

BOTSWANA GENERAL CERTIFICATE OF SECONDARY EDUCATION

TEACHING SYLLABUS

MATHEMATICS

Ministry of Education

Department of Curriculum Development and Evaluation

FOREWORD

The Ministry of Education is pleased to authorise the publication of this senior secondary syllabus which marks a watershed in the development of the public education system in Botswana and signals another milestone of progress in fulfilment of the goals set by the Revised National Policy on Education, Government Paper No. 2 of 1994.

In this era of widespread and rapid technological change and an increasingly inter-dependent global economy, it is essential that all countries foster human resources by preparing children adequately for their future. Survival in the coming millennium will depend on the ability to accommodate change and to adapt to environmental needs and emerging socio-economic trends. It is the wish of government to prepare Batswana for future growth and adaptation to ongoing change in the socio-economic context; specifically the transition from an agro-based economy to the more broadly based industrial economy which we are aiming at.

The senior secondary programme builds on the Ten Year Basic Education programme and seeks to provide quality

learning experiences. It aims to prepare our students for the world of work, further education and lifelong learning. However, secondary education must also pay attention to the all round development of the individual. It should provide not only for the acquisition of those skills needed for economic, scientific and technological advancement. It should also provide for the development of cultural and national identity and the inculcation of attitudes and values which nurture respect for one's self and for others.

Critical to the success of our secondary education programme is the recognition of individual talents, needs and learning styles. Hence, the role of the teacher in the classroom has changed. S/he must be a proficient manager and facilitator; a director of learning activities. S/he should be conscious of students' needs to take on board a measure of accountability and responsibility for their own learning. S/he must also take into account the widening range of ability of the student body and the different levels of achievement which they aspire to.

This means active participation for all and the creation of rich and diverse learning environments.

It is important then that we value the students' own experiences, build upon what they know and reward them for positive achievement. At the same time, we must be prepared to offer them guidance and counselling at all levels; assisting them to make the best decisions in keeping with their own interests, career prospects and preferences. In that way we shall prevail in nurturing at the roots of our system, the national ideals of democracy, development, self-reliance, unity and social harmony.

This syllabus document is the outcome of a great deal of professional consultation and collaboration. On behalf of the Ministry, I wish to record my appreciation and thank sincerely those who contributed to and were involved in the production of this syllabus.

P. T. Ramatsui

Permanent Secretary Ministry of Education

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The quality of this syllabus would not have been possible without invaluable contributions of Senior Secondary Mathematics teachers who participated in various consultative meetings held across the country in their own capacity and representatives of their schools and their regions - To them we say thank you, *le ka moso bagaetsho!*

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Introduction

Mathematics may be defined as the study of numbers, shapes and relationships between abstract entities logically bound together. Its methods are used to investigate, interpret, model and make decisions in everyday life, and they should also contribute to each individual's understanding of their environment. In the Senior Secondary Programme Mathematics will focus on extension and further acquisition of concepts and how these can be applied.

The Mathematics syllabus has been designed to meet the Senior Secondary Programme aims. The subject focuses on having every learner in the Senior Secondary School as its target. Content has been varied according to abilities and needs. In the case of the more able learners extended concepts and processes are provided. The syllabus includes selected major areas - Statistics and Probability, Geometry, Algebra, Measures and Numbers. It is expected that their interrelationship through a spiral process approach is adhered to to create an effective scheme.

The five major areas mentioned above stand as modules of this syllabus. Each module is subdivided into units. For each unit general objectives give rise to differentiated content specific objectives - core and extension. The specific objectives describe what students should be able to do in measurable terms.

An attempt should be made to relate each topic to practical situations. Teaching and learning materials should enable the learner to learn through discovery. Acquired knowledge and skills are to be applied to new situations. Project work will be an integral part of the learning activities and is to be assessed. Assessment will consist of school-based assessment and a terminal examination.

This syllabus was designed and developed in accordance with the recommendation of RNPE and on the basis that Mathematics has been allocated five periods per week on a 40 periods by 40 minutes per week timetable.

Rationale

Mathematics plays a key role in everyday life as a means of communication and because of its applications in every sphere of life. Mathematics facilitates understanding and acquisition of modern techniques and technologies that are necessary in managing and accommodating change. As

a product of human culture it is worthy to be studied for its own sake. Therefore this programme is designed to develop, to the maximum of each individual's potential; knowledge, skills and attitudes relevant to the society, the world of work and appreciation for mathematics as a subject.

Through learning mathematics students can develop intellectual growth, capacity to think and reason logically and critically. They can develop appreciation for patterns, structures and relationships; model and describe the world around them. They can increase their intellectual curiosity and imagination and develop problem solving capabilities and inquiry strategies. This will enhance the students' prospects of employment and/or further education. Mathematics will also prepare them for active and constructive participation in the society as citizens of tomorrow who can analyse and solve real life problems in day-to-day situations.

To achieve the stated aims and objectives the teaching and learning of Mathematics is to be based on a learner centred approach. A variety of methods are therefore to be used such as exposition and consolidation, discussions, practical work, problem solving activities and investigative work. Furthermore teaching and learning of mathematics should utilise modern technology such as the graphic calculators and computers that place mathematics in a realistic context.

This will offer a constructivist view on mathematics to learners, promote interest and motivation, and prepare the students effectively for the next century paving the way for future generations.

Aims of Senior Secondary Programme

On completion of the two year senior secondary programme learners should have:-

- 1. acquired knowledge, developed confidence and ability to assess their personal strengths and weaknesses and be realistic in choosing appropriate career/employment opportunities and or further education and training
- 2. developed skills to assist them in solving technical and technological problems as they relate to day- to-day life situations.
- 3. developed desirable attitudes and behavioural patterns in interacting with the environment in a manner that is protective, preserving and nurturing.
- 4. acquired attitudes and values, developed basic skills and understanding to allow for execution of rights and responsibilities as good citizens of Botswana and the world
- 5. developed information technology skills as well as an understanding and appreciation of their influence in the dayto-day activities
- 6. acquired knowledge, attitudes and practices that will ensure good family and health practices including awareness and management of epidemics (such as HIV/AIDS) that prepare them for productive life
- 7. developed pre-vocational knowledge and manipulative skills that will enable them to apply content learnt and attitudes and values developed to practical life situations in the world of work
- 8. developed an understanding and acquired skills in business, everyday commercial transactions and enterpreneurship

9. developed foundation skills such as problem solving, critical thinking, communication, inquiring, team work/interpersonal to help them to be productive and adaptive to survive in a changing environment.

Aims of Senior Secondary Mathematics

On completion of the two-year Senior Secondary Mathematics Programme learners should have:-

- 1. developed positive attitudes to mathematics, including confidence, enjoyment and perseverance
- 2. developed abilities to understand and use mathematics as a means of communication with emphasis on the use of clear expression and representation of ideas and facts
- 3. developed abilities to solve problems, model and describe varieties of practical life situations, present the solution clearly, check and interpret the results
- 4. developed abilities to produce and appreciate imaginative and creative work arising from mathematical ideas
- 5. developed willingness and abilities to work independently and co-operatively
- 6. developed abilities to apply mathematical concepts and skills in other disciplines of the curriculum
- 7. developed abilities to apply skills and knowledge in situations the students will meet in life
- 8. developed an understanding of the role mathematics plays in society
- 9. developed a feel for number, measure, shape, space, movement, position and carry out calculations and understand the significance of the results obtained
- 10.developed an appreciation of the role of mathematics in technology and the whole society
- 11.developed appreciation of patterns and relationships in mathematics as a subject
- 12. developed appreciation of the interdependence of different branches of mathematics

- 13. developed appreciation of the role of modern technology in mathematics
- 14.acquired appropriate mathematical knowledge, skills and attitudes to meet the challenges in the world of work and to function as well informed citizens
- 15.acquired basic mathematical knowledge and skills for further study at tertiary level pertinent to any discipline of the student's choice.

Assessment Objectives

In the following attainment skill areas learners are required to demonstrate their ability to:-

- 1. Computation
- 1.1 perform calculations with and/or without a calculating aid.
- 1.2 estimate, approximate and use appropriate degrees of accuracy.
- 1.3 use common systems of units.
- 2. Application
- 2.1 recognise, understand and apply appropriate mathematical procedures in a given situation.
- 2.2 recognise, understand and apply properties of shapes, positions, movements and transformations in 2 and/or 3 dimensions.
- 2.3 understand and apply relationships and their representation.
- 2.4 collect, process, represent (in tabular, graphical and diagrammatic forms) and interpret data.
- 2.5 use geometrical instruments.

- 3. Reasoning
- 3.1 recognise and justify generalisations of patterns and structures in a variety of situations and forms.
- 3.2 formulate problems into mathematical terms, select, apply and communicate appropriate techniques of solution and interpret the solutions in terms of the problems, e.g. investigation and project work.

Assessment

The scheme of assessment will consists of school based assessment and a terminal examination.

School based assessment in the form of tests (e.g. diagnostic, aptitude, achievement, oral, practice, attitude, performance), exercises, assignments, discussions, investigation and project work, and any other form of school based assessment will be undertaken by teachers to improve instruction and to guide progressions. School based assessment or course work will contribute towards the certification of the learner.

Terminal examination will be administered and its details will be provided by the examining body - Examinations, Research and Testing Division.

NUMBERS

Topics	General Objectives	Specific Objectives
	Learners should be able to:-	Learners should be able to:-
Numbers and op	erations	
Types of Numbers	understand concepts of numbers	 identify, represent and use numbers (whole numbers, natural (including zero), integer, rational, irrational, real)
		 demonstrate an understanding of application of classes of numbers (odd, even, rectangle, triangle, square, cube, prime, etc.) in a variety of situations
Calculator	acquire further knowledge and skills on the use of a calculator	- use efficiently the facilities of a scientific calculator
Patterns and Sequences	explore sequences	- complete a sequence - describe in words the pattern of a linear sequence
		- state in algebraic form a pattern of

Estimation and Approximation	understand and use the concept of estimation and approximation	a linear sequence - generate a sequence of numbers - use patterns of sequences to represent a given situation and solve problems - state in algebraic form a pattern of a simple non-linear sequence - approximate to specified number of decimal places, number of significant figures or place value in a given context - approximate to a reasonable degree of accuracy - estimate quantities - approximate computations - check reasonableness of results
Directed Numbers	use and apply directed numbers	- solve problems involving directed numbers in practical situations
Fractions	use and apply fractions	 solve problems involving fractions, percentages (including interest) and ratio/proportions in practical situations
Indices	acquire further	- interpret integral and simple

knowledge on indices and apply them	fractional indices - solve problems involving integral and simple fractional indices - simplify expressions involving fractional indices
	- interpret and solve problems involving fractional indices
express and use numbers in standard form	 express numbers in standard form to a specified number of decimal places or significant figures order numbers in standard form
	- solve problems involving numbers in standard form

MEASURES

Topics	General Objectives	Specific Objectives
	Learners should be able to:-	Learners should be able to:-
Mensuration		
Perimeter, Area, Volume, Mass and Density	develop further knowledge on measures and apply it	perimeters and areas of compound
Estimates		

Limits of Accuracy	understand and use limits of accuracy	 give appropriate upper and lower bounds for data given to a specified accuracy (e.g. measured lengths) obtain appropriate upper and lower bounds to solutions of simple problems (e.g. calculations of the perimeter or the area of rectangle) given data to a specified accuracy
Graphs		
Travel Graphs	acquire knowledge on travel graphs	- draw displacement-time and velocity-time graphs
		 interpret displacement-time and velocity-time graphs
		- calculate velocity using displacement-time graphs
		 calculate acceleration using velocity-time graphs (straight line segments)
		- calculate distance travelled as area under velocity-time graphs (straight line segments)
		- draw displacement-time graphs given velocity-time graphs and vice

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ALGEBRA

Topics	General Objectives	Specific Objectives
	Learners should be able to:-	Learners should be able to:-
Formulae		
Substitution	understand and use formulae	- substitute for variables and evaluate a specified variable in a given formula
Subject of the Formula		 change the subject of simple formula to a specified variable change the subject of the formula to a specified variable
Expressions		
Expressions	manipulate expressions	3.2.1.1 simplify algebraic expressions including binomial products
		- manipulate simple algebraic fractions
		- factorise expressions of the form $ax \pm ay$, $ax + bx + kay + kby$, $a^2x^2 - b^2y^2$, $x^2 + bx + c$ and

			 a²+2ab+b², where a, b, c and k are constants manipulate algebraic fractions factorise expressions of the form ax²+bxy+cy² and ax²+bx+c where a, b and c are constants
Equations and	inequalities		
Linear and Quadratic Equations	form and equations	solve	 solve fractional equations with numerical and/or one term linear algebraic denominator and at most two-term linear algebraic numerators in one variable form and solve linear equations in practical situations solve two simultaneous linear equations in two variables, using graphical, substitution and elimination methods
			 solve quadratic equations by factorisation, formula and graphical methods
			- solve fractional equations with numerical and linear algebraic denominators/numerators leading

	to a linear or quadratic equation - solve two simultaneous equations, one linear and the other quadratic equation - form and solve quadratic equations in practical situations
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		 form and solve linear simultaneous equations in practical situations solve quadratic equations by completing the square
		- derive the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, given
		$ax^2 + bx + c = 0$, where $a \neq 0$, b and c are constants
		 solve cubic equations by trial and improvement and/or graphical methods
Inequalities	Inequalities form and solve inequalities	 solve simple linear inequalities indicate the region containing the points whose coordinates satisfy at least one inequality of the form ax by < c, ax +by ≤ c, ax + by > c, ax + by ≥ c where a, b, and c are integers
		- form and solve linear inequalities in practical situations
		- solve quadratic inequalities algebraically and graphically

Functions		
Graphs	acquire knowledge on graphs	 find the equations of line graphs interpret equations of line graphs of the form y = mx + c graph functions of the form y = ax² + bx + c and y = ax³ recognise and interpret graphs of y = ax² + bx + c and y = ax³ graph functions of the form y = kx², where a = 2, 3; and k is a constant y = kx², where a = 1, 2, 3; and k is a constant y = ax, where a is a positive integer recognise and interpret graphs of the form, y = kx² where a = 2, 3; and k is a constant y = kx² where a = 2, 3; and k is a constant y = kx² where a = 1, 2, 3; and k is a constant y = ax, where a is a positive integer

		- estimate the gradient at a point by drawing a tangent
		- interpret the gradient at a point by drawing a tangent
		- interpret and apply translations in the context of a graphical representation
Graphs in Praction	al Situations	
Graphs	acquire knowledge on graphs	- draw graphs from practical situations
		- interpret graphs representing practical situations

GEOMETRY

Topics	General Objectives	Specific Objectives
	Learners should be able to:-	Learners should be able to:-
Geometrical Terr	ns and Relationships	
Geometrical terms and relationships	acquire further knowledge on geometrical terms and relationships	 use and interpret geometrical terms use properties of angles to calculate specified angles and/or length of line segments including angle properties of special triangles and quadrilaterals use properties of polygons to calculate specified angles and/or sides
Circles		
Angle properties of circle	acquire knowledge on properties of a circle	- calculate specified angles or line segments using the properties:- angle in a semi-circle is 90°, angle between tangent and radius at a point of contact is 90°, angle at the centre of a circle is twice the angle

		at the circumference subtended by the same arc, angles at the circumference subtended by the same arc are equal, opposite angles in a cyclic quadrilateral are supplementary - calculate specified angles or line segments using the property: angle between tangent and chord through the point of contact is equal to the angle subtended by the same chord at the circumference in the alternate segment
Symmetry properties of circle		- calculate specified angles or line segments using the properties :- equal chords are equidistant from the centre, chords equidistant from the centre are equal in length, perpendicular through the centre to a chord bisects the chord, tangents from an external point to the circle are equal in length
Symmetry		
Reflectional	understand and use	- recognise and describe line

symmetry	properties of symmetry	symmetry
Rotational symmetry		- recognise and describe rotational symmetry
Symmetry properties o	of	- recognise and use symmetry properties of plane shapes
polygons		 recognise and use symmetry properties of prisms (including cylinder) and pyramids (including cone)

Trigonometry	Trigonometry		
Bearings	understand and use bearings	- interpret and use three-figure bearings to describe journeys	
Trigonometric ratios and Pythagorean theorem	understand and apply trigonometric ratios and Pythagorean theorem	 apply Pythagorean theorem and the sine, cosine and tangent ratios for acute angles to calculate a side or an angle of a right-angled triangle including problems involving angles of elevation, depression and bearings solve problems using sine and cosine ratios for angles in the range of 0° to 180° solve problems using sine and cosine ratios for angles between 180° and 360° 	
Sine and Cosine Rule	understand and use sine and cosine rules	 solve problems using the sine rule (formula is to be provided) solve problems using the cosine rule (formula is to be provided) solve simple trigonometric problems in 3-D 	

Co-ordinate Geo	Co-ordinate Geometry		
Distances	understand and use coordinates in Cartesian plane	- calculate the distance between two points	
Coordinates of midpoint of the line segment		- calculate the coordinates of the midpoint of a line segment given the coordinates of its endpoints	
		 calculate the coordinates of the end point of a line segment given the coordinates of its midpoint and one endpoint 	
Loci in 2-D	Loci in 2-D		
Loci in 2-D	understand and use loci in 2-D	- draw and/or state the locus of points at a given distance from a given point	
		 draw and/or state the locus of points at a given distance from a given line 	
		 draw and/or state the locus of points equidistant from two given points 	
		4- draw and/or state the locus of points equidistant from two given	

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intersecting lines
 draw and/or state the locus of points equidistant from two given parallel lines
 use the locus of points at a given distance from a given point and the method of intersecting loci to locate specific points
 use the locus of points at a given distance from a given line and the method of intersecting loci to locate specific points
 use the locus of points equidistant from two given points and the method of intersecting loci to locate specific points
 use the locus of points equidistant from two given intersecting lines and the method of intersecting loci to locate specific points
 use the locus of points from two given parallel lines and the method of intersecting loci to locate specific points

		 use the construction of angle bisectors, perpendicular bisector of a line segment, the drawing of a line parallel to a given line in loci problems determine the locus of a point moving in a 2-D plane according to given rules (none being an inequality)
Transformations		
Reflection	understand and use transformations	- reflect simple plane figures on a co-ordinate grid
		- recognise and describe fully a reflection given a shape and its image on a co-ordinate grid
Rotation		 rotate simple plane shapes about any point through multiples of right-angles on a co-ordinate grid recognise and describe fully a rotation given a shape and its image on a co-ordinate grid
Translation		- translate plane figures on a co- ordinate grid

	- recognise and describe a translation given a shape and its image on a co-ordinate grid using a vector
Enlargement	 draw an enlargement of a simple shape on a co-ordinate grid recognise and describe fully an enlargement
	- make, use and interpret scale drawings e.g. maps
	 use the relationship between lengths of line segments and areas of similar plane shapes to solve problems
	 use the relationship between lengths of line segments, surface areas and volumes of similar solids to solve problems
	 describe a single transformation which is equivalent to a combination of two transformations

Vectors		
Vectors	understand and use vectors	- represent vectors by $\begin{pmatrix} x \\ y \end{pmatrix}$, a, \underline{a} , a or
		\overline{AB}
		- represent vectors by directed line segments
		- add and subtract vectors
		- multiply vectors by a scalar
		- calculate the magnitude of a vector
		- use the sum and the difference of
		two vectors to express given
		vectors in terms of two coplanar vectors

STATISTICS & PROBABILITY

Topics	General Objectives	Specific Objectives
	Learners should be able to:-	Learners should be able to:-
Data Handling		
Data Processing	collect and manipulate statistical data	 collect raw data (discrete and continuous) represent data in a frequency table construct and interpret a histogram (equal intervals), frequency polygon and cumulative frequency curve understand the reasons for organising, presenting data in a tabular or diagrammatic form and point out advantages and/or disadvantages that particular representations may have construct and interpret histograms with unequal intervals
Measures of Central	acquire further knowledge on	- find mean, median, mode and modal class

Tendency	measures of central tendency	 estimate median from a cumulative frequency curve and by linear interpolation from a cumulative frequency table interpret measures of central tendency (median, mean, mode) 	
Measures of Dispersion	acquire knowledge on measures of dispersion	 calculate range and interquartile range estimate quartiles and percentiles from a cumulative frequency curve and by linear interpolation from a cumulative frequency table calculate variance and standard deviation interpret measures of dispersion (interquartile range, variance and standard deviation) 	
Scatter Graphs	understand and use scatter diagrams	 plot and interpret scatter graphs draw the line of best fit by inspection find the equation of the line of best fit 	
Probability			

Simple Probability	understand and use probability	- understand and use the vocabulary of probability in practical situations
		- understand and use the probability scale
		- calculate probability of a single event
		- distinguish between theoretical and experimental probability
		 calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate
Mutually Exclusive and Independent		- understand the meaning of mutually exclusive and independent events.
Events		- use addition and multiplication of probabilities, as appropriate, in simple cases