

II. *A Discourse concerning a Method of Discovering the true Moment of the Sun's Ingress into the Tropical Signs.* By E. Halley.

IT may perhaps pass for a Paradox, if not seem extravagant, if I should assert that it is an easier matter to be assured of the moments of the Tropicks, or of the times of the Sun's entrance into *Cancer* and *Capricorn*, than it is to observe the true times of the Equinoctials or Ingress into *Aries* and *Libra*. I know the Opinion both of Ancient and Modern Astronomers to the contrary; *Ptolemy* says expressly, *Τὰς τῶν τροπικῶν τμήσεις δυσδιακρίτους εἶναι*; And *Ricciolus* begins his Chapter of the Solstitial Observations with these words, *Merito Snellius, in notis ad observationes Hassiacas, pronuncia- vit, Herculei esse laboris vitare in Solstitiis observandis errorem quadrantis diei*, and this because of the exceeding slowness of the change of the Sun's Declination on the day of the Tropick, being not a quarter of a Minute in 24 Hours. This indeed would make it very difficult, nor would any Instruments suffice to do it, were the moment of the Tropick to be determined from one single Observation. But by three subsequent Observations made near the Tropick, at proper intervals of time, I hereby design to shew a Method to find the moment of the Tropicks capable of all the exactness the most Accurate can desire; and that without any consideration of the Parallax of the Sun, of the Refractions of the Air, of the greatest Obliquity of the Ecliptick, or Latitude of the Place: All which are required to ascertain the times of the Equinoctials from Observation, and which being faultily assumed, have occasioned an Error of near three Hours in the times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*,

Kepler, the Vernal being so much later, and the Autumnal so much earlier than by the *Calculus* of those Famous Authors.

Now before we proceed, it will be necessary to premise the following *Lemmata*, serving to demonstrate this Method, *viz.*

1. That the motion of the Sun in the Ecliptick, about the time of the Tropicks, is so nearly equable, that the difference from equality is not sensible, from five days before the Tropick, to five days after: and the difference arising from the little inequality that there is, never amounts to above $\frac{1}{4}$ of a single Second in the Declination, and this by reason of the nearness of the *Apogæon* of the Sun to the Tropick of *Cancer*.

2. That for five Degrees before and after the Tropicks, the differences whereby the Sun falls short of the Tropicks, are as the Versed Sines of the Sun's distance in Longitude from the Tropicks, which Versed Sines in Arches under five Degrees, are beyond the utmost nicety of sense, as the Squares of those Arches. From these two follow a Third:

3. That for five days before and after the Tropicks, the Declination of the Sun falls short of the utmost Tropical Declination, by Spaces which are in duplicate proportion, or as the Squares of the Times by which the Sun is wanting of or past the moment of the Tropick.

Hence it is evident that if the Shadows of the Sun, either in the Meridian or any other Azimuth, be carefully observed about the time of the Tropicks, the Spaces whereby the Tropical shade falls short of, or exceeds those at other times, are always proportionable to the Squares of the Intervals of Time between those Observations and the true time of the Tropick, and consequently if the Line, on which the Limits of the shade is taken, be made the Axis, and the correspondent times from the Tropick expounded by Lines, be erected on

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their respective Points in the Axis as ordinates, the extremities of those Lines shall touch the Curve of a Parabola; as may be seen in the Figure: Where a, b, c, e , being supposed Points observed, the Lines aB, bC, cA, eF , are respectively proportional to the times of each Observation before or after the Tropical Moment in *Cancer*.

This premised, we shall be able to bring the Problem of finding the true time of the Tropick by three Observations, to this Geometrical one: having three Points in a Parabola A, B, C , or A, F, C given, together with the direction of the Axis, to find the Distance of those Points from the Axis. Of this there are two cases, the one when the time of the second Observation B is precisely in the middle time between A and C : In this case putting t for the whole time between A and C , we shall have Ac the interval of the remotest Observation A from the Tropick by the following Analogy.

As $2ac - bc$ to $2ac - \frac{1}{2}bc ::$ So is $\frac{1}{2}t$ or AE : to Ac the time of the remotest Observation A from the Tropick.

But the other case when the middle Observation is not exactly in the middle between the other two times, as at F , is something more operose, and the whole time from A to C being put $= t$, and from A to $F = s, ce = c$,

and $bc = b$, the Theorem will stand thus $\frac{ttc - bss}{2tc - 2bs} = Ac$

the time sought.

To illustrate this Method of Calculation it may perhaps be requisite to give an Example or two for the sake of those Astronomers that are less instructed in the Geometrical part of their Art.

Anno 1500 *Bernard Walther* in the Month of *June* at *Nuremburg* observed the Chord of the distance of the Sun from the *Zenith* by a large Parallacltick Instrument of *Ptolemy*, as follows:

June

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June 2. 45467. June 8. 44975.
June 9. 44934. and June 12. 44883.
June 16. 44990. June 16. 44990.

In both which cases the middle time is exactly in the middle between the extrems, and therefore in the former three, $ac=533$, $bc=477$ and t , the time between being 14 days, by the first Rule, the time of the Tropick will be found by this Proportion, as 589 to $827\frac{1}{2}$:: So $\frac{1}{2} t$ or 7 days to 9 days $20^h. 2'$. whence the Tropick *Anno* 1500 is concluded to have fallen *June* $11^d. 20^h. 2'$. In the latter three, ac is $=107$, and $bc=15$, and the whole interval of Time is 8 days $=t$; whence as 199 : to $206\frac{1}{2}$:: so is 4 days to $4^d. 3^h. 37'$. which taken from the *16th*. day at Noon, leaves $11^d. 20^h. 23'$. for the time of the Tropick, agreeing with the former to the third part of an hour.

Again, *Anno* 1636, *Gassendus* at *Marseilles* observed the Summer Solstice by a *Gnomon* of 55 Foot high, in order to determine the Proportion of the *Gnomon* to the Solstitial shade, and he hath left us these Observations, which may serve as an Example for the second Rule.

June 19. *St. N.* shadow 31766 parts, whereof the *Gnomon*
June 20. 31753 (was 89428.
June 21. 31751
June 22. 31759

These being divided into two sets of three Observations each, *viz.* the 19th. 20th. and 22th. and the 19th. 21th. and 22th. we shall have in the first three, $c=13$ and $b=7$, $t=3$ days, $s=1$, and in the second $c=15$ and $b=7$, $t=3$ and $s=2$. Whence according to the Rule, the 19th. day at Noon the Sun wanted of the Tropick a time proportionate to one day, as $ttc - ssb$ to $2tc - 2bs$, that is, as 110 to 64 in the first set, or

107 to 62 in the second set; that is, 1^d. 17^h. 15'. in the first, or 1^d. 17^h. 25'. in the second set: So that we may conclude the Moment of the Tropick to have been June 10^d. 17^h. 20'. in the Meridian of *Marseilles*.

Now that these two Tropical times thus obtained, will be found to confirm each others exactness from their near agreement, appears by the interval of time between them; viz. 1^d. 2^h. 30'. less than 136 *Julian* years: whereof 1^d. 1^h. 8'. arises from the defect of the length of the Tropical Year from the *Julian*, and the rest from the Progression of the Sun's *Apogæon* in that time; so that no two Observations made by the same Observer in the same place, can better answer each other, and that without any the least Artifice or Force in the management of them.

What were the Methods used by the Ancients to conclude the hour of the Tropicks, *Ptolemy* has no where delivered; but it were to have been wished that they had been aware of this, that so we might have been more certain of the moments of the Tropicks we have received from them, which would have been of singular use to determine the Question, Whether the Sun's *Apogæon* be fixt in the Starry Heaven; or if it move, what is the true motion thereof? It is certain, that if we take the Account of *Ptolemy*, the Tropick said to be observed by *Euctemon* and *Meton*, *Junii 27. manè*, Anno 432 *ante Christum*, can no ways be reconciled without supposing the Observation made the next day, or June 28th. in the Morning. And *Ptolemy's* own Tropick observed in the Third Year of *Antoninus*, Anno *Christi* 140, was certainly on the 23th. and not the 24th. day of June; as will appear to those that shall duly consider and compare them with the length of the Year deduced from the diligent and concordant Observations of those two great Astronomical *Genii*, *Hipparchus* and *Albatani*; established and confirmed by the concurrence

of all the Modern Accuracy. For these Observations give the length of the Tropical Year such as to anticipate the *Julian Account* only one day in 300 Years; but we are now secure that the said Period of the Sun's Revolution does anticipate very nearly three days in 400 Years; so that the *Tables of Ptolemy* founded on that Supposition, do err about a whole day in the Sun's Place, for every 240 Years. Which principal Error is so Fundamental a Point, does vitiate the whole Superstructure of the *Almagest*, and serves to convict its Author of want of Diligence, or Fidelity, or both.

But to return to our Method, the great Advantage we have hereby, is, that any very high Building serves for an Instrument, or the top of any high Tower or Steeple, or even any high Wall whatsoever, that may be sufficient to intercept the Sun, and cast a true shade: Nor is the Position of the Plane on which you take the shade, or that of the Line therein, on which you measure the Recess of the Sun from the Tropick, very material; but in what way soever you discover it, the said Recess will be always in the same Proportion, by reason of the smallness of the Angle, which is not six Minutes in the first five days: Nor need you enquire the height or distance of your Building, provided it be very great, so as to make the Spaces you measure large and fair. But it is convenient that the Plane on which you take the shade be not far from Perpendicular to the Sun, at least not very Oblique, and that the Wall which casts the shade, be straight and smooth at top, and its Direction nearly East and West, for Reasons that will be well understood by a Reader skilful in the Doctrine of the Sphere. And it will be requisite to take the Extremest greatest or least deviation of the shadow of the Wall, because the shade continues for a good time at a stand, without alteration, which will give the Observer leisure to be assured of what he does, and not be surprized.

surprized by the quick transient motion of the shade of a single Point at such a distance. The Principal Objection is, that the *Penumbra* or Partile shade of the Sun is in its extreame very difficult to distinguish from the true shade, which will render this Observation hard to determine nicely. But if the Sun be transmitted through a *Telescope*, after the manner used to take his *Species* in a *Solar Eclipse*, and the upper half of the Object-glass be cut off by a Paper pasted thereon, and the exact upper Limb of the Sun be seen just Emerging out of, or rather continging the *Species* of the Wall, (the Position of the *Telescope* being regulated by a fine Hair extended in the *Focus* of the Eye-glass,) I am assured that the limit of the shade may be obtained to the utmost exactness: And of this I design to give a *Specimen* by an Observation to be made in *June* next, by the help of the High Wall of *St. Paul's Church, London*, of which some following Transaction may give an Account. In the mean time what I have promised may suffice to set others at work, where such or higher Buildings are to be met with. I shall only Advertise, that the Winter-Tropick by this Method may be more certainly obtained than the *Summer's*, by reason that the same *Gnomon* does afford a much larger *Radius* for this manner of Observation.
