



$$\overline{gz^7 + hz^8 + iz^9 \& C} \Big| \overset{m}{=}$$

$$a^{m-3}b^3z^{m+3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4}b^4z^{m+4}$$

$$x^{m-2}bc + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3}b^2c$$

$$x^{m-1}d + \frac{m}{1} \times \frac{m-1}{1} a^{m-2}bd$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2}c^2$$

$$+ \frac{m}{1} a^{m-1}e$$

$$\frac{m-4}{5} \times \frac{m-5}{6} a^{m-6}b^6z^{m+6} \& C$$

$$\frac{m-3}{4} \times \frac{m-4}{1} a^{m-5}b^4c$$

$$\frac{m-2}{3} \times \frac{m-3}{1} a^{m-4}b^3d$$

$$\frac{m-2}{1} \times \frac{m-3}{2} a^{m-4}b^2c^2$$

$$\frac{m-1}{2} \times \frac{m-2}{1} a^{m-3}b^2e$$

$$\frac{m-1}{1} \times \frac{m-2}{1} a^{m-3}bcd f$$

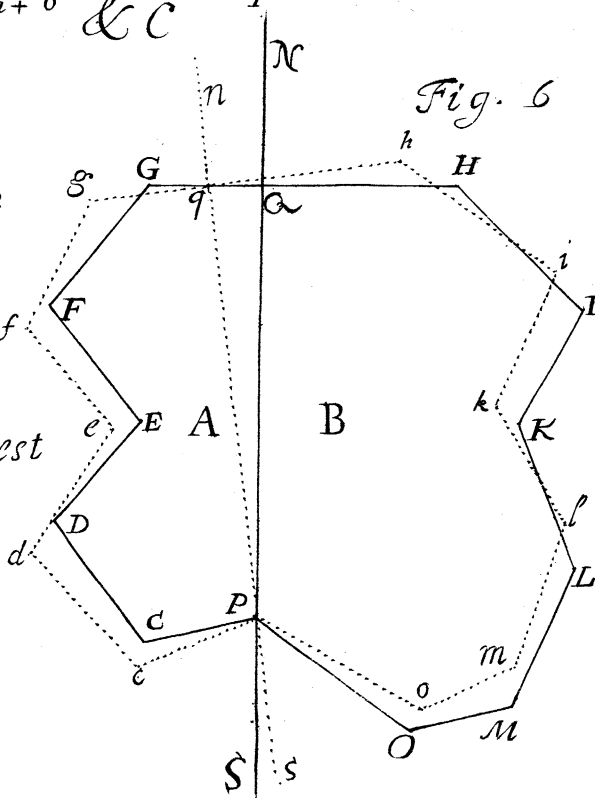
$$\frac{m}{1} \times \frac{m-1}{1} a^{m-2}bf$$

$$\frac{m-1}{2} \times \frac{m-2}{3} a^{m-3}c^3 \text{ West}$$

$$\frac{m}{1} \times \frac{m-1}{1} a^{m-2}ce$$

$$\frac{m}{1} \times \frac{m-1}{2} a^{m-2}d^2$$

$$+ \frac{m}{1} a^{m-1}g$$



swim somewhat deep in the Water, sometimes are catch'd, though not often. The Seamen have reached them with a Fisgig, a kind of barbed Iron, at the end of a Pole tyed fast to a Rope, and have made good cheer with them. But this is only my Conjecture, with which I end my Journal. *Deo Servatori Laus.*

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### III. *A Method of Raising an infinite Multinomial to any given Power, or Extracting any given Root of the same.* By Mr. Ab. De Moivre.

**T**IS about two Years since, that considering Mr. *Newton's* Theorem for Raising a Binomial to any given Power, or Extracting any Root of the same; I enquired, whether what he had done for a Binomial, could not be done for an infinite Multinomial. I soon found the thing was possible, and effected it, as you may see in the following Paper; I design in a little time to shew the Uses it may be applied to: In the mean while, those that are already vers'd in the Doctrine of Infinite Series, and have seen what Applications Mr. *Newton* has made of his Theorem, may of themselves derive several Uses from this.

I suppose that the Infinite Number Multinomial is  $ax + bzx + cx^3 + dx^4 + ex^5$  &c.  $m$  is the Index of the Power, to which this Multinomial ought to be Rais'd, or if you will, 'tis the Index of the Root which is to be Extracted: I say that this Power or Root of the Multinomial, is such a Series as I have express'd.

For the Understanding of it, it is only necessary to consider all the Terms by which the same Power of  $z$  is Multiplied; in order thereto I distinguish two things in each of these Terms; 1<sup>o</sup> The Product of certain Powers

Powers of the quantities,  $a, b, c, d,$  &c. 2° the *Uncie* (as *Oughtred* calls 'em) prefix to these Products. To find all the Products belonging to the same Power of  $z$ , to that Product, for instance, whose Index is  $m+r$  (where  $r$  may denote any integer Number) I divide these Products into several *Classes*; those which immediately after some certain Power of  $a$  (by which all these Products begin) have  $b$ , I call *Products* of the 1<sup>st</sup> *Classis*; For *Example*  $a^{m-4}b^3e$  is a Product of the 1<sup>st</sup> *Classis*, because  $b$  immediately follows  $a^{m-4}$ ; those which immediately after some Power of  $a$  have  $c$ , I call Products of the 2<sup>d</sup> *Classis*, so  $a^{m-3}ccd$  is a Product of the 2<sup>d</sup> *Classis*; Those which immediately after some Power of  $a$  have  $d$ , I call Products of the 3<sup>d</sup> *Classis*, and so of the rest.

This being done, I Multiply all the Products belonging to  $z^{m+r-1}$  (which precedes immediately  $z^{m+r}$ ) by  $b$  and Divide 'em all by  $a$ ; 2° I Multiply by  $c$  and Divide by  $a$ , all the Products belonging to  $z^{m+r-2}$ , Except those of the 1<sup>st</sup> *Classis*; 3° I Multiply by  $d$  and Divide by  $a$  all the Products belonging to  $z^{m+r-3}$ , Except those of the 1<sup>st</sup> and 2<sup>d</sup> *Classis*, 4° I Multiply by  $e$  and Divide by  $a$  all the Terms belonging to  $z^{m+r-4}$ , Except those of the 1<sup>st</sup>, 2<sup>d</sup> and 3<sup>d</sup> *Classis*, and so on, till I meet twice with the same term. Lastly, I add to all these Terms the Product of  $a^{m-1}$  into the Letter whose *Exponent* is  $r+1$ .

Here I must take notice that by the *Exponent* of a Letter, I mean the number which expresses what Place the Letter has in the *Alphabet*, so 3 is the *Exponent* of the Letter  $c$  because the Letter  $c$  is the 3<sup>d</sup> in the *Alphabet*.

It is evident that by this Rule, you may easily find all the Products belonging to the several Powers of  $z$ , if you have but the Product belonging to  $z^m$ , viz.  $a^m$ .

To

To find the *Unciæ* which ought to be prefixt to every Product, I consider the Sum of Units contained in the Indices of the Letters which compose it (the Index of *a* excepted) I write as many Terms of the Series  $m \times m - 1 \times m - 2 \times m - 3, \&c.$  as there are *Units* in the Sum of these Indices, this Series is to be the Numerator of a Fraction, whose Denominator is the Product of the several Series  $1 \times 2 \times 3 \times 4 \times 5, \&c. 1 \times 2 \times 3 \times 4 \times 5, \&c. 1 \times 2 \times 3 \times 4 \times 5 \times 6, \&c.$  the 1<sup>st</sup> of which contains as many Terms as there are *Units* in the Index of *b*, the 2<sup>d</sup> as many as there are *Units* in the Index of *c*, the 3<sup>d</sup> as many as there are *Units* in the Index of *d*, the 4<sup>th</sup> as many as there are *Units* in the Index of *e*, &c.

*Demonstration.*

To raise the Series  $ax + bzx + cx^3 + dx^4, \&c.$  to any Power whatsoever, write so many Series equal to it as these are *Units* in the Index of the Power demanded. Now it is evident that when these Series are so Multiplied, there are several Products in which there is the same Power of *z*, thus if the Series  $ax + bzx + cx^3 + dx^4 \&c.$  is rais'd to its Cube, you have the Products  $b^3z^6, abcz^6, aadz^6$ , in which you find the same Power  $z^6$ . Therefore let us consider what is the Condition that can make some Products to contain the same Power of *z*, the first thing that will appear in relation to it, is that in any Product whatsoever, the Index of *z* is the Sum of the particular Indices of *z* in the Multiplying Terms (this follows from the the nature of Indices) thus  $b^3z^6$  is the Product of  $bz^2, bz^2, bz^2$ , and the Sum of the Indices in the Multiplying Terms, is  $2 + 2 + 2 = 6$ ;  $abcz^6$  in the Product of  $ax, bzx, cx^3$ , and the Sum of them Indices of *z* in the Multiplying Terms is  $1 + 2 + 3 = 6$ ;  $aadz^6$  is the Product of  $ax, ax, dx^4$ , and the Sum of the

$Z \ Z \ Z \ Z$ 
Indices

Indices of  $z$  in the Multiplying Terms is  $1+1+4=6$ ; the next thing that appears is, that the Index of  $z$  in the Multiplying Terms is the same with the Exponent of the Letter to which  $z$  is joyn'd, from which two Considerations it follows that, *To have all the Products belonging to a certain Power of  $z$ , you must find all the Products where the Sum of the Exponents of the Letters which compose 'em shall always be the same with the Index of that Power.* Now this is the method I use to find easily all the Products belonging to the same Power of  $z$ , Let  $m+r$  be the Index of that Power, I consider that the Sum of the Exponents of the Letters which compose these Products must exceed by  $1$  those which belong to  $z^{m+r-1}$ , now because the excess of the Exponent of the Letter  $b$  above the Exponent of the Letter  $a$ , is  $1$ , it follows that if each of the Products belonging to  $z^{m+r-1}$  is Multipl'd by  $b$ , and Divided by  $a$ , you will have Products the Sum of whose Exponents will be  $m+r$ ; Likewise the Sum of the Exponents of the Letters which compose the Products belonging to  $z^{m+r}$  exceeds by  $2$  the Sum of the Exponents of the Letters which compose the Products belonging to  $z^{m+r-2}$ ; Now because the Exponent of the Letter  $a$  is less by  $2$  than the Exponent of the letter  $c$ , it follows, that if each Product belonging to  $z^{m+r-2}$  is Multipl'd by  $c$  and Divided by  $a$ , you will have other Products the Sum of whose Exponents is still  $m+r$ ; Now if all the Products belonging to  $z^{m+r-2}$  were Multiplied by  $c$  and Divided by  $a$ , you would have some Products that would be the same as some of those found before, therefore you must except out of 'em those that I have call'd Products of the 1<sup>st</sup> Classis; what I have said shows why all the Products belonging to  $z^{m+r-3}$ , except those of the 1<sup>st</sup> and 2<sup>d</sup> Classis must be Multipl'd by  $d$  and Divided by  $a$ : Lastly, you see the Reason why to all these Products is added

added the Product of  $a^{m-1}$  by the Letter whose Exponent is  $r+1$ ; 'Tis because the Sum of the *Exponents* is still  $m+r$ .

As for what relates to the *Unciæ*; observe that when you Multiply  $ax+bxz+cx^3+dz^4$ , &c. by  $ax+bxz+cx^3+dz^4$ , &c. each Letter  $a, b, c, d$ , &c. of the 2<sup>d</sup> Series is Multipl'd by each of the Letters  $a, b, c, d$ , &c. of the 1<sup>st</sup> Series; Thus the Letter  $a$  of the 2<sup>d</sup> Series is Multipl'd by the letter  $b$  of the 1<sup>st</sup>, and the letter  $b$  of the 2<sup>d</sup> Series is Multipl'd by the Letter  $a$  of the 1<sup>st</sup>; therefore you have the 2 Planes,  $ab, ab$  or  $2ab$ ; for the same reason you have  $2ac, 2ad$ , &c. Therefore you must prefix to each Plane of those that compose the Square of the Infinite Series  $ax+bxz+cx^3$ , &c. the Number which expresses how many ways the Letters of each Plane may be changed; Likewise if you Multiply the Product of the two preceding Series by  $ax+bxz+cx^3$ , &c. each Letter  $a, b, c, d$ , of the 3<sup>d</sup> Series is Multiplied by each of the Planes form'd by the Product of the 1<sup>st</sup> and 2<sup>d</sup> Series; Thus the Letter  $a$  is Multiplied by the Planes  $bc$  and  $cb$ ; the Letter  $b$  is Multiplied by  $ac$  and  $ca$ ; the Letter  $c$  is Multiplied by  $ab$  and  $ba$ ; therefore you have the 6 Solids,  $abc, acb, bac, bca, cab, cba$ , or  $6abc$ ; Therefore you must prefix to each Solid whereof the Cube of the Infinite Series is compos'd, the Number which expresses how many ways the Letters of each Solid may be Changed. And generally, *You must prefix to any Product whereof any Power of the Infinite Series  $ax+bxz+cx^3$ , &c. is compos'd the Number which expresses how many ways the Letter of each Product may be changed.*

Now to find how many ways the Letters of any Product, for instances  $a^{m-n}b^h c^p d^r$  may be changed; this is the Rule which is commonly given: write as many terms of the Series  $1 \times 2 \times 3 \times 4 \times 5$ , &c. as there are Units in the Sum of the Indices, viz.  $m-n+h+p+r$ , let this Series

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be

be the Numerator of a Fraction whose Denominator shall be the Product of the Series,  $1 \times 2 \times 3 \times 4 \times 5$ , &c.  $1 \times 2 \times 3 \times 4 \times 5$ , &c.  $1 \times 2 \times 3 \times 4 \times 5 \times 6$ , &c.  $1 \times 2 \times 3 \times 4 \times 5$ , &c. whereof the 1<sup>st</sup> is to contain as many Terms, as there Units in the 1<sup>st</sup> Index  $m-n$ ; the 2<sup>d</sup> as many as there are Units in the 2<sup>d</sup> Index  $b$ ; the 3<sup>d</sup> as many as there are Units in the 3<sup>d</sup> Index  $p$ ; the 4<sup>th</sup> as many as there are Units in the 4<sup>th</sup> Index  $r$ . But the Numerator and Denominator of this Fraction have a common Divisor, *viz.* the Series  $1 \times 2 \times 3 \times 4 \times 5$ , &c. continued to so many Terms as there are Units in the 1<sup>st</sup> Index  $m-n$ ; therefore let both this Numerator and Denominator be Divided by this common Divisor, then this new Numerator will begin with  $m-n+1$ , whereas t'other began with 1, and will contain so many Terms as there are Units in  $b+p+r$ , that is so many as there are Units in the Sum of all the Indices, excepting the 1<sup>st</sup>; as for the new Denominator, it will be the Product of 3 Series only, that is of so many as there Indices, excepting the 1<sup>st</sup>. But if it happens withal, that  $n$  be equal to  $b+p+r$  as it always happens in our *Theorem*, then the Numerator beginning by  $m-n+1$ , and being continued to so many Terms as there are Units in  $b+p+r$  or  $n$ , the last Term will be  $m$  necessarily, so if you invert the Series and make that the first Term which was the Last, the Numerator will be  $m \times m-1 \times m-2 \times m-3$ , &c. continued to so many Terms as there are Units in the Sum of the Indices of each Product, excepting the 1<sup>st</sup> Index. There remains but one thing to demonstrate which is, that, what I have said of Powers whose Index is an Integer, may be adapted to Roots, or Powers whose Index is a Fraction; but it appears at first sight why it should be so: for, the same Reason which makes me consider Roots under the Notion of Powers, will make me conclude, that



that whatever is said of one may be said of t'other. However I think to give sometime a more formal Demonstration of it.

See the Theorem *Fig. 5.*

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IV. *A Demonstration of an Error committed by common Surveyors in comparing of Surveys taken at long Intervals of Time arising from the Variation of the Magnetick Needle, by William Molyneux Esq; F. R. S.*

THE Variation of the Magnetick Needle is so commonly known, that I need not insist much on the Explication thereof, 'tis certain that the true Solar Meridian, and the Meridian shewn by a Needle, agree but in very few places of the World; and this too, but for a little time (if a Moment) together. The Difference between the true Meridian and Magnetick Meridian perpetually varying and changing in all Places and at all Times; sometimes to the Eastward and sometimes to the Westward.

On which account 'tis impossible to compare two Surveys of the same place, taken at distant times, by Magnetick Instruments (such as the *Circumferentor*, by which the *Down Survey*, or Sir *William Petty's Survey of Ireland* was taken) without due allowance be made for this Variation. To which purpose we ought to know the Difference between the Magnetick Meridian and true Meridian at that time of the *Down Survey*, and the said Difference at the time, when we make a New Survey to compare with the *Down Survey*.

Ent

Theorem Fig. 5. Transact. N<sup>o</sup> 230.

$$\overbrace{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 + hz^8 + iz^9 \& C}^m$$

$$= a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4} b^4 z^{m+4}$$

$$+ \frac{m}{1} a^{m-1} c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b d + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} c^2 + \frac{m}{1} a^{m-1} e$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 z^{m+5}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 d$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{2} a^{m-3} bc^2$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} be$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} cd$$

$$+ \frac{m}{1} a^{m-1} f$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} a^{m-6} b^6 z^{m+6} \& C$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{1} a^{m-5} b^4 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 d$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} \times \frac{m-3}{2} a^{m-4} b^2 c^2$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 e$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{1} a^{m-3} bcd$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bf$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} c^3 \text{ West}$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} ce$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} d^2$$

$$+ \frac{m}{1} a^{m-1} g$$

