UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE K 11 - 192

TITLE Quartimax Orthogonal Rotation of Factors (DOI Only)

DURATION .36 mp + ct = time in seconds

The first term is the time required for input and output where m is the number of factors, and p the number of variables in the matrix being rotated. The second term is the time required for computation. Only very rough estimates can be made of the number c of cycles required for convergence. As the number of factors increases, with a fixed number of variables, c increases rapidly. Centroid factors will usually require a larger c than principal axes factors. Approximate estimates of c may be taken from the following table:

NUMBER of FACTORS	4	6	8	12	16
c	4	7	12	21	36

The time t required for each cycle depends primarily on the number of factors and secondarily on the number of variables. The table below can be used for a rough estimation of t.

NUMBER of FACTORS	3	5	9	13	17	21
t (in seconds)	1	17	50	120	200	320

DESCRIPTION The aim of rotation may be taken as that of decreasing the complexity of the factorial description of the variables. The quartimax method of rotation provides for finding the orthogonal transformation which maximizes the variance of the squared factor loadings (thereby achieving a high degree of inequality among the factor loadings). This can be shown to be equivalent to maximizing the fourth powers of the loadings. In applying the method, two factors are operated upon at a time. The maximizing angle of transformation ϕ for two factors i and j is given by the equation:

$$\phi = 1/4 \arctan \frac{4 \sum_k f_{ki} f_{kj} (f_{ki}^2 - f_{kj}^2)}{\sum_k (f_{ki}^2 - f_{kj}^2)^2 - 4 f_{ki}^2 f_{kj}^2},$$

where f_{ki} is the matrix of factor loadings. This program obtains the quartimax solution by successive transformations of pairs of factors until the sum of fourth powers no longer increases. For a more detailed account of the method, see: J. O. Neuhaus and C. F. Wrigley, "The quartimax method: An approach to orthogonal simple structure," Brit. J. statist. Psychol., Nov. 1954.

OUTPUT — The output consists of a two-hole delay character punched at the end of each cycle of rotations and the set of rotated factors punched by columns with an N at the end of each factor.

TAPES USED

1. Program Tape. This is the same for all problems and may be copied from the library tape.
2. Parameter Tape. This is punched as follows:

00 6K
00 (p)F 00 (p)F
00 27K
00 (m)F 00 (m)F
22 278N

where (p) is the number of variables and (m) is the number of factors.

3. Matrix Tape. Multiply each element of the matrix of factor loadings by one-tenth and then punch the scaled matrix factor by factor in the form required for Illinois Library Routine N 3. The character N is punched after the last loading on each factor.

INSTRUCTIONS to ILLIAC OPERATOR

1. Program tape
2. Parameter tape — Bl. up
3. Data tape — Bl. down

If continuous punching has not begun when estimated

time has elapsed, raise the black switch to OBEY. Wait for Illiac to stop on the order 24 090: Then start by raising white switch. Continuous punching will begin immediately and last less than 4 minutes.

Program ends on OF from 116.

CAPACITY $(m) \cdot (p) \leq 745$

COMMENTS At the end of each complete cycle of rotations the increase in the sum of fourth powers, $\Sigma_i \Sigma_j f_{ij}^4$, is inspected to see if another cycle of rotations should be made. If the increase has been less than $T \times 2^{-45} \times 10^4$ then no further iterations will be made, where T is an integer less than 1,000. The tolerance, T, is taken to be 10 unless specified. If the tolerance desired is not ten then insert the following immediately before the "22 278N" on the parameter tape:

00 29K

00 F 00 (T) F

If it is necessary to complete the rotation during a second time period on the computer, use the following procedure. The black switch is raised to the "obey" position about five minutes before the first time period is to end; the computer will complete a cycle of rotations and then stop. Start with the white switch as soon as the computer stops and the rotated factors will be printed. Proceed in the second session as in the first, but using the rotated factor tape as the matrix tape.

K11 contains an old obsolete version of T2 which does not agree with the serial copy of T2. This old version is correct, however.

RT: 10/13/60

DATE November 17, 1955

PROGRAMMED BY J. O. Neuhaus

CHECKED BY R. J. Tuorey

APPROVED BY J. Nash

JON.
RJT: mge

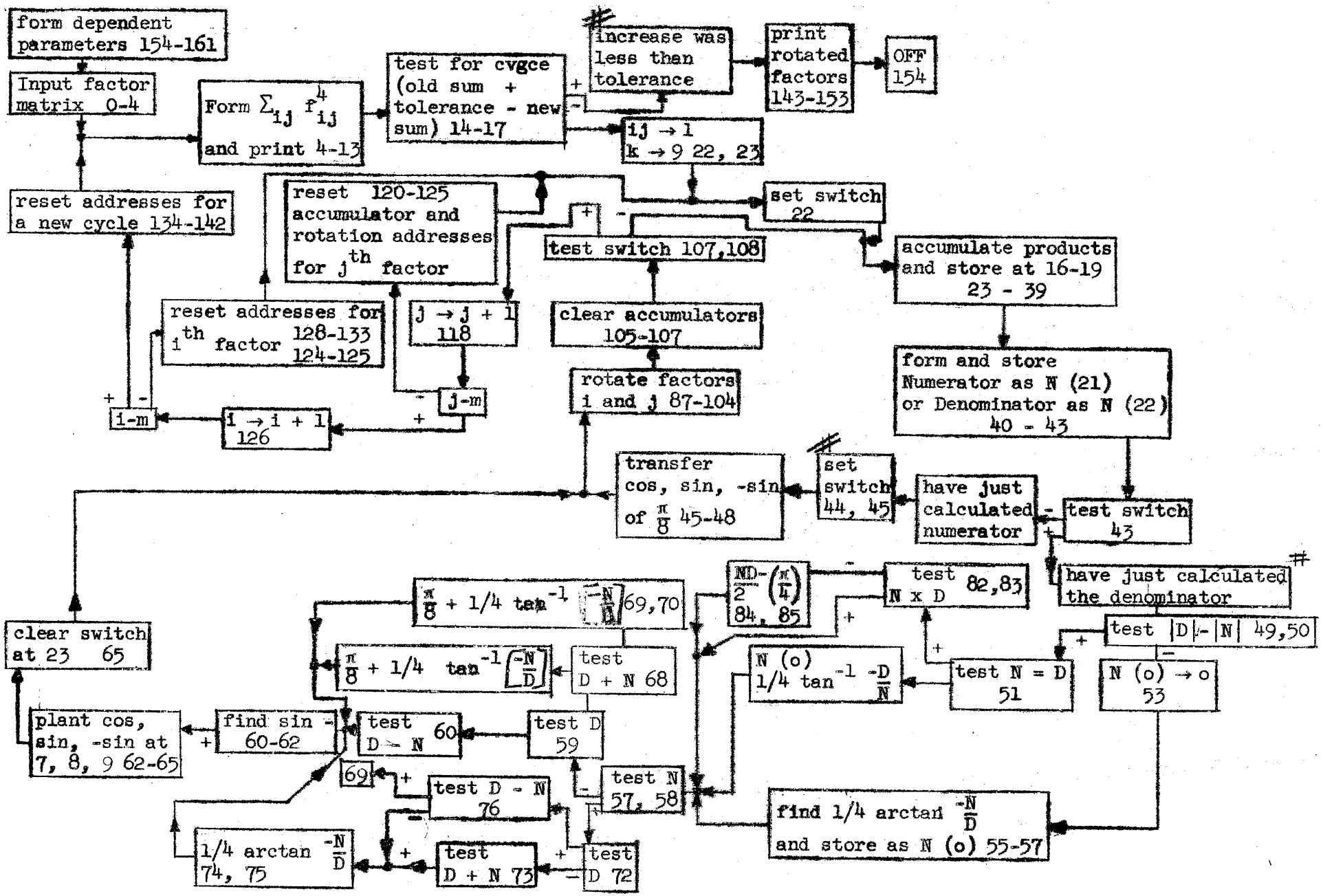
This program was prepared for the library by the Psychology Department.

USE OF STORAGE - QUARTIMAX

LOCATION	CONTENTS
5	00 1F 00 1F
6	00 pF 00 pF : variables (on parameter tape)
7	40 $\binom{279}{+mp}$ F 00 F; cos $\frac{\pi}{8}$
8	sin $\frac{\pi}{8}$
9	-sin $\frac{\pi}{8}$
10	previous $\sum_{i,j} f_{ij}^4$.
11	279×2^{-39} during input, last $\sum_{i\ j} f_{ij}^4$. during calculation
12	counter (accumulating variables)
13	i
14	j
15	binary switch (after forming sum)
16	$2\Sigma_k f_{ki}^2$
17	$2\Sigma_k f_{kj}^2$
18	$4\Sigma_k f_{ki}^3 f_{kj}$
19	$4\Sigma_k f_{ki} f_{kj}^3$
20	50 mp 00 F
21	temporary store for $\left(\sum_{k=1}^{p} f_{ki} f_{kj} (f_{ki}^2 - f_{kj}^2)\right)$ = Numerator
22	Denominator
23	binary switch
24	1/2 cos $\frac{\pi}{8}$
25	1/2 sin $\frac{\pi}{8}$
26	$\frac{\pi}{8}$

LOCATION	CONTENTS
27	m : factors (on parameter tape)
28	50 279F 75 279F
29	tolerance T
31	DNI Library Routine N - 3
52	SCP Library Routine P - 6
65	sin - cos Library Routine T - 1
95	arctan-Library Routine T - 2
124	body - Main Routine

FLOW CHART - QUARTIMAX



LOCATION	ORDER	NOTES PAGE 1
	Code X1	Decimal order input
	00 11K	
	00 F	
	00 279F	
	00 3K	Print parameter
	00 F	
	00 4F	
	00 29K	tolerance
	00 F	
	00 10F	
	00 5K	
	00 1F	
	00 1F	
	00 7K	
	40 SS	
	00 F	
	00 20K	
	50 SS	
	00 F	
	00 24K	
	00F004619397662 00J	1/2 cas $\frac{\pi}{8}$
	00F001913417161 50J	1/2 sin $\frac{\pi}{8}$
	00F003926990816 99J	$\frac{\pi}{8}$
	00 28K	
	50 SS	
	75 SS	
	00 31K	
Library Routine N 3		Decimal number sequence input
	00 52K	
Library Routine P 6		Single column print

LOCATION	ORDER	NOTES	PAGE 2
	00 65K		
	Library Routine T 1	Sine, cosine	
	00 95K		
	Library Routine T 2	Arctan	
	00 124K		
0	40 SS		
	50 L		
1	26 31F		
	L5 L		
2	L4 6F		
	46 L	Input factor matrix	
3	L0 7F		
	36 5L		
4	26 L		
	41 10F		
5	50 SS		
	75 SS		
6	00 2F		
	40 F		
7	50 F		
	75 F		
8	00 2F		
	L4 10F		
9	40 10F		
	L5 5L	Form sum of fourth powers $(\Sigma_{ij} f_{ij}^4)$	
10	L4 5F	test for convergence, store	
	40 5L		
11	L0 20F	as N(10)	
	32 12L		
12	26 5L		
	92 515F		
13	L5 11F		
	L4 29F		
14	L0 10F		
	32 16L		

LOCATION	ORDER	NOTES PAGE 3
15	L5 10F	
	40 11F	
16	24 20L	
	92 59F	
17	92 59F	
	92 135F	Transfer to print routine if started
18	26 143L	with white switch
	00 F	
19	00 F	
	00 F	
20	41 12F	$k \rightarrow 0$
	L5 5F	
21	40 13F	$i, j \rightarrow 1$
	40 14F	
22	49 15F	
	26 23L	
23	50 SS	
	75 SS	
24	00 1F	increase 2 $\Sigma_k f_{ki}^2 = N$ (16)
	40 F	
25	L4 16F	
	40 16F	
26	50 SS	
	75 SS	
27	00 1F	increase 2 $\Sigma_k f_{kj}^2 = N$(17)
	40 1F	
28	L4 17F	
	40 17F	
29	50 SS	
	75 SS	
30	40 2F	
	50 F	

LOCATION	ORDER	NOTES	PAGE 4
31	7J 2F		
	L4 18F	increase $4 \sum_k f_{ki}^3 f_{kj} = N(18)$	Accumulate
32	40 18F		Sums
	50 1F		
33	7J 2F		
	L4 19F	increase $4 \sum_k f_{ki} f_{kj}^3 = N(19)$	
34	40 19F		
	L5 23L		
35	L4 5F		
	40 23L		
36	46 29L		
	L5 26L		
37	L4 5F		
	40 26L	advance addresses	
38	42 29L		
	L5 12F		
39	L4 5F		
	40 12F		
40	L0 6F		
	36 42L	test for completion of sums	
41	26 23L		
	00 F		
42	L5 18F		
	L0 19F	store Numerator as N(19)	
43	40 21F	test switch	
	L1 15F		
44	36 49L	set switches	
	49 23F		
45	41 15F		
	L5 24F		
46	40 7F		
	L5 25F		
47	40 8F		
	L1 25F	store cos, sin, - sin of $\frac{\pi}{8}$ at 7, 8, 9	

LOCATION	ORDER		NOTES
48	40 9F		
	26 87L		
49	L7 21F	test (| Denon. | - | Numer.|)	
	L2 22F		
50	36 51L		
	26 53L		
51	40 F	test (N = D)	
	L1 F		
52	32 82L		
	26 77L		
53	41 F		
	50 F		
54	L1 21F	(-N) $\frac{\cdot}{\cdot}$ D	
	66 22F		
55	S5 F		
	50 55L		
56	26 95F	find 1/4 arctan	
	10 2F		
57	40 F	test N	
	L5 21F		
58	36 72L	test D	
	L5 22F		
59	36 68L	test D - N	Find the angle
	L0 21F		of rotation θ
60	32 66L		
	L5 F		
61	L0 26F	find sin [θ]	
	50 61L		
62	26 65F		
	L5 2F		
63	40 7F		
	L5 1F		

LOCATION	ORDER	NOTES PAGE 6
64	40 8F	
	L1 1F	store cos, sin, - sin
65	40 9F	
	41 23F	
66	26 87L	
	L5 F	
67	L0 26F	
	26 61L	
68	L4 21F	
	36 71L	
69	L5 26F	
	L4 26F	Find the angle
70	L4 F	of rotation θ
	22 61L	
71	23 69L	
	00 F	
72	L5 22F	
	32 75L	
73	L4 21F	
	32 74L	
74	22 60L	
	L5 F	
75	22 61L	
	L0 21F	
76	36 71L	
	22 74L	
77	41 F	
	50 F	
78	L1 22F	
	66 21F	
79	S5 F	
	50 79L	

LOCATION	ORDER	NOTES	PAGE 7
80	26 95F		
	10 2F		
81	40 F		
	L1 F		
82	26 57L		
	50 21F		
83	7J 22F		
	32 85L		
84	L1 26F		
	10 1F		
85	26 57L		
	L5 26F		
86	10 1F		
	26 57L		
87	41 2F		
	50 SS		
88	7J 7F		
	40 F		
89	50 SS		
	7J 8F		
90	L4 F		
	40 1F		
91	50 SS		
	7J 9F		
92	40 F		
	50 SS	find k^{th} component	
93	7J 7F	of the rotated vectors	
	L4 F		
94	40 3F		Rotate the i^{th}
	L4 3F		and j^{th} factors
95	40 SS		
	L5 1F		

LOCATION	ORDER	NOTES
96	L4 1F	
	40 SS	
97	F5 87L	
	42 87L	
98	42 96L	
	00 20F	
99	46 91L	
	F5 92L	
100	42 92L	
	00 20F	reset addresses
101	46 89L	
	46 95L	
102	L5 5F	
	L4 2F	
103	40 2F	
	L0 6F	
104	36 105L	
	22 87L	
105	41 16F	
	40 17F	
106	40 18F	
	40 19F	clear accumulators
107	40 12F	
	L1 23F	test switch
108	36 118L	
	L5 43L	
109	L4 5F	
	46 43L	
110	L5 23L	
	L0 6F	
111	40 23L	
	46 29L	

LOCATION	ORDER	NOTES	PAGE 9
112	42 87L		
	42 96L	reset addresses in	
113	46 91L		
	L5 26L	accumulation and	
114	L0 6F		
	40 26L	rotation routines	
115	42 29L		
	42 92L		
116	46 89L		
	46 95L		
117	26 23L		
	00 F		
118	L5 14F		
	L4 5F	$j \rightarrow j + 1$	
119	40 14F		
	L0 27F	test (i - m)	
120	36 126L		
	L5 23L		
121	L0 6F		
	40 23L	reset addresses in	
122	46 29L		
	42 87L	accum. and rotation	
123	42 96L		
	46 91L	routines for j^{th} factor	
124	L5 43L		
	L0 5F		
125	46 43L		Reset addresses,
	26 22L		increase counters,
126	L5 13F	$i \rightarrow i + 1$	and test to find
	L4 5F		position in pro-
127	40 13F		gram
	L0 27F	test (i - m)	

LOCATION	ORDER	NOTES	PAGE 10
128	36 134L		
	L5 13F		
129	L4 5F		
	40 14F		
130	L5 23L		
	L4 6F	reset addresses for	
131	40 26L		
	42 29L	i^{th} factor	
132	42 92L		
	46 89L		
133	46 95L		
	26 124L		
134	L5 28F		
	40 5L		
135	40 23L		
	46 29L		
136	42 87L		
	42 96L		
137	46 91L		
	L4 6F	reset addresses to	
138	40 26L		
	42 29L	prepare for a new cycle	
139	42 92L		
	46 89L		
140	46 95L		
	L5 43L		
141	L0 5F		
	46 43L		
142	22 4L		
	00 F		
143	41 7F		
	41 8F		

LOCATION	ORDER	NOTES PAGE 11
144	L5 SS	
	50 144L	
145	26 52F	
	L5 144L	
146	L4 5F	
	46 144L	Punch
147	L5 8F	Rotated
	L4 5F	Factors
148	40 8F	
	L0 6F	
149	36 150L	
	26 144L	
150	92 770F	
	92 129F	
151	L5 7F	
	L4 5F	
152	40 7F	
	L0 27F	
153	36 154L	
	22 143L	
154	0F F	
	41 11F	
155	50 6F	
	75 27F	
156	00 19F	
	L4 20F	form mp
157	46 20F	
	46 7F	and plant
158	L5 23L	Form
	L4 6F	dependent
159	40 26L	parameters
	42 29L	plant address of 1st

LOCATION	ORDER	NOTES	PAGE 12	K11
160	42 92L	loading in 2nd factor		
	46 89L			
161	46 95L			
	24 L			
	Sum check			
	24 999N			

JON.
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11/17/55