mairan) has assum'd to serve his intended Purpose -That bis Demonstrations relating to the Difference of the A-Etion of the Centrifugal Force, are of no Service to him. for reconciling the Experiments made on Pendulums, with Monf. Caffini's Measures; -because, when applied to Sir Isaac Newton's Principles, they will make Pendulums longer at the Æquator than at Paris. and when applied to Monf. Mairan's own Principles, they will make them a whole Inch (horter at the Æ. quator than at Paris, contrary to all Observations. which, at a Medium, make Pendulums but about two Lines or \_\_\_\_\_ of an Inch longer at the Æquator than at Paris. - That he has built his Demonstrations upon a wrong Notion of Gravity - And that he has not confider'd what is most material in the Effect of the Centrifugal Force, acting on Bodies descending by their Gravity, between the Æquator and the Poles, namely, the Alteration of their Line of Direction, which wou'd make them fallout of the Perpendicular towards the Æquator.

I fhall add one more Philosophical Argument, given me by a Friend, to whom I communicated my Thoughts on this Subject; because it is wholly independent on those Principles of Philosophy, concerning which, some of the Gentlemen that believe the oblong spheroidical Figure of the Earth, and the *English* Philosophers, are not yet agreed; and it is this.

If the Earth was of an oblong spheroidical Figure, higher at the Poles than the Æquator; the Axis of its Revolution, wou'd either go thro' one of its short Diameters, or be continually changing unless the faid Axis did exactly coincide with the Axis of the Figure.

DE-

## ( 299 )

#### DEMONSTRATION.

Suppose fuch an oblong Figure as A a \* fix'd to the Axis Pp at the Center C, but capable of moving freely round it towards P or towards p, yet fo as to be oblig'd to move with the Axis, when it is turned round. Suppofe now the Poles P and p to be fix'd, and the Body, thus conftituted, to be turn'd fwiftly round the Axis **P**p: then if the Angle ACP be oblique, and the Figure A D a E be oblong, the Parts A C and C a will acquire a Centrifugal Force, which will enlarge the Angle p C A, till it comes to be a right one. Befides this, a Velocity will be generated in the Motion, while A is going towards the Perpendicular  $\alpha$  C, which will make it go farther on towards P, as to B, with a Motion which will after that be retarded, till the Centrifugal Force has Strength enough to fend it back again the contrary Way; and to it will move continually with a reciprocal Motion, like the Ofcillation of a Pendulum; and if a little of this Motion be loft at every Ofcillation, then the oblong Figure ADaE will at last move quietly about its leffer Axis DE coinciding with Pp.

If A A did not at first exactly coincide with P p, the Centrifugal Force will have the above-mentioned Effect; and that this is not the Case in the Earth is more than probable, because the unequal Distribution of Sea and Land, besides the Phænomena of the Tides must make the Axis of its Gravity, and consequently the Axis of its Revolution, to differ from the Axis of the oblong Spheroid, if the Earth had such a Figure; without considering that every Earth-quake wou'd alter so nice an Equilibrium, which once lost, wou'd never be recover'd again. To leave nothing unexamin'd, relating to the Controverfy, I have again confider'd the Meafures and Obfervations, mentioned in the Account of the Meridian drawn thro' *France*, in the Memoirs of the *Royal Academy*, for the Year 1720; and I find them to want a great deal of the Accuracy requir'd in fo nice a Point, as determining the different Lengths of Degrees upon the Surface of the Earth. To prove my Affertion, I beg that the Reader will examine the following Tables, whereby it appears, that if any thing certain can be deduc'd from the faid Obfervations and Meafures, (either taken as they are, or reduc'd to the Level of the Sea, by the Rules given by Monf. *Caffini*\*) it will be in Favour of Sir *Ifaac Newton*'s Figure of the Earth, rather than theirs.

In the following Table, the first Column gives the Names of Places; the second, the Distances from *Paris*, according to the Measures taken by the *French* Gentlemen; the third, the Latitudes observed by the same; the fourth, the Latitudes, such as the measured Distances will give them, supposing the Earth spherical; the fifth, the Differences between these and the Latitudes observed, expressed in Seconds of a Degree, where when the Latitude computed, exceeds the Latitude observed, the Letter N (North) shews that Difference to be in Favour of Mons. *Calfini's* Figure, and the contrary Difference mark'd by the Letter S (South) is in Favour of Sir *Isaac Newton's* Figure.

<sup>\*</sup> Memoirs for the Year 1720. Vol. I. P. 1. Ch. 13.

( 301 )

| Names of<br>Places. | Diftances from<br><i>Paris</i> mealur'd. | Latitudes ob-<br>ferv'd. |     |       | Latitudes in a<br>ipherical Earth<br>computed from the<br>mealur'd Diftances. |     |       | Differences in<br>Seconds. |  |
|---------------------|--|--------------------------|-----|-------|---|-----|-------|----------------------------|--|
| Ι.                  | II.<br>Toifes.                           | 111.                     |     |       | IV.   |     |       | v.                         |  |
| Dunkirk.            | 125552                                   | 510                      | 21  | 25112 | 510   | 21  | 25112 | 011                        |  |
| Am iens,            | 60370                                    | 49                       | 53  | 48    | 49  | 53  | 48    | 011                        |  |
| Sourdon.            | 4997¢±                                   | 49                       | 42  | 42    | 49  | 42  | 52,I  | 10,1 N                     |  |
| Paris.              |  | 48                       | 50  | 10    | 48  | 50  | 20,3  | 10,3 N                     |  |
| Malvoisine.         | 18838                                    | 48                       | 30  | 47    | 48  | 30  | 32,1  | 14,9 S                     |  |
| Voulon.             | 67962                                    | 47                       | 39  | 17    | 47  | 38  | 53,6  | 23,4 S                     |  |
| Bourges.            | 100192                                   | 47                       | 4   | 31    | 47  | 04  | 58,7  | 27,7 N                     |  |
| S. Sauvier.         | 139934                                   | 46                       | 23  | 24    | 46  | 23  | 12    | 12,0 S                     |  |
| Croc.               | 169540                                   | 45                       | 51  | 43    | 45  | 52  | 4,6   | 21,6 N                     |  |
| Bort.               | 196484                                   | 45                       | 23  | 27    | 45  | 23  | 45,2  | 18,2 N                     |  |
| Aurillac.           | 223606                                   | 44                       | 55  | 13    | 44  | 55  | 14,5  | 1,5 N                      |  |
| Rodès.              | 256575                                   | 44                       | 20  | 54    | 44  | 20  | 35,1  | 19,9 S                     |  |
| Alby.               | 280612                                   | 43                       | 55  | 32    | 43  | 55  | 19    | 13,0 S                     |  |
| Carcassone.         | 321430                                   | 43                       | 12  | 55    | 43  | I 2 | 24,5  | 31,5 S                     |  |
| Collioure.          | 360604                                   | 42                       | 3 I | I 3 💈 | 42  | 31  | 13,8  | 0,                         |  |

In this Table it is to be observed that there is an equal Number of Differences mark'd N(North) and S(South) and if the Differences on each Side be added together, there will be 89",4 on the North Side, and 114,"7 on the South: This last agrees best with Sir *Isac New*ton's Figure, which must be supposed for the Correction of spreat a Difference.

In the next Table, the first Column gives the Names of Places; the fecond, the Latitudes observ'd; the third, the Distances in the Meridian from *Paris*, reduc'd to the Level of the Sea; the fourth, the Differences of the second Column express'd in Seconds of a Degree; the fifth, the Differences of the Numbers in the third Column; and the fixth, the Measure of a Degree by the fourth and fifth Columns compar'd.

Vol.XXXIII. U u

| ( 302 )  |   |  |  |   |  |   |   |
|--|---|--|--|---|--|---|---|
| I.<br>Names of Places.   |   | Π.   | -  | 111.  | IV.<br>Seconds<br>of a De-   | V.  | VI.   |
| Dunkirk.<br>Amiens.<br>Clermont.<br>The R. Obfervatory.<br>Voufon,<br>\$\sigma's. Sauvier.<br>Croc.<br>Bort.<br>Aurillac.<br>Rodes.<br>Alby. | 510<br>49<br>49<br>48<br>47<br>46<br>45<br>45<br>45<br>44<br>44<br>43 | 21<br>53<br>22<br>50<br>39<br>23<br>51<br>23<br>55<br>20<br>55 | 1911<br>56<br>57<br>10<br>17<br>24<br>43<br>46<br>13<br>53<br>32 | Toifes.<br>125454<br>60444<br>31028<br>0<br>67959<br>139937<br>169539<br>196480<br>223616<br>256474<br>280612 | gree.<br>4103"<br>1859<br>1967<br>4253<br>4553<br>1901<br>1677<br>1713<br>2060<br>1521 | Toifes.<br>65010<br>29416<br>31028<br>67959<br>71978<br>29602<br>26941<br>27136<br>32858<br>24138 | Toifes,<br>57040<br>56965<br>56787<br>57525<br>56912<br>56058<br>57834<br>57028<br>57422<br>57131 |
| Carcaffone,<br>Collioure.  | 43<br>42  | 55)<br>12<br>31  | 55<br>13   | 321430<br>36061 <b>4</b>  | 2557<br>2502   | 40818<br>39184  | 57468<br>56380  |

In this Table in the third Column, over-against St. Sauvier, the Number which was 139944 is corrected to make it 139937, to the Advantage of the oblong Figure. In the fixth Column, the Numbers appear fo irregular, as to be unfit to decide this Controverfy. Then if a Comparison be made between Dunkirk, St. Sauvier (which is very near the Middle of France, and almost in the Meridian of Paris) and Collioure, the Meafurement is absolutely in Favour of Sir Isac Newton's Theory; the mean Degree between Dunkirk and St. Sauvier being larger by about 64 Toifes, than between S. Sanvier and Collioure; and to reduce them even to an Equality, there must be a greater Alteration made in the Situation of those three Places, than it is reasonable to fuppofe their Observations to be capable of admitting. Here follows the Comparison.

| Dunkirk and Collioure    |                            | 157061  |
|--------------------------|----------------------------|---------|
| Dunkirk and Paris        |                            | 56960   |
|                          | A mean Degree is           | 57097   |
| Dunkirk and S. Sauvier   |                            | 57090,4 |
| S. Sauvier and Collioure |                            | 57026,5 |
| According                | g to Monf. <i>Picard</i> , | 57060   |
|                          | 2                          | То      |

To conclude, I will propose a Method of observing the Figure of the Shadow of the Earth in Lunar Eclipses, whereby the Difference between the Diameters in the oblong spheroidical Figure, if there be such an one as Mons. Cassing affirms (viz. of 96 to 95) may be discover'd.

Let  $P \not\equiv P \not\equiv *$  represent the Earth, seen from the Sun at the Time of the Summer Solftice; it is evident, that the fame Figure will express the Section of the Earth's Shadow at the Moon's Distance, as seen from the Earth. If EE represents the Eccliptick,  $\not\equiv \not\equiv$  will be the shortest Diameter of the Section; and if L L be taken for the Moon's Way, in a total and central Eclipse of the Moon, by observing the Time which is spent in the Passage of the Center of the Moon, thro' the Shadow, and reducing that Time to Seconds of a Degree of a great Circle of the Heavens, we shall have the least Diameter of the Shadow.

Again, let the fame Letters  $\dagger$  represent the fame Things, only here the Section of the Shadow is such, as the Earth will cast at the Aquinox, and the Eclipse of the Moon is here supposed partial, its Center just touching the Shadow. When the Moon's Center is got to c, if the Latitude of its Center or its Distance from the Ecliptick be observ'd, we shall have the Length c C nearly equal to the longest Semi-diameter of the Shadow.

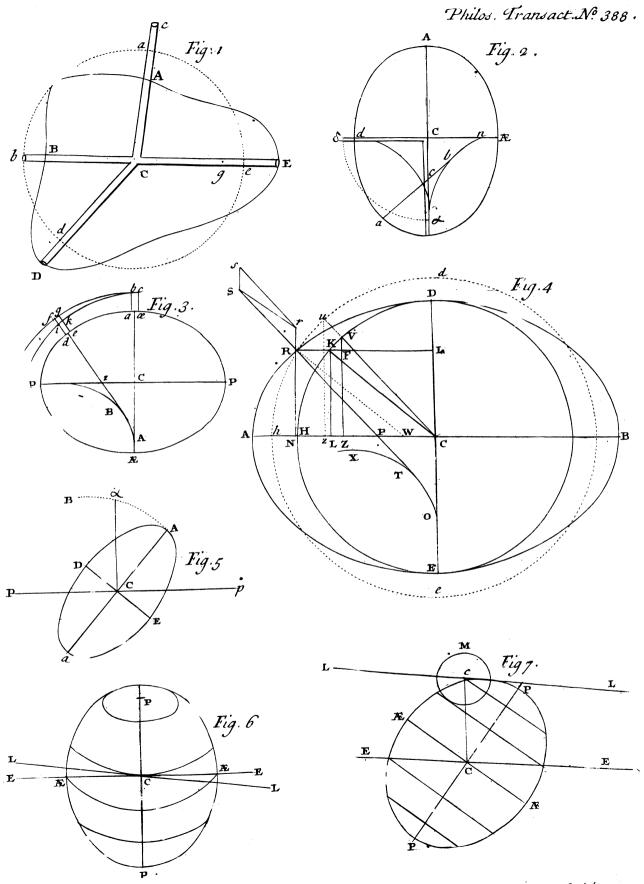
Now, comparing  $cC^*$  in this Figure to  $LC^*$  in the former (the Difference between cC and CP (Fig. 7.) and between CL and CÆ (Fig. 6.) not being worth notice) they ought to be to one another, as 96 to 95, which in fuch a Shadow will give a Difference of about 25" at a Medium, fensible enough to be observed, notwithstanding the *Penumbra*. If therefore those A-

<sup>\*</sup> Fig. VI.  $\dagger$  Fig. VII. \* Fig. VII. \* Fig. VI. U u 2 ftronomers

ftronomers who have Instruments nice enough, and fufficient Skill in the Management of them, to take Angles to 3 or 4 Seconds of a Degree, will observe what I have been mentioning in total and partial Eclipses of the Moon; by such Observations they will easily convince us, that the Figure of the Earth is such as Monf. Caffini supposes it, or convince him that he has been miftaken.

The Semi-diameter of the Earth's Shadow, when the Earth is in Perihelio, and the Moon in Apogeo is 38', or 2280'', without confidering the Encrease of the Shadow, on account of the Atmosphere of the Earth, which wou'd make it 39' or 2340" (allowing one Second for a Mile: ) and the Semi-diameter of the Shadow, when the Earth is in Aphelio, and the Moon in Perigzo is 461, 20", or 278011, which encreased on account of the Atmosphere of the Earth, will bring it to 47', 20" or 2840". Now if the Proportion of 95 to 96 be taken in both Cafes you will have these Analogies, { 95:96::2340":2364".6 } So that 2364",6 -2340'' = 24''.6 will be the Difference of the Semi-diameters, when the Section of the Shadow is the leaft, and 2869",8 - 2840" = 29",8 will be the Difference of Semi-diameters, when the Section of the Shadow is the greatest; the Sum of those Differences 24",6 + 29",8 halved, will give the Difference, when the Section of the Shadow is at a Medium = 27'' 4; from which if we take 211,4 because in Fig. 7. Cc is a little less than CP, and in Fig. 6. LC is something greater than A.C., we shall have C c in Fig. 7. to compare with LC in Fig. 6. which will exceed it by 25", if Mon (. Caffini's Figure of the Earth be the true one.

II. A



S. Parker Sculy