UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE H 1 - 71

TITLE	Inverse Interpolation, A Real Root of $f(x) = 0$ (DOI or SADOI)
TYPE	Closed
NUMBER OF WORDS	33
PARAMETERS	S3, address of auxiliary routine to compute $f(x)$
TEMPORARY STORAGE	10, 11, 12, 13, 14, 15, 16
ACCURACY	2^{-38} or better
DURATION	$[2(t+1) + n(t+6)]$ ms, where n = number of iterative linear interpolations to find root, and t = duration in ms of auxiliary routine which computes $f(x)$.
DESCRIPTION	Enter with the link address in Q and two values x_1 and x_2 of the argument of $f(x)$ in memory positions 10 and 12. (These two values of x and the contents of A upon entry will be lost by the routine so if needed must be stored). If there is an odd number (e.g., 1) of roots between x_1 and x_2, the routine will be left with one of them in A, if there is an even number (e.g., 0) of roots between x_1 and x_2, the routine will be left with $-1 = \|-1\|$ in A Multiple roots are treated as single roots. The auxiliary subroutine whose address is to be entered in S3 before read-in must replace x in A by $f(x)$ in A. This inverse interpolation routine uses storage locations 10-16 for quantities which must be preserved during the operation of the auxiliary subroutine so that other library subroutines which in general use lower memory positions as temporary storage may be utilized in the computation of $f(x)$
METHOD	If $(x_1)_n$ and $(x_2)_n$ are two arguments of $f(x)$ such that $f[(x_1)_n] \times f[(x_2)_n] < 0$, linear interpolation is used to find the next approximation to a root x_{n+1}. To make the iterative process second order, $f[(x_1)_n] \times f[x_{n+1}]$ is tested for sign. If it is positive, replace

$(x_1)_n$ by $(x_2)_n$, $f[(x_1)_n]$ by $f[(x_2)_n]$, $(x_2)_n$ by x_{n+1} , and $f[(x_2)_n]$ by $f[x_{n+1}]$. If it is negative, replace $(x_2)_n$ by x_{n+1}, $f[(x_2)_n]$ by $f[(x_{n+1})]$, $f[(x_1)_n]$ by (1/2) $f[(x_1)_n]$, and leave $(x_1)_n$ unaltered. The process is repeated until either $|(1/2)\ f(x_{n+1})| = 0$ or $|(x_1)_n - (x_2)_n| \leq 2^{-38}$.

DATE December 11, 1953 rt. 1/23/59
CODED BY J. N. Snyder
APPROVED BY J. Nash

lgr

LOCATION	ORDER		NOTES PAGE 1
	00 K(H1)		
0	S5 F		
	L4 4L		Plant link addresses
1	42 30L		
	42 31L		
2	L5 10F		
	50 2L		
3	26 S3		1/2 $f[(x_1)_0]$ to 11
	10 1F		
4	40 11F		
	00 1F		Waste
5	L5 12F		
	50 5L		
6	26 S3		1/2 $f[(x_2)_0]$ to 13
	10 1F		
7	40 13F		
	50 13F		
8	75 11F		Test $f[(x_1)_0]$ $f[(x_2)_0]$
	36 31L		
9	L5 11F	from 29	
	L0 13F		
10	40 14F		
	50 10F		
11	75 13F		
	40 16F		Linear interpolation
12	S1 F		
	50 12F		
13	74 11F		
	L0 16F		
14	66 14F		
	S5 F		
15	40 14F		
	50 15L		
16	26 S3		1/2 $f(x_{n+1})$ to 5
	10 1F		
17	40 15F		
	L3 15F		Test - $\lvert f(x_{n+1})\rvert$

LOCATION	ORDER		NOTES
18	36 30L		
	L5 10F		
19	L0 12F		
	40 16F		
20	51 7L		Test $2^{-38} - \|(x_1)_n - (x_2)_n\|$
	00 2F		
21	L2 16F		
	36 30L		
22	50 15F		
	75 11F		Test $f(x_{n+1})$ $f[(x_1)_n]$
23	32 25L		
	L5 11F		
24	10 1F		Replace 1/2 $f[(x_1)_n]$ by 1/2 $f[(x_1)_n]$
	40 11F		
25	22 27L		
	L5 12F	from 23	Replace $(x_1)_n$ by $(x_2)_n$
26	40 10F		
	L5 13F		Replace 1/2 $f[(x_1)_n]$ by 1/2 $f[(x_2)_n]$
27	40 11F		
	L5 14F	from 25	Replace $(x_2)_n$ by x_{n+1}
28	40 12F		
	L5 15F		Replace 1/2 $f[(x_2)_n]$ by 1/2 $f(x_{n+1})$
29	40 13F		
	26 9L		Repeat cycle
30	L5 14F	from 18, 21	Put x_{n+1} in A and leave
	22 ( )F	by 1	
31	L5 32L		Put -1 in A and leave
	22( )F	by 1	
32	80 F		
	00 F		= -1