

I. *The Construction of a Quadratrix to the Circle, being the Curv described by its Equable Evolution.*

1. **B**Y the *Equable Evolution* of a Circle, I mean such a gradual approach of its Periferie to *Rectitude*, as that all its parts do *together*, and *equally* evolve or unbend: or so that the same Line becomes successively a less and less Arc of a reciprocally greater Circle.

2. Let A H K A (Fig. 6.) be the Periferie of a Circle. A E a Tangent to the point A. Let this Circular Line be suppos'd cut or divided at A, and then to unbend (like a *Spring*) its upper end remaining fixt to its Tangent A E, whilst the other parts do *Equally Evolve* or extend themselves thorough all the degrees of less Curvature (as in ABD, AMC, &c.) till they become straight in coincidence with the Tangent A E.

3. Let AMC be the *Evolving Curv* in any middle position between its first and last. Joyn the fixt end A, and the moving end C, by the Chord-line AC, intersecting the first Circle at H. I say that AMC is a *like* Segment to A n H, cut off in the first Circle by the Chord A H. For, by the supposition of AMC is the Arc of a Circle, having A E a Tangent common both to it and A n H, and both Arcs are terminated in the same Right-line A C.

4. Hence the Curv ADCE (describ'd by the moving end of the Periferie in its Evolution) may be thus constructed. Let the Circle AHKA be by bisections divided into any number of equal parts. Let H be one of the points of such division. Then say, as the number of equal parts in the Arc A n A: is to the number of parts in the whole Periferie AHKA :: so is the Chord A H: to a fourth Line, which let be A C in A H produc'd. So is C a point in the Curve A D C E.

5. Dem. Upon A C describe A M C, an Arc *like* to the Arc A n H. Whence— A H : A C :: A n H : A M C. But by construction, A H : A C :: A n H : perif: A H K A, therefore is the Arc AMC equal to the whole Periferie AHKA
and

and like to the Arc $A n H$. Consequently AMC represents the Evolving Periferie, in a position like to the Arc $A n H$, and C is the describing point.

6. After the same manner may be found other points thro which the Curv may be drawn. But here (as in the old *Quadratrix* of *Dinostratus*) the point E cannot be precisely determined but the Curv may be brought so near it, that its flexure or tendency will so lead to the point E , that AE shall be near enough to the truth for common uses.

7. Supposing the point E found, a *Tangent* to any point of the Curv may be drawn: and supposing a *Tangent* drawn, the point E may be determined; the property of the *Tangent* being this, that supposing RT a *Tangent* to the point C and CA, CE , drawn from C to each end of the rectify'd Circle, the Angle ACT (the lesser angle that AC makes with the tangent) is equal to the tangent made by the 2 Lines drawn from C .

8. c be a point in the *Quadratrix* indefinitely near to C ; and draw Ac intersecting $AHKA$ in h , and AMC in o . To Ac as a chord, draw the Arc Amc like unto the Arc Ana . To the point c of the Arc Amc draw the *Tangent* $CL = AE$, and joyn LA : So is oC an indefinitely little particle of the Arc coincident with its *Tangent*.

9. Because of the like Segments $AnhA, AMoA, AmcA$, as chord Ac : to chord lo : : So is Arc $Amc (= AMC)$: to Arc AMo . Or $Ac : Ao :: Amc (= AMC) : AMo$. And dividing $Ac \cdot Ao (= co) : Ao :: Amc - AMo (= Co) : AMo$. That is, $co : Ao :: Co : AMo$. and alternately, $co : Co :: Ao : AMo$. Put AC for Ao , and AMC for AMo (as differing infinitely little) and then 'tis $co : Co :: AE : AMC$. But by construction $CL = AE = AMC$ whence $co : Co :: AC : CL$ and the Angle $LCA = Coc$. (oc being infinitely near to AC , is therefore parallel to it.) and therefore Coc, ACL are like *Triangles*.

10. Because of $CL = AE$, Ang. $EAC = LCA$. (CL and EA being *Tangents* to the two ends of the same circular Arch AMC , make equal Angles with its chord AC .) and AC common to both, the *Triangles* EAC, ACL are like and equal: therefore are all three Coc, ACL, EAC like *Triangles*. Whence it follows, that the Angle ACE (in the *Triangle* EAC) is equal to the Angle ocC (in the *Triangle* coC .) But $ocC = ACT$ because oc and AC are parallel; therefore Ang. $ACE = ACT$. QED.