# Steel Design (CIVI 306) 

By:
Michael Terk

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## C O N N E X I O N S

Rice University, Houston, Texas

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## Chapter 1

## Compression Members

### 1.1 Deriving Euler's equation ${ }^{1}$

### 1.1.1 Derivation of Euler's equation

Start with the differential equation giving the deflected shape of an elastic member subjected to bending.

$$
\begin{align*}
M & =-\left(E I \frac{d y}{d}\right)  \tag{1.1}\\
& =P y
\end{align*}
$$

Set equal to zero.

$$
\begin{equation*}
E I \frac{d y}{d}+P y=0 \tag{1.2}
\end{equation*}
$$

Divide everything by $E I$.

$$
\begin{equation*}
\frac{d y}{d}+\frac{P}{E I} y=0 \tag{1.3}
\end{equation*}
$$

Set the variable, $\alpha^{2}$

$$
\begin{equation*}
\alpha^{2}=\frac{P}{E I} \tag{1.4}
\end{equation*}
$$

then, plug that in to get:

$$
\begin{equation*}
\frac{d y}{d}+\alpha^{2} y=0 \tag{1.5}
\end{equation*}
$$

Since this is a second order, linear, ordinary differential equation with constant coefficients, it solves to:

$$
\begin{align*}
y & =A \sin (\alpha x) \\
& =B \cos (\alpha x) \tag{1.6}
\end{align*}
$$

Take the boundary condition that $x=0$ and $y=0$ to solve for $B$

$$
\begin{align*}
y(0) & =0 \\
& =A(0)  \tag{1.7}\\
& =B(1)
\end{align*}
$$

[^0]\[

$$
\begin{equation*}
B=0 \tag{1.8}
\end{equation*}
$$

\]

Now, take the boundary conditions $x=L$ and $y=0$.

$$
\begin{align*}
y(L) & =0  \tag{1.9}\\
& =A \sin (\alpha L)
\end{align*}
$$

Since $A$ cannot equal zero:

$$
\begin{equation*}
\sin (\alpha L)=0 \tag{1.10}
\end{equation*}
$$

Take the sine inverse of both sides, and $\alpha L$ can be $0, \pi, 2 \pi$, etc. So...

$$
\begin{equation*}
\alpha L=n \pi \tag{1.11}
\end{equation*}
$$

Solve for $\alpha^{2}$

$$
\begin{equation*}
\alpha^{2}=\frac{n^{2} \pi^{2}}{L^{2}} \tag{1.12}
\end{equation*}
$$

Set the two $\alpha^{2}$ 's equal and solve for $P$.

$$
\begin{equation*}
P_{\mathrm{cr}}=\frac{n^{2} \pi^{2}(E I)}{L} \tag{1.13}
\end{equation*}
$$

Assume that $n=1$
Now we can solve for $F_{\text {cr }}$ using this equation.

$$
\begin{align*}
F_{\text {cr }} & =\frac{P_{\text {cr }}}{A_{g}} \\
& =\frac{\pi^{2} E}{L^{2}} \frac{I}{A}  \tag{1.14}\\
& =\frac{\pi^{2} E r^{2}}{(k L)^{2}}
\end{align*}
$$

where:

$$
\begin{equation*}
r=\sqrt{\frac{I}{A}} \tag{1.15}
\end{equation*}
$$

### 1.2 Example finding $\mathrm{Fcr}^{2}$

### 1.2.1 Problem

A W12 X 72 is used as a column. It is 10 feet long and the steel strength is 50 ksi . Find the maximum compressive load it can hold.

[^1]
### 1.2.2 Givens

The first section of the Manual will give the properties for the W12 X 72 column. $A_{g}, I_{x}$, and $I_{y}$ are found on page 1-20 and 1-21.

- W12 X 72
- $1=10 \mathrm{ft}$
- $F_{y}=50 \mathrm{ksi}$
- $A_{g}=21.2$
- $I_{x}=597$
- $I_{y}=195$
- take $K=1$


### 1.2.3 Solution

The equations and AISC guidelines for solving this and other columns and compression member problems can be found in the Manual starting on page 16.1-27.

1. First, find $r$ (governing radius of gyration about the axis of buckling, in.):

$$
\begin{align*}
r & =r_{y} \\
& =\sqrt{\frac{195}{21.1}}  \tag{1.16}\\
& =3.04
\end{align*}
$$

2. Section E2 of the Specifications section gives the equations to find the design compressive strength.

- $A_{g}=$ gross area of member, square inches
- $F_{y}=$ specified minimum yield stress, ksi
- $E=$ modulus of elasticity, ksi
- $K=$ effective length factor
- $l=$ laterally unbraced length of member, in.

3. Next find the design compressive strength by first finding the value for $\lambda_{c}$.

$$
\begin{align*}
\lambda_{c} & =\frac{K l}{r \pi} \sqrt{\frac{F_{y}}{E}}  \tag{1.17}\\
& =0.519
\end{align*}
$$

4. Since this is less than 1.5 the equation:

$$
\begin{equation*}
F_{\mathrm{cr}}=0.658^{\lambda_{c}}{ }^{2} F_{y} \tag{1.18}
\end{equation*}
$$

can be used for Fcr (Section 1.1) and the column is considered short and stocky.
5. Therefore,

$$
\begin{equation*}
F_{\mathrm{cr}}=44.7 \mathrm{ksi} \tag{1.19}
\end{equation*}
$$

### 1.2.4 Answer

Now we can use the equation

$$
\begin{align*}
\phi P_{n} & =\phi F_{\mathrm{cr}} A_{g}  \tag{1.20}\\
& =802
\end{align*}
$$

where

$$
\begin{equation*}
\phi=0.85 \tag{1.21}
\end{equation*}
$$

So 802 k is the maximum load the column can sustain. needs figure and help on question

### 1.3 Example finding capacity ${ }^{3}$

### 1.3.1 Problem

Find the capacity of a W14 X 74 column of A36 steel and a length of 20 feet.

### 1.3.2 Givens

The first section of the Manual will give the properties for the W14 X 74 column. $A_{g}, I_{x}, I_{y}, r_{y}$, and $r_{x}$ are found on pages 1-18 and 1-19.

- W14 X 74
- $l=20 \mathrm{ft}$
- $F_{y}=36 \mathrm{ksi}$
- $A_{g}=21.8$
- $I_{x}=795$
- $I_{y}=134$
- $r_{x}=6.04$
- $r_{y}=2.48$


### 1.3.3 Solution

The equations and AISC guidelines for solving this and other columns and compression member problems can be found in the Manual starting on page 16.1-27.

1. First find the slenderness ratio to determine about which axis the bending will occur.

$$
\begin{align*}
\frac{K l}{r_{x}} & =\frac{1 \times 20 \times 12}{6.04}  \tag{1.22}\\
& =39.7 \\
\frac{K l}{r_{y}} & =\frac{1 \times 20 \times 12}{2.48}  \tag{1.23}\\
& =96.77
\end{align*}
$$

2. Since $\frac{K l}{r_{y}}$ is greater than $\frac{K l}{r_{y}}$, the $F_{\text {cry }}$ will be less than $F_{\text {crx }}$ and $F_{\text {cry }}$ will be the governing factor. The bending will be about the y-axis. Now, we can solve for $F_{\text {cr }}$ and $P_{n}$.
3. Section E2 of the Specifications section gives the equations to find the design compressive strength.

- $A_{g}=$ gross area of member, square inches
- $F_{y}=$ specified minimum yield stress, ksi
- $E=$ modulus of elasticity, ksi
- $k=$ effective length factor
- $l=$ laterally unbraced length of member, in.

[^2]4. Next find the design compressive strength by first finding the value for $\lambda_{c}$.
\[

$$
\begin{align*}
\lambda_{c} & =\frac{K l}{r \pi} \sqrt{\frac{F_{y}}{E}}  \tag{1.24}\\
& =1.085
\end{align*}
$$
\]

5. Since this is less than 1.5 the equation:

$$
\begin{equation*}
F_{\mathrm{cr}}=0.658^{\lambda_{c}}{ }^{2} F_{y} \tag{1.25}
\end{equation*}
$$

can be used for $F_{\text {cr }}$.
6. Therefore,

$$
\begin{equation*}
F_{\mathrm{cr}}=21.99 \mathrm{ksi} \tag{1.26}
\end{equation*}
$$

### 1.3.4 Answer

Now we can use the equation

$$
\begin{align*}
\phi P_{n} & =\phi F_{\text {cr }} A_{g}  \tag{1.27}\\
& =408
\end{align*}
$$

where

$$
\begin{equation*}
\phi=0.85 \tag{1.28}
\end{equation*}
$$

So 408 k is the maximum load the column can sustain.

### 1.4 Width Thickness Ratio ${ }^{4}$

In order to use section E2 (page 16.1-27) of the Manual, the width-thickness ratios must be less than $\lambda_{r}$ This means the member is slender.

### 1.4.1 Width Thickness Ratio

For compact members: $\frac{b}{t} \leq \lambda_{p}$. For non-compact members: $\lambda_{p} \leq \frac{b}{t} \leq \lambda_{r}$.
In this case, $\frac{b}{t}$ is the Width Thickness Ratio. The specifications for the widths of different shaped members, $b$ can be found on page 16.1-12 of the Manual. The $\mathbf{t}$ refers to the thickness of the member. Also, on pages 16.1-14 and 16.1-15, you will find the Limiting Width-Thickness Ratios for Compression Elements tables. These tables will give the an equation for $\lambda_{p}$ (compact) and for $\lambda_{r}$ (non-compact) depending on a description of the element.

### 1.4.2 Stiffness

"Two types of elements must be considered: unstiffened elements, which are unsupported along one edge parallel to the direction of the load, and stiffened elements, which are supported along both edges." LRFD Steel Design Second Edition: William T. Segui, 1999

For example, an L-shaped member has two unstiffened elements because each member is only supported (or connected) at one end. Also, a C-shaped member has two unstiffened elements and one stiffened element. The web is stiffened because it is supported on both sides as opposed to the flanges which are unstiffened .

## Exercise 1.1

(Solution on p. 16.)
How many unstiffened and stiffened elements would an I-shaped member have?

[^3]
### 1.4.3 B5: Local Buckling

One example of using the section B5: LOCAL BUCKLING 1. Classification of Steel Sections on page 16.1-12 in the AISC Steel Manual is with an I-shaped member. Since the flanges are unstiffed, and it is I-shaped, use a width, $b$, from part (a). Therefore, $b$ can be taken as half the full-flange width, $b_{f}$. This means the Width Thickness Ratio is: $\frac{b_{f}}{t}$. Then, since the web is a stiffened element, use part (a) of the stiffened elements section and use the distance $h$ for the "width." This gives the Width Thickness Ratio as: $\frac{h}{t}$.

### 1.4.4 Values for Width Thickness Ratio

An easy way of finding values for $\frac{b}{t}$ can be found on page 16.1-150. Here, Table 6: Slenderness Ratios of Elements as a Function of $F_{y}$ From Table B5.1, gives values for the ratios that are given in formula form in Table B5.1 (16.1-14). This is useful in cutting back on calculating errors.

For example, the flanges of I-shaped sections in pure compression with $F_{y}=50 \mathrm{ksi}$, Table 6 gives the value of 15.9 for:

$$
\begin{align*}
\lambda_{r} & =0.56 \sqrt{\frac{E}{F_{y}}}  \tag{1.29}\\
& =15.9
\end{align*}
$$

NOTE: Values for the ratios $\frac{b_{f}}{2 t_{f}}$ and $\frac{h}{t_{w}}$ are tabulated in the dimensions and properties tables in Part 1 of the Manual

### 1.5 Effective Length ${ }^{5}$

### 1.5.1 Define effective length

The equations for critical buckling load include the variable KL which is the effective length. $K$ is the effective length factor. Values for $K$ vary depending on the load and type of supports of a member. A listing of the values can be found in the Manual on page 16.1-189 in Table C-C2.1. For instance, the value for $K$ with the condition that both ends of a column are rotation free and translation fixed (pinned) is 1.0.

### 1.5.2 Technical vs. recommended values of $K$

"Two values for $K$ are given: a theoretical value and a recommended design value to be used when the ideal end condition is approximated. Hence, unless a 'fixed' end is perfectly fixed, the more conservative design values are to be used. Only under the most extraordinary circumstances would the use of the theoretical values be justified. Note, however, that the theoretical and recommended design values are the same for conditions (d) and (f) in the Commentary Table C-C2.1. The reason is that any deviation from a perfectly frictionless hinge or pin introduces rotational restraint and tends to reduce $\mathbf{K}$. Therefore use of the theoretical values in these two cases is conservative." LRFD Steel Design Second Edition: William T. Segui, 1999
note: The larger the effective length, the less strength there is in a column. So, if there is a choice of effective lengths, the larger value will give the more conservative strength value.

[^4]
### 1.5.3 Actual length vs. effective length

Sometimes the actual length of a member differs from the effective length. This is true when a member is supported somewhere in the middle in addition to at the two ends. The effective length then, is the length from one support to another. Also, a member can be supported two different ways in two different axes. For example, a column can be supported at the top in the bottom while looking at it in the x-direction, but braced in the middle when looking at it from the y-direction. We refer to the distance between the supports in the y-direction and the x-direction as $L_{y}$ and $L_{x}$, respectively.

### 1.5.4 Using KL(x)

The design strengths given in the column load tables beginning on page 4-21 are based on the effective length with respect to the y-axis. A procedure was developed (as follows) to use $K_{x} L$ in the tabulated values.

The tablulated values in chapter 4 of the Manual are in terms of the $y$-axis being the stong axis. This means they are based on the values of KL being equal to $K_{y} L$. However, if a situation occurs where one would need the values of KL with respect to the x-axis, the following procedure can be used.

The KL as tabulated is equal to either $K_{y} L$ or $\frac{K_{x} L}{\frac{Y_{x}}{r_{y}}}$. We can obtain $\frac{K_{x} L}{\frac{Y_{x}}{r_{y}}}$ by:

1. $\frac{\mathrm{KL} y}{r_{y}}$
2. $y=\frac{r_{y}}{r_{x}}$
3. $\mathrm{KL} y=\mathrm{KL} \frac{r_{y}}{r_{x}}$
4. $\frac{K_{x} L}{\frac{1}{r_{y}}}=\mathrm{KL} y$

### 1.6 Example with different effective lengths ${ }^{6}$

### 1.6.1 Problem

A W12 X 65 column, 24 feet long, is pinned at both ends in the strong direction, and pinned at the midpoint and the ends in the weak direction. The column has A36 steel.

### 1.6.2 Method 1 - with column tables

### 1.6.2.1 Number 1 - Find effective length

Since the x -direction is the strong one and the y -direction is the weak one, then:

$$
\begin{align*}
& L_{x}=24  \tag{1.30}\\
& L_{y}=12 \tag{1.31}
\end{align*}
$$

Notice that the effective length in the y-direction is half the total length of the member because there is a lateral support at the midpoint.

Looking at the Manual on page 16.1-189 shows that the $K$ value for a column pinned at both ends is 1.0. Since the column is pinned at the ends and at the middle,

$$
\begin{align*}
& K_{x}=1  \tag{1.32}\\
& K_{y}=1 \tag{1.33}
\end{align*}
$$

[^5]Now we can say that:

$$
\begin{align*}
K_{x} L_{x} & =1 \times 24  \tag{1.34}\\
& =24 \\
K_{y} L_{y} & =1 \times 12  \tag{1.35}\\
& =12
\end{align*}
$$

### 1.6.2.2 Number 2-Finding the capacity

Since, the steel is A36, you cannot use the column tables from Chapter 4 of the Third Edition Manual as the values are all given in terms of $F_{y}=50 \mathrm{ksi}$. However, in the Second Edition, in Chapter 3, the column tables give information for terms of $F_{y}=36 \mathrm{ksi}$.

From page 3-24 of the Second Edition Manual, the capacity for a W12 X 65 column with $K_{y} L_{y}=12$ is 519 kips.

Then to find $K_{x} L_{x}$ in terms of $r_{y}, K_{x} L_{x}$ must be divided by: $\frac{r_{x}}{r_{y}}$. This gives:

$$
\begin{align*}
\frac{K_{x} L_{x}}{\frac{r_{x}}{r_{y}}} & =\frac{24}{1.75}  \tag{1.36}\\
& =13.71
\end{align*}
$$

This is close enough to 14 , that we can then look in the tables for the KL value of 14 , or interpolate for 13.71) and find the capacity for the W12 X 65 member. The capacity is 497 kips.

### 1.6.3 Method 2 - with buckling formulas

If you do not have the tables for A36 steel, you must use the formulas on page 16.1-27 of the Manual.

### 1.6.3.1 Number 1 - Show the width-thickness ratio

In order for the equations in section E2 of the Manual to apply, the width-thickness ratio must be $\lambda_{r}$.

$$
\begin{equation*}
\frac{b_{f}}{2 t_{f}}<\lambda_{r} \tag{1.37}
\end{equation*}
$$

The value for $\frac{b_{f}}{2 t_{f}}$ (9.92) can be found on page 16.1-21, as well as the value for $\frac{h}{t_{w}}$ (24.9). The formula for $\lambda_{r}$ can be found on page 16.1-14/15. Then, the value for that formula can be found on page 16.1-150.

The flanges are unstiffened and in pure compression, so the formula is:

$$
\begin{equation*}
\frac{b_{f}}{2 t_{f}}=9.92<0.56 \sqrt{\frac{E}{F_{y}}}=15.9 \tag{1.38}
\end{equation*}
$$

The web is stiffened and in compression, so the formula is:

$$
\begin{equation*}
\frac{h}{t_{w}}=24.9 \leq 1.49 \sqrt{\frac{E}{F_{y}}}=42.3 \tag{1.39}
\end{equation*}
$$

Another way to easily find the formulas for $\lambda_{r}$ is to go to page 16.1-183 and look at the picture of the I-shaped member. The arrows point to either the flange or the web and formulas correspond to the arrows giving the axial compression formulas that you need for that element of the member.

### 1.6.3.2 Number 2-Compute slenderness ratios

The slenderness ratios can be found for both the x -axis and the y-axis. We know $K$, and $L$, and $r$ can be found in the properties section of the Manual on page 1-20.

$$
\begin{align*}
\frac{K_{x} L_{x}}{r_{x}} & =\frac{24 \times 12 \times 1}{5.28}  \tag{1.40}\\
& =54.54 \\
\frac{K_{y} L_{y}}{r_{x}} & =\frac{12 \times 12 \times 1}{3.02}  \tag{1.41}\\
& =47.68
\end{align*}
$$

Then, using Table 3-36 on page 16.1-143 of the Manual and interpolation, we can determine that $\phi_{c} F_{\text {cr }}=26.21$, and that $\phi_{c} P_{n}=\phi_{c} F_{\text {cr }} A_{g}=500 k$

### 1.6.4 Answer

The capacity of the W12 X 65 column is 500 kips.

### 1.7 Slenderness Ratio ${ }^{7}$

### 1.7.1 Definition

## Definition 1.1: Slenderness ratio

The ratio of the effective length of a column to the radius of gyration of the column, both with respect to the same axis of bending
Manual of Steel Construction Third Edition - AISC, 2001.
In algebra form, the slenderness ratio is:

$$
\begin{equation*}
\frac{K L}{r} \tag{1.42}
\end{equation*}
$$

### 1.7.2 More on the topic

The variable that governs $F_{\text {cr }}$ is the slenderness ratio. The larger the slenderness ratio, the less strength there is in a column. This means the capacity decreases as the slenderness ratio increases.

The AISC recommendation from section B7 are:

$$
\begin{equation*}
\frac{K L}{r} \leq 200 \tag{1.43}
\end{equation*}
$$

### 1.7.3 Where you see slenderness ratio

The slenderness ratio shows up when comparing the strength of columns and more specifically in the design strength formula variable, $\lambda_{c}$ :

$$
\begin{equation*}
\lambda_{c}=\frac{K L}{r \pi} \sqrt{\frac{F_{y}}{E}} \tag{1.44}
\end{equation*}
$$

[^6]
### 1.8 Equations for $\mathbf{P}^{8}$

### 1.8.1 What is P ?

The abbreviation, $P$, is used to describe the axial load on a member.

### 1.8.2 Basic requirements

The basic requirements for compression members are covered in Chapter E of the AISC Specification (page 16.1-27). The relationship between loads and strength takes the form:

$$
\begin{equation*}
P_{u} \leq \phi_{c} P_{n} \tag{1.45}
\end{equation*}
$$

where:

- $P_{u}=$ sum of factored loads
- $P_{n}=$ nominal compressive strength $A_{g} F_{\text {cr }}$
- $F_{\text {cr }}=$ critical buckling stress
- $\phi_{c}=$ resistance factor for compression members $=0.85$


### 1.8.3 Design strength equation

The design strength equation,

$$
\begin{equation*}
P_{n}=A_{g} F_{\mathrm{cr}} \tag{1.46}
\end{equation*}
$$

has the variable $F_{\mathrm{cr}}$ which is a function of $\lambda_{c}$, the slenderness parameter. The equation for $\lambda_{c}$ is:

$$
\begin{equation*}
\lambda_{c}=\frac{K l}{r \pi} \sqrt{\frac{F_{y}}{E}} \tag{1.47}
\end{equation*}
$$

The equation for $F_{\text {cr }}$, then depends on the value of $\lambda_{c}$. For instance:

- For $\lambda_{c} \leq 1.5$

$$
\begin{equation*}
F_{\mathrm{cr}}=0.658^{\lambda_{c}{ }^{2}} F_{y} \tag{1.48}
\end{equation*}
$$

- For $\lambda_{c}>1.5$

$$
\begin{equation*}
F_{\mathrm{cr}}=\frac{0.877}{\lambda_{c}{ }^{2}} F_{y} \tag{1.49}
\end{equation*}
$$

### 1.8.4 Slenderness parameter

The slenderness parameter incorporates the material properties. If it is less than 1.5 , the compression member is said to be elastic. Then, if the value is over $1.5, F_{\text {cr }}$ must be reduced to account for the effects of initial crookedness.

Put in figure p96.

[^7]
### 1.9 Effective Length and Frames ${ }^{9}$

### 1.9.1 A different effective length for frames

Effective length can be found easily on isolated columns by using Table C-C2 in the Commentary of the Specification Section of the Manual. However, this table will not work very well with rigid frames. Columns in a frame are not independent, they are continuous. The buckling of one member will affect all the members around it. Therefore, the end conditions necessary for using Table C-C2 are not sufficient. It is important to account for the degree of restraint by connecting members of a column in a frame.

A frame can be unbraced or braced, where unbraced means horizontal displacement in possible. A frame can also have sidesway.
"The rotational restraint provided by beams, or girders, at the end of a column is a function of rotational stiffnesses of the members intersecting at the joint." LRFD Steel Design Second Edition - William T. Segui, 1999. The restraint is proportional to $\frac{E I}{L}$.

$$
\begin{align*}
G & =\frac{\sum \frac{E_{c} I_{c}}{L_{c}}}{\sum \frac{E_{g} I_{g}}{L_{g}}}  \tag{1.50}\\
& =\frac{\sum \frac{I_{c}}{L_{c}}}{\sum \frac{I_{g}}{L_{g}}}
\end{align*}
$$

$K$, then depends on the ratio of column stiffness to the girder stiffness at each end.

### 1.9.2 G and K relationships

$K$ is relatively small when a slender column is connected to a girder of large cross section. This is because the girder effectively prevents rotation and acts as a fixed end. The $G$ value for this case is rather small too. $K($ or $G)$ is relatively large when the ends of very stiff columns are connected to rather flexible beams. This is because the ends of the column can more freely rotate and approach the pinned condition.
"The relationship between $G$ and $K$ has been quantified in the Jackson-Mooreland Alignment Charts (Johnston, 1796), which are reproduced in Figure C-C2.2 in the Commentary. To obtain a value of $K$ from one of the nomograms, first calculate the value of $G$ at each end of the column, letting one value be $G_{A}$ and the other be $G_{B}$. Connect $G_{A}$ and $G_{B}$ with a straight line, and read the value of $K$ on the middle scale. The effective length factor obtained in this manner is with respect to the axis of bending, which is the axis perpendicular to the plane of the frame. A separate analysis must be made for buckling about the other axis. Normally the beam-to-column connections in this direction will not transmit moment, sidesway is prevented by bracing, and $K$ can be taken as 1.0 .

### 1.9.3 Value of G for pinned support

$G$ can be taken as 10 at a pinned support because at a pin connection, the situation is just like a very stiff column attached to infinitely flexible girders. This means the girders have zero stiffness. Then, the ratio of column stiffness to girder stiffness would be infinite for a perfectly frictionless hinge. This end condition can only be approximated in practice, so the discussion accompanying the alignment chart recommends that $G$ be taken as 10 .

### 1.10 Example finding $K$ for frames ${ }^{10}$

### 1.10.1 Problem

Find the $K$-value for all columns of the following frame. All columns are oriented with the web in the plane of the paper, (the plane of buckling).

[^8]

Figure 1.1: (not drawn to scale)

### 1.10.2 Section Properties

The section properties can be found in the Dimensions and Properties section of the Manual. The lengths are found on Figure 1.1.

W8 X 31

- $I=110 \mathrm{in} .^{4}$
- $L=15 \mathrm{ft}$

W14 X 30

- $I=291 \mathrm{in}^{4}$
- $L=20 \mathrm{ft}$

W18 X 50

- $I=800 \mathrm{in}^{4}$
- $L=30 \mathrm{ft}$ and $L=20 \mathrm{ft}$


### 1.10.3 Solution

Since the columns and beams are oriented with the web as the plane of buckling, the axis of bending is the x-axis. Therefore, all values of $I$, above, are with respect to the x-axis.


Figure 1.2: An example of a column bending in the $x$-direction.

## For each column:

1. First we must determine the $G$ values for each corner of the frame with the equation:

$$
\begin{equation*}
G=\frac{\sum \frac{I_{c}}{L_{c}}}{\sum \frac{I_{g}}{L_{g}}} \tag{1.51}
\end{equation*}
$$

where, $c$ is for column and $g$ is for girder or beam.
2. Then we use Fig. C-C2.2a and Fig. C-C2.2b in the Manual to find the $K$ values for each column. If a column is sidesway inhibited, it is braced against any sideways movement; if a column is sidesway uninhibited, it is not braced in the sideways direction.

## Column AB

- $G_{A}=10$ because it is a pinned support.
- $G_{B}=\frac{\frac{110}{15}}{\frac{800}{30}}=0.274$
- $A B$ is sidesway inhibited (the support at $J$ braces any sideways motion of the column) so lining up the value of $G_{A}$ and $G_{B}$ on 16.1-191 gives: $K=0.77$


## Column CD

- $G_{C}=10$ because it is a pinned support.
- $G_{D}=\frac{\frac{110}{15}+\frac{110}{12}}{\frac{800}{30}+\frac{800}{20}}=0.2475$
- $C D$ is sidesway inhibited (the support at $J$ braces any sideways motion of the column) so lining up the value of $G_{C}$ and $G_{D}$ on 16.1-191 gives: $K=0.76$


## Column GF

- $G_{F}=1$ because it is a pinned support that cannot move sideways, so it is like a fixed support (?check on this).
- $G_{G}=\frac{\frac{110}{15}+\frac{110}{12}}{\frac{800}{20}+\frac{800}{20}}=0.206$
- $G F$ is sidesway inhibited (the support at $J$ braces any sideways motion of the column) so lining up the value of $G_{G}$ and $G_{F}$ on 16.1-191 gives: $K=0.67$


## Column ED

- $G_{D}=0.2475$ from before.
- $G_{E}=\frac{\frac{110}{12}}{\frac{291}{20}}=0.63$
- $E D$ is sidesway uninhibited (there is no sideways bracing for the top portion of the frame) so lining up the value of $G_{E}$ and $G_{D}$ on 16.1-192 gives: $K=1.15$


## Column GH

- $G_{G}=0.206$ from before.
- $G_{H}=0.63$ from before.
- $G H$ is sidesway uninhibited (there is no sideways bracing for the top portion of the frame) so lining up the value of $G_{G}$ and $G_{H}$ on 16.1-192 gives: $K=1.14$


### 1.11 Flexural-Torsional Buckling ${ }^{11}$

### 1.11.1 Definitions

There are three ways a compression member can buckle, or become unstable. These are flexural buckling, torsional buckling, and flexural-torsional buckling.

## Definition 1.2: Flexural buckling

This type of buckling can occur in any compression member that experiences a deflection caused by bending or flexure. Flexural buckling occurs about the axis with the largest slenderness ratio, and the smallest radius of gyration.
Definition 1.3: Torsional buckling
This type of buckling only occurs in compression members that are doubly-symmetric and have very slender cross-sectional elements. It is caused by a turning about the longitudinal axis. Torsional buckling occurs mostly in built-up sections, and almost never in rolled sections.

## Definition 1.4: Flexural-torsional buckling

This type of buckling only occurs in compression members that have unsymmetrical cross-section with one axis of symmetry. Flexural-torsional buckling is the simultaneous bending and twisting of a member. This mostly occurs in channels, structural tees, double-angle shapes, and equal-leg single angles.

### 1.11.2 Where to find information for flexural-torsional information

The Manual provides specifications for flexural-torsional buckling in the Specification section, Section E3 (p. 16.1-28), and Appendix E3 (p. 16.1-94. Section E3 is specifically for double-angles and tee-shaped compression member whose elements have width-thickness ratios less than $\lambda_{r}$.

Torsional variables can be found in the Dimensions and Properties section of the Manual in the first section. Torsional properties start on page 1-89 and Flexural-torsional properties on page 1-96.

[^9]
### 1.11.3 Center of Gravity and Shear Center

## Definition 1.5: Shear center

"The shear center is that point through which the loads must act if there is to be no twisting, or torsion, of the beam." LRFD Steel Design Second Edition - William T. Segui

The shear center is always located on the axis of symmetry, therefore, if a member has two axes of symmetry, the shear center will be the intersection of the two axes. Channels have a shear center that is not located on the member; the value, $e_{0}$, tabulated in the Manual is the distance from the channel to the shear center.

## Definition 1.6: Center of gravity

The center of gravity is the point at which all moments generated from the mass of the element equal zero.
For members like an I-shaped member, the center of gravity and the shear center are the exact same point where the two axes of symmetry intersect. for channels, the shear center and the center of gravity are different, which creates a couple and makes the twisting that causes torsional buckling.

### 1.11.4 Design strength for double-angle and tee-shaped compression members

Double-angles and tee-shaped members with a width-thickness ratio less than $\lambda_{r}$ should use the formula:

$$
\begin{gather*}
\phi_{c}=0.85  \tag{1.52}\\
P_{n}=A_{g} F_{\mathrm{crft}} \tag{1.53}
\end{gather*}
$$

where the "ft" of $F_{\text {crft }}$ stands for "flexural-torsional," and is expressed as:

$$
\begin{equation*}
F_{\mathrm{crft}}=\frac{F_{\mathrm{cry}}+F_{\mathrm{crz}}}{2 H}\left(1-\sqrt{1-\frac{4 F_{\mathrm{cry}} F_{\mathrm{crz}} H}{\left(F_{\mathrm{cry}}+F_{\mathrm{crz}}\right)^{2}}}\right) \tag{1.54}
\end{equation*}
$$

Here, $F_{\text {crz }}$ is expressed as:

$$
\begin{equation*}
F_{\mathrm{crz}}=\frac{G J}{A\left(r_{0}{ }^{*}\right)^{2}} \tag{1.55}
\end{equation*}
$$

where

- $r_{0}{ }^{*}=$ the polar radius of gyration about the shear center, in.
- $G=\frac{E}{2 \times(1+\vartheta)}$
- $J=$ torsional stiffness
- $H=1-\frac{y_{0}{ }^{2}}{\left(r_{0}\right)^{2}}$
- $y_{0}=$ distance between shear center and centroid, in.
- $F_{\text {cry }}=$ equation given in Section E2 for flexural buckling about the y-axis of symmetry.


## Solutions to Exercises in Chapter 1

## Solution to Exercise 1.1 (p. 5)

Four unstiffened elements (the flanges) and one stiffened element (the web). In this case, each half of one of the flanges consists of two elements joined at the web.

## Chapter 2

## Tension

### 2.1 Introduction to Tension Members ${ }^{1}$

### 2.1.1 Tension

Tension refers to the tensile force that can act on a member.


Figure 2.1: An example of a member under tension.

In order to design tension members, it is important to analyze how the member would fail under both yielding and fracture. These are the limit states. The limit state that produces the smallest design strength is considered the controlling limit state.

[^10]
### 2.1.2 Where to find information about tension members

The AISC Manual of Steel Construction lists the requirements for designing tension members in Chapter D of the Specifications Section (page 16.1-24).

### 2.2 Yielding Limit State ${ }^{2}$

### 2.2.1 Introduction to Yielding

The limit state of yielding must be considered in tension members to prevent failure from deformation. "To prevent excessive deformation, initiated by yielding, the load on the gross section must be small enough that the stress on the gross section is less than the yield stress $F_{y} .{ }^{\prime \prime}$

### 2.2.2 Equations for Yielding

$$
\begin{align*}
\phi_{t} & =0.90  \tag{2.1}\\
P_{n} & =F_{y} A_{g} \tag{2.2}
\end{align*}
$$

Here the design strength of the tension members is $\phi_{t} P_{n}$ where:

- $A_{g}=$ gross area of member, in. ${ }^{2}$
- $F_{y}=$ specified minimum yield stress, ksi


### 2.3 Fracture Limit State ${ }^{3}$

### 2.3.1 Introduction to Fracture

The limit state of fracture must be considered in tension members to prevent failure from breaking. "To prevent fracture, the stress on the net section must be less than the tensile strength $F_{u} . "$

### 2.3.2 Equations for Fracture

$$
\begin{gather*}
\phi_{t}=0.75  \tag{2.3}\\
P_{n}=F_{u} A_{e} \tag{2.4}
\end{gather*}
$$

The resistance factor is smaller for fracture than it was for yielding because failure in fracture is more sudden and serious than in yielding. Here the design strength of the tension members is $\phi_{t} P_{n}$ where:

- $A_{e}=$ effective net area, in. ${ }^{2}$
- $F_{u}=$ specified minimum tensile stress, ksi

[^11]
### 2.4 Staggered Holes

### 2.4.1 Failure due to Staggered Holes

When a member has staggered bolt holes, a different approach to finding $A_{e}$ for the fracture limit state is taken. This is because the effective net area $\left(t \times w_{n}\right)$ is different as the line of fracture changes due to the stagger in the holes. The test for the yielding limit state remains unchanged (the gross area is still the same).

### 2.4.2 Failure Lines

The net width now must account for the change in direction of the line of fracture. First, look at different ways a tension member with staggered holes can fracture. These pictures depict the different lines of failure. When analyzing a member like this, it is important to find all the lines of failure and then determine which line of failure is the weakest cross section. That cross section will be taken as the net width, $w_{n}$.


Figure 2.2

[^12]
### 2.4.3 Net Width

In order to find the net width, first the variables $s$ and $g$ must be known. They are shown in Figure 2.3.
$\qquad$


Figure 2.3

$$
\begin{equation*}
w_{n}=w_{g}-N \phi_{d}+\frac{s^{2}}{4 g} \tag{2.5}
\end{equation*}
$$

where:

- $w_{n}=$ net width
- $w_{g}=$ gross width
- $\phi_{d}=\phi_{b}+\frac{1}{16}+\frac{1}{16}$ where: $\phi_{b}=$ diameter of the bolt
- $N=$ number of bolts in cross section
- $s=$ longitudinal center-to-center spacing of any two consecutive holes, in.
- $g=$ transverse center-to-center spacing between fastener gage lines, in.
- The term $\frac{s^{2}}{4 g}$ is added for every non-straight segment


### 2.5 Effective Net Area With Shear Lag ${ }^{5}$

### 2.5.1 Why use the factor U ?

"Shear lag occurs when some elements of the cross section are not connected, as when only one leg of an angle is bolted to a gusset plate. The consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed. Lengthening the connected region will reduce this effect. Research by Munse and Chesson (1963) suggests that shear lag be accounted for by using a reduced, or effective, net area. Because shear lag affects both bolted and welded connections, the effective net area concept applies to both types of connections." LRFD Steel Design Second Edition - William T. Segui

[^13]

Figure 2.4: An L-shaped member that is only bolted on one side. This type of member will have shear lag.

### 2.5.2 Bolted Sections

The equation for the effective net area is:

$$
\begin{gather*}
A_{e}=A_{n} U  \tag{2.6}\\
U=1-\frac{x^{*}}{\ell} \leq 0.9 \tag{2.7}
\end{gather*}
$$

where:

- $A_{e}=$ effective area, in. ${ }^{2}$
- $A_{n}=$ net area, in. ${ }^{2}$
- $U=$ reduction coefficient
- $x^{*}=$ connection eccentricity, in.


Figure 2.5: An example of the connection eccentricity for an L-shaped member.

- $\ell=$ length of the connection in the direction of loading, in.


### 2.5.3 Welded sections

The equation for the effective net area depends on the type of weld

1. When the tension is transmitted only by a longitudinal weld connecting the member to something other than a plate, or the weld is both longitudinal and transverse:

$$
\begin{gather*}
A_{e}=A_{g} U  \tag{2.8}\\
U=1-\frac{x^{*}}{\ell} \leq 0.9 \tag{2.9}
\end{gather*}
$$

2. When the tension is transmitted only by a transverse weld:

$$
\begin{gather*}
A_{e}=A U  \tag{2.10}\\
U=1.0 \tag{2.11}
\end{gather*}
$$

3. When the tension is transmitted to a plate only by longitudinal welds along both edges at the end of the plate:

$$
\begin{equation*}
A_{e}=A_{g} U \tag{2.12}
\end{equation*}
$$

where

- For $\ell \geq 2 w$ $\qquad$ $U=1.00$
- For $2 w>\ell>1.5 w$ $U=0.87$
- For $1.5 w>\ell>w \ldots \ldots . . . . . . . . . . . . U=0.75$


## where:

- $A_{e}=$ effective area, in. ${ }^{2}$
- $A_{g}=$ gross area, in. ${ }^{2}$
- $A=$ area of directly connected elements, in. ${ }^{2}$
- $U=$ reduction coefficient
- $x^{*}=$ connection eccentricity, in.
- $\ell=$ length of the longest weld, in.


## Chapter 3

## Beams

### 3.1 Introduction to Beams ${ }^{1}$

### 3.1.1 Introduction

In order to design a beam in accordance with the AISC code for steel design, 6 limit states must be considered. These are yielding, Lateral-Torsional Buckling, Web Local Buckling, Flange Local Buckling, Shear Capacity, and Serviceability. Only when a beam satisfies these limit states can it be considered safe for public use.

### 3.1.2 The six limit states

- Yielding is the most common limit state and the first to address. It refers to the strength of the beam to resist the largest possible moment that can be applied to the beam. Basically, it limits the beam from bending. Yielding depends on the load, the supports, the span of the beam, and the strength of the steel.
- Lateral-Torsional Buckling, the second limit state refers to the beam's ability to hold up against torsion, or twisting in the lateral direction. This limit state compares the lateral bracing to a maximum allowable bracing length. With adequate bracing, the beam will not twist into failure.
- The third limit state, Web Local Buckling refers to the strength of the web of a member in a beam to resist failure. Basically, the width and thickness of the web must be large enough to withstand the loading conditions. This means the width-thickness ratio must fall between certain limits so the web does not collapse or fail.
- The fourth limit state, Flange Local Buckling, is just like Web Local Buckling, except the limits are for the flanges of a member in a beam. It refers to the strength of the flange of a member to resist failure. The width and thickness of the flange must be large enough to withstand the loading conditions. This means the width-thickness ratio must fall between certain limits so the flange does not collapse or fail.
- Shear Capacity, the fifth limit state, usually is not the controlling limit state, except for beams with very small lateral spans. The shear in the web of a beam must be limited so it does not exceed the maximum allowable shear.
- Serviceability, the final limit state, refers to the beam's deflection. The beam must be serviceable and not deflect so much that vibrations can be a problem and should not deflect to a noticeable angle that people can detect and feel uncomfortable with.

[^14]
### 3.2 Yielding ${ }^{2}$

### 3.2.1 Introduction to Yielding

The first limiting state to check in a beam is that of yielding. The yielding equations take in to account the stress on a member and the bending moment.

### 3.2.2 Equations to determine yielding

### 3.2.2.1 The flexural design strength as determined by the yielding limiting state is:

$$
\begin{equation*}
\phi_{b} M_{n} \tag{3.1}
\end{equation*}
$$

where:

$$
\begin{align*}
& \phi_{b}=0.90  \tag{3.2}\\
& M_{n}=M_{p} \tag{3.3}
\end{align*}
$$

### 3.2.2.2 the plastic moment corresponds to:

$$
\begin{equation*}
M_{p}=F_{y} Z \leq 1.5 M_{y} \tag{3.4}
\end{equation*}
$$

where, the moment corresponding to onset of yielding at the extreme fiber from an elastic stress distribution is:

$$
\begin{equation*}
M_{y}=F_{y} S \tag{3.5}
\end{equation*}
$$

note: $Z$ and $S$ are given in Part 1 of the Manual for each member.

### 3.3 Flange Local Buckling ${ }^{3}$

### 3.3.1 Introduction to Flange Local Buckling

The fourth limit state for beams is Flange Local Buckling, or FLB for short. It is exactly the same as Web Local Buckling, except the width-thickness ratio is in terms of the flange and not the web. This type of buckling occurs when the width-thickness ratio is not large enough to withstand the moment on the beam. The way to prevent this type of buckling is to limit the with-thickness ratio.

The limits can be computed for flange local buckling. The width-thickness ratio is compared to $\lambda_{p}$ and $\lambda_{r}$. Then the maximum moment can be calculated.

[^15]
### 3.3.2 Equations to determine FLB



Figure 3.1: The graph illustrates the options for FLB

When,

$$
\begin{equation*}
\frac{b}{t}<\lambda_{p} \tag{3.6}
\end{equation*}
$$

there is no FLB and the cross section is compact because,

$$
\begin{gather*}
M_{n}=M_{p}  \tag{3.7}\\
M_{p}=F_{y} Z \leq 1.5 M_{y} \tag{3.8}
\end{gather*}
$$

from the yielding state.
When,

$$
\begin{equation*}
\lambda_{p}<\frac{b}{t}<\lambda_{r} \tag{3.9}
\end{equation*}
$$

the graph is linear, and therefore a linear interpolation between $M_{p}$ and $M_{y}$ is used for the maximum moment.

And finally, when $\frac{b}{t}>\lambda_{r}$ the graph is non-linear, the flange is non-slender, and there is an equation to find the maximum moment in Appendix F of the Specification section of the Manual (page 16.1-96).

### 3.4 Web Local Buckling ${ }^{4}$

### 3.4.1 Introduction to Web Local Buckling

The third limit state for beams is Web Local Buckling, or WLB for short. This type of buckling occurs when the width-thickness ratio is not large enough to withstand the moment on the beam. The way to prevent this type of buckling is to limit the with-thickness ratio.

The limits can be computed for web local buckling. The width-thickness ratio is compared to $\lambda_{p}$ and $\lambda_{r}$ . Then the maximum moment can be calculated.

### 3.4.2 Equations to determine WLB



Figure 3.2: The graph illustrates the options for WLB

[^16]When,

$$
\begin{equation*}
\frac{b}{t}<\lambda_{p} \tag{3.10}
\end{equation*}
$$

there is no WLB and the cross section is compact because,

$$
\begin{gather*}
M_{n}=M_{p}  \tag{3.11}\\
M_{p}=F_{y} Z \leq 1.5 M_{y} \tag{3.12}
\end{gather*}
$$

from the yielding state.
When,

$$
\begin{equation*}
\lambda_{p}<\frac{b}{t}<\lambda_{r} \tag{3.13}
\end{equation*}
$$

the graph is linear, and therefore a linear interpolation between $M_{p}$ and $M_{y}$ is used for the maximum moment.

And finally, when $\frac{b}{t}>\lambda_{r}$ the graph is non-linear, the web is non-slender, and there is an equation to find the maximum moment in Appendix F of the Specification section of the Manual (page 16.1-96).

### 3.5 Lateral Torsional Buckling ${ }^{5}$

### 3.5.1 Introduction to Lateral Torsional Buckling

The second limit state for beams is Lateral Torsional Buckling, LTB for short. LTB occurs when the compression portion of a cross section is restrained by the tension portion and the deflection due to flexural buckling is accompanied by torsion or twisting. The way to prevent LTB is to have adequate lateral bracing at adequate intervals along the beam. The limit state is the interval of the bracing.

For each cross section, it is possible to compute limits for LTB. The laterally unbraced length, $L_{b}$, is compared to two limit states, $L_{p}$ and $L_{r}$. Depending on which limit state $L_{b}$ sits at, the maximum moment for the beam can be calculated.

[^17]
### 3.5.2 Equations to determine LTB



Figure 3.3: This graph illustrates the LTB options

- When,

$$
\begin{equation*}
L_{b}<L_{p} \tag{3.14}
\end{equation*}
$$

there is no LTB because

$$
\begin{gather*}
M_{n}=M_{p}  \tag{3.15}\\
M_{p}=F_{y} Z \leq 1.5 M_{y} \tag{3.16}
\end{gather*}
$$

from the yielding limit state.

- When

$$
\begin{equation*}
L_{p}<L_{b}<L_{r} \tag{3.17}
\end{equation*}
$$

the graph is linear, and therefore a linear interpolation between $M_{p}$ and $M_{y}$ is used for the maximum moment:

$$
\begin{equation*}
M_{n}=C_{b}\left(M_{p}-\left(M_{p}-M_{r}\right) \frac{L_{b}-L_{p}}{L_{r}-L_{p}}\right) \leq M_{p} \tag{3.18}
\end{equation*}
$$

- And finally, when

$$
\begin{equation*}
L_{b}>L_{r} \tag{3.19}
\end{equation*}
$$

the graph is non-linear, and there is an equation to find the maximum moment in Chapter F of the Specification section of the Manual (page 16.1-34):

$$
\begin{equation*}
M_{n}=M_{\mathrm{cr}} \leq M_{p} \tag{3.20}
\end{equation*}
$$

### 3.6 Serviceability ${ }^{6}$

### 3.6.1 Introduction to Serviceability

The final limit state for beams is serviceability. "A serviceable structure is one that performs satisfactorily, not causing any discomfort or perceptions of unsafety for the occupants or users of the structure" LRFD Steel Design Second Edition - William T. Segui. A beam is serviceable when it satisfies certain deflection limits. This is so there are no visible sags or deflections that lead to a flexible beam. If the beam is too flexible, it could have a problem with vibrations.

### 3.6.2 Deflection limits

The deflection of a beam depends on the loading and support conditions. A list of deflection formulas can be found in Part 5 of the Manual starting on page 5-162. After finding the deflection for the beam in question, compare it to the maximum allowable total (service dead load plus service live load) deflections. These are dependent on the function of the beam.

- Plastered construction:

$$
\begin{equation*}
\frac{L}{360} \tag{3.21}
\end{equation*}
$$

- Unplastered floor construction:

$$
\begin{equation*}
\frac{L}{240} \tag{3.22}
\end{equation*}
$$

- Unplastered roof construction:

$$
\begin{equation*}
\frac{L}{180} \tag{3.23}
\end{equation*}
$$

where, $L$ is the span length.

[^18]

Figure 3.4: An example of a deflection on a simply supported beam. Here, $\Delta=\frac{P L^{3}}{48 E I}$ at the point of the load.

### 3.7 Shear Capacity ${ }^{7}$

### 3.7.1 Introduction to Shear Capacity

The fifth limit state for beams is Shear Capacity. The shear capacity of a beam is the maximum amount of shear the beam can withstand before failure. Usually the shear capacity is the controlling limit state on short spans with large loads.

### 3.7.2 Equations for Shear Capacity

The shear strength relationship is:

$$
\begin{equation*}
V_{u} \leq \phi_{v} V_{n} \tag{3.24}
\end{equation*}
$$

where

- $V_{u}=$ maximum shear based on the controlling combination of factored loads
- $\phi_{v}=0.9=$ resistance factor for shear
- $V_{n}=$ nominal shear strength

The design shear strength then can be found depending, first, on whether or not the web is stiffened. If the web of a singly or doubly symmetric beam is unstiffened and $\frac{h}{t_{w}} \leq 260$, then Chapter F2 in the Specification Section of the Manual (page 16.1-35) can be used to define $V_{n}$, otherwise, Appendix F2 in the Manual (page 16.1-102) can be used.

As long as the shear strength, $V_{u}$, of a beam satisfies the maximum shear strength, $\phi_{v} V_{n}$, value, shear is not a limiting factor of the beam design. If the shear strength, does exceed the maximum allowable shear, a different load or a different cross-section must be chosen.

[^19]
### 3.8 The Modification Factor, $\mathrm{Cb}^{8}$

### 3.8.1 Introduction to $\mathbf{C b}$

$C_{b}$ is a modification factor used in the equation for nominal flexural strength when determining LateralTorsional Buckling. The reason for this factor is to allow for non-uniform moment diagrams when the ends of a beam segment are braced. The conservative value for $C_{b}$ can be taken as 1.0 as according to Chapter F2a of the Specification Section of the Manual (page 16.1-32) for all cases. Also, the Manual gives a table of values for $C_{b}$ for some loading conditions of simply supported beams. This is located in Section 5 of the Manual (page 5-35) and is named, Table 5-1.

### 3.8.2 Equation for $\mathbf{C b}$

If the value of $C_{b}$ is not given in Table 5-1, the equation in Chapter F2a of the Specification Section of the Manual (page 16.1-32), given here, can be used to find $C_{b}$.

$$
\begin{equation*}
C_{b}=\frac{12.5 M_{\max }}{2.5 M_{\max }+3 M_{A}+4 M_{B}+3 M_{C}} \tag{3.25}
\end{equation*}
$$

where:

- $M_{\max }=$ absolute value of maximum moment in the unbraced segment, kip-in.
- $M_{A}=$ absolute value of maximum moment at quarter point of the unbraced segment, kip-in.
- $M_{B}=$ absolute value of maximum moment at centerline of the unbraced segment, kip-in.
- $M_{C}=$ absolute value of maximum moment at three-quarter point of the unbraced segment, kip-in.

[^20]
## Glossary

## C Center of gravity

The center of gravity is the point at which all moments generated from the mass of the element equal zero.

## F Flexural buckling

This type of buckling can occur in any compression member that experiences a deflection caused by bending or flexure. Flexural buckling occurs about the axis with the largest slenderness ratio, and the smallest radius of gyration.

## Flexural-torsional buckling

This type of buckling only occurs in compression members that have unsymmetrical cross-section with one axis of symmetry. Flexural-torsional buckling is the simultaneous bending and twisting of a member. This mostly occurs in channels, structural tees, double-angle shapes, and equal-leg single angles.

## S Shear center

"The shear center is that point through which the loads must act if there is to be no twisting, or torsion, of the beam." LRFD Steel Design Second Edition William T. Segui
Slenderness ratio
The ratio of the effective length of a column to the radius of gyration of the column, both with respect to the same axis of bending

## T Torsional buckling

This type of buckling only occurs in compression members that are doubly-symmetric and have very slender cross-sectional elements. It is caused by a turning about the longitudinal axis. Torsional buckling occurs mostly in built-up sections, and almost never in rolled sections.

## Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. Ex. apples, § 1.1 (1) Terms are referenced by the page they appear on. Ex. apples, 1

1 1-18, 4
1-19, 4
B B7, 9
beam, § $3.1(23)$, § $3.3(24), \S 3.4(26)$, § $3.5(27)$, § 3.6(29), § 3.8(31)
bolt, § 2.4(19)
buckling, § 3.3(24), § 3.4(26), § 3.5(27), § 3.8(31)

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## Steel Design (CIVI 306)

Design of steel members, connections, and assemblies. Behavior of steel members as related to design.


#### Abstract

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