XIV. *On Friction between Surfaces moving at Low Speeds.* By FLEEMING JENKIN, *F.R.SS.L. & E., Professor of Engineering in the University of Edinburgh, and* J. A. EWING.

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[PLATE 20.]

THE following paper contains an account of an investigation of the Friction between surfaces in motion under circumstances such as do not appear to have been examined before.

The general subject of Friction has received the attention of many writers, beginning with AMONTONS in 1699. In 1799 COULOMB began to investigate it, and in 1781 he communicated to the Academy of Sciences a paper containing the result of his experiments, entitled "Mémoire sur la théorie des machines simples," which is published in vol. x. of the 'Savans Étrangers.' COULOMB pointed out the necessity of distinguishing between the friction which resists the relative movement of surfaces already in motion, or what is now called kinetic friction, and the friction which tends to prevent surfaces at rest from being set in motion, or what is now called static friction. He found that with two dry metallic surfaces there was no difference between the static and kinetic friction. In the other cases which he examined there was a more or less considerable difference, the static being always greater than the kinetic. He found also that the static friction depended on the length of time during which the surfaces were at rest; a prolongation of the time of rest had the effect of increasing the friction, and the rate of this increase varied much in different cases. When the intensity of pressure was great the static friction reached its highest value in a shorter time than when the pressure was small. COULOMB also examined the influence of velocity on the kinetic friction; but his means of observing this seem to have been somewhat rough. He was, however, able to point out that generally, although subject to several exceptions, friction is independent of the extent of surface in contact, directly proportional to the pressure, and independent of the velocity.

In 1784 VINCE laid before the Royal Society the results of some experiments, which, although not very conclusive, agreed in the main with those of COULOMB.

The Philosophical Transactions for 1829 contain a paper on friction by G. RENNIE, in which an interesting account is given of the early history of the subject. RENNIE'S experiments with hard surfaces such as those of metal or wood confirmed, as far as they went, the conclusions of COULOMB.

One part of RENNIE'S work is of special interest on account of the light which has been thrown upon it by the comparatively recent progress of scientific theory. He observed that between ice and ice and between ice and steel (the temperature of the atmosphere being about 28° Fahr.) the coefficient of friction *diminished* largely as the pressure increased. The friction was far from regular; but the diminution of the coefficient under increased pressure was very marked, and amounted in one case to a change from 0·125 to 0·018. This result, which seems to be in complete accordance with J. THOMSON'S discovery of the effect of pressure in lowering the melting-point of ice (Trans. R. S. E. 1849), does not appear ever to have had the attention directed to it that it deserves.

Our most definite knowledge of the phenomena of friction is due to MORIN, who executed from 1830 to 1834 an elaborate series of experiments, the results of which were communicated to the Paris Academy of Sciences, and published in four memoirs. MORIN introduced the system of automatically registering the motion of a body along a horizontal plane surface under the action of certain forces, one of which was the friction to be measured, and he applied this method with great care to the determination of the friction between a large number of different substances. He not only confirmed the laws enunciated by COULOMB, but showed that all the numerous exceptions which COULOMB had mentioned conformed to the general laws when tested by the new and more accurate methods. The results of MORIN have been accepted as conclusive, and the work of subsequent experimenters has been practical rather than directly scientific in its object.

MORIN agreed with COULOMB in distinguishing between static and kinetic friction, and although he did not observe that the time of rest affected the result to nearly so great an extent as COULOMB had affirmed, he found that the static value was usually greater than, but sometimes sensibly equal to, the kinetic. He also noticed that in many cases a slight shock was enough to destroy the distinction between the two.

It occurred to us that instead of there being an abrupt change from the static to the kinetic value of friction at the instant in which motion begins, there might possibly be continuity between the two kinds, and hence that in those cases in which the static coefficient considerably exceeded the kinetic, the latter would be affected by changes of the velocity when the velocity was very small, in such a way as to increase as the velocity diminished. The experiments of MORIN showed that this change in the kinetic value, if it took place at all, must have been confined to very low velocities, so low that his method of observation did not enable him to detect it. The question of whether the friction is affected by changes of velocity under a velocity of, say, 0·01 foot per second, is left by the researches of MORIN an entirely open one; for the length of time which elapsed in his experiments between the instant at which motion began or ended and that at which the velocity was only 0·01 foot per second must have been far too short to allow any definite measurement to be made during it of the rate of acceleration of the moving body.

By means of an apparatus which differs essentially from any previously employed, we have been able to make definite measurements of the friction between surfaces whose relative velocity varied from about 0·01 foot per second down to about 0·0002 foot per second, and have found that in those cases in which the static coefficient largely exceeds the kinetic, the kinetic friction gradually increases as the velocity diminishes between those limits, so that in all probability there is continuity between the two kinds.

In designing an apparatus for the purpose of carrying out this inquiry the following requirements suggested themselves:—That the velocity of the rubbing surfaces (besides being exceedingly low) should change very gradually, so that the acceleration might be capable of measurement at velocities differing only slightly from one another, and differing as little as possible from rest. That the change should be from motion to rest rather than from rest to motion, so as to avoid any jerk at the instant of passage from one to the other state. That no force whatever except the friction to be measured should take part in the action; especially that the means adopted of registering the motion should be such as to cause no retardation. That the surfaces in contact should not change progressively during the motion, but should be periodically restored several times during the motion, so that any change observed to take place in the friction might not be due to a specific change in the surfaces: this last condition might perhaps be more generally expressed by saying that the whole apparatus should periodically return to exactly the same configuration. It was also desirable that a very small change in the velocity should cause a complete change of the rubbing surfaces.

The present inquiry being entirely limited to the question of what influence velocity has on friction at very low speeds, we did not consider it necessary to provide means of measuring the friction at speeds exceeding about 0·01 foot per second, and that is the highest velocity to which our determinations extend. Further, in order to make the apparatus as well conditioned as possible for this particular purpose, it was not arranged so that the intensity of pressure on the rubbing surfaces could be either measured or varied.

The apparatus with which the experiments were conducted, and which was designed with a view to the fulfilment of the above conditions, is shown in fig. 1, Plate 20. A is a disk of cast iron turned on both sides as well as round the circumference, so as to be exactly cylindrical, and weighing 86·2 lbs. Its diameter is nearly 2 feet and its thickness $\frac{3}{4}$ inch. The disk is supported by means of a steel spindle (*a*), the ends of which, resting on the bearings *b b*, are equal cylinders with a diameter of only about 0·1 inch. The small ends of the spindle are shown more clearly in fig. 2, which gives a section of the apparatus in a vertical plane through the axis of the disk, and also in figs. 3 & 4, which give a full-size side elevation and vertical section through the centre of one bearing. The bearings (*b b*) which carry the ends of the axle (*a*) were made successively of the various materials whose friction against the steel axle it was desired to determine. These pieces are fixed by screws to strong iron uprights forming part of the frame B. The bearings consist of rectangular notches, the lower surfaces of both being in the same horizontal plane and each pair of corresponding sides in the same vertical plane.

The breadth of the notch is considerably greater than the diameter of the axle. Thus when the disk is caused to revolve in the direction of the arrow in fig. 1 the axle rolls along the bottom of the notch until it reaches the vertical side on the right hand, and then any subsequent motion can only occur by the sliding of the circumference of the axle upon the bottom and side of each bearing. To prevent the disk from sliding laterally in the event of the bearings ceasing to be exactly level, steel end-plates (*c c*) are provided which can be adjusted by means of screws so as just to touch the ends of the spindle without sensible pressure, and these are slightly rounded at the ends so that their points of contact with the end-plates may lie as nearly as possible in the geometrical axis of the axle. The end-plate is omitted for the sake of clearness in fig. 1, but it is shown in fig. 3, displaced, however, from its proper vertical position in order to allow the end of the axle and the bearing to be fully seen. There was no difficulty in adjusting the end-plates so that their pressure on the axle should cause an infinitesimally small retardation of the motion of the disk, in comparison with that due to the friction of the axle upon the bottom and side of its bearings. Owing to the great weight of the disk this friction was so considerable that the resistance of the air could safely be neglected so long as the velocity of rotation of the disk did not greatly exceed the greatest velocity which occurred in our experiments. When, therefore, the disk was caused to revolve by the temporary application of any force, and was then left to itself, it gradually came to rest in virtue of one cause only—the friction of the axle upon its bearings. By observing the rate of (negative) acceleration of the disk throughout its motion the value of the friction could be determined for all velocities of the rubbing surfaces from the greatest or initial velocity down to the lowest velocity at which the acceleration could be measured.

In order to determine the rate of retardation of the disk at all times throughout its revolution it was necessary to devise some means of recording with great exactness the angular distances moved through by it during successive short intervals of time. This was effected by recording the linear distances moved through by its circumference during successive semiperiods of oscillation of a short pendulum. To obtain a permanent record of these spaces without introducing any new source of retardation whatever, such as would be introduced if a pencil or brush were caused to press either continuously or intermittently against any part of the moving disk, we adopted the method of recording which Sir WILLIAM THOMSON invented for the purpose of registering the arrival of electrical impulses through long submarine cables, and which has found practical application in his siphon recorder. The recording apparatus is shown on the right-hand side in fig. 1. A pendulum (C) is supported on a horizontal knife-edge (*d*) attached to a fixed stand (D), and is capable of oscillating in a plane perpendicular to that of the disk, its position of rest being directly opposite the middle of the circumference of the disk. The knife-edge (*d*) enters a hollow cylinder on the top of the pendulum-rod, and this hollow cylinder is of greater internal diameter at the centre than at the ends, so that the knife-edge bears against it only at the ends. The pendulum is therefore free to oscillate about the line joining these two bearing points. Its period can be altered by shifting up or down the bob *e*. At *f* a small cradle is soldered to the pendulum-rod, and carries a fine glass siphon (*g*), the shorter end of which dips into a box (*h*) containing ink (aniline blue dissolved in water). The longer end (*i*) of the siphon is bent in the manner shown in fig. 1, and stands at a distance of rather less than one tenth of an inch from a strip of paper (E E) two and a half inches broad which is fastened round the circumference of the disk, and projects about three quarters of an inch on each side of it. The lateral stiffness of the paper makes it assume a cylindrical form when placed round the disk, and the ends are fastened together so as to make the cylinder complete. The breadth of the paper is somewhat greater than the maximum amplitude of oscillation of the point *i* of the siphon fixed to the pendulum. In order to make the ink run through the siphon and be deposited on the strip of paper, the ink is continually maintained in a state of electrification. This is effected as follows:—The ink-box (*h*) stands on an insulating rod of vulcanite (*k*), and has fixed to it a small horizontal brass plate (*m*). At a short distance above *m* is the point of a brass rod (*l*) which slides up and down in a V-groove cut in the side of *n*, another rod of vulcanite forming the bracket to which the knife-edge of the pendulum is secured. The rod *l* is pressed into the notch on *n* by the spring *o*. To the top of the rod *l* a wire is fastened which leads to an inductive electrical machine. The machine which we employed was identical in construction with the "mouse-mill" which is used to electrify the ink in the siphon recorder. (See Sir W. THOMSON'S 'Electrostatics and Magnetism,' and 'Journal of the Society of Telegraph Engineers,' vol. v. 1877.) The plate *m* becomes electrified by aërial convection from the point of the rod *l*. The rate of electrification of the ink may be varied by raising or lowering the rod *l*.

When the siphon becomes electrified to a certain extent, the attraction so developed between its point *i* and the paper strip is sufficient to cause the long limb of the siphon to bend until the point *i*, or rather the particle of ink projecting from it, just touches the paper. When this takes place a very small drop of ink is deposited on the paper, and the siphon, ink-holder, and plate *m* are instantaneously diselectrified. The point *i* then recedes from the paper, drawn back by the elasticity of the long limb, until the electrification (continuously communicated by the rod *l*) is again sufficient to cause an advance of *i* towards the paper, when another drop of ink is deposited, and so on. The point of the siphon is by these means kept in a state of rapid vibration towards and from the paper, every advance being accompanied by the deposit of a particle of ink. The rate of electrification is adjusted (by moving the rod *l* up or down) so that the time taken to recharge the siphon, inkholder, and plate corresponds to the period of vibration due to the elasticity of the glass. If the electrification be too rapid the point *i* will be checked in its recession from the paper before the completion of its semiperiod of free vibration. If, on the other hand, the electrification be too slow, the impulse given by the new accumulation of electricity following each discharge will not come soon enough. The adjustment of the rate of electrification is a matter requiring some attention, and it is sometimes difficult to see whether the rate is too great or too small, since both errors have much the same effect in preventing the siphon from vibrating properly. The deposit of ink must be made in a succession of particles by the rapid vibration of the siphon in the manner described, and not as a continuous "brush" or "glow."

In our experiments the length of the longer limb of the siphon was about $3\frac{1}{2}$ inches, and its period of vibration when under the influence of the electrification was about 0·028 second; in other words the particles of ink were deposited on the paper at intervals of 0·028 second of time. The distance between successive spots was in some cases as great as $\frac{3}{4}$ of an inch. The thickness and length of the siphon were both much greater than those of the siphons used in the siphon recorder, and the period of vibration was consequently much greater also. We at first attempted to use siphons of the same size as those used in the recorder, but owing to the much greater speed of the paper in our apparatus, as well as the necessity of keeping the siphon-point at a greater distance from the paper, so that any irregularity in the paper band might pass without scraping against the point, we found it to be necessary to use much longer and coarser tubes.

When the pendulum is at rest, and the disk is caused to revolve, if the rod l be continuously electrified the vibrations of the siphon deposit a series of particles of ink forming a dotted straight line in the centre of the paper strip. If when the disk is revolving the pendulum is made to oscillate, a curved line will be traced out by the drops of ink, crossing the central straight line at intervals depending on the velocity of revolution of the disk. The distances between the successive points of intersection of the curve with the central line are the distances described by the circumference of the disk in equal intervals of time, these intervals being equal to the semiperiod of oscillation of the pendulum.

The experiments were conducted as follows:—A pair of bearings ($b\ b$) were selected of the material whose friction against steel was to be measured. Rectangular notches were cut in them, and they were secured in their places on the uprights as shown in figs. 1 & 2, and exactly levelled. The disk with its spindle was then set on them and the end-plates ($c\ c$) adjusted. A strip of strong paper (E E) 6·3 feet long and 2·5 inches broad was stretched round the periphery and its ends secured by gum, one end overlapping the other for an inch or two. The paper band was continuous except for this junction. The pendulum was then caused to oscillate, and the electrical machine in connexion with l was put in action. When the siphon was vibrating rapidly and the ink was being freely deposited on the paper, the disk was set revolving in the direction of the arrow (fig. 1) by means of an impulse given by hand during only a short part of a revolution. It was then left to itself, and usually performed from three to eight complete revolutions before coming to rest. The motion of the point of the siphon relatively to the paper on the disk was registered in the form of an undulating curve of dots crossing the central position at intervals which gradually diminished as the velocity of the disk became less. Fig. 5 shows a short portion of the paper strip on which the dotted curve formed by the deposited drops of ink has been completed by a line drawn through the dots. The portion marked 8 was the first to be traced by the siphon; then 7, which was traced one complete revolution later, and so on down to 1, which was the final portion of the curve traced just as the disk was coming to rest. P is the point at which the siphon first crossed the central position after the motion of the disk had ceased.

When an unguent was used both bearings were well supplied with it just before the disk was set revolving; and when necessary the supply was kept up during the revolution. The total time during which the motion of the disk lasted never exceeded one minute.

The pendulum was next allowed to come to rest, and while the disk was slowly turned by hand, the central line was traced out by the siphon. The strip of paper was then cut across at one place and removed from the disk.

The distances from P (fig. 5) to the successive points at which the curve crossed the central line were measured by means of a rule graduated to six-hundredths of a foot, and the differences between the successive values were found. These differences, which may be called Δs, are the distances described by a point in the circumference of the disk during a half beat, Δt, of the pendulum. $\frac{\Delta s}{\Delta t}$ is the mean velocity during the time Δt, and this mean is the actual velocity at the middle of the time Δt, provided that the acceleration is uniform during that time. Now even if the force due to friction were to change very considerably with changes in the velocity, this force, and therefore the acceleration, would be sensibly constant during the short interval Δt, during which the velocity undergoes exceedingly little change. Hence we are completely justified in taking the successive values of $\frac{\Delta s}{\Delta t}$ as accurately representing the velocities at times differing by Δt; and since Δt is constant, the successive values of Δs are proportional to these velocities.

A curve was next drawn, as O A B in fig. 6, in which the ordinates were the successive values of Δs, and the abscissæ differed by the constant quantity Δt. This curve expressed the velocity as a function of the time, and would be a straight line when the acceleration was uniform. When the acceleration was greater at low than at high velocities, the curve would be convex upwards, as shown in the figure, and the acceleration at any point, such as A, would be proportional to the tangent of the inclination of the tangent at that point, or $\frac{\delta y}{\delta x}$.

This means of finding the value of the acceleration at any point, in which the method of tangents was only once used, was obviously much more accurate than if the first curve plotted had been simply one connecting the distances moved through by the disk with the times, and the method of tangents had been applied to that curve in order to enable a second curve to be drawn connecting the velocities with the times.

A single numerical example will suffice to explain the method of calculation by which, in each experiment, the acceleration due to friction was deduced from the graphic record. The particular case chosen is that of steel rubbing against bearings of beech, without any unguent. The curve is a comparatively short one, extending over only three complete revolutions of the disk. Column I. Table I. gives the distance (measured in six-hundredths of a foot) from a point which corresponds in this example to the point P in fig. 5, to the successive points in which the curve crosses the central line. Column II. gives the differences between these successive values, or what we have called Δs—that is, the distance moved through by a point in the circumference of the disk in times equal to the half-period of the pendulum.

TABLE I.

I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.	I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.	I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.
8	8	1533	163	5692	315
24	16	1705	172	6021	329
49	25	1887	182	6357	336
85	36	2077	190	6703	346
128	43	2275	198	7054	351
183	55	2483	208	7419	365
244	61	2701	218	7787	368
317	73	2925	224	8173	386
397	80	3161	236	8561	388
487	90	3402	241	8964	403
586	99	3659	257	9368	404
694	108	3921	262	9788	420
809	115	4195	274	10211	423
937	128	4475	280	10648	437
1071	134	4768	293	11093	445
1217	146	5063	295	11542	449
1370	153	5377	314		

s for one revolution $=3718\ \frac{\text{feet}}{600}$.

When the curve is drawn corresponding (in this example) to that shown in fig. 6, the irregularities in the successive values of Δs, which are due to the fact that the central line has not been exactly central, disappear, and the curve turns out to be exactly straight. Hence between the limits of velocity to which this experiment extends the friction is perfectly constant. The tangent of the inclination of the line is measured and found to be ·01515, δy (see fig. 6) being expressed in feet, and δx in terms of the unit Δt. To find $\frac{d^2s}{dt^2}$, the acceleration in the direction of motion of a point in the circumference of the disk in feet and seconds, we must divide this quantity by $(\Delta t)^2$ in seconds. Throughout all the experiments Δt was 0·3571 second. Hence for this example

$\frac{d^2s}{dt^2}=0\cdot1188$ foot per second. If ω be the angular velocity of the disk, the angular acceleration $=\frac{d\omega}{dt}=\frac{1}{R}\frac{d^2s}{dt^2}$, where R is the radius, or 0·9857 foot. The moment, M′, of the couple due to friction, measured in absolute kinetic units, is $I\frac{d\omega}{dt}$, where I is the moment of inertia of the disk about its axis. Since the mass of the disk is 86·2 lbs., and its radius of gyration 0·697 foot, I=41·9. Hence, in the above example, M′=5·05. To reduce this to M, the moment of the couple due to friction where the force is expressed in terms of the gravitation unit, we must divide by 32·2; hence M=0·157.

The value of this couple having been obtained, the coefficient of friction, μ, expressing the ratio of the force due to friction to the normal pressure, remains to be found. During the revolution of the disk the axle presses against one side and the bottom of each bearing in the manner shown in fig. 7. If P_v be the pressure against the bottom, P_h the pressure against the side, and W the weight of the disk, we have

$$P_h=\mu P_v$$

and

$$P_v=W-\mu P_h=W-\mu^2 P_v.$$

Hence

$$P_v=\frac{W}{1+\mu^2} \text{ and } P_h=\frac{\mu W}{1+\mu^2}.$$

The couple due to friction (M) is $\mu P_v r+\mu P_h r$, where r is the radius of the axle.

We have therefore a quadratic equation for determining μ:

$$\mu^2(Wr-M)+\mu Wr=M;$$

or, to put it in a form better suited for arithmetical work,

$$\mu^2+\mu\frac{Wr}{Wr-M}=\frac{Wr}{Wr-M}-1.$$

This equation has only one positive root.

It has here been assumed that μ is the same for both the places where sliding occurs. Even in cases where this assumption is not quite warrantable (as when the bearings are made of wood, and the motion is at the bottom in a plane perpendicular to the fibres, and at the side in the direction of the fibres), the amount of error so introduced into the determination of μ will be exceedingly small; for, since P_v is much greater than P_h, the value of μ, as above determined, is very approximately that corresponding to the bottom surface.

Substituting for W and r their numerical values, viz. W=86·2 lbs. and r=·004135 foot, we have

$$\mu^2+\mu\frac{0\cdot3564}{0\cdot3564-M}=\frac{0\cdot3564}{0\cdot3564-M}-1.$$

In the example cited M=0·157; whence

$$\mu=0\cdot366.$$

The greatest and least values of the relative velocity of the sliding surfaces in this experiment are determined as follows:—The greatest value of Δs (Table I.) is 449 six-hundredths of a foot, and this distance was passed through by a point in the circumference of the disk in Δt, or 0·3571 second. The ratio of the radius of the disk to that of the axle is 238·4. Hence the greatest velocity of sliding between the circumference of the axle and the bearings is 0·0088 foot per second. The lowest limit of velocity of sliding down to which the observation can be said to extend depends, of course, upon how near to velocity $=0$ in the curve shown in fig. 6 the tangent can be drawn by which the acceleration is determined. In this case any change in the inclination of the curve could certainly be observed down to as low a velocity as that corresponding to $\Delta s=10$. This ordinate corresponds to a velocity of sliding of 0·0002 foot per second. Hence the above determination of μ extends from a velocity of 0·0088 foot per second, as the higher limit, down to 0·0002 foot per second, as the lower limit, and between these two limits the value of μ remains perfectly constant.

This limit of 0·0002, or *one five-thousandth of a foot per second*, as the least relative velocity of the sliding surfaces for which the determination of the coefficient of friction is definite, is approximately the same for all the experiments that follow. In every case the determination of μ is definite for as low a velocity as this. In some cases it is definite for even lower velocities. The higher limit of velocity to which the experiments extend varies in different cases, and is stated in each.

This exceedingly low limit of velocity could only be secured by making the diameter of the axle very small. As a result of this, the state of balance of the disk was rarely perfect. Although the disk was turned with the greatest care, so as to be truly cylindrical, it proved, when placed in its bearings, to be slightly heavier on one side than on the other. This irregularity was easily removed by applying a small counterpoise near the periphery; but we found that the state of balance so produced was not permanent. This was, no doubt, due to an almost infinitesimal bending of the axle from time to time. Although this irregularity could not be observed by the eye when the disk was revolving, it became apparent when the curve corresponding to fig. 6 was drawn, its effect on the curve being to introduce an undulation whose period was that of one revolution of the disk. Of course, if the friction was uniform throughout the movement, tangents drawn to the curve at corresponding phases in the successive undulations should be parallel. In this way the irregularity due to a want of balance in the disk was eliminated, and the friction determined throughout the whole curve. The example which has been given above is one of the comparatively rare cases in which the disk was for the time being in a state of sensibly perfect balance.

The cases examined were as follows:—Steel on steel, steel on brass, steel on polished agate, steel on greenheart, and steel on beech; in each case under the three different conditions, dry, oiled, and wet with water. The oil made use of was the fine oil employed by watchmakers. In order that no trace of unguent might be present when dry or wet surfaces were being examined, the oil was removed from the metallic surfaces by washing them with solution of caustic potash; while in the case of wood the friction was first determined when the surfaces were dry; then they were wetted with water, and the friction again measured; and, lastly, when the water had dried off, oil was applied. In every case the same specimens were used in all three determinations. When the bearings were of wood the fibres were vertical, and the coefficient of friction was approximately that corresponding to motion in a plane normal to the fibres. The following is a summary of the results:—

I. *Steel on Steel. Dry.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0057 foot per second . . $\mu=0{\cdot}337$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0089 foot per second . . $\mu=0{\cdot}350$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0086 foot per second . . $\mu=0{\cdot}365$

Mean . . . . $\mu=0{\cdot}351$

In this series, although the friction remained sensibly uniform during the time each experiment lasted, there was a progressive increase of the coefficient, probably due either to a tearing of the surface or to chemical action. The time that elapsed between the successive experiments was great relatively to the time taken up by any one of them, and a good deal of motion took place between the surfaces in each interval. We have observed several cases in which there is a similar progressive increase, but none in which there is a progressive decrease in the value of μ.

II. *Steel on Steel. Oiled.*

In this case the friction appeared to be somewhat *less* at the lower than at the higher velocities. This result is quite anomalous; we have observed no other instance in which the same thing occurs. The change, however, in the value of μ is not great, and it is possible that this result may be due to error of observation; we therefore state it with caution, although the different observations show a remarkably close agreement amongst themselves.

1. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}119$
   For velocity 0·0046 foot per second . . . . $\mu=0{\cdot}130$

Here 0·0046 foot per second is the greatest velocity observed.

2. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}116$
   For velocity 0·0060 foot per second . . . . $\mu=0{\cdot}130$

From this to the greatest velocity, 0·0065 foot per second, μ remains sensibly constant.

3. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}119$
   For velocity 0·0043 foot per second . . . . $\mu=0{\cdot}130$

Here 0·0043 is the greatest velocity observed.

4. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}116$
   For velocity 0·0040 foot per second . . . . $\mu=0{\cdot}127$

From this to the greatest velocity observed, 0·0059 foot per second, μ remains sensibly constant.

Mean value of μ at velocity 0·0002 foot per second . .	0·118
,, ,, ,, about 0·0050 foot per second . .	0·129

It is to be observed in connexion with these values, and, indeed, in connexion with our experiments with unguents generally, that, owing to the very great intensity of pressure on the small bearing-surfaces of the axle, the unguent must have been to a great extent forced out, so as to leave the surfaces in the state described by MORIN as "unctuous."

III. *Steel on Steel. Wet with water.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0058 foot per second . . $\mu=0{\cdot}178$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0052 foot per second . . $\mu=0{\cdot}205$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . . $\mu=0{\cdot}241$

Mean . . . . $\mu=0{\cdot}208$

The remarks made in Case I. apply here. On comparing these values with those for dry steel on steel, we see that the presence of water here diminishes the friction.

IV. *Steel on Brass. Dry.*

1. Friction very irregular, but apparently unaffected by velocity. From velocity 0·0002 foot per second to greatest velocity observed, 0·0059 foot per second . . $\mu=0{\cdot}180$
2. Very irregular. From velocity 0·0002 to 0·0053 . . . $\mu=0{\cdot}202$
3. Irregular. From velocity 0·0002 to 0·0056. . . . . $\mu=0{\cdot}202$

Mean . . . . $\mu=0{\cdot}195$

Although the friction was very irregular here, it seemed to be quite independent of the velocity. The coefficient is strikingly less than in the case of dry steel surfaces.

V. *Steel on Brass. Oiled.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0044 foot per second . $\mu=0{\cdot}146$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . $\mu=0{\cdot}146$

Mean . . . . $\mu=0{\cdot}146$

Here the coefficient appears to be entirely unaffected by change of velocity.

VI. *Steel on Brass. Wet with water.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0048 foot per second . $\mu=0{\cdot}106$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0045 foot per second . $\mu=0{\cdot}102$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0041 foot per second . $\mu=0{\cdot}107$

Mean . . . . $\mu=0{\cdot}105$

The coefficient here is remarkably low, indicating, when compared with the preceding, that, at least when the intensity of pressure is very great, water is a better unguent than oil for surfaces of steel and brass.

VII. *Steel on Polished Agate. Dry.*

1. Friction not very regular, but apparently independent of velocity. From velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . . $\mu=0{\cdot}168$
2. Ditto. From velocity 0·0002 foot per second to greatest velocity observed, 0 0053 foot per second . . . . . $\mu=0{\cdot}191$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0065 foot per second . $\mu=0{\cdot}240$

Mean . . . . $\mu=0{\cdot}200$

The remarks made in Case I. apply here.

VIII. *Steel on Polished Agate. Oiled.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . μ=0·106
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0041 foot per second . μ=0·107
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . μ=0·107

Mean . . . . μ=0·107

IX. *Steel on Polished Agate. Wet with water.*

The determinations here are not perfectly definite, but appear to agree in showing that the friction increases slightly at very low velocities.

Taking the mean of three observations, we have

For velocity 0·0002 foot per second . . . . μ=0·166
For velocity 0·0060 foot per second . . . . μ=0·146

The greatest velocity observed was 0·0069 foot per second, and it is doubtful whether μ was not still sensibly diminishing at that velocity.

X. *Steel on Greenheart. Dry.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0058 foot per second . . μ=0·212
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0047 foot per second . . μ=0·213
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . . μ=0·221

Mean . . . . μ=0·215

XI. *Steel on Greenheart. Oiled.*

In this case there was a very marked increase of friction as the velocity diminished. From a velocity of 0·0002 to about 0·005 foot per second, the friction gradually diminished, and at the latter velocity it appeared to have become nearly constant.

1. For velocity 0·0002 foot per second . . . . μ=0·123
   For velocity 0·005 foot per second . . . . μ=0·094
   Greatest velocity observed 0·0064 foot per second.
2. For velocity 0·0002 foot per second . . . . μ=0·126
   For velocity 0·004 foot per second . . . . μ=0·098
   Greatest velocity observed 0·0042 foot per second.
3. For velocity 0·0002 foot per second . . . . μ=0·124
   For velocity 0·005 foot per second . . . . μ=0·097
   Greatest velocity observed 0·0050 foot per second.

Mean value of μ for velocity of 0·0002 foot per second . . .	0·124
Mean value of μ for higher limit of velocity	0·096

This change in the value of μ seems to be far greater than experimental errors can account for. The three (independent) observations given above agree closely both as regards the initial and final values of μ.

The specimens of greenheart which were used in the above experiments were new, that is, their surfaces were freshly cut at the time these experiments were made. They were then laid aside without having the oil which had been put on them removed, and after an interval of six months the observation was repeated, fresh oil being applied. Two determinations were made, giving results which agreed closely in all respects. These results were

For velocity 0·0002 foot per second	μ=0·077
For velocity 0·004 foot per second	μ=0·062

Greatest velocity observed 0·0055 foot per second.

Here it is noticeable that although the prolonged exposure to oil had altered the bearing-surface of the wood so as greatly to diminish the coefficient of friction, the effect of change of velocity was the same as before. From velocity 0·004 foot per second upwards the coefficient underwent little or no appreciable change.

XII. *Steel on Greenheart. Wet with water.*

In this case there was an equally marked increase of friction at low speeds.

1. For velocity 0·0002 foot per second	μ=0·290
For velocity 0·005 foot per second	μ=0·232
Greatest velocity observed 0·0066 foot per second.	
2. For velocity 0·0002 foot per second	μ=0·292
For velocity 0·005 foot per second	μ=0·243
Greatest velocity observed 0·0056 foot per second.	
3. For velocity 0·0002 foot per second	μ=0·290
For velocity 0·006 foot per second	μ=0·237
Mean value of μ for velocity 0·0002 foot per second . . .	0·291
Mean value of μ for higher limit of velocity	0·237

The diminution of μ as the velocity increases amounts to about twenty per cent. of its value at the lower limit. This proportion is almost identical with that observed in the case of steel rubbing on oiled greenheart.

In both of the foregoing cases (XI. & XII.) in which this marked change took place in the value of μ, the experiments were at first made under the following conditions:—The oil (or water) was liberally supplied to the bearings just before the disk was caused to revolve, and the supply was not renewed during the revolution of the disk. It was suspected that the increase of μ might be due to the absorption of the unguent by the wood during the time that the disk was revolving. To test whether this was the case, the experiments were in both cases repeated, the supply of unguent being kept up continuously during the revolution of the disk. Precisely the same change in μ was observed under these circumstances as under the circumstances in which the experiments were at first made.

As a further confirmation of the result already stated for Case XII., the following determinations of μ were made with a new pair of greenheart bearings, but with the same steel axle.

4. For velocity 0·0002 foot per second . . . . μ=0·253
   For velocity 0·007 foot per second . . . . μ=0·181
   Greatest velocity observed 0·007 foot per second.

5. For velocity 0·0002 foot per second . . . . μ=0·262
   For velocity 0·0056 foot per second . . . . μ=0·203
   Greatest velocity observed 0·0056 foot per second.

6. For velocity 0·0002 foot per second . . . . μ=0·262
   For velocity 0·008 foot per second . . . . μ=0·207
   Greatest velocity observed 0·0106 foot per second.

Mean value of μ for velocity 0·0002 foot per second . . . . 0·259
Mean value of μ for higher limit of velocity . . . . 0·197

Here the change is equally marked; the diminution is rather more than twenty per cent. of the higher value. It will be observed that although in each set of determinations the separate observations agree well, the values obtained from the second set are considerably lower than those obtained from the first, no doubt on account of some specific difference in the bearing-surfaces. Taking a general mean from the two sets of observations, we have

Value of μ for velocity 0·0002 foot per second . . . . . . 0·275
Value of μ for higher limit of velocity . . . . 0·217

Under this set of conditions μ appears to undergo little change when the velocity exceeds about 0·006 foot per second.

XIII. *Steel on Beech. Dry.*

1. Friction perfectly uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0088 foot per second . . . . μ=0·366
2. Friction perfectly uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0089 foot per second . . . . μ=0·367

Mean . . . . =0·3665

This result differs from that of Case X. only in the magnitude of the coefficient.

XIV. *Steel on Beech. Oiled.*

The results obtained in the corresponding experiment with greenheart appear again here.

1. For velocity 0·0002 foot per second . . . . $\mu=0·126$
   For velocity 0·005 foot per second . . . . $\mu=0·100$
   Greatest velocity observed 0·0057 foot per second.
2. For velocity 0·0002 foot per second . . . . $\mu=0·126$
   For velocity 0·005 foot per second . . . . $\mu=0·101$
   Greatest velocity observed 0·0055 foot per second.
3. For velocity 0·0002 foot per second . . . . $\mu=0·126$
   For velocity 0·0037 foot per second . . . . $\mu=0·110$
   Greatest velocity observed 0·0037 foot per second.

In the last experiment it appears that the higher limit of the velocity was not high enough to enable μ to assume its minimum value. We therefore reject it in estimating the means, which are as follows:—

Mean value of μ for velocity 0·0002 foot per second . . . . 0·126
Mean value of μ for higher limit of velocity . . . . 0·101

The change in the value of μ occurs here below a velocity of 0·005 foot per second. The total change is almost exactly twenty per cent. of the higher value.

XV. *Steel on Beech. Wet with water.*

Here again we have an equally unmistakable increase of the coefficient of friction as the speed diminishes.

1. For velocity 0·0002 foot per second . . . . $\mu=0·344$
   For velocity 0·007 foot per second . . . . $\mu=0·254$
   Greatest velocity observed 0·007 foot per second.
2. For velocity 0·0002 foot per second . . . . $\mu=0·353$
   For velocity 0·008 foot per second . . . . $\mu=0·290$
   Greatest velocity observed 0·008 foot per second.
3. For velocity 0·0002 foot per second . . . . $\mu=0·370$
   For velocity 0·0074 foot per second . . . . $\mu=0·292$
   Greatest velocity observed 0·0074 foot per second.

The curves for these observations connecting μ with the velocity indicate pretty clearly that the greatest velocity observed was insufficient to bring μ to its least value. Taking, however, the means of the observations as they stand, we have

Mean value of μ for velocity 0·0002 foot per second . . . . 0·356
Mean value of μ for highest velocity observed . . . . 0·279

A comparison of all the above results shows that, omitting the doubtful cases of oiled steel (II.) and wet agate (IX.), the coefficient of friction remained entirely unaffected by changes in the velocity except where the bearings were made of wood, and that even in these circumstances no change of the coefficient could be detected so long as the surfaces were dry; but when oil or water was present the coefficient of friction increased in a very marked manner as the velocity diminished, this increase of the coefficient of friction taking place under a limit of velocity of about 0·01 foot per second. It is to be observed that the cases in which this increase of the coefficient at low speeds occur are precisely those in which COULOMB and MORIN have found that there is a very marked difference between the static and kinetic values of the coefficient of friction. In the case of dry metal surfaces both these writers are agreed that no difference can be detected between the friction of rest and that of motion; and we find that in such cases no change takes place in the value of μ as the velocity varies. MORIN also found that the friction between unctuous metallic surfaces was the same or nearly the same for rest as for motion, a result which agrees with the absence of almost any change of the coefficient of friction in our investigation of Cases II. & V. The friction between wet metallic surfaces is not spoken of by COULOMB, and received comparatively little attention from MORIN, who does not say whether he found any difference between its static and kinetic values. We have not seen any account of experiments on the friction between metals and agate. As to the remaining sets of conditions, MORIN says that he found no sensible difference between the static and kinetic values of the friction of dry metals on wood (Mémoire i. pp. 104, 106). In fact, out of all the cases which we have examined, the only ones in which there is known to be a marked difference between the friction of rest and that of motion are those in which steel slides on oiled or wetted surfaces of wood, and in these cases, and these only, we have detected a very considerable increase in the coefficient of friction at very low speeds.

Although the varying want of balance of the disk, which has been already referred to, was never so great as to throw any doubt on the fact that the rate of acceleration did change in these particular cases (and we are unable to see any cause for the change of the rate of acceleration except a change in the coefficient of friction), still it was considerable enough to make the determination of the precise way in which the coefficient changed with changes in the velocity a matter of the greatest difficulty. We have been able to draw only very roughly curves connecting the coefficient μ with the relative velocity of the sliding surfaces. These all agree in showing that $\frac{d\mu}{dv}$, or the rate of change of the coefficient of friction relatively to change of velocity, becomes greater as the velocity becomes less. This being so it is perfectly possible that at velocities below 0·0002 foot per second, which is the lowest limit to which our observation may be taken as extending, a considerable further increase of the coefficient may take place before motion entirely ceases.

It is in fact highly probable that in those cases in which the static coefficient of friction is greater than the kinetic (that is, the coefficient which is observed when the surfaces are moving at a moderate speed), the latter gradually increases when the velocity becomes extremely small, so as to pass without discontinuity into the former. The experiments of COULOMB show that the friction between surfaces at rest is itself not constant, but increases as the time of rest is prolonged. It seems doubtful whether this result is due to a real change in the static coefficient or not; but if it is we may suppose that not only does the coefficient of friction increase continuously as the state of the surfaces changes from motion to rest, but continues for a time to increase after the latter state has been reached. To prove this with certainty is probably impossible, both because of the difficulty of observing the rate of retardation of moving surfaces down to the point at which motion wholly ceases, and because of the impossibility of measuring the static coefficient between surfaces which have not been in contact for a finite length of time; but the results of the experiments which have been stated above seem to give a strong colour of probability to the hypothesis.

The friction of steel on greenheart wetted with water (Case XII.) has special interest. MORIN has endeavoured to explain the excess of the friction of rest over that of motion, by supposing that the unguent present is more or less expelled when the surfaces are at rest. This view may perhaps in some cases be correct, but it is certainly not always tenable. With wetted steel and greenheart surfaces the static coefficient is very much greater than the kinetic, although the presence of water has an effect the reverse of that of an unguent (compare X. & XII.). It seems impossible that the expulsion of what has the effect of increasing the friction should give rise to a further increase.

We found that our apparatus was not suited for determining the value of the friction between surfaces at rest. Attempts were made to use it for this purpose, but the results were not satisfactory. The static coefficients showed a fair general agreement with those given above; the most marked divergences appeared in the cases where the kinetic values increased at low speeds. In these the static values of the coefficients were considerably greater than even the greatest kinetic values.

In making practical deductions from experiments on friction, it is to be observed that the phenomena are so dependent on apparently insignificant variations of the conditions of the observation that it is hazardous to state, for the guidance of the engineer, results which are obtained under conditions greatly different from those met with in practice. We therefore only draw attention to two points, both of which seem to be novel:—(1) The excellence of greenheart, whether dry or oiled, as a material for bearings; and (2) the very small friction between steel and brass when the surfaces are wetted with water.

We regret that the laboriousness of the measurements and calculations which have had to be made in connexion with each observation has prevented the number of cases examined from being more numerous than they are. For the same reason, the experiments, of which the above is an account, although for the most part made in the summer of 1876, have only now been prepared for publication.

Table II. gives a synopsis of the results. In Column IV. the value of μ is given, corresponding to the lowest velocity to which the observations extended. Columns V. and VI. show the change in the value of μ wherever any was observed, Column V. giving the changed value of μ, and Column VI. the velocity at which the change appeared to be sensibly complete. Columns V. and VI. are left blank wherever no change was detected in the value of μ from the least velocity up to the greatest velocity observed, which is given in Column VII.

TABLE II.

I. Number.	II. Surfaces.	III. Condition.	IV. μ at velocity 0·0002 ft. per sec.	V. μ changes to	VI. At velocity (ft. per sec.).	VII. Greatest velocity to which observation extends (ft. per sec.).	VIII. Remarks.
I.	Steel on Steel.	Dry.	0·351	......	......	0·0089	
II.	,, ,,	Oiled.	0·118	(?) 0·129	0·005	0·0065	Condition of surfaces rapidly changing, so as to increase μ.
III.	,, ,,	Wet with water.	0·208	......	......	0·0058	
IV.	Steel on Brass.	Dry.	0·195	......	......	0·0059	Very irregular.
V.	,, ,,	Oiled.	0·146	......	......	0·0064	
VI.	,, ,,	Wet with water.	0·105	......	......	0·0048	
VII.	Steel on Agate.	Dry.	0·200	......	......	0·0064	Condition of surfaces rapidly changing, so as to increase μ.
VIII.	,, ,,	Oiled.	0·107	......	......	0·0054	
IX.	,, ,,	Wet with water.	0·166	(?) 0·146	0·006	0·0069	
X.	Steel on Greenheart.	Dry.	0·215	......	......	0·0064	
XI.	,, ,,	Oiled.	0·124	0·096	0·005	0·0064	
	,, ,,	,,	0·077	0·062	0·004	0·0055	After prolonged exposure to oil.
XII.	,, ,,	Wet with water.	0·275	0·217	0·006	0·0106	
XIII.	Steel on Beech.	Dry.	0·366	......	......	0·0089	
XIV.	,, ,,	Oiled.	0·126	0·101	0·005	0·0057	Change in μ still going on at greatest velocity observed.
XV.	,, ,,	Wet with water.	0·356	0·279	0·008	0·0080	

Fig. 1.

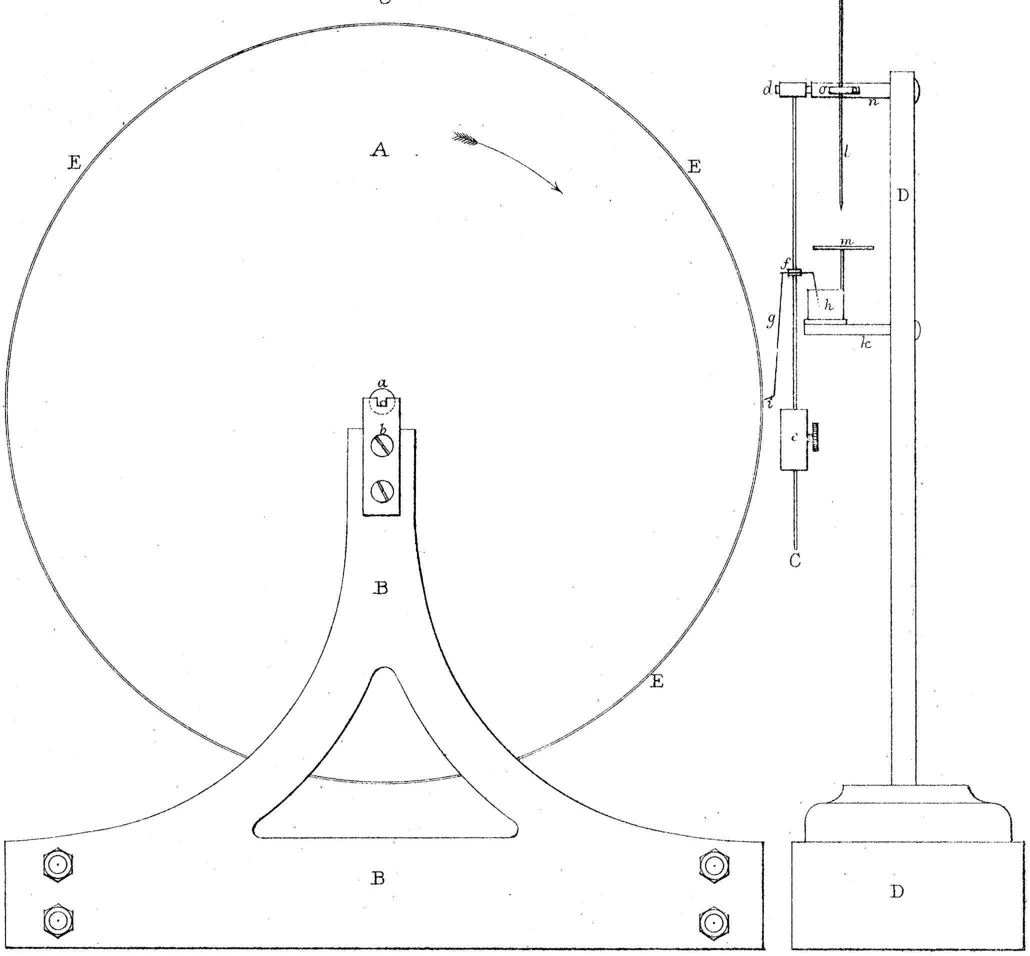


Fig. 2.

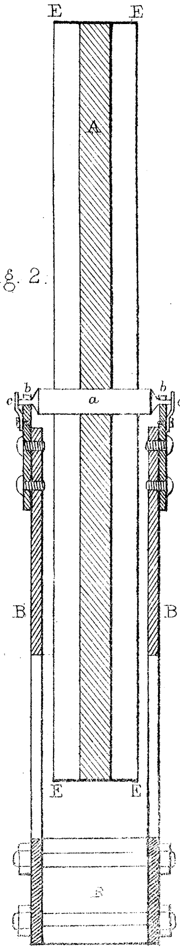


Fig. 3.

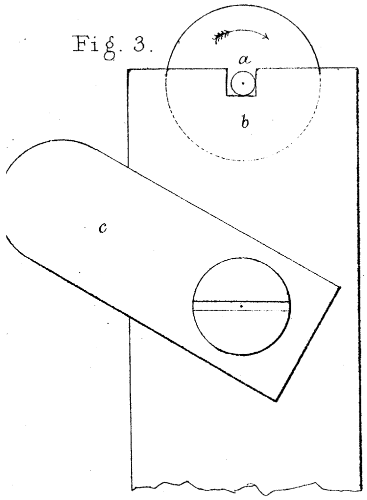


Fig. 4.

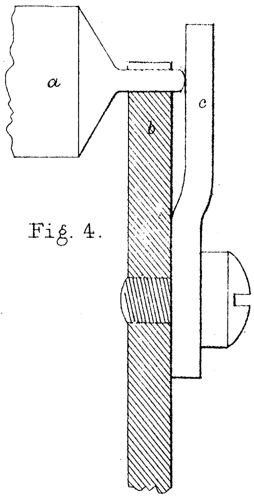


Fig. 7.

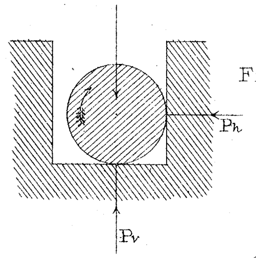


Fig. 6.

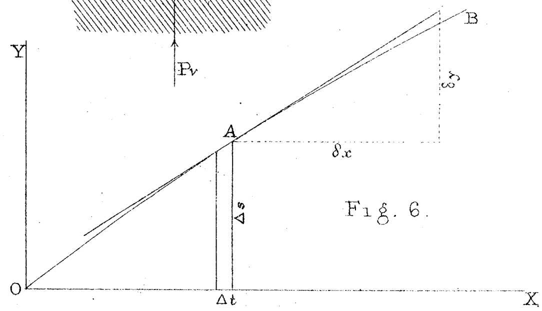
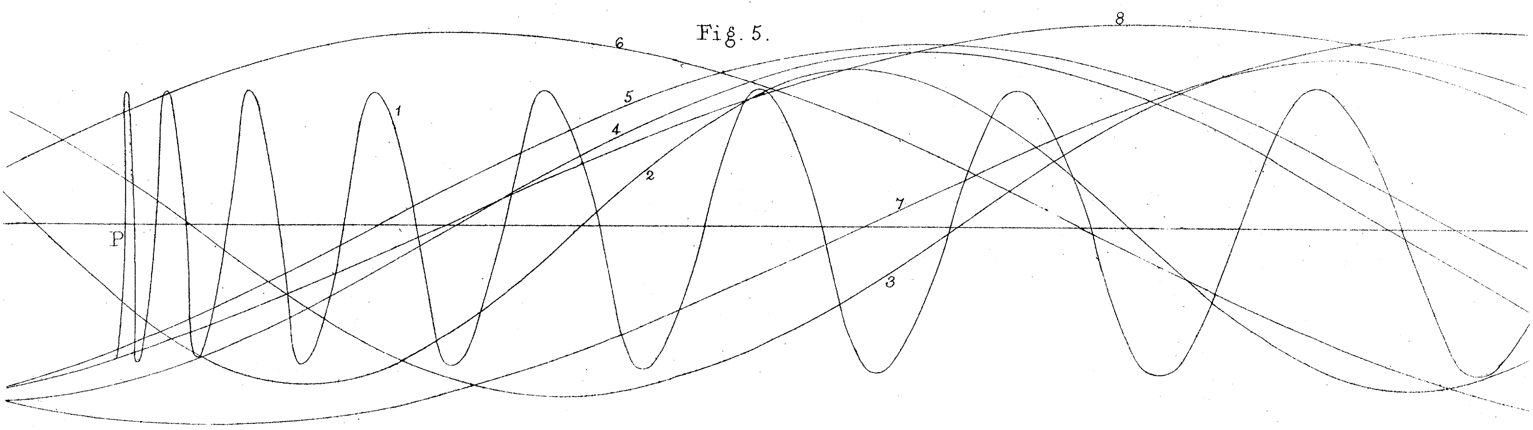


Fig. 5.



W. West & C^o. lith.