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DIGITAL COMPUTER LABORATORY
STATISTICAL LIBRARY

KSL 5.02 - 306

TITLE: Multiplication of a matrix by its transpose; either
 $AA^T=C$ or $A^T A=C^*$

TYPE: Entire program

CAPACITY: If $j \leq 64$ max $i = 160$
If $j \leq 128$ max $i = 80$

ACCURACY: 10 decimal places

DESCRIPTION: The routine will read a matrix, A_{ij} , and form either
 $AA^T = C$ or $A^T A = C^*$ as specified by the parameter tape.
Features of the routine are:

1. The input matrix A, can be rescaled by any power of ten prior to multiplication, (provided the elements remain within the range of the machine $-1 \leq a_{ij} < +1$) enabling the user to adjust the scaling on C either to prevent overflow or for other reasons. The machine also checks for overflow of elements of C. (See section on stops and error diagnosis).
2. The output matrix, C or C*, will be symmetric. The parameter tape specifies four output options:
triangular in row or column form,
or square in row or column form.
3. If overflow should occur, proper switching allows a new scale parameter to be inserted and the problem redone without need to re-read data.

METHOD OF USE:

	<u>Stops</u>
1. Master tape	34087
2. Parameter tape	240K6
3. Matrix A tape	24087

At stop 24087, a new parameter tape can be read by raising black switch. To change the scaler only, move white switch up and down, then raise black switch. (See section on stops and error diagnosis).

PARAMETER TAPE PREPARATION:

There are four parameters with terminating symbols and they must appear in the following order:

dJ tF xL +sS.

dJ: d is the number of decimal places desired in the output matrix.

tF: When $t = 0$, $A^T A$ will be formed
when $t = 1$, AA^T will be formed

xL: x determines the form of the output matrix C or C*.
If $x = 0$, the output will be in triangular form by columns punched with a carriage return after each element and two carriage returns at the end of each row.

If $x = 1$, the output will be in triangular form punched by rows across the page; two carriage returns separate rows, and if the row is too wide for the page, appropriate carriage returns are inserted.

If $x = 2$, the output will be square by columns with a carriage return after each element. An N and two carriage returns are punched after each row.

If $x = 3$, the output will be in square form by rows across the page with an N and two carriage returns after each row; if the row is too wide for the page, appropriate carriage returns are inserted.

+sS: +s or -s is the integer exponent of ten, the
or

oS: scaling constant. Each element of A is multiplied by 10^{+s} prior to multiplication. If no rescaling on A is desired, this should appear as oS. (NO SIGN should be indicated).

DATA TAPE PREPARATION:

The elements of matrix A are punched by rows as signed fractions with an N at the end of each row. If an F is punched instead of an N, the computer will stop on either

300K8 or 300SS. By raising black switch reading of A is resumed. After the final N, a J must be punched to indicate the end of the matrix.

STOP AND ERROR DIAGNOSIS:

STOPS	LOC.	DIAGNOSIS
34087	3F4	Master tape has read correctly; black switch to read the parameter tape.
240K6	OK4	End of parameter tape; black switch to read in data; white switch to skip data input.
300K8	OK8	F termination, first row; black switch to continue.
300SS	OSK	F termination, subsequent rows; black switch to continue.
24087	13J	<u>End of problem</u> (a) Black switch to begin a new problem with a different parameter tape. (b) To rescale only raise white switch up and down and then raise black switch to read in new scaling parameter. (c) If just new data is to be read in using former parameters, move white switch up and down twice, then black switch to read in new data.
FF001	OSF	Number of elements in rows are not equal. Check data tape.
FF002 003	11F,171 122,175	Overflow in an element of C. To read in a new scaling parameter -sS only, move white switch up and down; then to skip re-reading the data white switch again.
FF004 005	13K 13J	Sum check failure in matrix A or A^T . White switch three times; re-insert the data, and then black switch to start program.
FF006	3F5	Master tape sum check failure, re-read master tape.

DURATION IN SECONDS: AA^T square form
 $i^2(.02d + .002j + .12) + i (.005dj + .018j + .03) +40$
triangular form
 $i^2(.01d + .002j + .07) + i (.005dj + .018j + .03) +40$
 $A^T A$ square form
 $j^2(.02d + .002i + .12) + j (.005di + .018i + .015) +40$
triangular form
 $j^2(.01d + .002i + .07) + j(.005di + .018i + .015) +40$

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LOCATION			ORDER	NOTES	PAGE 1	KSL 5.02		
Abs.	Rel.	Sym.						
3	0		003K					
			00F 00F	d				
			00F 00F	0, 1 $A^T A$, AA^T				
			00F 00F	0, 1, 2, 3 form of output				
			00F 00F	J				
			00F 00F	65 or 129				
			00F 00F	Sum of elements of A				
			00F 00F	I				
		10	10		00F 00F	2560 - 129 or 65		
					8511F 00F	$[A_{il}]$ or $[A_{lk}]$		
	00F 00F			i				
	00F 00F			j				
	00F 00F			k				
	00F 00F			Sum of elements of A				
	00F 00F			Sum of elements of B				
	8511F 00F			$[A_{ki}]$ or $[A_{lj}]$				
	00F 00(1)							
	00F 00F			sum box				
20	20		00F 00F	tally of row elements				
			00F 00F	65/d + 2 or 1				
			00F 00F	Scaling factor				
			00F 00F	21(N12)				
			7J22F 40600F					
			S5F 40600F					
			00K					
		26	0	(1)	00F 001F			
				(2)	00F 002F			
				(6)	00F 006F			
(9)	00F 009F							
30	(10)			00F 0010F				
	(65)			00F 0065F				
	(129)			00F 00129F				
	(600)			00F 00600F				
	(2560)			00F 002560F				

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		PAGE 2 KBL 5.02
35	9	(-1)	80F 00F	
		(D1)	001F 001F	
		(TOD)	8611F 00F	
		(FRD)	8511F 00F	
		(END)	40(TRC) L5600F	
			00K	
40		(N12)	00K	
79		(P16)		
			00K	
135	0	(IN)	192F 402F	Read first three parameters
		(PAR)	41F 814F	
			L0(10) 36(STO)	
			L4(10) 50F	
			74(10) S5F	
140			40F 22(PAR)	
		(STO)	42(STA) 92575F	
		(STA)	L5F 40F	
			L52F L42F	
			402F 36(PAR)	
	10	(SC)	41F 814F	Read in scaling parameter
			L0(10) 36(INK)	
			4122F 22(DATA)	
		(INK)	40F 411F	
		(IND)	814F L0(10)	
150			32(INE) L4(10)	
			501F 74(10)	
			S5F 401F	
		(INE)	26(IND) L11F	
	20		402F 50(10)	Compute 10 ^S
		(INH)	F52F 402F	
			32(INF) 75(10)	
		(INF)	26(INH) S5F	
			402F L3F	

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		PAGE 3 KSL 5.02
159	25		36(ING) 41F	
160	26		50F L5(1)	
			662F S5F	Compute 10^{-8}
			4022F 22(DATA)	
		(ING)	L52F L4(-1)	
	30	(DATA)	4022F 24(DAT)	Black switch to read in data
			22(PA) 26(DAT)	White switch to skip data
		(DAT)	50600F 50(DAT)	Read first row
			26(N12) L0(2)	
		(TES)	30(TES) 416F	Stop for F at end of first row
			L521(N12) 4023F	
170			1020F L0(600)	
			426F L56F	Store J
			L0(65) 32(DRU)	
			L5(65) 407F	Store 65 or 129 into L [7]
	40	(DRU)	221(DRU) L5(129)	
			407F L5(6)	
			42(SCKB) L5(1)	Set I = 1
			409F 418F	
		(SCH)	50600F 50(SCH)	
			26(SCK) L5F	
180			408F L5(2560)	Store sum first row
			L07F 4010F	Store 2560 - 129 or 65
			L5(600) L46F	
			42(END) 26(TRA)	
	50	(INA)	50600F 50(INA)	
			26(N12) L0(2)	Read in rest of data
		(TIS)	301(TIS) L4(1)	Stop on F
			32(PA) L521(N12)	
			L023F 40F	
			L3F 32(INB)	
190		(INB)	FF1F 26(INC)	FF1 - wrong number of elements in row
		(INC)	50600F 50(INC)	Sum row and add to sum check box
			26(SCK) L5F	

LOCATION			ORDER	NOTES	PAGE 4	KSL 5.02
Abs.	Rel.	Sym.				
193	59		L48F 408F			
	60		F59F 409F	I = I + 1		
		(TRA)	L5(600) 42(TRB)			
			L510F L47F			
			4010F L4(TOD)			
		(TRB)	40(TRC) L5600F			
		(TRC)	8611F 00F	Store row starting at L [600] to drum		
200			F5(TRC) 40(TRC)			
			F5(TRB) 42(TRB)			
			L0(END) 36(INA)			
		(PA)	22(TRB) L55F			
	70		0039F 36(PF)			
			L53F L4(2)			
			007F 40F			
			L5(65) 66F	Form 65/d + 2		
			S5F 1032F			
			4021F 261(PF)			
210		(PF)	L5(1) 4021F			
			L53F 0020F			
			46(PRB) L321F	Store d into print routine		
		(PG)	4020F 92139F			
	80		L54F L0(1)			
			36(AT) 26(TA)	Switch to AA ^T or A ^T A		
			00K	Drum transfer subroutine		
216	0	(DR)	K5F 42(DRA)			
			46(DRB) 40F			
			F5F 42(DRD)			
		(DRD)	42(RET) L5F			
220		(DRA)	46(DRF) L5F			
			42(DRC) 26(DRB)			
		(DRB)	L4F 42(FIN)			
		(DRF)	L5F 40(DRE)			
		(DRE)	8511F 00F			

LOCATION			ORDER	NOTES
Abs.	Rel.	Sym.		PAGE 5 KSL 5.02
225	9	(DRC)	JOF 40F	<p>[] Planted from (TA) or (AT) determines whether subroutine reads row or column</p> <p>Sum check of matrix rows subroutine</p> <p>Either 6 or 9 planted from (AT) or (TA)</p> <p>AA^T</p> <p>Transfer row A_{i1} to position starting L [600]</p> <p>Sum elements in row A_{i1}</p>
	10	(DRK)	L5(DRE) L4F 40(DRE) L5(DRC) 42(DRC) L0(FIN)	
		(RET)	36(DRE) 22F	
		(FIN)	50F 40F	
			00K	
231	0	(SCK)	K5F 42(RES) 1020F 42(SCKA) 42(SUM) 41F	
		(SCKA)	L5F L4F 40F F5(SCKA) 42(SCKA) L5(SCKA)	
		(SCKB)	L0(SUM) L06F	
		(RES)	36(SCKA) 22F	
		(SUM)	75F L4F	
240	0	(AT)	00K L5(600) L46F 4224F 4225F L518F 42(DRK) L5(6) 42(SCKB) L5(FRD) L4(2560) 4011F L5(1) 4012F 4115F	
		(ATJ)	L5(1) 4014F L5(FRD) L4(2560) 4017F L5(600)	
250	10		42(ATP) 421(ATP) 42(ATQ) 421(ATQ)	
		(ATA)	506F 50(ATA) 26(DR) 00600F 0011F 4116F	
		(ATB)	50600F 50(ATB)	

LOCATION			ORDER	NOTES	PAGE 6	KSL 5.02		
Abs.	Rel.	Sym.						
256	16		26(SCK) L5F L415F 4015F L322F 32(ATS)	Multiply row by scaling factor				
260	20	(ATU) (ATP)	L522F 32(ATQ) 1039F 75600F S5F 40600F F5(ATP) 42(ATP) F51(ATP) 421(ATP) L025F 32(ATS)					
		(ATQ)	26(ATU) 50600F 7J22F 40600F F5(ATQ) 42(ATQ) F51(ATQ) 421(ATQ) L024F 32(ATS)					
270	30	(ATS)	22(ATQ) L5(P) 46(ATE) 42(ATE)		Transfer row A_{kl} to position starting L [800]			
		(ATC)	506F 50(ATC) 26(DR) 00800F 0017F 4119F					
		(ATD)	50800F 50(ATD) 26(SCK) L5F L416F 4016F L5(1) 4013F			Sum elements in row A_{kl}		
		(ATE)	50600F 7J800F			Multiply $A_{ij} A_{kj}$		
280	40		40F 361(ATY) L519F 32(ATZ)		Start of test for overflow			
		(ATZ)	262(ATZ) L4F 4019F 26(ATW) L4F 36(ATY) 4019F 26(ATW)					
		(ATY)	FF2F 26(SC) L519F 32(ATX) L4F 4019F					
		(ATX)	26(ATW) L4F					

LOCATION			ORDER	NOTES	PAGE 7	KSL 5.02
Abs.	Rel.	Sym.				
290	50		322(ATX) FF3F 26(SC) 4019F (ATW) L513F L06F 36(PRA) L5(ATE) L4(DL) 40(ATE) F513F 4013F	Print C _{ik}		
		(ATO)	26(ATE) L55F L0(2) 36(A) L514F L012F 361(ATH) 221(A)			
300	60	(A)	L514F L09F 32(ATH) L517F L47F 4017F F514F 4014F			
		(ATH)	22(ATS) 92770F 92135F 92519F L321F 4020F L512F L09F 32(ATK) L511F L47F 4011F F512F 4012F	Print N at end of row Reset row tally		
310	70	(ATK)	26(ATJ) L58F L015F 40F L3F 32(ATL)	Start of sum checks		
		(ATL)	FF4F L58F L016F 40F L3F 32(ATM)			
		(ATM)	FF5F 24(IN) 24(SC) 24(DAT)	End of problem - stop 24087		
		(PRA)	50F 50(PRA)			
320	80	(P)	26(PR) 22(ATO) 00600F 00800F			

LOCATION			ORDER	NOTES	PAGE 8	KSL 5.02
Abs.	Rel.	Sym.				
322	0	(TA)	00K L5(600) L49F 4224F 4225F L5(6) L4(1) 42(DRK) L5(9) 42(SCKB) L5(FRD) L4(2560) 4011F L5(1) 4014F	$A^T A$		
330	10	(TAJ)	4115F L5(1) 4013F L5(FRD) L4(2560) 4017F L5(600) 42(TAP) 421(TAP) 42(TAQ) 421(TAQ) 42F			
		(TAA)	509F 50(TAA) 26(DR) 00600F 0011F 4116F	Transfer column A_{1k} to position starting L[600]		
		(TAB)	50600F 50(TAB) 26(SCK) L5F L415F 4015F	Sum elements in column A_{1k}		
340	20	(TAU)	L322F 32(TAS)			
		(TAP)	1039F 75600F S5F 40600F F5(TAP) 42(TAP) F51(TAP) 421(TAP) L025F 32(TAS)	Multiply column by scaling factor		
		(TAQ)	26(TAU) 50600F 7J22F 40600F			
350	30		F5(TAQ) 42(TAQ) F51(TAQ) 421(TAQ) L024F 32(TAS)			
		(TAS)	22(TAQ) L5(P)			

LOCATION			ORDER	NOTES	PAGE 9	KSL 5.02
Abs.	Rel.	Sym.				
354	32		46 (TAE) 42 (TAE)			
		(TAC)	509F 50 (TAC)	Transfer column A_{ij} to position starting L[800]		
			26 (DR) 00800F			
			0017F 4119F			
		(TAD)	50800F 50 (TAD)	Sum elements of columns A_{ij}		
			26 (SCK) L5F			
360			L416F 4016F			
			L5 (1) 4012F			
	40	(TAE)	50600F 7J800F	Multiply $A_{ik} A_{ij}$		
			40F 361 (TAY)	Start of test for overflow		
			L519F 32 (TAZ)			
		(TAZ)	262 (TAZ) L4F			
			4019F 26 (TAW)			
			L4F 36 (TAY)			
			4019F 26 (TAW)			
		(TAY)	FF2F 26 (SC)			
370			L519F 32 (TAX)			
			L4F 4019F			
	50	(TAX)	26 (TAW) L4F			
			322 (TAX) FF3F			
			26 (SC) 4019F			
		(TAW)	L512F L09F			
			36 (PRD) L5 (TAE)	Print C^*_{kj}		
			L4 (D1) 40 (TAE)			
			F512F 4012F			
		(TAO)	26 (TAE) L55F			
380			L0 (2) 36 (B)			
			L513F L014F			
	60		361 (TAH) 221 (B)			
		(B)	L513F L06F			
			32 (TAH) L517F			
385	63		L4 (1) 4017F			
			F513F 4013F			
		(TAH)	22 (TAS) 92770F	Print N at end of row		

LOCATION			ORDER	NOTES	PAGE 10	KSL 5.02
Abs.	Rel.	Sym.				
388	66		92135F 92519F L321F 4020F	Reset tally for rows		
390	70		L514F L06F 32 (TAK) L511F L4(1) 4011F F514F 4014F			
		(TAK)	22 (TAJ) 22 (ATK)	Transfers to sum check routine in (AT)		
		(PRD)	50F 50 (PRD)			
			26 (PR) 22 (TAO)			
			00K	Print subroutine		
397	0	(PR)	K5F 42 (PZ)			
			L520F 36 (PRC)			
		(PC)	92963F L519F			
400		(PRB)	50F 50 (PRB)			
			26 (P16) F520F			
			4020F 22 (PZ)			
		(PRC)	92131F L321F			
			4020F 92519F			
		(PZ)	26 (PC) 22F			
			00996K			
996	0		L3F 34 (IN)			
			FF6F 26 (IN)			
			K30758F 490403F			
			26L 261N			