The analysis multiplication coefficient of P-N junction

by

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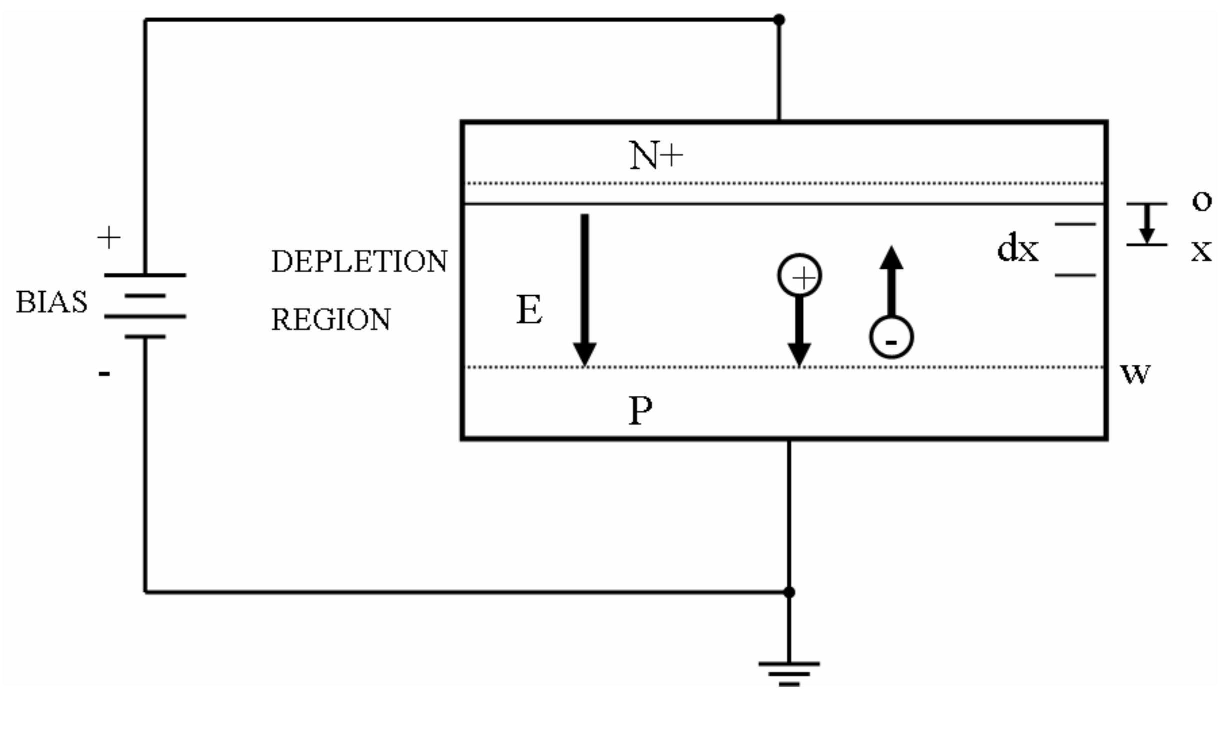
1. Analysis multiplication coefficient (增值係數) of P-N junction

The total number of electron-hole pairs [1] created in the depletion region due to a single electron-hole pair initially generated at a distance x from the junction is given by：

$$M(x) = 1 + \int_0^x \alpha_n M(x)\, dx + \int_x^w \alpha_p M(x)\, dx \qquad \text{Eq.(1)}$$

In order to perform the avalanche breakdown analysis, assume that an electron-hole pair is generated at a distance x from the junction as illustrated in the next figure.



M(x) is a total number of electron-hole pairs treated in the depletion region. The solution of Eq.(1) is showed as follows：

$$M(x)-1=\int_0^x \alpha_n M(x)\,dx+\int_x^w \alpha_p M(x)\,dx$$

$$M(x)-1=M(x)\left[\int_0^x \alpha_n\ dx+\int_x^w \alpha_p\ dx\right]$$

$$\frac{M(x)}{M(x)}-\frac{1}{M(x)}=\frac{M(x)\left[\int_0^x \alpha_n\ dx+\int_x^w \alpha_p\ dx\right]}{M(x)}$$

$$1-\frac{1}{M(x)}=\int_0^x \alpha_n\ dx+\int_x^w \alpha_p\ dx=\int_0^w \alpha\ \ dx$$

$$M(x)=M(0)\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right] \qquad \text{Eq.(2)}$$

$$M(0)=\left\{1-\int_0^w \alpha_p \exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]dx\right\}^{-1} \qquad \text{Eq.(3)}$$

M(0) is the total number of electron-hole pairs at the edge of the depletion region.

By substituting Eq.(3) into Eq.(2), a solution of Eq.(4) is showed

as followsS：

$$M(x)=M(0)\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]$$

$$M(x)=\left\{1-\int_0^w\alpha_p\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]dx\right\}^{-1}\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]$$

$$M(x)=\frac{1}{1-\int_0^w\alpha_p\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]dx}\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]$$

$$M(x)=\frac{\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]}{1-\int_0^w\alpha_p\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]dx} \qquad \text{Eq.(4)}$$

To solution $\int_0^x\left(\alpha_n-\alpha_p\right)dx$, if we define $\alpha_n=\alpha_p$

then $\int_0^x\left(\alpha_n-\alpha_p\right)dx=\int_0^x 0\,dx=0$ and $\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]=\exp 0=1$

therefore $M(x)=\dfrac{\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]}{1-\int_0^w\alpha_p\exp\left[\int_0^x\left(\alpha_n-\alpha_p\right)dx\right]dx}=\dfrac{1}{1-1}=\text{infinity}$

Using the solution of Eq.(1).

$$1-\frac{1}{M(x)}=\int_0^x \alpha_n\ dx+\int_x^w \alpha_p\ dx=\int_0^w \alpha\ \ dx$$

If we set the M(x) tending to infinity, then $\frac{1}{M(x)}\cong 0$

Therefore, we get Eq.(5) and Eq.(6).

$$\int_0^w \alpha_p \exp\left[\int_0^x \left(\alpha_n-\alpha_p\right)\, dx\right]dx=\int_0^w \alpha_p dx=1 \qquad \text{Eq.(5)}$$

$$\int_0^w \alpha_{\mathrm{p}}\ dx=1 \qquad \text{Eq.(6)}$$

The empirical expression of the multiplication coefficient [2] can be calculated by using ：

$$M(V)=\frac{1}{1-\left(\frac{\mathrm{V}}{\mathrm{V_B}}\right)^n}$$

The multiplication coefficient [1]can be calculated by using：

$$M_n=\frac{1}{1-\left(\frac{\mathrm{V}}{\mathrm{V_A}}\right)^4} \qquad \text{Eq.(7)}$$

for the case of a $\mathrm{N^+}$/P diode, and

$$M_P=\frac{1}{1-\left(\frac{\mathrm{V}}{\mathrm{V_A}}\right)^6} \qquad \text{Eq.(8)}$$

for the case of a P^{+}/N diode.

In these equations, V_A is the avalanche breakdown voltage and V is the applied reverse bias supported by the junction. These expressions for the multiplication coefficient are useful for analysis of the blocking voltage capability of devices containing internal open base transistors. Some examples of these devices are thyristors and IGBTs.

2. References：

[1]. B.JAYANT BALIGA , Power semiconductor devices , pp.69-70 , PWS Publishing Company, 1996.

[2]. S. L. Miller, “Ionization rates for holes and electrons in silicon,” Phys. Rev., vol. 105, number 4 , pp.1246-1249, Bell Telephone Laboratories , Murray Hill , New Jersey , February , 1957.

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