A Letter from Mr. John Collins *to the Reverend and Learned* Dr. John *Wallis* Savilian *Professor of* Geometry *in the* University *of* Oxford, *giving his thoughts about some* Defects *in* Algebra.

TO describe the *Locus* of a cubick Æquation.

A Cardanick Æquation convenient for the purpose, (*viz.* such as shall have the dioristick limits rational) must have the Coefficient of the roots to be the triple of a square number such is $a^3 - 48a = N$.

Assume a rank of roots in Arithmetical progression, and raise resolvends thereto $a^3 - 48a = N$ or resolvends.

Such are

R	N
1	1------48=47
2	8- ---96=88
3	27---144= 17
4	64 --192=128
5	125--240=115
6	216- -288=72
7	343- -336=+7
8	512-384=+128
9	729-432=+297

Draw a *Base* line and a perpendicular thereto, and from O in the *Base* line prick the negative *resolvends* downwards, and the affirmative ones upwards, and raise their roots upon them as ordinates, a *Curve* passing through the same is one Moity of the *Curve* or *Locus* on the right hand for affirmative roots and the other moity on the left hand is described in the same manner by assuming a rank of negative roots, and raising resolvends thereunto. The *Curve Fig.* 4. may give a resemblance of the thing.

And 16 the third part of the Coefficient of the roots cubed is equal to the square of 64 half the *resolvend*, or *dioristick* limit.

Which in composing of *Cardans* canon is always substracted from the square of half the *absolute*, as in the example following.

If I were to find the root belonging to the *resolvend* 297

The square of half thereof is ———— $22052\frac{1}{4}$

The square of 64 half the dioristick Limit — 4096

The difference is $17956\frac{1}{4}$

And the rule is $148\frac{1}{2} + \sqrt{17956\frac{1}{4}}$.

$148\frac{1}{2} + \sqrt{17956\frac{1}{4}}$.

That is in a quadratick Æquation, if 297 were the sum of the two roots and 64 the root of the *Rectangle*: then if from the square of half the sum, the rectangle be subducted, there remains the square of half the difference of the

Z roots,

roots, and giving them an universſal *Cube* root, it is.

$\sqrt[3]{148\frac{1}{2}+\sqrt{17956\frac{1}{4}}}+\sqrt[3]{148\frac{1}{2}--\sqrt{17856\frac{1}{4}}}$ 9 to 9 the root ſought.

In the former Scheme *Q. B.* and *Q P.* may ſignifie the roots of *Cardans Binomials* that run infinitely upward, and terminate at *Q* as is mentioned in *Section the 5th.* And if they can be continued downwards, probably they will terminate at *O.* and *R.* The *touch* line in Section 2*d.* may here be repreſented by the line 9 *S.* and the *Chord* line between 9 and 8 by *T.* from whence tis plain that any root between 9 and 8 found near, may be limited by Approximations of *Majus* and *Minus*.

As to CARDANS RULES

1 The deſcription of the *Locus* is before handled.

2 The *touch* line affording approaches by an Æquation derived out of that propoſed is before deſcribed, and the method of drawing is mentioned by Dr. *Wallis* in the Tranſactions.

3. The Limits are of two kinds (*viz.*) either the *Baſe* limits when the reſolvend is *O.* and the æquation falls a degree lower: or the *dioriſtick* limits whereby a pair of roots gain or looſe their poſſibility, as is before deſcribed.

4 *Cardans* canons are but the ſum of the roots of a ſolid quadratick æquation ariſing out of half the *dioriſtick* limit as the v of the rectangle, and the reſolvend as the ſumm

5 If the roots of thoſe *binomials* are ſeparately prickt down as ordinates on their *reſolvends*, they beget *curves* infinitely continued upward, and meeting in a point biſecting the root that is equal to a pair of equal roots, when the æquation is juſt limited, or dioriſtick as aforeſaid in the Figure at *Q.*

6 If theſe *binomials* are prickt down as ordinates to their reſolvends, Mr. *Newton* upon ſudden thoughts, ſuppoſed they may deſcribe both ſides of an *Hyperbole*.

7 If ſo they cannot be continued downwards, but by the method in *Mercators Logarithmotechnia*: moſt numbers of a conſtant habitude belonging to any arithmetical progreſſion, may by aid of the differences, and a Table of Figurative numbers (yea, and I add otherwiſe) be continued upward or downward, and if theſe run downward they will probably end both in the *baſe* limits at O and *R*.

8 If theſe binomial *curves* be continued downward, and ſeparately found ſhould always added make the root of a cubick Æquation capable of 3 roots: then *Cardans* impoſſible or negative roots are prov'd poſſible, and we only in ignorance how to extract them.

9 Aſſume any root within the limits of 3 poſſible roots, and raiſe a reſolvend to it, and when you have done, by *Cardan's* Rules improved, you may find that root, and, with a little va-

rying

rying the ſame,both the other roots (as in the Poſtſcript): for every number or magnitude capable of a *cube* root, is capable of two more, ſee *Section the* 11*th*. following.

10 If the roots in the former Section, be aſſumed in Arithmetical progreſſion, and the æquation with its ſeveral Reſolvends be depreſſed, there will come out a regular Series of Quadratick Æquations, whence an eaſie method will riſe of writing down ſuch ranks as multiplied by an Arithmetical progreſſion, ſhall always beget the ſame cubick æquation, the Reſolvend only varying.

11 Let the roots of this ſeries of quadraticks be found as uſual in binomials, let theſe binomials be cubed,and then let it be obſerved, whether the reſults are conſtant portions of the ſquare of the Reſolvend and of the dioriſtick limit: and if ſo, *Cardans* Rules will have their defect ſupplyed.

12 In breaking a biquadratick, 'tis aſſerted that by leaving the Reſolvend at liberty, it may be infinitely and rationally done, without the Aid of the ſeparating cubick Æquation.

13 But ſuppoſing ſuch ſeparating cubick in ſtore, of which *Bartholinus* in his dioriſtick hath given us great furniture in *Species*, why may not ſeveral roots of that æquation be aſſumed rational, and thence the biquadratick broken into as many pairs of quadratick æquations?

14 May not from hence a method ariſe of writing down 2 Series of quadraticks that multiplied together ſhall always beget the ſame biquadratick Nomes, the Reſolvend only varying?and hence the *Locus* of the æquation is eaſily deſcribed.

15 Here again (as in the 11) if the binomial roots of theſe quadraticks be ſquaredly ſquared, and thoſe reſults are conſtant portions of the cube of the Reſolvend, and the dioriſtick limit; it will be certain there may be general ſurd Canons for æquations of the 4*th*. dimenſion, and *Monſieur Cluverius* (now at *London*) poſitively aſſerts he hath a general method to obtain them for all Dimenſions.

16 As *Cardans* are ſurd canons deriv'd from the Reſolvend, and dioriſtick limit, ſo it were worthy diſquiſition, whether other ſurd Canons (of which many are fitted to particular caſes by your ſelf, *Leibnitz* and others) do not ariſe out of the limits of thoſe particular caſes and æquations, and whether the glimpſe of a general Method might thence be deriv'd for all other æquations, though encumbred with negative quantities? which Mr. *Gregory*, a little before his death, ſaid he had attained.

17 The Learned Dr. *Pell* hath often aſſerted that after the Limits of an æquation are once obtain'd, then it is ea-

 ſy

ſy to find all the roots to any Reſolvend offer'd.

Now for inſtance (according to *Huddens* method) in a biquadratick æquation, you muſt multiply all the terms beginning with the higheſt, and ſo in order by 4, 3, 2, 1. and the laſt term or Reſolvend by 0. whereby it is deſtroyed, and you come to a cubick Æquation, the ſame as *Harriot* uſes to take away the penultimtae Term of the biquadratick, the roots whereof being found, and as roots having Reſolvends raiſed thereto in the biquadratick Æquation, are the dioriſtick Limits thereof.

18 And if this eaſy method were known, we may come down the Ladder to the bottom, and fall into irrational quantities, and aſcend again. Againſt which aſſymetry, an Æquation might be aſſumed low, as a rational quadratick, and thence a cubick Æquation formed, whoſe limits ſhould be found by aid of the quadratic Æquation, and out of that cubick a Biquadratick Æquation, whoſe limits ſhould be found by the aid of that cubick Æquation, *&c.*

19 Æquations may be ſo continued of two Nomes, that both the dioriſtick and baſe limits, ſhould be rational, then ſuppoſing ſuch Æquation incomplete, the increaſing or diminiſhing the roots, fills up all the vacant places.

Q. Whether or in what place one or both ſorts of Limits ſhall looſe their rationality? And what is the nature of the roots thus drawn? in this I think you have already determined in divers of your ſurd Canons.

20 What Dr. *Pells* method mention'd in *Section* 17 ſhould be I cannot gueſs, unleſs it be either.

To make ſurd Canons. Or good approaches.

Or that raiſing Reſolvends out of aſſumed roots, thoſe ſhould make a ſtore from whence to derive the roots of the Reſolvend offered.

Or making quadratick Æquations out of the dioriſtick and baſe limits, thoſe might be interpoled, by aid of a Table of figurate numbers, or otherwiſe thereby, as in quadratick Æquations to attain two roots of a biquadratick at once. which if performed the greateſt difficulties are overcome, and why ſhould not this ſeem probable, in regard the *Curve* or *Locus*, be the Æquation what it will, makes indented porches.

21 Suppoſe I ſhould propound two cubick or biquadratick Æquations, in both whereof all the ſigns are +. It is propounded out of theſe two, to derive a third Æquation, whoſe root ſhall be the Summ, Difference, or Rectangle of the Roots of the two Æquations propounded. This Mr *Gregory* a little before his death writ word he had obtained and in the following Series for finding the Moity of a Hyperbolick Logarithm I ſuppoſe made uſe of. From

From a number propos'd ſubſtract an Unit, let that be Numerator, and to it add an Unit, let that be Denominator, and call that fraction N.

Then $\frac{N}{1} + \frac{N^3}{3} + \frac{N^5}{5} + \frac{N^7}{7} + \frac{N^9}{9} + \frac{N^{11}}{11} + \frac{N^{13}}{13}$, &c. is Equal to half the Hyperbolick Logarithm ſought.

EXAMPLE in the Number 2.

		N	
The Fraction is $\frac{1}{3}$	1,	3333333 =	3333333
	3,	370370 =	123456
	5,	41152 =	8230
The Rank N is eaſily	7,	4572 =	653
made by dividing ev'ry	9,	508 =	56
preceding number by 9.	11,	56 =	5
	13,	6 =	0

3465733
2

6931466 which is The Hyperbolick Logarithm of 2 ſought.

I want time to conſider the premiſes, but hope you will, (in regard you ſeem to think it ſtrange that any difficulties ſhould remain about Cubicks that are not preſently reſolved) your conſiderations wherein will be very acceptable and worthy publick view.

Other Series in Print of *Mercators*, &c. diſpatch not as this doth neither thereby can the Logarithm of 2 be eaſily made, but by making the Logarithms of ſuch mixt numbers or fractions that multiplied together make the reſult 2 juſt as $2 \times 1\frac{1}{2} = 3$; whence having and finding that of $1\frac{1}{2}$, you preſently have the Logarithm of 3.

22 A *Cardanick* Æquation that is a Cubick one wanting the ſecond term, may be multiplied or divided by a rank of continual proportionals, ſo as to render the coefficient of the roots canonick, that is, to make it the ſame with the Æquations of the Table, that find the Sine, Tangent, or Secant of the third part of that arch to which any Sine, Tangent, or Secant is propounded, and ſo finding the roots in the tables, thoſe ſought are thence obtained by Multiplication or Diviſion. Yea, and the coefficient of the roots may in like manner be rendred an Unit, and then the Reſolvends ſought in a table of the ſums or differences of the Cubes of numbers and their roots, ſhall help you to ſuch roots, as multiplied or divided as aforeſaid ſhall be the true ones ſought.

23 It is an enquiry worth conſideration, whether two of the roots of a biquadratick may not be kept conſtant, and

the

the rest be encreased or diminished, either Arithmetically, or by multiplication and division in a known *Ratio*? certainly regular Progressions will arise, though as yet, we cannot encrease the true roots of an Æquation without as much diminishing the Negative nor can we multiply or divide the roots without we alter all of them. and consequently cannot reduce coefficients to such habitudes as are desirable.

24 It is a pleasant concinnity out of a root to raise a Resolvend without raising any of the Powers of the root, and at the same time without a thorough binomial Division to depress the Æquation a degree lower.

EXAMPLE.

Let the Æquation be $a^4 + 10a^3 + 6a^2 + 20a = 1072$.

Let the root be 4, the resolvend is thus raised by adding the coefficients as you go, and multiplying by the root, thus $+4 + 10 = 14 \times 4 = 56 + 6 = 62 \times 4 = 248 + 20 = 268 \times 4 = 1072$. with the same work the Æquation may be depressed without Division.

EXAMPLE.

Let the Æquation be as before, and place the root with the former products underneath respectively, the summ is the depressed Æquation.

$$a^4 + 10a^3 + 6a^2 + 20a - 1072 = 0$$
$$+4 \qquad +56 \quad +248 \quad +1072$$

The sum $a^4 + 14a^3 + 62a^2 + 268a = 0$. that is divided by a.

$a^3 + 14a^2 + 62a + 268 = 0$. which is the under Æquation sought found without Division

25 It's conceived that all Æquations may be so regulated as to be reduced to as many Arithmetical Progressions of multipliers in whole numbers, as the Æquation hath dimensions, whereof one of the progressions shall be a Series of roots: hence the raising Resolvends by tentative work is rendred Logarithmetical For Example write down any 3 arithmetical Progressions, *viz.*

R H

$1 \times 6 \times 3 = 18$ I say the Rank H are the Resolvends or
$2 \times 7 \times 5 = 70$ *Homogenea Comparationis* of a cubick Æquation,
$3 \times 8 \times 7 = 168$ whose roots are the Rank R. This cubick Æ-
$4 \times 9 \times 9 = 324$ quation is easily attained out of the differen-
$5 \times 11 \times 10 = 550$ ces of the Rank R. for out of the Rank R in

any Æquation proposed raise separately the respective powers (with regard to their Coefficients) and out of the three a^3, a^2, a ranks so raised compose their respective differences, and they shall be the same with the differences of the rank of Resolvends or *Homogenea Comparationis* here noted by H.

If

If such Æquation be encombred with fractions they are all removed at once, by multiplying most conveniently, by the least number that is divisible by the Denominators of such fractions, hence also the infinite Series before mentioned (and others) are reduceable to Logarithms.

26 Where Æquations have all their terms affected with the same sign both + or −) *Mr. Newton and Mr. Gregory* deceased have affirmed they are all reduceable to some pure high power, which is of singular use in the infinite Series. And a Learned *German* where this cannot be done, hath asserted that they may be reduced to a higher power, with a variable Coefficient. which is the root sought with a common addend or subducend. And even this would render an easy tentative Logarithmetical way for attaining the root.

27 If but one Root of an Æquation can be found at a time, then questionless a better Method is not yet attained, then what is mentioned in the printed proposal about Printing Mr. *Bakers* Treatises therein mentioned.

28 Lastly, as to Constructions for Æquations, the following Probleme seems to be universal.

Any two analytick *Curves* (*viz.*) such as wherein the Habitude between the Base and Ordinate may be expressed by an Æquation being given in Magnitude and Position, and from the points of their intersection ordinates let fall to the *Axis* of either figure, or upon parallels to the said *Axis*, the inquiry is of what Æquation those ordinates are the roots? *Dr. Barrow* liked the proposition as well grounded, and left a discourse about doing it in the conick Sections, in which there are 3 cases, either the axes are parallel or being produced concur, beyond the vertexes of the figures without; or otherwise intersect within the figures. *Mr. Gregory* entred on the same contemplation, but death deprived us of the benefit of his thoughts.

Of Analytick (*alias* Geometrick) *Curves* there are innumerable sorts, of which I shall mention one or two kinds,

Between an Arithmetical Progression and its squares, or between its squares and its cubes, or its cubes and Biquadratics, there may be interpoled as many Arithmetical or Geometrical means as you please: and thence *Loca* or *Curves* deriv'd, which some call *Paraboloids* or *Parabolasters*, see *Gregories Geometriæ pars universalis* printed in *Italy* in Quarto.

Postscript explaining Section the 9th.

After you have obtained the Cube roots of *Cardans* Binomials, according to *Van Schoten*, in *Des Cartes* or *Kersey*, if you change the Sines of the rational parts of those roots, as also the

the Sines of the Radical Parts, and multiply those parts by 3, the results are also roots of the cubick Æquation first propofed.

EXAMPLE.

$aaa - 21a - 20 = 0$

The cube Roots of the Binomials are $+2\frac{1}{2}+\sqrt{-\frac{3}{4}}$
$+2\frac{1}{2}-\sqrt{-\frac{3}{4}}$

Their summ is the Root fought $= +5$

And the other two Roots are $-2\frac{1}{2}+\sqrt{2\frac{1}{4}}$
$-2\frac{1}{2}-\sqrt{2\frac{1}{4}}$

Also in this Æquation $a^3 - 60a - 32 = 0$

The Binomial Roots are $+4+\sqrt{-4}$
$+4-\sqrt{-4}$

Hence the Root fought is $+8$

And the other two roots are $-4+\sqrt{+12}$
$-4-\sqrt{+12}$.

ADVERTISEMENT.

These papers were fent by Mr. *Collins* to Dr. *Wallis* in a Letter of 3 *Octob.* 1682, (with this Character, *I have sent you herewith my thoughts about some defects in Algebra:*) and are a Copy of what he had written to some other (but I know not whom) to whom he speaks all along in the second person, whereas of others he speaks in the third person. And he did intend (had he lived longer) to perfect it further; by omitting some things which (though here he notes as defects) he found after to be done already, and supplying some others. But he lived not to perfect it, and therefore (that it be not lost) we here give it as we found it.

OXFORD,

Printed at the *THEATER*, and are to be sold by *Moses Pitt*, at the *Angel*, and *Samuel Smith*, at the *Princes Arms* in St. *Paul's Church-yard LONDON.* 1684.

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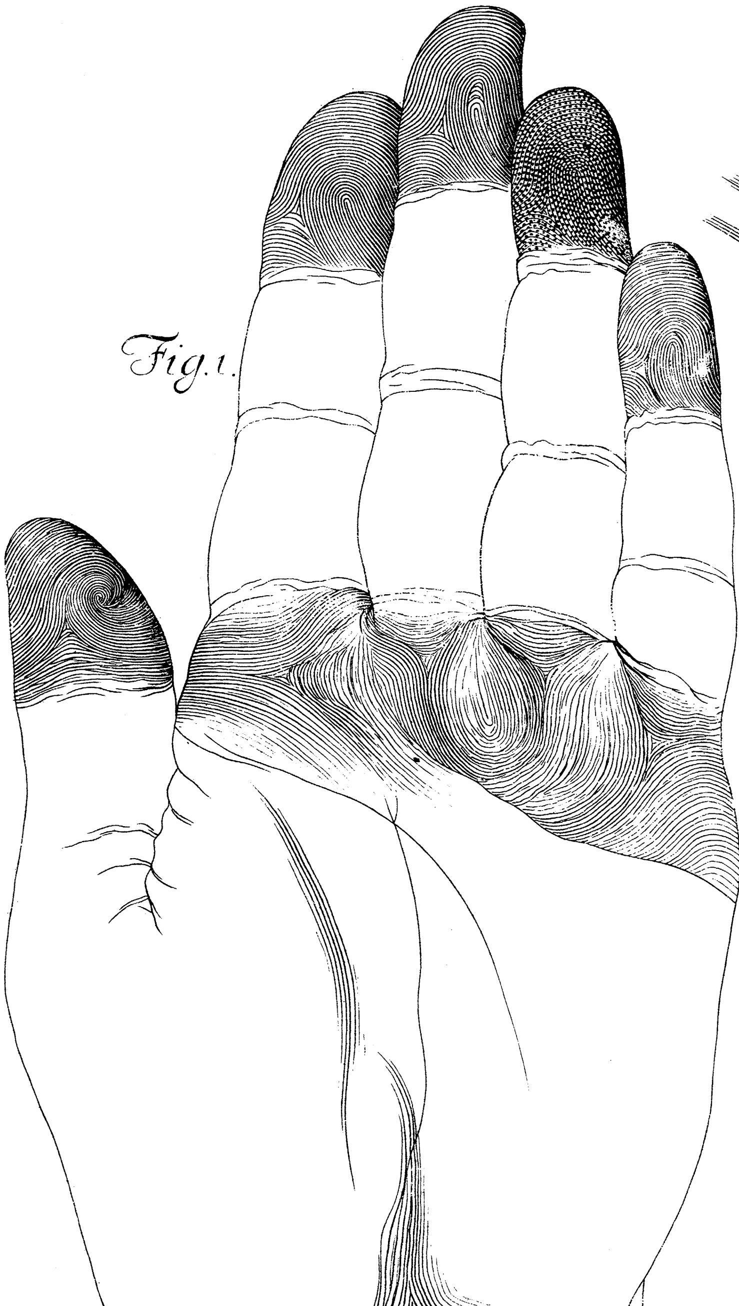
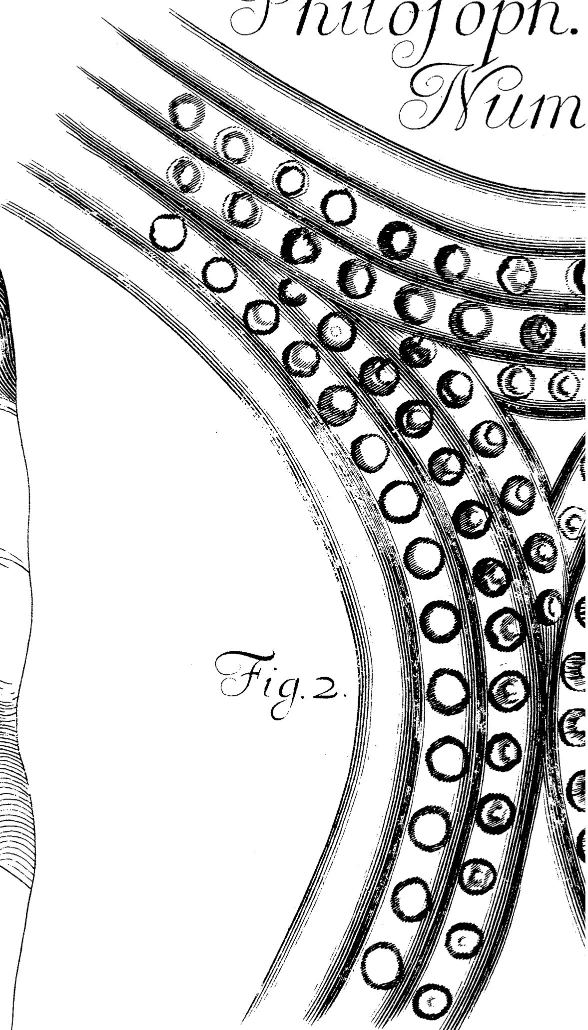


Fig.1.



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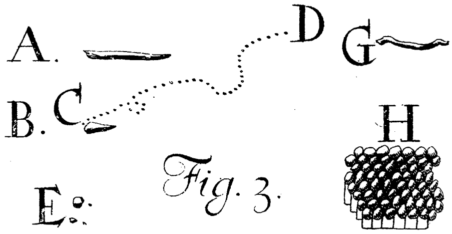

Fig.3.

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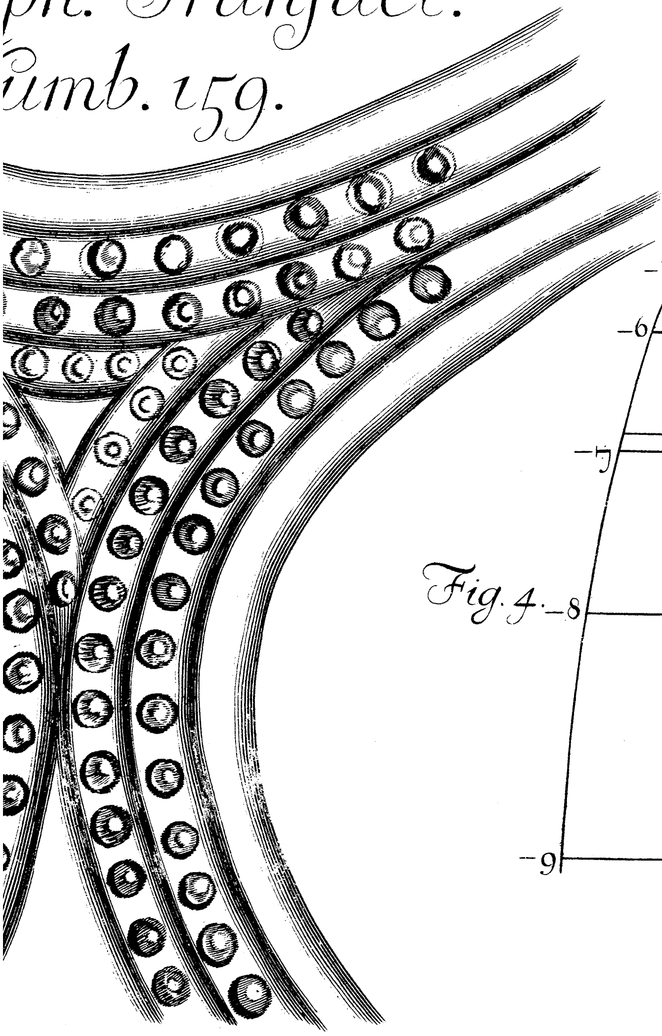

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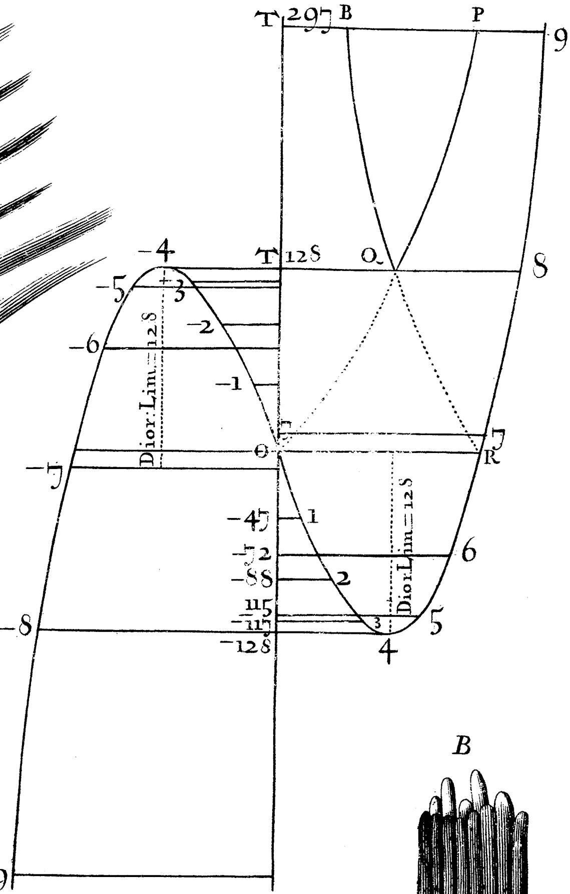

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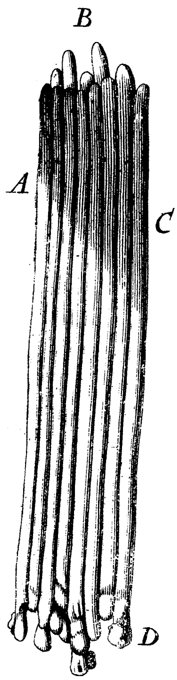

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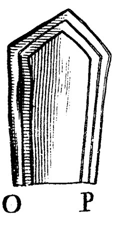

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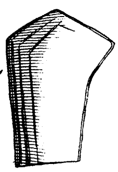

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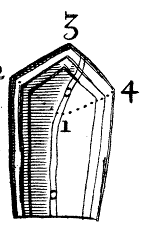

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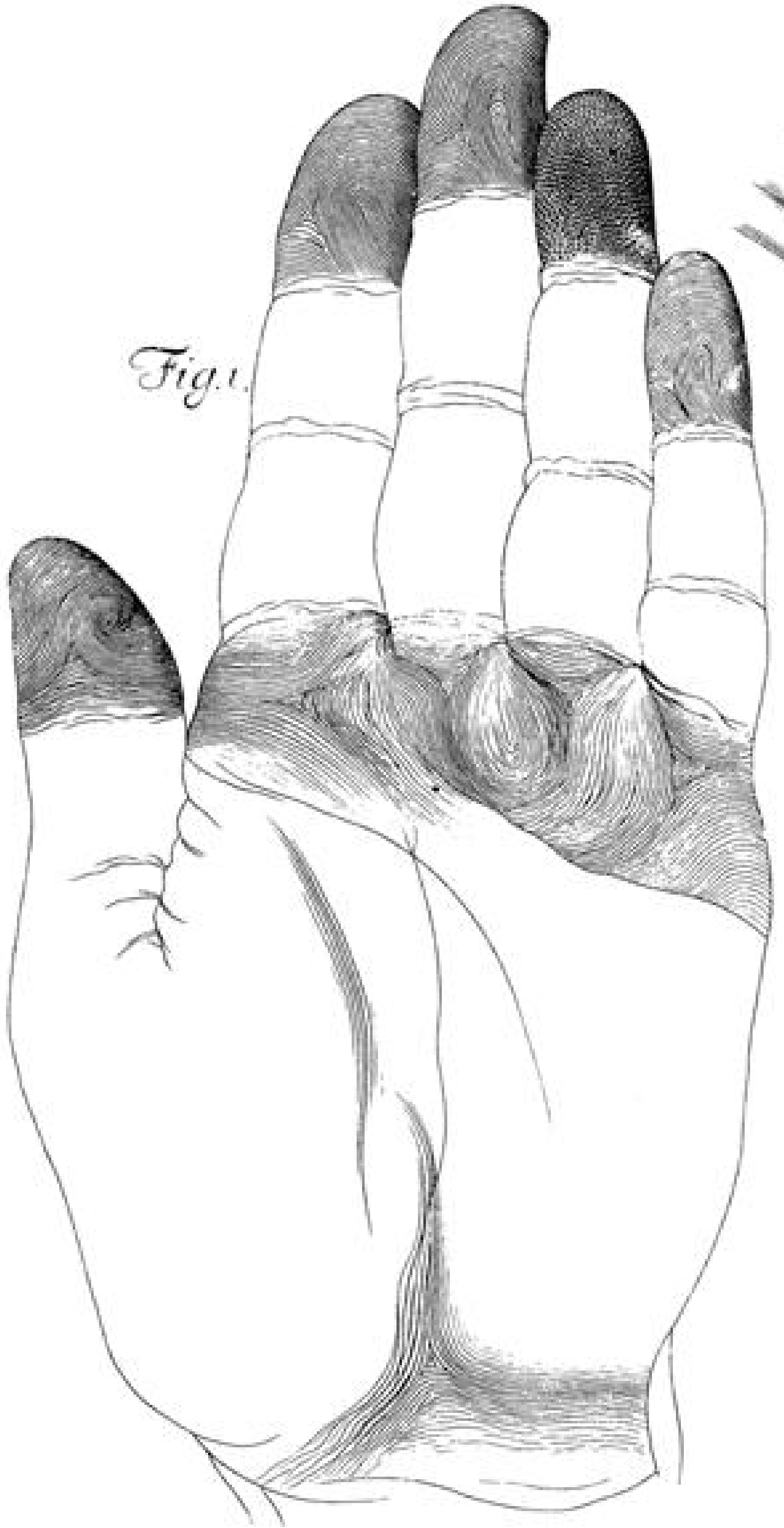

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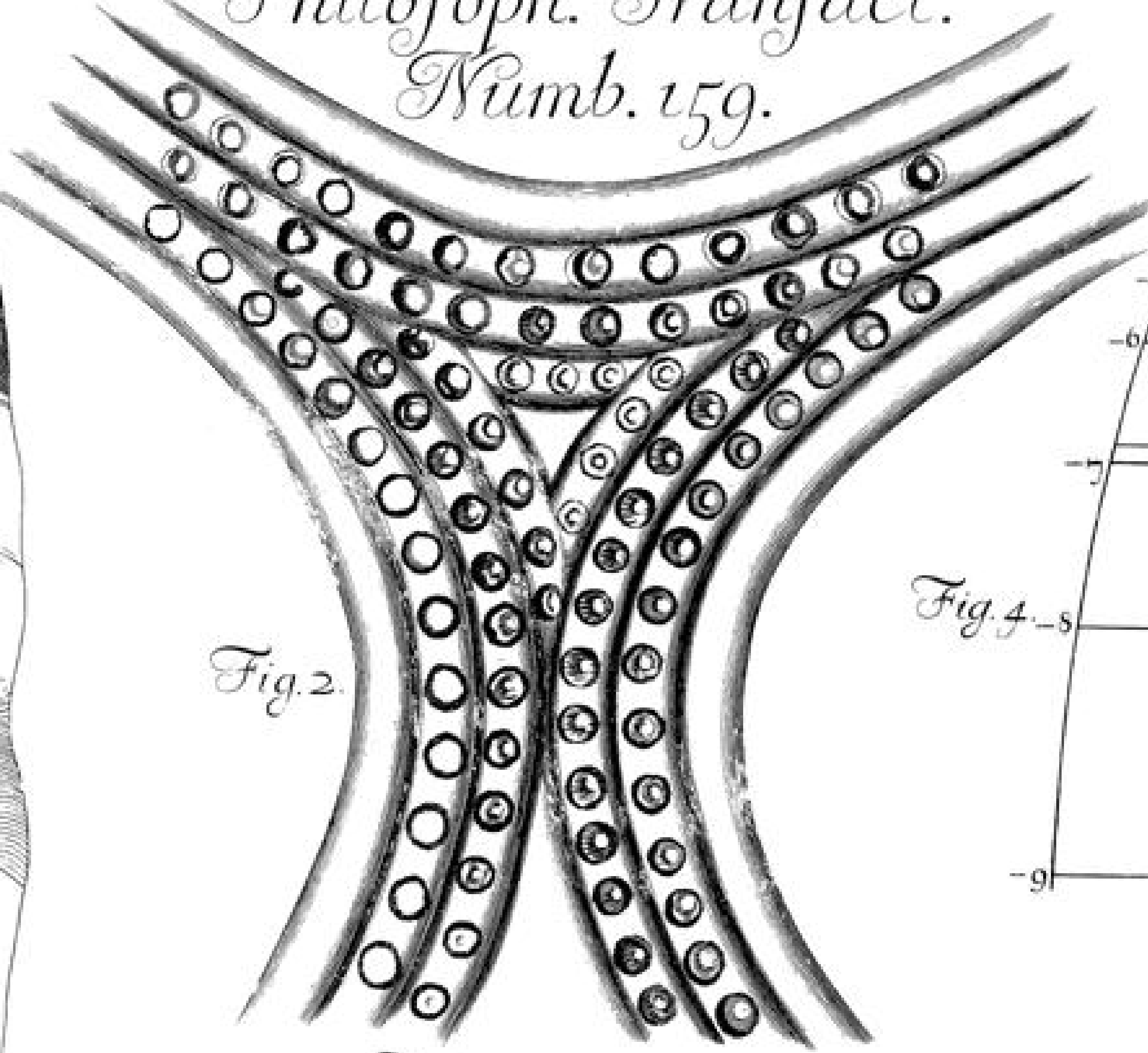

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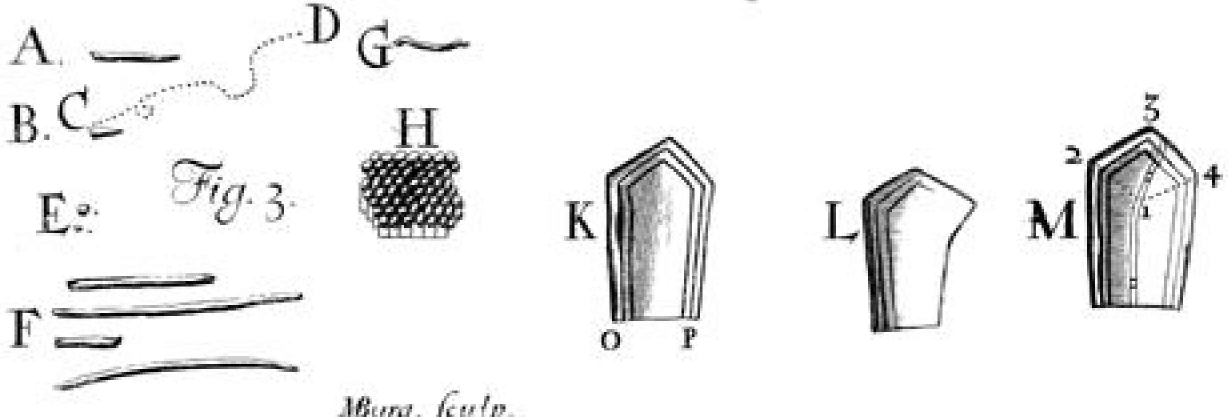

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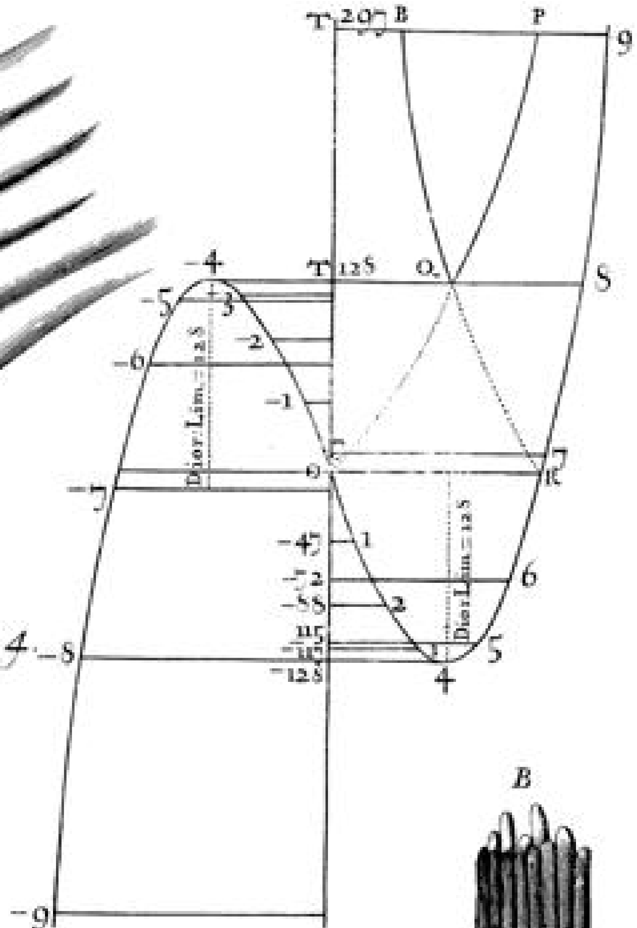

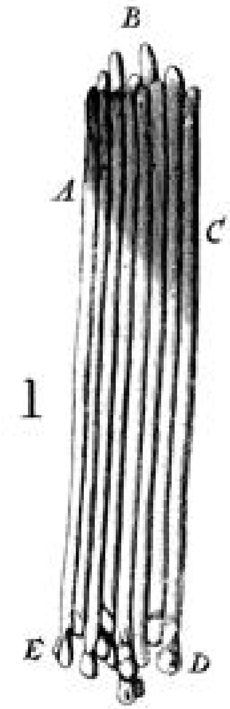

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