REPUBLIC OF BOTSWANA

# BOTSWANA GENERAL CERTIFICATE OF SECONDARY EDUCATION 

## TEACHING SYLLABUS

## MATHEMATICS

Ministry of Education
Department of Curriculum Development and Evaluation

## FOREWORD

Critical to the success of our secondary education programme is the recognition of individual talents, needs and learning styles. Hence, the role of the teacher in the classroom has changed. S/he must be a proficient manager and facilitator; a director of learning activities. S/he should be conscious of students' needs to take on board a measure of accountability and responsibility for their own learning. S/he must also take into account the widening range of ability of the student body and the different levels of achievement which they aspire to. This means active participation for all and the creation of rich and diverse learning environments.

It is important then that we value the students' own experiences, build upon what they know and reward them for positive achievement. At the same time, we must be prepared to offer them guidance and counselling at all levels; assisting them to make the best decisions in keeping with their own interests, career prospects and preferences. In that way we shall prevail in nurturing at the roots of our system, the national ideals of democracy, development, self-reliance, unity and social harmony.

This syllabus document is the outcome of a great deal of professional consultation and collaboration. On behalf of the Ministry, I wish to record my appreciation and thank sincerely those who contributed to and were involved in the production of this syllabus.


P. T. Ramatsui<br>Permanent Secretary<br>Ministry of Education

## ACKNOWLEDGEMENT

The Curriculum Development Division wishes to acknowledge and pass its sincere gratitude to all colleagues from - University of Botswana; Examination, Research and Testing Division; Department of Non-Formal Education; Department of Secondary Education; Department of Primary Education; Department of Teacher Training and Development; who in their own special way have collaborated and contributed in the planning, development and production of this syllabus.
Special gratitude is give to the following members of the Senior Secondary Mathematics Task force for their invaluable commitment, dedication and contributions to the development of the syllabus.

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| B K Ramolefhe | Department of Non-Formal Education |
| J Deurwaardeur | Tonota College of Education |
| B Raboijane | Lobatse Senior Secondary School |
| M Mokakapadi | Madiba Senior Secondary School |
| S M Mooketsi | Francistown Senior Secondary School |
| C J Setlhong | St Joseph’s College |
| W Motswagole | Mosele-wa-pula Community Junior Secondary School |
| M Thuto | Faculty of Science - UB |

The quality of this syllabus would not have been possible without invaluable contributions of Senior Secondary Mathematics teachers who participated in various consultative meetings held across the country in their own capacity and representatives of their schools and their regions - To them we say thank you, le ka moso bagaetsho!

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## Introduction

Mathematics may be defined as the study of numbers, shapes and relationships between abstract entities logically bound together. Its methods are used to investigate, interpret, model and make decisions in everyday life, and they should also contribute to each individual's understanding of their environment. In the Senior Secondary Programme Mathematics will focus on extension and further acquisition of concepts and how these can be applied.

The Mathematics syllabus has been designed to meet the Senior Secondary Programme aims. The subject focuses on having every learner in the Senior Secondary School as its target. Content has been varied according to abilities and needs. In the case of the more able learners extended concepts and processes are provided. The syllabus includes selected major areas - Statistics and Probability, Geometry, Algebra, Measures and Numbers. It is expected that their interrelationship through a spiral process approach is adhered to to create an effective scheme.

The five major areas mentioned above stand as modules of this syllabus. Each module is subdivided into units. For each unit general objectives give rise to differentiated content specific objectives - core and extension. The specific objectives describe what students should be able to do in measurable terms.

An attempt should be made to relate each topic to practical situations. Teaching and learning materials should enable the learner to learn through discovery. Acquired knowledge and skills are to be applied to new situations. Project work will be an integral part of the learning activities and is to be assessed. Assessment will consist of school-based assessment and a terminal examination.

This syllabus was designed and developed in accordance with the recommendation of RNPE and on the basis that Mathematics has been allocated five periods per week on a 40 periods by 40 minutes per week timetable.

## Rationale

Mathematics plays a key role in everyday life as a means of communication and because of its applications in every sphere of life. Mathematics facilitates understanding and acquisition of modern techniques and technologies that are necessary in managing and accommodating change. As
a product of human culture it is worthy to be studied for its own sake. Therefore this programme is designed to develop, to the maximum of each individual's potential; knowledge, skills and attitudes relevant to the society, the world of work and appreciation for mathematics as a subject.

Through learning mathematics students can develop intellectual growth, capacity to think and reason logically and critically. They can develop appreciation for patterns, structures and relationships; model and describe the world around them. They can increase their intellectual curiosity and imagination and develop problem solving capabilities and inquiry strategies. This will enhance the students' prospects of employment and/or further education. Mathematics will also prepare them for active and constructive participation in the society as citizens of tomorrow who can analyse and solve real life problems in day-to-day situations.

To achieve the stated aims and objectives the teaching and learning of Mathematics is to be based on a learner centred approach. A variety of methods are therefore to be used such as exposition and consolidation, discussions, practical work, problem solving activities and investigative work. Furthermore teaching and learning of mathematics should utilise modern technology such as the graphic calculators and computers that place mathematics in a realistic context. This will offer a constructivist view on mathematics to learners, promote interest and motivation, and prepare the students effectively for the next century paving the way for future generations.

## Aims of Senior Secondary Programme

On completion of the two year senior secondary programme learners should have:-

1. acquired knowledge, developed confidence and ability to assess their personal strengths and weaknesses and be realistic in choosing appropriate career/employment opportunities and or further education and training
2. developed skills to assist them in solving technical and technological problems as they relate to day- to-day life situations.
3. developed desirable attitudes and behavioural patterns in interacting with the environment in a manner that is protective, preserving and nurturing.
4. acquired attitudes and values, developed basic skills and understanding to allow for execution of rights and responsibilities as good citizens of Botswana and the world
5. developed information technology skills as well as an understanding and appreciation of their influence in the day-to-day activities
6. acquired knowledge, attitudes and practices that will ensure good family and health practices including awareness and management of epidemics (such as HIV/AIDS) that prepare them for productive life
7. developed pre-vocational knowledge and manipulative skills that will enable them to apply content learnt and attitudes and values developed to practical life situations in the world of work
8. developed an understanding and acquired skills in business, everyday commercial transactions and enterpreneurship
9. developed foundation skills such as problem solving, critical thinking, communication, inquiring, team work/interpersonal to help them to be productive and adaptive to survive in a changing environment.

## Aims of Senior Secondary Mathematics

On completion of the two-year Senior Secondary Mathematics Programme learners should have:-

1. developed positive attitudes to mathematics, including confidence, enjoyment and perseverance
2. developed abilities to understand and use mathematics as a means of communication with emphasis on the use of clear expression and representation of ideas and facts
3. developed abilities to solve problems, model and describe varieties of practical life situations, present the solution clearly, check and interpret the results
4. developed abilities to produce and appreciate imaginative and creative work arising from mathematical ideas
5. developed willingness and abilities to work independently and co-operatively
6. developed abilities to apply mathematical concepts and skills in other disciplines of the curriculum
7. developed abilities to apply skills and knowledge in situations the students will meet in life
8. developed an understanding of the role mathematics plays in society
9. developed a feel for number, measure, shape, space, movement, position and carry out calculations and understand the significance of the results obtained
10. developed an appreciation of the role of mathematics in technology and the whole society
11. developed appreciation of patterns and relationships in mathematics as a subject
12. developed appreciation of the interdependence of different branches of mathematics
13. developed appreciation of the role of modern technology in mathematics
14. acquired appropriate mathematical knowledge, skills and attitudes to meet the challenges in the world of work and to function as well informed citizens
15. acquired basic mathematical knowledge and skills for further study at tertiary level pertinent to any discipline of the student's choice.

## Assessment Objectives

In the following attainment skill areas learners are required to demonstrate their ability to:-

## 1. Computation

1.1 perform calculations with and/or without a calculating aid.
1.2 estimate, approximate and use appropriate degrees of accuracy.
1.3 use common systems of units.

## 2. Application

2.1 recognise, understand and apply appropriate mathematical procedures in a given situation.
2.2 recognise, understand and apply properties of shapes, positions, movements and transformations in 2 and/or 3 dimensions.
2.3 understand and apply relationships and their representation.
2.4 collect, process, represent (in tabular, graphical and diagrammatic forms) and interpret data.
2.5 use geometrical instruments.

## 3. Reasoning

3.1 recognise and justify generalisations of patterns and structures in a variety of situations and forms.
3.2 formulate problems into mathematical terms, select, apply and communicate appropriate techniques of solution and interpret the solutions in terms of the problems, e.g. investigation and project work.

## Assessment

The scheme of assessment will consists of school based assessment and a terminal examination.
School based assessment in the form of tests (e.g. diagnostic, aptitude, achievement, oral, practice, attitude, performance), exercises, assignments, discussions, investigation and project work, and any other form of school based assessment will be undertaken by teachers to improve instruction and to guide progressions. School based assessment or course work will contribute towards the certification of the learner.
Terminal examination will be administered and its details will be provided by the examining body Examinations, Research and Testing Division.

## NUMBERS

| Topics | General Objectives | Specific Objectives |
| :---: | :---: | :---: |
|  | Learners should be able to:- | Learners should be able to:- |
| Numbers and operations |  |  |
| Types of Numbers | understand concepts of numbers | - identify, represent and use numbers (whole numbers, natural (including zero), integer, rational, irrational, real) <br> - demonstrate an understanding of application of classes of numbers (odd, even, rectangle, triangle, square, cube, prime, etc.) in a variety of situations |
| Calculator | acquire further knowledge and skills on the use of a calculator | - use efficiently the facilities of a scientific calculator |
| Patterns and Sequences | explore sequences | - complete a sequence <br> - describe in words the pattern of a linear sequence <br> - state in algebraic form a pattern of a linear sequence <br> - generate a sequence of numbers <br> - use patterns of sequences to represent a given situation and solve problems <br> - state in algebraic form a pattern of a simple nonlinear sequence |
| Estimation and Approximation | understand and use the concept of estimation and approximation | - approximate to specified number of decimal places, number of significant figures or place value in a given context <br> - approximate to a reasonable degree of accuracy <br> - estimate quantities <br> - approximate computations <br> - check reasonableness of results |
| Directed Numbers | use and apply directed numbers | - solve problems involving directed numbers in practical situations |
| Fractions | use and apply fractions | - solve problems involving fractions, percentages (including interest) and ratio/proportions in practical situations |
| Indices | acquire further knowledge on indices and apply them | - interpret integral and simple fractional indices <br> - solve problems involving integral and simple fractional indices <br> - simplify expressions involving fractional indices <br> - interpret and solve problems involving fractional indices |
|  | express and use numbers in standard form | - express numbers in standard form to a specified number of decimal places or significant figures <br> - order numbers in standard form <br> - solve problems involving numbers in standard form |

## MEASURES

| Topics | General Objectives | Specific Objectives |
| :---: | :---: | :---: |
|  | Learners should be able to:- | Learners should be able to:- |
| Mensuration |  |  |
| Perimeter, Area, Volume, Mass and Density | develop further knowledge on measures and apply it | - solve practical problems involving perimeters and areas of compound shapes <br> - apply the formula $\mathrm{A}=\frac{1}{2} \mathrm{ab} \sin \mathrm{C}$ <br> - calculate surface area and volume of cylinders, prisms, pyramid, cones and spheres (for cone, pyramid and sphere the formula are to be given) <br> - solve practical problems involving density, volume and mass. <br> - solve practical problems involving compound shapes and solids <br> - derive the formula $A=\frac{1}{2} a b \sin C$ |
| Estimates |  |  |
| Limits of Accuracy | understand and use limits of accuracy | - give appropriate upper and lower bounds for data given to a specified accuracy (e.g. measured lengths) <br> - obtain appropriate upper and lower bounds to solutions of simple problems (e.g. calculations of the perimeter or the area of rectangle) given data to a specified accuracy |
| Graphs |  |  |
| Travel Graphs | acquire knowledge on travel graphs | - draw displacement-time and velocity-time graphs <br> - interpret displacement-time and velocity-time graphs <br> - calculate velocity using displacement-time graphs <br> - calculate acceleration using velocity-time graphs (straight line segments) <br> - calculate distance travelled as area under velocitytime graphs (straight line segments) <br> - draw displacement-time graphs given velocitytime graphs and vice versa |

## ALGEBRA

| Topics | General Objectives | Specific Objectives |
| :---: | :---: | :---: |
|  | Learners should be able to:- | Learners should be able to:- |
| Formulae |  |  |
| Substitution | understand and use formulae | - substitute for variables and evaluate a specified variable in a given formula |
| Subject of the Formula |  | - change the subject of simple formula to a specified variable <br> - change the subject of the formula to a specified variable |
| Expressions |  |  |
| Expressions | manipulate expressions | 3.2.1.1 simplify algebraic expressions including binomial products |
|  |  | - manipulate simple algebraic fractions <br> - factorise expressions of the form $a x \pm a y$, $a x+b x+k a y+k b y, a^{2} x^{2}-b^{2} y^{2}, x^{2}+b x+c$ <br> and $a^{2}+2 a b+b^{2}$, where $a, b, c$ and $k$ are constants <br> - manipulate algebraic fractions <br> - factorise expressions of the form $a x^{2}+b x y+c y^{2}$ and $a x^{2}+b x+c$ where $a, b$ and $c$ are constants |
| Equations and Inequalities |  |  |
| Linear and Quadratic Equations | form and solve equations | - solve fractional equations with numerical and/or one term linear algebraic denominator and at most two-term linear algebraic numerators in one variable <br> - form and solve linear equations in practical situations <br> - solve two simultaneous linear equations in two variables, using graphical, substitution and elimination methods <br> - solve quadratic equations by factorisation, formula and graphical methods <br> - solve fractional equations with numerical and linear algebraic denominators/numerators leading to a linear or quadratic equation <br> - solve two simultaneous equations, one linear and the other quadratic equation <br> - form and solve quadratic equations in practical situations |


|  |  | - form and solve linear simultaneous equations in practical situations <br> - solve quadratic equations by completing the square <br> - derive the formula $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$, given $a x^{2}+b x+c=0$, where $\boldsymbol{a} \neq 0, b$ and $c$ are constants <br> - solve cubic equations by trial and improvement and/or graphical methods |
| :---: | :---: | :---: |
| Inequalities | form and solve inequalities | - solve simple linear inequalities <br> - indicate the region containing the points whose coordinates satisfy at least one inequality of the form $a x+b y<c, a x+b y \leq c, a x+b y>c$, $a x+b y \geq c$ where $a, b$, and $c$ are integers <br> - form and solve linear inequalities in practical situations <br> - solve quadratic inequalities algebraically and graphically |
| Functions |  |  |
| Graphs | acquire knowledge on graphs | - find the equations of line graphs <br> - interpret equations of line graphs of the form $y=m x+c$ <br> - graph functions of the form $y=a x^{2}+b x+c$ and $y=a x^{3}$ <br> - recognise and interpret graphs of $y=a x^{2}+b x+c$ and $y=a x^{3}$ <br> - graph functions of the form $y=k x^{\frac{1}{a}}$, where $a=$ 2, 3; and $k$ is a constant $y=\frac{k}{x^{a}}$, where $a=1,2,3$; and $k$ is a constant $y=a^{x}$, where $\boldsymbol{a}$ is a positive integer <br> - recognise and interpret graphs of the form, $y=k x^{\frac{1}{a}}$ where $a=2,3$; and $k$ is a constant $y=\frac{k}{x^{a}}$, where $a=1,2,3$; and $k$ is a constant $y=a^{x}$, where a is a positive integer |


|  |  | - estimate the gradient at a point by drawing a <br> tangent <br> - interpret the gradient at a point by drawing a <br> tangent <br> - interpret and apply translations in the context of a <br> graphical representation |
| :--- | :--- | :--- |
| Graphs in Practical Situations |  |  |
| Graphs | acquire knowledge on graphs | - draw graphs from practical situations <br> - interpret graphs representing practical situations |

## GEOMETRY

| Topics | General Objectives | Specific Objectives |
| :---: | :---: | :---: |
|  | Learners should be able to:- | Learners should be able to:- |
| Geometrical Terms and Relationships |  |  |
| Geometrical terms and relationships | acquire further knowledge on geometrical terms and relationships | - use and interpret geometrical terms <br> - use properties of angles to calculate specified angles and/or length of line segments including angle properties of special triangles and quadrilaterals <br> - use properties of polygons to calculate specified angles and/or sides |
| Circles |  |  |
| Angle properties of circle | acquire knowledge on properties of a circle | - calculate specified angles or line segments using the properties:- angle in a semi-circle is $90^{\circ}$, angle between tangent and radius at a point of contact is $90^{\circ}$, angle at the centre of a circle is twice the angle at the circumference subtended by the same arc, angles at the circumference subtended by the same arc are equal, opposite angles in a cyclic quadrilateral are supplementary <br> calculate specified angles or line segments using the property: angle between tangent and chord through the point of contact is equal to the angle subtended by the same chord at the circumference in the alternate segment |
| Symmetry properties of circle |  | - calculate specified angles or line segments using the properties :- equal chords are equidistant from the centre, chords equidistant from the centre are equal in length, perpendicular through the centre to a chord bisects the chord, tangents from an external point to the circle are equal in length |
| Symmetry |  |  |
| Reflectional symmetry | understand and use properties of symmetry | - recognise and describe line symmetry |
| Rotational symmetry |  | - recognise and describe rotational symmetry |
| Symmetry properties of polygons |  | - recognise and use symmetry properties of plane shapes <br> - recognise and use symmetry properties of prisms (including cylinder) and pyramids (including cone) |


| Trigonometry |  |  |
| :---: | :---: | :---: |
| Bearings | understand and use bearings | - interpret and use three-figure bearings to describe journeys |
| Trigonometric ratios and Pythagorean theorem | understand and apply trigonometric ratios and Pythagorean theorem | - apply Pythagorean theorem and the sine, cosine and tangent ratios for acute angles to calculate a side or an angle of a right-angled triangle including problems involving angles of elevation, depression and bearings <br> - solve problems using sine and cosine ratios for angles in the range of $0^{\circ}$ to $180^{\circ}$ <br> - solve problems using sine and cosine ratios for angles between $180^{\circ}$ and $360^{\circ}$ |
| Sine and Cosine Rule | understand and use sine and cosine rules | - solve problems using the sine rule (formula is to be provided) <br> - solve problems using the cosine rule (formula is to be provided) <br> - solve simple trigonometric problems in 3-D |
| Co-ordinate Geometry |  |  |
| Distances | understand and use coordinates in Cartesian plane | - calculate the distance between two points |
| Coordinates of midpoint of the line segment |  | - calculate the coordinates of the midpoint of a line segment given the coordinates of its endpoints <br> - calculate the coordinates of the end point of a line segment given the coordinates of its midpoint and one endpoint |
| Loci in 2-D |  |  |
| Loci in 2-D | understand and use loci in 2D | - draw and/or state the locus of points at a given distance from a given point <br> - draw and/or state the locus of points at a given distance from a given line <br> - draw and/or state the locus of points equidistant from two given points <br> 4-draw and/or state the locus of points equidistant from two given intersecting lines <br> - draw and/or state the locus of points equidistant from two given parallel lines <br> - use the locus of points at a given distance from a given point and the method of intersecting loci to locate specific points <br> - use the locus of points at a given distance from a given line and the method of intersecting loci to locate specific points <br> - use the locus of points equidistant from two given points and the method of intersecting loci to locate specific points |
|  |  | - use the locus of points equidistant from two given intersecting lines and the method of intersecting loci to locate specific points |


|  |  | - use the locus of points from two given parallel lines and the method of intersecting loci to locate specific points <br> - use the construction of angle bisectors, perpendicular bisector of a line segment, the drawing of a line parallel to a given line in loci problems <br> - determine the locus of a point moving in a 2-D plane according to given rules (none being an inequality) |
| :---: | :---: | :---: |
| Transformations |  |  |
| Reflection | understand and use transformations | - reflect simple plane figures on a co-ordinate grid |
|  |  | - recognise and describe fully a reflection given a shape and its image on a co-ordinate grid |
| Rotation |  | - rotate simple plane shapes about any point through multiples of right-angles on a co-ordinate grid <br> - recognise and describe fully a rotation given a shape and its image on a co-ordinate grid |
| Translation |  | - translate plane figures on a co-ordinate grid <br> - recognise and describe a translation given a shape and its image on a co-ordinate grid using a vector |
| Enlargement |  | - draw an enlargement of a simple shape on a coordinate grid <br> - recognise and describe fully an enlargement <br> - make, use and interpret scale drawings e.g. maps <br> - use the relationship between lengths of line segments and areas of similar plane shapes to solve problems <br> - use the relationship between lengths of line segments, surface areas and volumes of similar solids to solve problems <br> - describe a single transformation which is equivalent to a combination of two transformations |


| Vectors |  |  |
| :---: | :---: | :---: |
| Vectors | understand and use vectors | - represent vectors by $\binom{x}{y}, \mathbf{a}, \underline{a}$, a or $\xrightarrow[\mathbf{A B}]{ }$ <br> - represent vectors by directed line segments <br> - add and subtract vectors <br> - multiply vectors by a scalar <br> - calculate the magnitude of a vector <br> - use the sum and the difference of two vectors to express given vectors in terms of two coplanar vectors |


| Topics | General Objectives | Specific Objectives |
| :---: | :---: | :---: |
|  | Learners should be able to:- | Learners should be able to:- |
| Data Handling |  |  |
| Data Processing | collect and manipulate statistical data | - collect raw data (discrete and continuous) <br> - represent data in a frequency table <br> - construct and interpret a histogram (equal intervals), frequency polygon and cumulative frequency curve <br> - understand the reasons for organising, presenting data in a tabular or diagrammatic form and point out advantages and/or disadvantages that particular representations may have <br> - construct and interpret histograms with unequal intervals |
| Measures of Central Tendency | acquire further knowledge on measures of central tendency | - find mean, median, mode and modal class <br> - estimate median from a cumulative frequency curve and by linear interpolation from a cumulative frequency table <br> - interpret measures of central tendency (median, mean, mode) |
| Measures of Dispersion | acquire knowledge on measures of dispersion | - calculate range and interquartile range <br> - estimate quartiles and percentiles from a cumulative frequency curve and by linear interpolation from a cumulative frequency table <br> - calculate variance and standard deviation <br> - interpret measures of dispersion (interquartile range, variance and standard deviation) |
| Scatter Graphs | understand and use scatter diagrams | - plot and interpret scatter graphs <br> - draw the line of best fit by inspection <br> - find the equation of the line of best fit |
| Probability |  |  |
| Simple Probability | understand and use probability | - understand and use the vocabulary of probability in practical situations <br> - understand and use the probability scale <br> - calculate probability of a single event <br> - distinguish between theoretical and experimental probability <br> - calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate |
|   <br> Mutually Exclusive <br> and Independent <br> Events  |  | - understand the meaning of mutually exclusive and independent events. <br> - use addition and multiplication of probabilities, as appropriate, in simple cases |

