tubes 11. This means either that the tubes in this set are more sensitive to voltage change than the others, or that they have a higher residual dispersion. In either case, this is a significant interaction. It is indicated by a pattern of outages.

In the range table is a single outage under tubes 5. This is a "wild shot."

In Part B, showing outages of averages, we see that the higher-numbered tubes give higher gain and that the lower-numbered resistors give lower gain. These are main effects.

It has been mentioned that data of this type could be studied by the analysis of variance. The control-chart method has its advantages. For one thing, it requires less work. Some experience indicates that the control chart method takes less than half the time required for an analysis of variance.

More important is the ability of the control chart to pinpoint troubles and to test the validity of the assumptions made.

A large number of outages on our control charts, arranged in some logical pattern, gives us good indication of both the existence and the nature of real differences or real interactions. A fair number of outages, scattered over the whole experiment in an irregular fashion, usually indicates poor experimentation—sloppy work. A small number of outages, particularly if they can be traced back to a few pieces of primary data, indicates that these observations were abnormal—"wild shots."

The analysis of variance assumes a stable residual dispersion and then forgets it. The control chart sets a trap for instability and waves a red flag when it appears. The analysis of variance assumes that certain interactions are possible and ignores all others. The control chart method makes an initial assumption of no interaction and then highlights any interaction that comes along.

The control chart continually invites us to examine the quality of our experimentation and furnishes us with the means for making that examination.

Statistical Evaluation of Life Expectancy of Vacuum Tubes Designed for Long-Life Operation*

ELEANOR M. McELWEE†

Summary—**Life-test data on subminiature vacuum tubes designed for 5,000 hours are analyzed statistically and an equation is derived for the curve of life survival percentages. Correlation of individual types to the general curve is found to be extremely high. Controls are determined for normal 500-hour life tests which assure rated long-life quality and are presented as a method for evaluating life expectancy before completion of long-life tests. Life test samples of lots of tubes released by this 500-hour plan were continued in operation for 5,000 hours, and the results are shown to be satisfactory.**

IN RECENT YEARS, the expanding field of industrial applications of vacuum tubes has contributed to an increased demand for greater reliability over a longer period of time. In reponse to this demand, engineers throughout the industry have attempted to design a line of electron tubes which might safely be rated far beyond the customary 500 hours used to evaluate the life of radio receiving tubes. The innovations in design and processing which produced the longer-life tubes, although undoubtedly of tremendous interest, are not within the scope of this paper. The problem with which we are concerned is one introduced by the development of such tubes—that of evaluating "long-life" quality within a reasonable length of time. It is obviously impractical for tube manufacturers to conduct life tests for 5,000 hours before release of lots of tubes, or for customers to wait seven to eight months for delivery. The efforts of many engineers were therefore applied to the search for a life test plan which would effectively measure 5,000-hour quality within the normal 500-hour life test period.

The life of a vacuum tube is commonly understood to be the length of time it will operate within a specified range of characteristics. Thus a tube is considered to have reached the end of life when it becomes inoperable for any reason, or when its characteristics fall outside the end point limits specified for the particular type. The normal life rating of 500 hours did not guarantee, however, that each tube had a minimum life of 500 hours, nor even that the average life of a group of tubes would be 500 hours. As defined in the JAN-1A specification, a 500-hour rating guarantees an aggregate useful life of at least 80 per cent of the total rated life. For example, a group of five tubes would have a total rated life of 2,500 tube hours. These tubes would pass the JAN specification if their total operation within specified limits was 2,000 tube hours or better. This total figure could be amassed in any one of a number of ways; e.g., by all tubes operating for 400 hours, by one failure immediately and four good to 500 hours, by two tubes good to 250 hours and three good to 500 hours, and so on. What is actually required is an average life of at least 80 per cent of the rating for any group of tubes life-tested for

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the specified time. For the purpose of better understanding of this paper, it will be assumed that a 5,000-hour life rating is applied in the same manner; i.e., that the 80 per cent limit will apply to a 5,000-hour, rather than a 500-hour test point, and that the average life of any group of tubes must be at least 4,000, rather than 400 hours.

It was apparent in the beginning that there were two approaches to the problem of a shorter life test: (1) an accelerated test which would be equivalent to 5,000 hours of normal operation, or (2) statistical controls on a normal 500-hour life test which would adequately predict 5,000-hour quality. Considerable time and effort were expended in the attempt to set up a reliable accelerated test. Various changes were made in voltages, currents and/or power dissipations in the hope of discovering a test exactly ten times as rigorous as normal operation. Unfortunately, it was impossible to determine test conditions which would accelerate normal tube failures without introducing contributory factors not present in normal operation. Several tests were found satisfactory for individual types of failures; e.g., cycling tests to determine the quality of heaters, immersion test for air leaks, fatigue test for shorts or poor welds, etc. However, it remained impossible to obtain satisfactory correlation between emission deterioration resulting from any accelerated test, and that resulting from normal operation. Consequently, the emphasis was transferred to statistical analysis in the hope of determining a consistent pattern to which the quality control method could be applied.

The first step in the statistical analysis of data was logically a survey of the occurrence of failures in operation with relation to time. In order to include results on as many tubes as possible, the initial survey was made on life test samples of early subminiature indirectly heated cathode-type vacuum tubes, rated for normal 500-hour operation. Failures per 500-hour period were listed for a heterogeneous group of 1,864 vacuum tubes, and the average life percentage[1] at the end of each period was calculated in accordance with the JAN specification, as shown in Table I. The ratio between these percentages seemed to indicate that they would follow the exponential curve $y=ab^x$, where $y=$ average life percentage, $x=$ hours of life expressed in thousands of hours, and a and b are constants denoting the y intercept and the slope of the line, respectively. In order to determine the goodness of fit of the empirical curve $y=ab^x$, or the straight line $\log y = \log a + x \log b$, the values of a and b were found by the statistical method of least squares.[2] Then the calculated equation becomes

$$y = 93.1(0.875)^x.$$

By substituting the given values of x, computed values of y are obtained, and y residuals are found by subtraction. From the statistical formula for the standard error of estimate,

$$S_y = \left[\frac{\sum (y \text{ observed} - y \text{ computed})^2}{\text{the number of observations}}\right]^{1/2}.$$

$$S_y = (4.05406)^{1/2}.$$

The index of correlation of the curve is determined by the formula

$$\rho_{xy} = \left(1 - \frac{S_y^2}{\sigma_y^2}\right)^{1/2},$$

where S_y is the standard error of estimate of the curve, and σ_y is the standard deviation of the observed values of y. Substituting,

$$\rho_{xy} = \left(1 - \frac{4.05406}{173.333}\right)^{1/2}$$

$$\rho_{xy} = 0.988.$$

The high degree of correlation obtained was a positive indication that the empirical equation $y=ab^x$ was a close representation of these data, at least. In order to verify the results of this first experiment, the same method was followed with two additional groups of data. For a group of 1,240 vacuum tubes of various types, most of which were experimental tubes designed for a longer life rating, the calculated curve was $y=98.2\ (0.966)^x$. The index of correlation with observed data was 0.998. For a group of 130 tubes of six types released as 5,000-hour tubes, the equation of the calculated curve was $y=98.3\ (0.973)^x$; the index of correlation was 0.995. Both the observed data and the calculated curve are plotted for each group in Fig. 1.

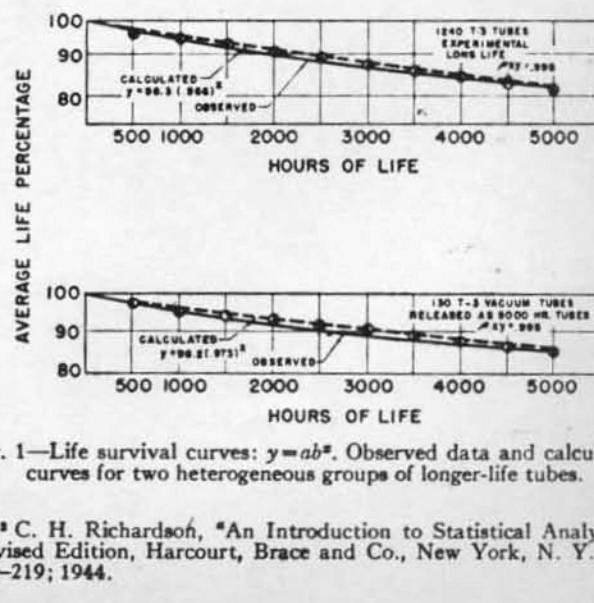

Fig. 1—Life survival curves: $y=ab^x$. Observed data and calculated curves for two heterogeneous groups of longer-life tubes.

[1] Average life percentage at X hours $=$

$$\frac{\Sigma \text{ (life hours for each tube*)}}{X \text{ hours (number of tubes started)}} \times 100.$$

E.g., if 5 tubes were started on life, one failed at 700 hours, 4 remained good past 1,000 hours, the average life percentage at 1,000 hours would be

$$\frac{700 + 4(1000)}{5(1000)} \times 100 = 94 \text{ per cent.}$$

* The life for any individual tube shall be a maximum of X hours.

[2] C. H. Richardson, "An Introduction to Statistical Analysis," Revised Edition, Harcourt, Brace and Co., New York, N. Y., pp. 210–219; 1944.

With the acceptance of the curve $y = ab^x$ as a general pattern for life survival percentages, there remained two essential points to be determined: (1) Could a universal value of the constant a be assumed that would satisfy all types of tubes? (2) If the value of the constant b were calculated from observed 500-hour results, how closely would the predicted percentages approximate actual life test operation? The answer to the former question at first seemed evident. Since only good tubes are subjected to life test, it was assumed that the y intercept of the curve would be 100 per cent, and therefore the equation would be $y = 100b^x$. Accordingly, the equation was checked with observed data, but it was noted that actual life test results beyond 1,500 or 2,000 hours were in all cases better than predicted percentages. Further analysis of data revealed that the rate of failure during the first 500-hour period of operation was higher than the rate of failure for any succeeding 500-hour period. The data seemed to indicate, in fact, that the rate of failure beyond 500 hours would be fairly constant, and would be approximately half that of the first 500-hour period. To compensate for this phenomenon, it was decided to use the value 99 for the constant a. In order to check the validity of predicted percentages, the same groups of data used previously were checked with percentages calculated from the equation $y = 99b^x$, the value of b being determined in each case by the observed 500-hour results. The correlation indices for the three groups were 0.975, 0.996 and 0.948, respectively. Curves for all three groups are plotted in Fig. 2. As an additional check on the general fit of the curve $y = 99b^x$, several types of 5,000-hour tubes were analyzed for correlation between observed data and the straight line based on the 500-hour percentage for each type. The index of correlation with the straight line was 0.986 for triode oscillators, 0.968 for radio-frequency pentodes and 0.976 for audio and video power amplifiers. Observed and calculated curves for each type are shown in Fig. 3.

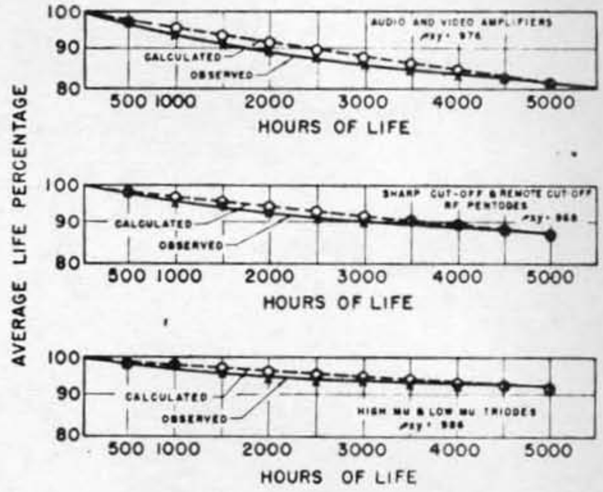

Fig. 3—Life survival curves: 5,000-hour tubes. Curves showing the correlation of observed and calculated percentages for several types of Premium Subminiature Tubes. Life-test conditions for types indicated were as follows:

	Ef	Eb	Rk	$Ec2$	Rg	Ehk
audio beam power tube	6.3	110	270Ω	110	500K	117Vrms
video amplifier	6.3	150	100Ω	100	500K	117Vrms
sharp cutoff pentode	6.3	100	150Ω	100	1 meg	117Vrms
semi-remote cutoff pentode	6.3	100	120Ω	100	1 meg	117Vrms
high mu triode	6.3	150	680Ω	—	1 meg	117Vrms
low mu triode	6.3	100	150Ω	—	1 meg	117Vrms

The correlation between predicted percentages and actual results was in all cases so high that the equation $y = 99b^x$ was accepted as the basis for all future statistical controls to be applied to life tests. This pattern of failure was a definite departure from the normal curve expected, and posed additional problems in the development of statistical controls. Earlier experience with incandescent and fluorescent lamps, and with tungsten filament-type vacuum tubes, indicated a certain wear-out point around which regular sigma limits could be plotted, as on a normal Gaussian type curve. A recent

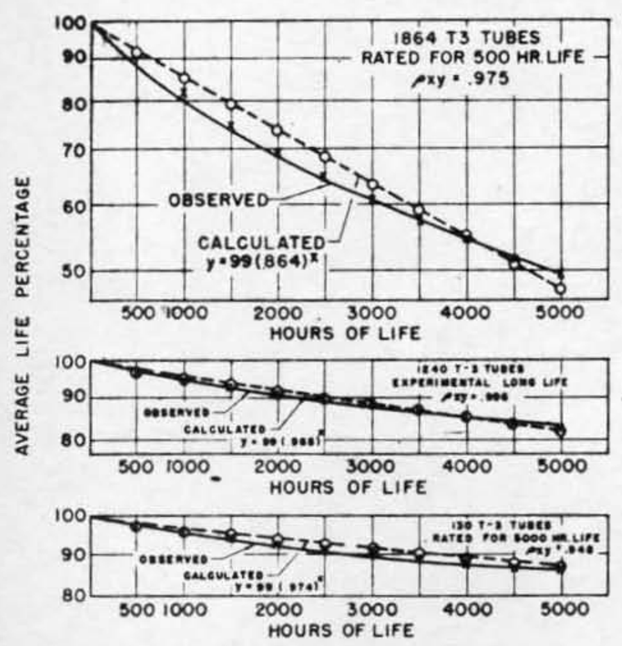

Fig. 2—Life survival curves: $y = 99b^x$. Correlation of observed data with curves calculated from 500-hour percentages for three heterogeneous groups of tubes.

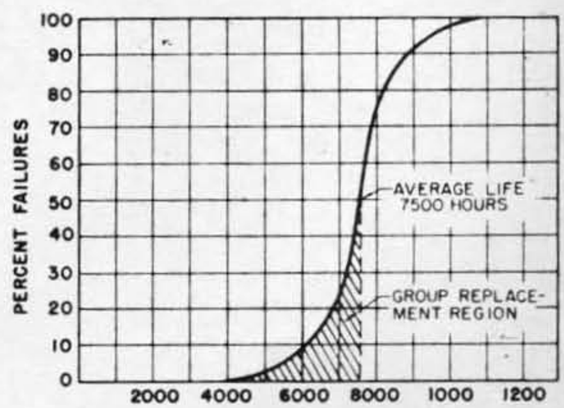

Fig. 4—Life failure curve for fluorescent lamps. Fluorescent lamp life data plotted as a normal Gaussian type curve.

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advertising bulletin for improved fluorescent lamps showed this type of curve plotted for failure percentages, with the average life marked at 7,500 hours, and standard deviations of 1,000 hours counted off on either side, as in Fig. 4. With such data, engineers can plan on a minimum life for each lamp, or an optimum replacement point for any group of lamps. Manufacturer's advertisements recommend the replacement of panels of lamps after 5,500 hours, a point at which a maximum of 2.5 per cent of the lamps will have failed. The 20 per cent, or 50 per cent, or 90 per cent failure points could be located just as easily.

Unfortunately, failure data for subminiature vacuum tubes do not follow a normal distribution, and conventional measures of central tendency and dispersion are not applicable to the problem of determining proper control limits. Therefore, it became necessary to devise a system of controls which might be applied to the exponential curve $y=99b^x$.

The first step in the process of setting up controls was to determine, from the 80 per cent—5,000 hour specification and the calculated $y=99b^x$ curve, a minimum limit to be applied at 500 hours. This 500-hour percentage was found to be 96.9 per cent. Then from accumulated data on subminiature tubes, 133 sample life tests of five tubes each were chosen which passed this 96.9 per cent—500-hour limit. Of these tests, not one failed to meet an 80 per cent—5,000-hour limit at the conclusion of the specified life test. These results led naturally to the conclusion that the minimum limit calculated was well chosen, and that the modified 500-hour test would serve as an adequate control on 5,000-hour quality.

Although the choice of a 500-hour limit was the solution to the original problem, it raised a new question of equal importance to manufacturer and customer. This new topic was the probability of release of a lot of tubes which would fail to meet the 5,000-hour life specification. In order to calculate the range of probability of such an occurrence, the five-tube sample life tests mentioned above were used to plot a frequency distribution of 5,000-hour percentages, as shown in Fig. 5. The average 5,000-hour percentage was found to be 89.4 per cent, with 2-sigma limits of 80.3 per cent and 98.5 per cent. These limits on the sample distribution were changed to limits on the universe or parent population by use of the formula

$$\sigma_{\text{sample}} = \left(\frac{N-1}{N}\right)^{1/2} \sigma_{\text{universe}},$$

resulting in new 2-sigma limits of the universe of 80.1 per cent and 98.7 per cent. Statistically speaking, 95 per cent of all released lots of tubes will fall within these limits. Conversely, 2.5 per cent of all released lots may fall on either side of these limits. To use a phrase common to all fields in which quality control is applied, it seems safe to assume an acceptable quality level (AQL) of 2.5 per cent at 5,000 hours.

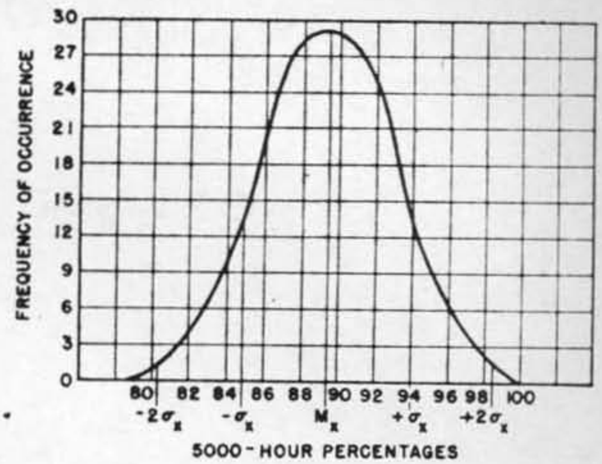

Fig. 5—Distribution of 5,000 hour percentage: 133 sample life tests, 5 tubes each. Range of average life percentages at 5,000 hours for a heterogeneous group of 133 sample life tests.

It would not be reasonable to assume that this life test plan will work equally well for all types of vacuum tubes, made by various manufacturers, until sufficient data to 5,000 hours has been collected and analyzed. The data included in this paper represent only subminiature indirectly heated cathode-type vacuum tubes made at the Kew Gardens Development Laboratory of Sylvania Electric Products Inc. Whether other tube types, or even the same types manufactured elsewhere, would produce equivalent results is a question which only the comparison of actual data will answer. Experience shows that the plan may be applied only to tubes which are designed for long-life operation, are conservatively rated, and are carefully controlled during production. To the writer's knowledge, there has been only one other published indication of an exponential curve of life percentages versus time, a life curve on repeater tubes published by the Bell Telephone Laboratories.[3] It is to be hoped that long-life data may be collected throughout the industry, and that universal life test specifications may be agreed upon by manufacturers and customers. The 500-hour test specification included in this paper was developed with the co-operation of the Bureau of Ships, the chief customer for the tube types represented, and was accepted by them for these particular types as manufactured in Kew Gardens, L. I., N. Y.

There remains one important point not yet mentioned: what kind of guarantee can be given to the customer? What will the manufacturer do if a group of tubes fails to meet the specified life rating in actual operation? Unfortunately, there is no satisfactory answer as yet. For subminiature long-life tubes, there are certain applications, such as hermetically sealed assemblies, where replacement of tubes is impossible. In many other applications for which subminiature tubes have been specially designed, replacement is difficult and expensive. In some cases, the failure of a tube may

[3] D. K. Gannett, "Determination of the average life of vacuum tubes," *Bell Tel. Lab. Rec.*; August, 1940.

TABLE I

STATISTICAL EVALUATION OF LIFE EXPECTANCY OF VACUUM TUBES DESIGNED FOR LONG-LIFE OPERATION

1,864 T-3 Vacuum Tubes Rated for 500 Hours Life

Hours of Life	*Number of Failures*	*Average Life Percentage at End of Period*
0– 500	298	92.0
500–1,000	153	82.0
1,001–1,500	113	74.8
1,501–2,000	84	69.2
2,001–2,500	72	64.8
2,501–3,000	75	61.0
3,001–3,500	43	57.2
3,501–4,000	65	54.8
4,001–4,500	36	51.4
4,501–5,000	53	49.5

cause the destruction of the entire unit. What the customer requires, therefore, is not a replacement guarantee for tubes which prove unsatisfactory, but a certain degree of assurance of reliability of operation. The plan proposed is an illustration of the application of statistical analysis to this difficult quality control problem. Although this plan may not be the perfect answer to the customers' requirements, it is a step in the right direction. It is at least a foundation for future improvements.

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Magnetic Recording with AC Bias*

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***Summary*—The function of alternating-current (ac) bias in magnetic recording is analyzed in a manner similar to that used to explain amplitude modulation. Certain simplifying assumptions are made to facilitate manipulation of mathematical expressions. The analytical results are compared with experimental observations of harmonic distortion, amplitude of fundamental, spurious recorded frequencies, frequency response, difficulty of erasure, and the like.**

INTRODUCTION

IN A MODULATOR for amplitude modulation (AM) radio transmission, an audio frequency and a much higher "carrier" frequency are combined in a nonlinear impedance. The output contains the two original frequencies, their sum, their difference, certain harmonics of each depending upon the character of the nonlinear impedance, and sums and differences of harmonics and fundamentals. A value of carrier amplitude must be selected for the particular nonlinear element to provide linearity and sufficient output in the desired band, which includes the carrier frequency and the carrier-audio sum and difference frequencies. A band-pass filter (tank circuit) is provided to attenuate undesired frequencies. The need for selecting a particular carrier amplitude is most obvious in the case of grid modulation.

In like manner, the action of alternating current (ac) bias in magnetic recording may be explained. The desired audio frequency and a much higher "bias" frequency are simultaneously fed into a nonlinear recording system. The recording contains the audio frequency, the bias frequency, and in addition to these, certain harmonics of each, and sums and differences of harmonics and fundamentals. A value of bias amplitude must be selected for the particular nonlinear recording characteristic to provide linearity and sufficient output in the desired audio band. Self-demagnetization in the recording medium and limited playback head resolution provide a low-pass filter which attenuates undesired (higher than audio) frequencies.

With the shapes of nonlinear recording characteristics in general use, this "bias" technique provides greatly reduced harmonic distortion of the audio, as compared to direct current (dc) bias or no bias.

This technique is capable of improving linearity of response for desired frequencies in a variety of nonlinear systems, whether for transmission or recording.

SCHEME FOR ANALYSIS

The transfer characteristics for a magnetic recording material is the B_r–H curve,[1] (see Fig. 1). Such a curve may be plotted from data taken by single dc exposures or from data taken in the symmetrically cyclically magnetized condition (SCMC). Curves plotted in these two ways are very similar, though not identical. A convenient set of measuring equipment for the SCMC case has been described by Wiegand and Hansen.[2]

* Decimal classification: R365.35. Original manuscript received by the Institute, April 28, 1950; revised manuscript received, August 31, 1950.

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[1] Marvin Camras, "Graphical analysis of linear magnetic recording using high-frequency excitation," PROC. I.R.E., vol. 37, pp. 569–573; May, 1949.

[2] D. E. Wiegand and W. W. Hansen, "A 60-cycle hysteresis loop tracer for small samples of low-permeability material," *Trans. AIEE*, vol. 66; 1947.