



Philosophical Transactions

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& lettere Etrusche. Delle quali vi mostrero io la copia à vostro piacere, come à me la mostrò & diede il dottissimo & parimente umanissimo Piero Vettori nostro, diligentissimo investigatore delle cose antiche; insieme con lo Alfabeto Etrusco, che all'ora non era fuori. De statuis istis transcripta hæc esse volui, tum ut, me Auctore, inquirere non tardares, quorum forte in porticibus, aliisve in locis admirandæ etiam nunc stant, tum præcipue ut scires, an forte Iconum, quas jam in magno fatis numero possidet R. Soc. prototypi, capita sua veneranda interillas tollant. De hac re doctos quibuscumq; per Italiam literarum commercium habes, speciatim *Hetruscos* & *Umbros* facile poteris percontari. Ex quibus, si quæ expectationi nostræ respondeant acceperis, fac quæso, ut ea quam primum cognoscam. Vale, Vir optime, ac eruditissime, & redamare perge integerrimum tui cultorem *Hickesum*. Faxit Deus, ut diutissime vivas doctorum omnium delicia, & bonarum literarum, is qui es, maximus semper adjutor, & patronus. Iterum vale.

IV. *The Theory of Musick reduced to Arithmetical and Geometrical Proportions, by the Reverend Mr Tho. Salmon.*

S I R,

HAVING had the honour last week of making the trial of a Musical experiment before the Society at *Gresham College*, it may be necessary to give a farther account of it; that the Theory of Musick, which is but little known in this Age, and the practice of it, which is arriv'd at a very great excellency, may be fixed upon the sure foundations of Mathematical certainty. The Propositions, upon which the Experiment was admitted, were: That Musick consist-
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ed in Proportions, and the more exact the Proportions, the better the Musick : That the Proportions offer'd were the same that the ancient *Grecians* us'd : That the Series of Notes and Half Notes was the same our Modern Musick aim'd at : which was there exhibited upon finger-boards calculated in Mathematical proportion. This was demonstrated upon a Viol, because the Strings were of the greatest length, and the proportions more easily discern'd ; but may be accommodated to any Instrument, by such mechanical contrivances as shall render those sounds, which the Musick requires.

To prove the foregoing Propositions, two Viols were Mathematically set out, with a particular Fret for each String, that every Stop might be in a perfect exactness : Upon these, a Sonata was perform'd by those two most eminent Violists, Mr *Frederick* and Mr *Christian Stefkins*, Servants to her Majesty ; whereby it appear'd, that the Theory was certain, since all the Stops were owned by them, to be perfect. And that they might be prov'd agreeable to what the best Ear and the best Hand performs in Modern practice, the famous *Italian*, Signior *Gasperini*, plaid another Sonata upon the Violin in Confort with them, wherein the most compleat Harmony was heard.

The full knowledge and proof of this Experiment may be found in the two following Schemes, wherein Musick is set forth, first Arithmetically and then Geometrically : The Mathematician may, by casting up the proportions, be satisfied, that the five sorts of Half-Notes here set down, do exactly constitute all those intervals, of which our Musick does consist. And afterwards he may see them set forth upon a Monochord, where the measure of all the Notes and Half-Notes comes exactly to the middle of the String. The Learned will find that these are the very proportions which the old *Greek* Authors have left us in their Writings, and the Practical Musician will testify, that these are the best Notes he ever heard.

Figure the 1st, containing the proportions set out Arithmetically.

		An Eighth		$\frac{1}{2}$
		A Seventh		$\frac{8}{15}$
		A Sixth		$\frac{3}{5}$
		A Fifth		$\frac{2}{3}$
		A Fourth		$\frac{3}{4}$
		A Greater Third		$\frac{4}{5}$
		Tone Major : Tone Minor		$\frac{8}{9}$
		Hemitone		$\frac{9}{10}$
		Tone Major		$\frac{15}{16}$
		Tone Minor		$\frac{16}{9}$
		Tone Major		$\frac{8}{9}$
		Tone Minor		$\frac{9}{10}$
		Tone Major		$\frac{17}{18}$
		Hemitone		$\frac{18}{17}$
		Tone Major		$\frac{16}{17}$
		Tone Minor		$\frac{17}{18}$
		Tone Major		$\frac{18}{19}$
		Tone Minor		$\frac{19}{18}$
		Tone Major		$\frac{17}{18}$
		Tone Minor		$\frac{18}{19}$
		Tone Major		$\frac{16}{17}$
		Tone Minor		$\frac{17}{18}$
		Tone Major		$\frac{15}{16}$
		Tone Minor		$\frac{16}{15}$
		Tone Major		$\frac{14}{15}$
		Tone Minor		$\frac{15}{14}$
		Tone Major		$\frac{13}{14}$
		Tone Minor		$\frac{14}{13}$
		Tone Major		$\frac{12}{13}$
		Tone Minor		$\frac{13}{12}$
		Tone Major		$\frac{11}{12}$
		Tone Minor		$\frac{12}{11}$
		Tone Major		$\frac{10}{11}$
		Tone Minor		$\frac{11}{10}$
		Tone Major		$\frac{9}{10}$
		Tone Minor		$\frac{10}{9}$
		Tone Major		$\frac{8}{9}$
		Tone Minor		$\frac{9}{8}$
		Tone Major		$\frac{7}{8}$
		Tone Minor		$\frac{8}{7}$
		Tone Major		$\frac{6}{7}$
		Tone Minor		$\frac{7}{6}$
		Tone Major		$\frac{5}{6}$
		Tone Minor		$\frac{6}{5}$
		Tone Major		$\frac{4}{5}$
		Tone Minor		$\frac{5}{4}$
		Tone Major		$\frac{3}{4}$
		Tone Minor		$\frac{4}{3}$
		Tone Major		$\frac{2}{3}$
		Tone Minor		$\frac{3}{2}$
		Tone Major		$\frac{1}{2}$
		Tone Minor		$\frac{2}{1}$

An Octave with a greater Third. $\frac{17}{18}$ C $\frac{16}{17}$ D $\frac{15}{16}$ E $\frac{14}{15}$ F $\frac{13}{14}$ G $\frac{12}{13}$ a $\frac{11}{12}$ b $\frac{10}{11}$ c $\frac{9}{10}$ d $\frac{8}{9}$ e $\frac{7}{8}$ f $\frac{6}{7}$ g $\frac{5}{6}$ h $\frac{4}{5}$ i $\frac{3}{4}$ k $\frac{2}{3}$ l $\frac{1}{2}$ m

$\frac{17}{18}$ A $\frac{16}{17}$ B $\frac{15}{16}$ C $\frac{14}{15}$ D $\frac{13}{14}$ E $\frac{12}{13}$ F $\frac{11}{12}$ G $\frac{10}{11}$ a $\frac{9}{10}$ b $\frac{8}{9}$ c $\frac{7}{8}$ d $\frac{6}{7}$ e $\frac{5}{6}$ f $\frac{4}{5}$ g $\frac{3}{4}$ h $\frac{2}{3}$ i $\frac{1}{2}$ k

The Explication of the first Figure.

Between the two lowest Lines, you have the Series of all the 12 half Notes in an Octave, from *Are* to *Alamire*, which added together make an Octave or exact Duple Proportion: The several parts also added together make all those intervals of which it is constituted. As for example, the two half Notes from *A* to $A \times \frac{47}{18}$, and from $A \times \frac{47}{18}$ to $B \frac{67}{12}$ make a Major Tone $\frac{8}{3}$; to which if an Hemitone from *B* to $C \frac{11}{2}$ be added, you have a lesser Third $\frac{1}{2}$.

In like manner between the two next lines, you have the series of all the 12 half Notes, in an Octave from *C fa ut* to *C sol fa ut*: the two first Tones added together make a greater Third: and so you may add a Tone or Hemitone till you arrive at every interval in the Octave, which so call'd because eight sounds are required for expressing those seven gradual steps whereby we commonly ascend to it.

It may be also observ'd, that the proportions falling upon the same Notes in two Keys, one finger-board will be sufficient for both.

TIS acknowledg'd by all that are acquainted either with Speculative or Practical Musick, that every interval is divided into two parts, whereof one is greater than the other: An Eighth $\frac{1}{8}$ into a Fifth $\frac{3}{4}$ and a Fourth $\frac{3}{8}$. Again, a Fifth $\frac{3}{4}$ into a greater Third $\frac{4}{3}$ and a lesser Third $\frac{1}{2}$. Thus also a greater Third $\frac{4}{3}$ must be divided into a Tone Major $\frac{8}{3}$ and a Tone Minor $\frac{9}{16}$. The Lesser Third (to comply with the practice of Musick) is rather compounded of, than divided into a Tone Major $\frac{8}{3}$ and an Hemitone, which is its complement, $\frac{11}{12}$.

Three Tones Major, two Tones Minor, and two of the foresaid Hemitones, placed in the order found in the Scheme, exactly constitute the practical Octave; which is so call'd because it consists of eight sounds, that contain the seven gradual intervals. But it is also necessary

to set down the Divisions of the whole Tones, which are the true Chromatick half Notes, because there is great use of them in Practical Musick.

To make all our whole Notes, and all our half Notes of an equal size, by falsifying the proportions, and bearing with their imperfections, as the common practice is, may be allow'd by such Ears as are vitiated by long custom: But it certainly deprives us of that satisfactory pleasure which arises from the exactness of sonorous numbers; which we should enjoy, if all the Notes were truly given according to the Proportions here assign'd.

It is very easie to satisfy our selves in the Arithmetical Scheme, by those operations which *Gassendus* has set down in his Manuduction to the Theory of Musick, Tom. V. pag. 635. As for example, his rule for Addition is, That two Proportions being given, if the Greater number of one be multiplied by the Greater number of the other, and the Lesser by the Lesser, the two numbers produc'd exhibit the compounded Proportions. Thus take a Practical Fifth $\frac{3}{2}$ and a Practical Fourth $\frac{4}{3}$ for the two Proportions given, multiply 3 by 4 and you have 12, then multiply 2 by 3 and you have 6: which compounded proportion of 12 to 6 makes the Practical Octave $\frac{2}{1}$.

Thus, according to his Arithmetical operations of Addition, Substraction, Multiplication or Continuation, and Division, is our whole System proved, which for the more easie application to Practical Musick, shall be also set forth Geometrically upon the 6 strings of a Viol. See *Figure the 2d.*

This Mathematical fixing of the Frets enables every Practitioner, who stops close to them, to give the Proportions of the Notes in a greater exactness, than can be done upon the Bass-Violin or Violin itself: since they may be set forth more perfectly by a pair of Compasses dividing a line, than the nicest Ear can direct.

Though the Frets for the several Strings do not stand in a straight line, and the places are also shifted in different Keys, yet the Ear naturally directs the Fingers to them: insomuch that those persons, who have all their lives time been accusom'd to stop upon Frets that go quite cross the Finger-boards of their Instruments, do with very little practice fall right upon these. Such is the power of a Musical Genius, as may be undeniably proved by those that play upon the Violin; who, when they change the Key, fall upon the right Stops, tho' they have no visible direction where to stop, nor time to alter, by the Ear, the Note they first pitched upon.

By this Standard of Regular Proportions may the Voice be formed to sing the purest Notes; they are all the same in Vocal and Instrumental Musick; if then the Instrument which governs the Voice be perfect, the Ear will of necessity bring it to perfection. It is a great pity that a good natural Voice should be taught to sing out of Tune, as it must do, if it be guided by an imperfect Instrument; and this may be the reason why so few attain to that melody, which is so much valued; but since we now know wherein perfection lies, a constant practice will come to the attainment of it. The dividing Wholes into Chromatick Hemitones is very necessary, but very difficult for the Voice to be broken to: If it learns from an Instrument whose whole Notes and whose half Notes are supposed to be equal, the sound must needs be very uncertain and unharmonical; whereas the proportions truly fixed, wou'd bring it to a perfection in the nicest and most charming part of Musick.

The Chromatick Hemitones are the smallest Intervals our Modern Musick aims at, tho' the Ancients had their Enharmonic quarter Notes, which they esteem'd their greatest excellency: These may also in time be recover'd, since we know their proportions; for as the Diatonick Tone is divided into Chromatick Hemitones, so after the same manner may the Chromatick Hemitones be divided into those least Enharmonic Intervals, which were ever made use of.

But if we go no further, yet this Experiment demonstrates the true Theory of Musick, and brings the practice of it to the greatest perfection.

V. *Part of a Letter from the late Sir Philip Skippon, Kt, to the late Reverend Mr John Ray, concerning the Bones of a Humane Foetus voided thro' an Impostume in the Groin. Communicated to the Publisher, by Mr Samuel Dale.*

London, January 22. 1662.

Yesterday in the Afternoon my Cozen *Horsnel* and my self visited a Woman 66 years old, in *Drury-lane*, who had a Child con'ined in her *Uterus* about 28 years ago; She bore two Children after this, one lived 11 years, and the other 6: About 8 years ago an *Impostume* broke out in the Right *Jagres*, and then several Bones of a Dead Child were expell'd, (some of them I have by me.) She hath a great Swelling now in that *Groin*, where she feels somewhat very hard, which she suspects are Bones. I took a particular account of all Circumstances; but this is proportionable to the narrow extent of a Letter, and indeed is the substance of all.

The 2d Figure, wherein the Proportions of Musick are described Geometrical-ly.

The Explication.

These six lines represent the six Strings of the Viol in the common Tuning.

The sounding part of each String from the Nut to the Bridge is suppos'd to be 30 inches long; the two middle Strings C and E are drawn out to 15 inches, the half of the whole.

'Tis easie to measure every interval with a pair of Compasses. Suppose you are to take the 20th part of the String G; 'tis an inch and a half for the first half Note: If you take the whole Note from G to A, 'tis the tenth part, and must be 3 inches.

After these are taken away, your String will be but 27 inches long, so that if you advance one Note, or a Major Tone further, you must take a 9th part of it, which will be 3 inches more, whereby you arrive at a greater Third, being the fifth part of the whole String. Thus the series of all the Notes may be demonstrated.

All the Strings are Unison at the stops where the tuning requires: So that though the Proportions be carried on as far as the frets allow, yet the string is open the same with the stop of that string to which it is tuned; and accordingly the series of the Notes proceeds as if they were all upon a Monochord.

Dd	G	C	E	a	d
$\text{Dd} \frac{19}{20}$	$\text{G} \frac{19}{20}$	$\text{C} \frac{17}{18}$	$\text{F} \frac{15}{16}$	$\text{a} \frac{17}{18}$	$\text{d} \frac{19}{20}$
$\text{Ee} \frac{18}{19}$	$\text{A} \frac{18}{19}$	$\text{D} \frac{16}{17}$	$\text{F} \frac{17}{18}$	$\text{b} \frac{16}{17}$	$\text{e} \frac{18}{19}$
$\text{Ff} \frac{15}{16}$	$\text{A} \frac{17}{18}$	$\text{D} \frac{19}{20}$	$\text{g} \frac{16}{17}$	$\text{c} \frac{15}{16}$	$\text{f} \frac{15}{16}$
$\text{Ff} \frac{17}{18}$	$\text{B} \frac{16}{17}$	$\text{E} \frac{18}{19}$	$\text{g} \frac{19}{20}$	$\text{c} \frac{17}{18}$	$\text{f} \frac{17}{18}$
$\text{G} \frac{16}{17}$	$\text{C} \frac{15}{16}$	$\text{F} \frac{15}{16}$	$\text{a} \frac{18}{19}$	$\text{d} \frac{16}{17}$	$\text{g} \frac{16}{17}$
$\text{G} \frac{19}{20}$	$\text{C} \frac{17}{18}$	$\text{F} \frac{17}{18}$	$\text{a} \frac{17}{18}$	$\text{d} \frac{19}{20}$	$\text{g} \frac{19}{20}$
$\text{A} \frac{18}{19}$	$\text{D} \frac{16}{17}$	$\text{g} \frac{16}{17}$	$\text{b} \frac{16}{17}$	$\text{e} \frac{18}{19}$	$\text{a} \frac{18}{19}$

d

$$d \frac{19}{20}$$

$$e \frac{189}{1910}$$

$$f \frac{15}{16}$$

$$f \frac{17}{18}$$

$$g \frac{16}{17} \frac{3}{4}$$

$$g \frac{19}{20}$$

$$a \frac{18}{19}$$

$$\begin{array}{cccccc}
 \text{A} & \frac{18}{19} & & \text{D} & \frac{16}{17} \frac{2}{3} & & \text{g} & \frac{16}{17} \frac{2}{3} & & \text{b} & \frac{16}{17} \frac{2}{3} & & \text{e} & \frac{18}{19} \frac{2}{3} & & \text{a} & \frac{18}{19}
 \end{array}$$

This Calculation serves but for two Keys A and C, which are called Natural, because they have no essential flats or sharps.

But because the Composer begins upon any Key, and the series of Notes must take its *terminus à quo* from thence; the Instrument-maker can provide such movable Finger-boards as will serve exactly for every Key. They are taken out and put in upon the Neck of the Viol, with as much ease, as you pull out and thrust in the Drawer of a Table.

Three, or at most five of them will be sufficient to accommodate all the Keys that are made use of.

$\frac{18}{19}$	$\frac{16}{17} \frac{2}{3}$	$\frac{16}{17} \frac{2}{3}$	$\frac{16}{17} \frac{2}{3}$	$\frac{18}{19} \frac{2}{3}$	$\frac{18}{19}$
$\frac{19}{20}$	$\frac{18}{19} \frac{3}{5}$	$\frac{17}{18}$	$\frac{17}{18}$	$\frac{15}{16} \frac{15}{24}$	$\frac{17}{18}$
$\frac{17}{18}$	$\frac{16}{17} \frac{5}{9}$	$\frac{16}{17} \frac{5}{9}$	$\frac{16}{17} \frac{5}{9}$	$\frac{17}{18}$	$\frac{17}{18}$
$\frac{16}{17} \frac{15}{20}$	$\frac{16}{17} \frac{15}{20}$	$\frac{16}{17} \frac{15}{20}$	$\frac{16}{17} \frac{15}{20}$	$\frac{16}{17} \frac{15}{20}$	$\frac{16}{17} \frac{15}{20}$
$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$

a 18|
19|

The 2d Figure, wherein the Proportions of Musick are described Geometrically.

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	$\text{E c} \frac{18}{19} \frac{9}{10}$	$\text{A} \frac{18}{19} \frac{9}{10}$	$\text{D} \frac{16}{17} \frac{8}{9}$	$\text{F} \frac{17}{18}$	$\text{b} \frac{16}{17}$	$\text{c} \frac{18}{19} \frac{9}{10}$
	$\text{F f} \frac{15}{16}$	$\text{A} \frac{17}{18}$	$\text{D} \frac{19}{20}$	$\text{g} \frac{16}{17} \frac{5}{6}$	$\text{c} \frac{15}{16} \frac{5}{6}$	$\text{f} \frac{15}{16}$
	$\text{F f} \frac{17}{18}$	$\text{B} \frac{16}{17} \frac{4}{5}$	$\text{E} \frac{18}{19} \frac{4}{5}$	$\text{g} \frac{19}{20}$	$\text{c} \frac{17}{18}$	$\text{f} \frac{17}{18}$
	$\text{G} \frac{16}{17} \frac{3}{4}$	$\text{C} \frac{15}{16} \frac{3}{4}$	$\text{F} \frac{15}{16} \frac{3}{4}$	$\text{a} \frac{18}{19} \frac{3}{4}$	$\text{d} \frac{16}{17}$	$\text{g} \frac{16}{17} \frac{3}{4}$
	$\text{G} \frac{19}{20}$	$\text{C} \frac{17}{18}$	$\text{F} \frac{17}{18}$	$\text{a} \frac{17}{18}$	$\text{d} \frac{19}{20}$	$\text{g} \frac{19}{20}$
	$\text{A} \frac{18}{19}$	$\text{D} \frac{16}{17} \frac{2}{3}$	$\text{g} \frac{16}{17} \frac{2}{3}$	$\text{b} \frac{16}{17} \frac{2}{3}$	$\text{c} \frac{18}{19} \frac{2}{3}$	$\text{a} \frac{18}{19}$
			$\text{g} \frac{19}{20}$	$\text{c} \frac{15}{16} \frac{15}{24}$		
			$\text{a} \frac{18}{19} \frac{3}{5}$	$\text{c} \frac{17}{18}$		
			$\text{a} \frac{17}{18}$	$\text{d} \frac{16}{17} \frac{5}{9}$		
			$\frac{16}{17} \frac{8}{15} \text{d}$	$\frac{19}{20}$		
			$\frac{15}{16}$	$\text{c} \frac{18}{19}$		