Exercici proposat 1

a)

Donat el següent senyal:

$$x(t) = -3 - 2\cos\left(12t - \frac{\pi}{2}\right) - \frac{2}{3}\sin\left(3t + \frac{\pi}{4}\right) - \cos(6t)$$

a) Calculeu la pulsació fonamental ω_0, el període fonamental T_0 i la freqüència fonamental f_0 del senyal $x(t)$.

b) Calculeu i dibuixeu els coeficients del desenvolupament en sèrie de Fourier a_k corresponents al senyal $x(t)$.

SOLUCIÓ

En aquest cas el més fàcil és utilitzar el seu equivalent en exponencials complexes, i per inspecció extraure els coeficients. Per tant, expandim en exponencials recordant la fórmula de Euler:

$$\boxed{e^{j\omega t} = \cos\omega t + i\sin\omega t}$$

$$x(t) = -3 - 2\left[\frac{1}{2}\left(e^{j\left(12t-\frac{\pi}{2}\right)} + e^{-j\left(12t-\frac{\pi}{2}\right)}\right)\right] - \frac{2}{3}\left[\frac{1}{2j}\left(e^{j\left(3t+\frac{\pi}{4}\right)} + e^{-j\left(3t+\frac{\pi}{4}\right)}\right)\right] - \left[\frac{1}{2}\left(e^{j(6t)} + e^{-j(6t)}\right)\right]$$

Ara agrupem termes:

$$x(t) = -3 - e^{j\left(12t-\frac{\pi}{2}\right)} - e^{-j\left(12t-\frac{\pi}{2}\right)} - \frac{1}{3j}e^{j\left(3t+\frac{\pi}{4}\right)} + \frac{1}{3j}e^{-j\left(3t+\frac{\pi}{4}\right)} - \frac{1}{2}e^{j(6t)} - \frac{1}{2}e^{-j(6t)}$$

$$x(t) = -3 + \left[\left(-e^{-\frac{j\pi}{2}}\right)\cdot\left(e^{12t}\right)\right] + \left[\left(-e^{\frac{j\pi}{2}}\right)\cdot\left(e^{-j12t}\right)\right] + \left[\left(-\frac{1}{3j}e^{\frac{j\pi}{4}}\right)\cdot\left(e^{j3t}\right)\right] + \left[\left(\frac{1}{3j}e^{-\frac{j\pi}{4}}\right)\cdot\left(e^{-j3t}\right)\right]$$
$$+ \left[\left(-\frac{1}{2}\right)\cdot\left(e^{j6t}\right)\right] + \left[\left(-\frac{1}{2}\right)\cdot\left(e^{-j6t}\right)\right]$$

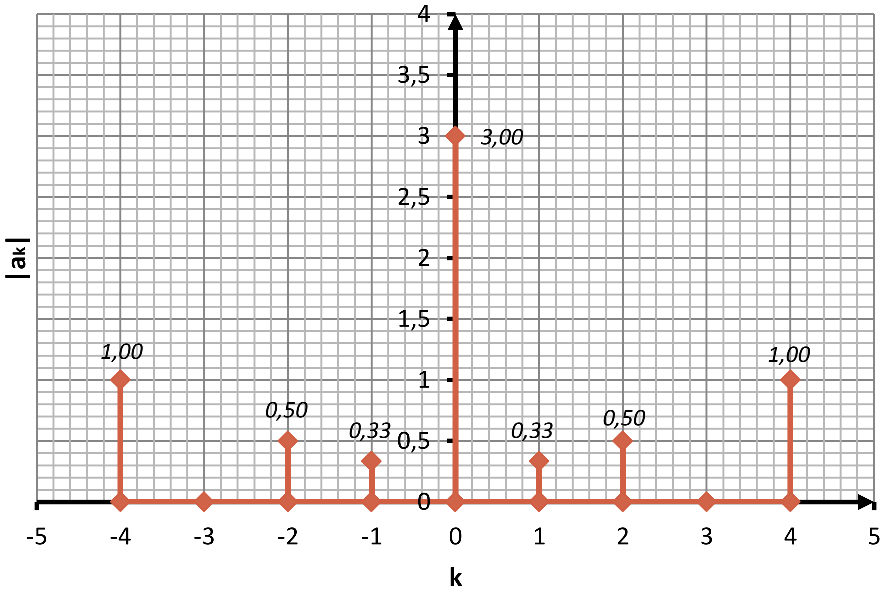
Així tenim que els coeficients són:

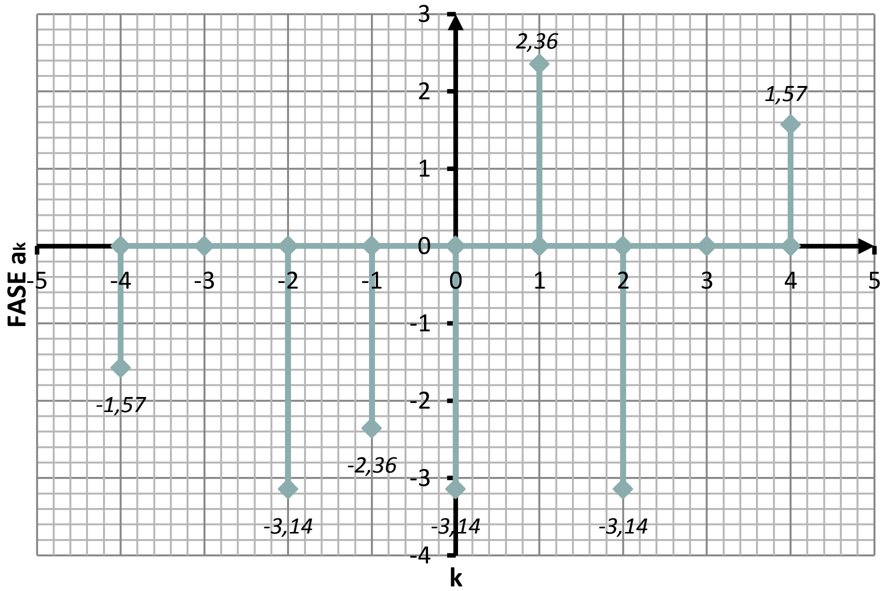
$\boldsymbol{a_0} = -3 = \boldsymbol{3e^{-j\pi}}$	
$\boldsymbol{a_1} = -\frac{1}{3j}e^{\frac{j\pi}{4}} = -\frac{1}{3j}\frac{\sqrt{2}}{2}(1+j) = \frac{\sqrt{2}}{6}(-1+j) = \boldsymbol{\frac{1}{3}e^{\frac{3\pi}{4}}}$	$\boldsymbol{a_{-1}} = \frac{1}{3j}e^{-\frac{j\pi}{4}} = \frac{1}{3j}\frac{\sqrt{2}}{2}(1-j) = \frac{\sqrt{2}}{6}(-1-j) = \boldsymbol{\frac{1}{3}e^{-\frac{3\pi}{4}}}$
$\boldsymbol{a_2} = -\frac{1}{2} = \boldsymbol{\frac{1}{2}e^{-\pi}}$	$\boldsymbol{a_{-2}} = -\frac{1}{2} = \boldsymbol{\frac{1}{2}e^{-\pi}}$
$\boldsymbol{a_3} = \boldsymbol{0}$	$\boldsymbol{a_{-3}} = \boldsymbol{0}$
$\boldsymbol{a_4} = -e^{-\frac{j\pi}{2}} = j = \boldsymbol{1e^{\frac{\pi}{2}}}$	$\boldsymbol{a_{-4}} = -e^{\frac{j\pi}{2}} = -j = \boldsymbol{1e^{-\frac{\pi}{2}}}$

Observem que tenim la següent pulsació, freqüència i període fonamentals:

$$\boxed{\omega_0 = 3\,rad/s;\ f_0 = \frac{3}{2\pi}\ Hz;\ \ T_0 = \frac{2}{3}\pi\ s}$$

Si representem l'amplitud i la fase dels coeficients de la SF obtenim:

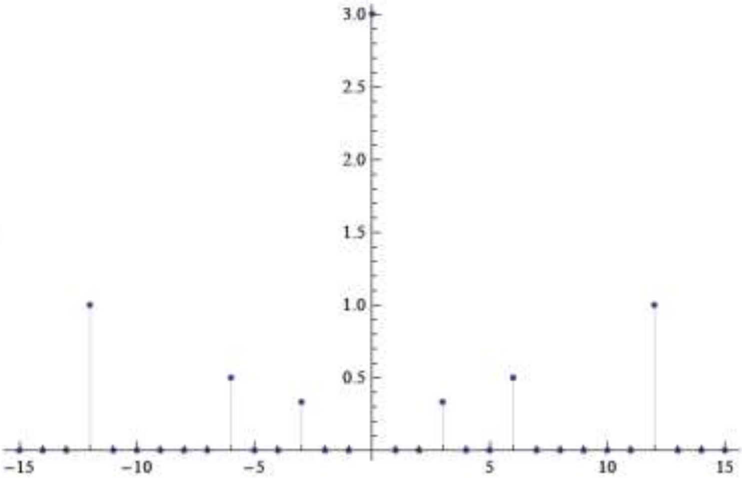




Podem comprovar el resultat mitjançant el *Wolfram Mathematica*:

```
In[49]:= FourierCoefficient[
           -3 - 2 * Cos[12 * t - (Pi / 2)] - (2 / 3) * Sin[3 * t + (Pi / 4)] - Cos[6 * t], t, n];
         DiscretePlot[Abs[%], {n, -15, 15}, PlotRange → All] // Quiet
```

Out[50]=




```
In[48]:= FourierCoefficient[
           -3 - 2 * Cos[12 * t - (Pi / 2)] - (2 / 3) * Sin[3 * t + (Pi / 4)] - Cos[6 * t], t, n]
```

Out[48]=

$$\begin{cases} -3 & n == 0 \\ -\frac{1}{2} & n == -6 \;||\; n == 6 \\ -i & n == -12 \\ i & n == 12 \\ -\frac{1}{3}\,(-1)^{1/4} & n == -3 \\ \frac{1}{3}\,(-1)^{3/4} & n == 3 \\ 0 & \text{True} \end{cases}$$

In[35]:= $N\left[-\frac{1}{3}\,(-1)^{1/4}\right]$

Out[35]= -0.235702 - 0.235702 i

```
In[40]:= Abs[-0.23570226039551584` - 0.2357022603955158` i]
Out[40]= 0.333333
In[41]:= Arg [-0.23570226039551584` - 0.2357022603955158` i]
Out[41]= -2.35619
```

In[42]:= $N\left[\frac{1}{3}\,(-1)^{3/4}\right]$

Out[42]= -0.235702 + 0.235702 i

```
In[43]:= Abs[-0.23570226039551578` + 0.2357022603955159` i]
Out[43]= 0.333333
In[44]:= Arg[-0.23570226039551578` + 0.2357022603955159` i]
Out[44]= 2.35619
```