Trap. rule (simple) fig. p.7-1

$[a, b]$, $\begin{cases} x_0 = a \\ x_1 = b \end{cases}$ $n = 1$

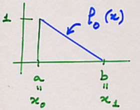
$$f_1(x) = p_1(x) = \sum_{i=0}^{1} l_i(x) f(x_i)$$

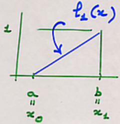
$$= l_0(x) f(x_0) + l_1(x) f(x_1)$$

($x_0 = a$, $x_1 = b$)

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{for } x = x_1 \end{cases}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \begin{cases} 0 & x = x_0 \\ 1 & x = x_1 \end{cases}$$





(1) $\ell_i(x_j) = \delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

Kronecker delta.

$\ell_0(x)$, $\ell_1(x)$ } linear funcs $\Rightarrow p_1(x)$ is a lin. comb. of ℓ_0 and ℓ_1, and must be therefore lin.

$$I = \int_a^b f(x)\,dx \cong I_1 = \int_a^b p_1(x)\,dx$$

poly — 1st order (lin.)

$$= \left(\int_a^b \ell_0(x)\,dx\right) f(x_0) + \left(\int_a^b \ell_1(x)\,dx\right) f(x_1) \quad (2)$$

HW: Use (2) to obtain (1) p. 7-1.

Simpson's rule (simple)

$[a, b]$ $x_0 = a$ $x_1 = \frac{a+b}{2}$

$x_2 = b$

(1) $f_2(x) = p_2(x) = c_2 x^2 + c_1 x^1 + c_0$ 8-3

c_0, c_1, c_2 = 3 unknowns

(2) $p_2(x_i) = f(x_i) \quad i = 0, 1, 2$

3 eqs, 3 unknowns c_2, c_1, c_0

(Method 1) Method 2: Use (1) - (2) p. 7-3 to get $p_2(x)$

(3) $$p_2(x) = \sum_{i=0}^{n=2} l_i(x) f(x_i)$$

Equiv. of Meth 1 and Meth 2:

(4) $$p_2(x_j) = \sum_{i=0}^{2} \underbrace{l_i(x_j)}_{= \delta_{ij}} f(x_i) = f(x_j) \quad \leftarrow \text{HW}$$

$j = 0, 1, 2$

(4) ≡ (2)

$$l_0(x) = \prod_{\substack{j=0 \\ j \neq i=0}}^{n=2} \frac{x - x_j}{x_0 - x_j} = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$i = 0$

It can be verified that:

$$\ell_0(x_0) = 1 \,, \quad \ell_0(x_1) = \ell_0(x_2) = 0$$

$$\ell_i(x_j) = \delta_{ij} \qquad i,j = 0,1,2$$