6889
Memorandum M-1343

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Digital Computer Laboratory
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SUBJECT: DESCRIPTION OF BASIC TRACK-WHILE-SCAN AND INTERCEPTION PROGRAM

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Abstract: The track-while-scan and interception program used in the initial experiments of the 6889 Air Defense Group is described. The note covers such subjects as the source and nature of the incoming data, the logical aspects of the track-while-scan action, and the computations used for computing the interception course. A flow diagram of the computer program is presented and discussed in detail; a coded program corresponding to the flow diagram is included but not analysed.

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I. INTRODUCTION

1.1 Scope of the Note

This note is intended as a general introduction to the basic program evolved by the 6889 Air Defense Group for the initial experiments and demonstrations of the use of a high-speed digital computer in an air defense system. The specific program discussed herein was developed during the latter part of 1950 and was successfully operated early in 1951; the program has proved to be so fundamental in nature that it has been used as the basis for the majority of succeeding programs. Since this note covers only the developments and concepts of the basic program, it is essentially a record of the state of affairs as of the early summer of 1951.

The remainder of this section of the note gives a general introduction to the first 6889 air defense experiments. Section II describes the source and the nature of the data used in the experiments. Section III discusses the general philosophy and logic of various parts of the track-while-scan action of the program; Section IV has a similar discussion regarding the interception equations. The material of Sections II, III, and IV is brought into unity in Section V which includes and discusses the flow diagram for the complete track-while-scan and interception program. A copy of the orders and data for the program are included as an Appendix.

1.2 Initial Objectives of Experiments

The objective of the first air defense experiments and studies was the use of the Whirlwind Computer to perform the necessary computational and data-processing functions associated with:

a) automatic track-while-scan (TWS); that is, the automatic tracking and display of selected aircraft using data obtained from a continuously-rotating search radar,

and b) the automatic track-while-scan of selected aircraft and the computation of the heading instructions necessary to guide one aircraft -- the interceptor -- on a collision course with a second aircraft -- the target. These interception computations were to be such that they could be used for the mid-course phase of the interception, leading to a closing phase under the direction of airborne intercept (AI) radar.

In each of these tasks, manual methods were to be used for the initial selection and designation of the aircraft to be tracked and for the transmission of computed heading instructions; all other tasks were to be performed automatically.

1.3 Equipment and Data

The computer programs written for the tasks listed above were to be used with Whirlwind in its initial stages of operations. This restricted the programs to the initial storage capacity and operating speeds. This capacity was 256 16-binary digit electrostatic storage registers, with an average time per single operation of between 50 and 100 μseconds. Thirty-two registers of test storage were also available; of these, 27 were toggle switch registers which were used for reading in programs from punched-paper tape, the other five were flip-flop registers with associated indicator lights. The flip-flop registers and their indicator lights could be used for inserting information into the computer or for reading particular results of computer computations.

The primary source of data for the computer was to be an MEW search radar operated by the Air Force Cambridge Research Center (AFCRC) at the Bedford Airport, some 12 miles northwest of Boston. Inasmuch as the radar had no means of altitude determination, the track-while-scan and interception operations were to be carried out in only two dimensions.

As a means of transmitting this radar data to Whirlwind and presenting it in a digital form, use was to be made of a prototype digital radar relay (DRR) link designed and tested in conjunction with the MEW radar by AFCRC. Although not designed specifically for extended operational use, the relay link was pressed into service because of its immediate availability.

The transmission of heading instructions from Whirlwind to the interceptor aircraft was to be by means of a voice radio link. Aircraft for the roles of interceptor and target were supplied by the Instrumentation Laboratory at MIT and by the AFCRC.

1.4 Progress of Work and Results

Work on the formulation of the computer programs and the construction of terminal equipment necessary to meet the two objectives of 1.2 was begun in the summer of 1950. Preliminary tests of programs with actual and simulated radar data were made in the late summer of that year, using only the 32 registers of test storage. Extensive operational tests were first made possible when 256 electrostatic storage registers became available for use by the computer applications groups in November of 1950.

During the fall of 1950, testing with actual radar data was hindered by marginal operation of the relay link and the radar set. These troubles were investigated and eliminated during the winter months, and early in the spring of 1951 a series of actual flight tests with radar data were held. These tests established that the computer with a storage capacity of 256 registers could successfully track five aircraft* or could track two aircraft, guiding one on a collision-course interception with the other. About ten interception flight tests were attempted and completed through June of 1951; the final separations of the target and interceptor aircraft as their paths crossed averaged between 500 and 1500 yards. (These interceptions, being completely of a collision course nature, had no provisions for placing the interceptor in an advantageous position behind or to the side of the target aircraft.)

1.5 Extension of Basic Programs

Along with the basic ideas developed and used for tracking-while scanning, it has been possible to use the computer to perform other related operations. Among these have been a limited form of automatic acquisition -- with the computer selecting and initiating tracking of aircraft -- and a processing of radar information to permit a direct and immediate typing of position coordinates and velocities **during the simultaneous tracking of two aircraft.**

Studies relating to the use of the Bedford radar have been continued during 1951 in an attempt to improve and extend the quantity and quality of the track-while-scan and interception functions.

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* A program to track nine aircraft was written and operated with some success. This program required that the tracked aircraft were not grouped together with less than a certain minimum angular separation. A program which eliminated this restriction and which had improved operating characteristics turned out to have a tracking capacity of five aircraft.

efforts are presently being made at providing heading instructions to permit the interceptor to make any desired type of approach to the target in the closing and final phases. A good deal of effort is also being put into an improvement of the methods employed for smoothing the quantized radar data and deriving values of aircraft velocity. New and improved programs have been written to take advantage both of an initial increase in the available electrostatic storage registers from 256 to 304, and of expected further increases to more than 608 registers.

II. [illegible]

2.1 The Radar Set

The primary source of data for the computer is the MEW radar set situated at the Bedford Airport. This radar is operated with a speed of rotation of 4 rpm and has a vertical fan-shaped beam 1.2° wide (measured between half-power points) and 2.5° in elevation (from the ground to the upper half-power point). The radar provides only range[*] and azimuth information, having no provisions for measuring the altitude or elevation angle of an aircraft.

Attempts at eliminating or suppressing radar returns from stationary targets by means of a mercury delay-line type of moving target indicator (MTI) proved unsuccessful, and at the present time no MTI is used. This results in a situation in which the radar video (echoes) contains a good deal of ground returns. These ground returns, or ground clutter, mask any aircraft returns at ranges of less than 10 to 20 miles.[**] Although the theoretical maximum range of the radar is upwards of 200 miles, the operational range seems to be limited to between 70 and 80 miles for moderate-sized aircraft flying at altitudes less than 10,000 feet.

There has as yet been no opportunity to make a detailed test of radar coverage at all points of the compass or at altitudes above 10,000 feet. The visibility of the radar is probably affected by the relatively poor site at Bedford, where the radar is located in a shallow valley. Coverage to the west is affected by the Monadanock range of hills and mountains at about 40 miles distance. For the most part, all flight tests have been restricted to the area north and east of Bedford.

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[*] Actually the radar measures slant range. The difference between the slant range and projected range is sufficiently small, especially in comparison with quantization errors (See Section 2.2.2), and is neglected in computer computations.

[**] Unless otherwise stated, in the remainder of this paper all distance measurements are assumed to be in nautical miles. One statute mile = .87 nautical miles.

2.2 DATA ENCODING AND TRANSMISSION

2.2.1 General Description

The range of an object giving a radar echo is determinable from a measurement of the elapsed time between the transmission of a pulse of energy from the radar and the reception of an echo from a reflecting object; the azimuth of the object giving the echo can be determined from the angular position of the directional radar antenna. Auxiliary equipment at Bedford converts the analog range and azimuth quantities to a digital form consisting of a group of parallel binary digits. This digital information is used to key a number of audio frequency oscillators and the paralleled outputs of the oscillators are multiplexed and transmitted over an ordinary telephone line to the Barta Building. At the receiving end of the telephone line, the multiplexed signals are filtered out, shaped into binary voltage pulses, and are introduced into Whirlwind through one of the flip-flop registers of test storage.

2.2.2 Quantization

The conversion of analog measurements to a digital form requires that the data be restricted to certain discrete levels; when this is done, the data can then be represented in numerical form by a limited number of digits. The effect of restricting the data to discrete levels or units is known as quantization and manifests itself as an uncertainty in the data of 1/2 the smallest level or unit of measurement. Quantization is quite similar to round-off effects in numerical computations.

In converting the analog data to digital form at Bedford, ranges are measured to the closest mile, azimuths to the closest 1/256 of a revolution. The range quantization corresponds to an uncertainty of ± 1/2 mile, while the azimuth quantization of 1/256 of a revolution or 1.4° corresponds to a tangential uncertainty of ± 1/2 mile at 40 miles, ± 1 mile at 80 miles. A convenient way of visualizing the range and azimuth quantization is to consider a polar grid -- consisting of radial lines and concentric circles -- placed on a map. The effect of the quantization of the data is to report aircraft as being only at the various intersections of the grid.

Inasmuch as the azimuth unit is 1/256th of a revolution, 256 different azimuths are possible and eight binary digits are required for complete specification. (The azimuth unit of 1/256 of a revolution or 1.4° was chosen due to its correspondence to the antenna beam width of 1.2°.) It was originally planned to use eight binary digits for range and to send ranges up to 255 miles.

This led to the choice of a one-mile unit as the smallest describable increment of range. In actual operation, however, only ranges up to 127 miles are transmitted; nevertheless, a one-mile unit is used and only seven binary digits are employed to specify a range.

2.2.3. Analog and Digital Storage Tubes

The pulse repetition frequency of the MEW radar is 300 cps, and within the space of one beamwidth an aircraft should give about 12 echoes or returns. If the returns from 12 outgoing pulses were superimposed, the returns from the actual targets on successive scans would overlap and reinforce at the corresponding ranges, while the noise and random returns would be spread out and would have little correlation. In order to take advantage of this fact and to increase the signal-to-noise ratio of the returns, returns are integrated over one beamwidth by means of a video storage tube at the radar site. The integration is performed with the raw radar video, before the data is quantized, and in this way the integration performs a filtering action and provides a means of video enhancement.

The rate at which information can be transmitted over the telephone line is limited, and with the techniques used in the present DRR only one piece of information -- either a range or azimuth -- can be transmitted each 1/50th of a second. The radar, on the other hand, may produce useful information at a widely varying rate; in the worst case with two aircraft along the same azimuth and but one mile apart in range, video returns or echoes are received 12.3 μseconds apart. After the quantization process, ground clutter and storm clouds appear as returns at successive ranges and azimuths, and under such conditions it is not unusual to have echoes along a single azimuth at four or five consecutive one-mile intervals. Although at peak rates the quantized radar information may be produced at 12.3 μsecond intervals, over a complete rotation of the radar the average rate of information falls below the 1/50 second rate which can be handled by the DRR.

As a means of properly handling returns at successive ranges and azimuths, it was necessary to install a digital storage tube at the Bedford end of the relay link. This storage tube was used to store all radar information as rapidly as it was received and quantized. The tube had a capacity of 32 pieces of information, and either range and azimuth information could be read out from it onto the telephone line at 1/50 second intervals. This digital storage tube was used in conjuction with the DRR until January, 1951, when it was removed due to poor reliability. Since that time, the data transmission has been such that only one quantized range -- that corresponding to the earliest or closest return -- can be transmitted at each quantized azimuth. Under such a situation, ground clutter or nearby aircraft would tend to shade or blank-out aircraft at larger ranges; in order to prevent this from being too serious, a human operator at Bedford inspects and gates out the video returns from the ground clutter. This gate, which may be adjusted in range as the antenna rotates, is usually manipulated so as to blank out the video at ranges of less than 15 to 25 miles.

The adjustable range gate is only a temporary expedient, and some storage may be reintroduced into the DRR in order to permit data to be transmitted for two or more aircraft at the same azimuth. The computer programs which have been written are of such a form that they will operate satisfactorily under conditions in which the storage at Bedford is or is not in use.

2.2.4 Timing

Information is transmitted over the telephone line each 1/50 of a second. Whenever there is no range or azimuth information available to be sent at a particular 1/50 second interval, a zero range or null signal is transmitted.

Prior to the removal of the digital storage tube at Bedford, whenever there was radar information to be transmitted, the azimuth information was sent at one 1/50 second interval and at succeeding intervals of 1/50 second the range or ranges at that azimuth were sent. These were followed by zero ranges until the next sequence of azimuth, range, etc. were available to be sent. If the radar received no returns during a sector of the scan, only zero ranges were transmitted in that period.

Since the removal of the digital storage tube, there is, of course, only one range per azimuth.* At the present time, every other 1/50 second transmission is an azimuth -- the current azimuth of the antenna; after each azimuth either a range or zero range is sent. Since there are 1/2 x 15 x 50 or 375 azimuth

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* Actually, this is only one range per 12-pulse integration.

transmission times and but 256 different azimuths, about half the azimuths are transmitted twice per scan.

2.[illegible].5 Coding

The transmission system from Bedford has 10 parallel binary channels, the information sent in each channel being either a 0 or a 1. Consider these channels as having outputs at Whirlwind numbered from 0 to 9.** Output #0 is the timing channel and receives a 0 each 1/50th of a second. Output #1 is used to distinguish between ranges and azimuths; an azimuth is indicated by a 1, a range by a 0. Outputs #2 through 9 receive the parallel digital range or azimuth information with the following conventions:

Azimuth:

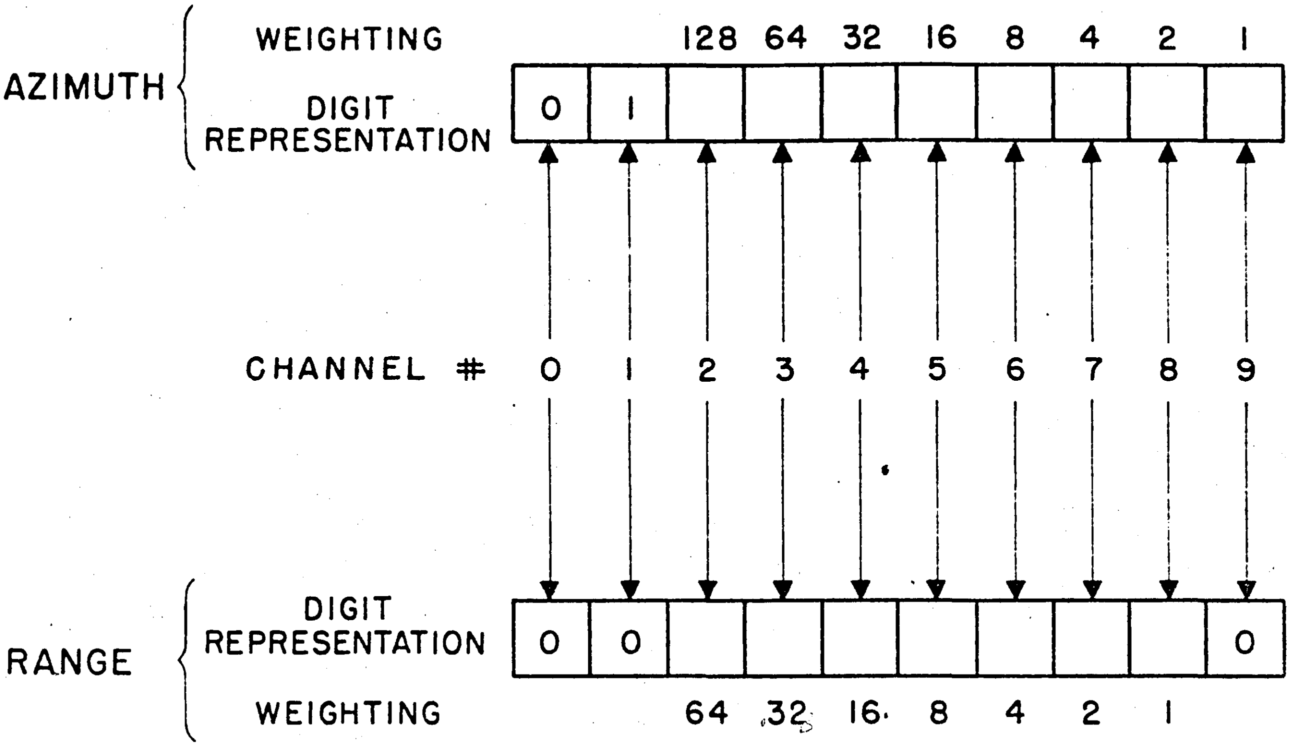
> The digital information is a binary number from 0 up to 255 representing the number of 1/256 parts of a complete revolution. Output #9 is weighted as 1, #8 as 2, #7 as 4, ...#2 as 128. An azimuth of 0 corresponds to true north, 64 is west, 128 is south, 192 is east.

Ranges:

> The digital information is expressed in nautical miles, and extends from 0 through 127. Output #9 is not used for range transmission and contains a 0, output #8 is weighted as 1, #7 as 2, #6 as 4......... #2 as 64.

The transmission of ranges and azimuths is summarized in Figure [illegible]. It should be noted that 1's at outputs # 1 through 9 correspond to an azimuth of 255/256, just east of north. Zeros at outputs #1 through 9 is the zero range or null indication mentioned above.

** The terminology output is used here in place of channel so that the outputs can be numbered in accordance to the numbering of the digit positions in Whirlwind to which they are connected. The channel numbers, as assigned by AFCRC, do not correspond to the output numbers.



DIGITAL CODING OF RANGES AND AZIMUTHS
FOR DATA LINK FROM BEDFORD

FIG. 1

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2.3 Introduction of Data Into Whirlwind

A piece of information -- azimuth, range, or zero range -- is received at the Whirlwind end of the telephone line each 1/50 of a second. The data, in the form of multiplexed signals, is filtered, the pulses are shaped, and the data is then introduced into Flip-Flop Register 4 of test storage. The transfer of the information is done with video pulses, and is synchronized so that the data is not introduced while the computer is reading into or out of the flip-flop register.*

The arrival of the data is, in general, non-synchronous with the program action of the computer, and since there is no buffer storage at the input of the computer to store or save this data, it becomes necessary for the computer to inspect the flip-flop register oftener than each 1/50 of a second so that data is not lost. At these times the computer must remove, operate upon, and store internally any data discovered during the inspection.

The computer initially sets all digits of the input flip-flop register (FF4) to 1's. This condition represents -0 to the computer, and this negative condition is used as a no-input indication. The ten data outputs from the telephone lines are connected to the correspondingly numbered digits of the input flip-flop. In addition, the timing channel (output #0) is also connected to digits 10 through 15. The action of an input from the telephone line is to reproduce the binary digit information in digits 1 - 9, while changing digits 0, 10, 11, 12, 13, 14, 15 to the zero condition. The changing of digit 0 to a 0 effectively makes the flip-flop appear positive and hence is used as an indication by the computer that data has been received.

2.4 Specialized Characteristics of the Quantized Radar Data

Mention has already been made of the large number of echoes received from stationary targets such as ground clutter. During inclement weather, returns are received from storm clouds and in general there is a certain amount of apparent radar return which results from noise within the radar receiver or from atmospheric conditions. Some of this noise manages to survive the integration process and is quantized; other incorrect information is transmitted as a result of encoding errors at Bedford or errors in transmission.

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* For details of the synchronization, see M-2068 and E-387

Another characteristic of the data which must be noted is the possibility of a lack of reports on certain aircraft during successive scans of the antenna. These misses may be due to the vertical position of the aircraft with respect to the radar beam, the distance of the aircraft, or the type of reflecting surface which the aircraft presents. The latter factor is manifested by a higher probability of miss when an aircraft makes a turn.

The quantization of the radar returns requires that a definite choice be made as to whether a particular analog measurement should be quantized to one discrete level or the next. The characteristics of the encoding equipment at Bedford are such that when the digital storage tube is in use, targets whose actual range falls close to half-way between two successive quantized values may be reported at both ranges. This phenomenon of multiple range returns is nonexistent under the present mode of operation without the storage tube. *

The action of the video storage tube together with the width of the radar beam and the attendant side-lobes tend to create multiple azimuth returns wherein a single aircraft will be reported at the same range but at more than one consecutive azimuths. This phenomenon is rather common and may involve more than two consecutive azimuths when the aircraft in question is at a relatively low range and the radar echo is fairly strong.

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* See 2.2.4 Apparent Multiple range returns are possible when the same azimuth is transmitted at successive azimuth transmission intervals.

3. TRACK-WHILE-SCAN

3.1 General Considerations

3.1.1 Definition of Terms

For convenience in the remaining sections of this note, it will be necessary to define two terms which shall be used in the description of the track-while-scan activity of the computer.

By the term *correlation* is intended the means by which the radar data corresponding to a particular aircraft is identified on successive scans of the radar antenna. The term *smoothing* is used to describe the process by which the radar data pertaining to a particular aircraft is manipulated and used on successive antenna scans so as to permit a determination of the velocity components of the aircraft. In the sense in which it is used in this note, *track-while-scan* implies both the correlation and smoothing of the radar data.

3.1.2 Coordinate Systems and Displays

In accordance with the discussion of Section 2.2, radar data is made available to Whirlwind in polar coordinates in terms of ranges and azimuths. This data is converted to (x, y) coordinates by the computer both for use in the track-while-scan function and as a means of implementing suitable displays. The conversion from (r, θ) to (x, y) coordinates is accomplished by means of a computer subprogram which calculates the sines and cosines of incoming azimuth angles.

A primary reason for the coordinate conversion is to enable a computer-controlled display of all or part of the received radar data. The scopes presently available for display purposes at Whirlwind permit a proper PPI display only when the deflection voltages are proportional to the (x, y) coordinates of the information. These (x, y) positions, available in the computer in a digital form, are converted to proportional analog voltages for the scope deflections by means of digital-analog decoders. The PPI presentation is under the control of the computer which displays the incoming data in azimuth sequence, but the scope is intensified and a spot illuminated *only* for each piece of radar data; this latter fact makes the display different from normal radar PPI displays inasmuch as the faintly-traced rotating line signifying the rotation of the antenna is missing and there is but a single intensity of display, with no half-tones.

For the general purposes of track-while-scan and interceptions, the computer generates two separate displays, either of which or both may be presented on long-persistence cathode ray tubes. On one display, *the D scope display*, all of the incoming radar data is shown; a second display, *the F scope display*, is used as a means of presenting only a filtered picture containing only those selected aircraft which the computer is tracking. Either display or both may be shown on a single oscilloscope depending on the position of a switch.

A display scope, specially designed and constructed at AFCRC, has also been installed at the Barta Building where it can be used to directly display (r, θ) data arriving over the telephone line or data previously recorded from the telephone line on Magnecorder Tape. This display scope, commonly referred to as the Digital PPI, does not require processing of the data by the computer and provides, among other things, a handy means of checking the incoming radar data. However, for the purposes of a selective or filtered display, the computer-controlled presentation and the additional scopes and decoders are necessary.

A further reason for the conversion to and use of data in (x, y) coordinate form is that it permits simpler programming for obtaining smoothed values of velocities and for calculating interception courses. The simpler programming is primarily due to the fact that an aircraft flying a straight-course at constant ground speed has constant x and y velocity components. A straight-line constant-speed path does not, however, produce constant velocity components in a radial or tangential direction. Higher order derivatives in (r, θ) coordinates, i.e., accelerations, complicate the smoothing of velocities and the computation of the headings for an interception course.

One disadvantage which enters into the use of (x, y) coordinates arises from the effects of the (r, θ) quantization. The range quantization is fixed, while the azimuth quantization, in terms of distance rather than angle, varies with range and above 20 miles the quantization in azimuth is effectively worse than that in range. The coordinate conversion process tends to mix the two quantizations, with the resultant quantization in (x, y) coordinates varying sinusoidally with position. The mixing of quantization has an effect on the smoothing of velocities since the generally smaller quantization error in range is mixed with the larger error in azimuth to produce an intermediate amount of quantization in both x and y coordinates.

A second mode of display is used by the computer to present the results of the computation of the proper heading angle to be transmitted to the pilot of the interceptor aircraft. This angle is displayed in the indicator lights associated with one of the flip-flop storage registers. The angle is initially calculated in binary form, but for purposes of display it is converted to binary-coded decimal form with reference to magnetic north. (Each decimal digit of the heading angle, expressed in degrees, is represented by its direct binary equivalent.) Recently a direct decimal display of the [illegible] angle has been made possible by means of a relay converter connected to the binary-coded indicator lights.

3.2 Correlation

3.2.1 The Problem

As defined in Section 3.1.1, the correlation function is that of determining on successive scans which piece of incoming data corresponds to an aircraft being tracked. The incoming data consists of a sequence of azimuths and ranges, or after conversion, a sequence of (x, y) positions. From this data must be extracted those reports which appear to correspond most closely to an aircraft being tracked. The method of correlation must be sufficiently sophisticated so that the computer is able to make a proper choice when the radar receives echoes from two or more aircraft closely spaced. The computer similarly must not be confused when the DRR transmits multiple returns from a single aircraft. On the other hand, the computer must be able to recognize and take proper steps when the radar misses an aircraft on one or more scans.

The correlation process relies heavily upon predicted values of position as supplied by the smoothing action of the computer. The two processes are interrelated: The correlation process must supply the proper data for the smoothing, while the smoothing process, in turn, prepares for the correlation on the next scan. Once started, the sequence is self-perpetuating, but proper action is required for the initiation of tracking on the target. That is to say, on some scan the computer must be informed that it is to start tracking a particular aircraft. In the interception problem, the computer must also be informed initially of the identity of the aircraft: target or interceptor.

3.2.2 Use of Velocity Information

Under optimum conditions -- those in which there are few aircraft in the area, these aircraft are fairly well separated, there is little ground clutter, and the aircraft do not travel very far in terms of quantizing units from scan to scan -- the correlation function can be carried out without any knowledge of the velocity of the aircraft. Correlation is accomplished by expecting and looking for the aircraft during one scan of the radar at the same position or within a small area about the position where it was reported on the last scan.

Under more realistic conditions, however -- conditions in which there is a high density of aircraft with crossing aircraft paths, a good deal of ground clutter, and high aircraft speeds -- the correlation function is not so easily performed. In this case, a knowledge of the aircraft's velocity permits a prediction to be made of the expected position of the aircraft and enables the computer to select one and possibly the best of a number of pieces of radar data which might correspond to the aircraft. In general, the knowledge of an aircraft's velocity components eases the correlation problem since it presents additional information with which to carry out the assigned tasks.

As noted above, the velocity components produced as a result of a smoothing process enable a short-range prediction of position to be made, this prediction then simplifying the correlation process. With reliable velocity information, the computer is able to predict positions of an aircraft for some longer times in the future; this is reliable only if the aircraft is travelling a straight path. Predictions of future positions are necessary for the computation of a collision course in which the interceptor is directed on a straight-line path which will enable it to reach a point in space at the same time as the target aircraft (see Section 4.2).

3.2.3 Initial Correlation of Data -- The Time Counter

Assume that at some time during a scan of the antenna, the computer has been able to predict the probable position of a tracked aircraft during the next scan. The initial step in the correlation process, then, is to screen the incoming information during the next scan and pick out those pieces of data which correspond closely enough to this predicted position to warrant consideration.

One method of attack would be to inspect <u>every</u> piece of incoming information, checking its coordinates with the predicted coordinates of the aircraft in question. Obviously the inspection of each piece of incoming information is unncessary, since we need inspect only those pieces of data whose azimuths fall close to the predicted azimuth of the aircraft. That is to say, we need only inspect those pieces of data received when the antenna is pointing approximately in the direction of the aircraft.

When the above-mentioned method was first investigated and considered, the digital storage tube was still being used as a part of the DRR. As noted in Section 2.2.4, the characteristics of the data at that time were such that azimuths and ranges were sent only when the coordinates of a radar return were being transmitted to the computer. Thus the azimuth of the antenna was made available to the computer only when a radar return was being transmitted, and this azimuth value would not necessarily correspond to the present position of the antenna due to the delay of the digital storage tube. A further unfortunate characteristic of the digital storage tube was that if the information to be sent accumulated beyond the capacity of the digital storage tube, the excess information was written in over the previously-stored information. This overwriting caused the stored information to be in error, and erroneous azimuths and ranges were transmitted over the telephone line. The overloading of the digital storage tube occurred frequently enough to make it necessary that the computer not use the transmitted azimuths of the DRR as an indication of the present orientation of the antenna.

The overloading of the digital storage tube was but one of the reasons why no attempt was made to use angular position of the antenna directly as a means of data correlation; if it had been the only reason, certain changes could have been made in the digital storage system to eliminate overloading.* As discussed in Section 3.1.2, the use of (x, y) coordinates was necessitated for several reasons, and if correlation with azimuths were to be carried out there would be a necessity to convert the predicted (x, y) position of the aircraft to (r,θ) coordinates. Such a conversion would have been fairly costly in computer storage. In addition, a comparison of the azimuths of the tracked aircraft and of the incoming data is not a simple operation and certain care is necessary to handle the discontinuity in azimuths as the antenna passes north. These reasons, and others which will become evident in succeeding sections, pointed to another method of determining when incoming radar data should be correlated with the predicted position of a tracked aircraft. The method adopted at that time (see below) still appears to be the most acceptable one even under the latest operating conditions with the digital storage tube removed and the DRR sending all azimuths, each azimuth being transmitted in close relationship to the antenna position.

In brief, it was desired to set up a small angular search sector centered on the azimuth of a tracked aircraft, and all data arriving in this sector was to be compared with the position of that aircraft. The bounds and position of the sector were established by counting the pieces of information arriving over the DRR. As noted, one piece of information -- azimuth, range, or zero range -- is received each 1/50 second. Thus in 15 seconds or one antenna rotation, 750 pieces of information are received and 750 counts can be registered by the computer. If a storage register referred to as a <u>time counter</u> were set to 0 when the data corresponding to the <u>tracked aircraft</u> was received during one

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* These changes were under consideration at the period prior to the removal of the digital storage tube at Bedford.

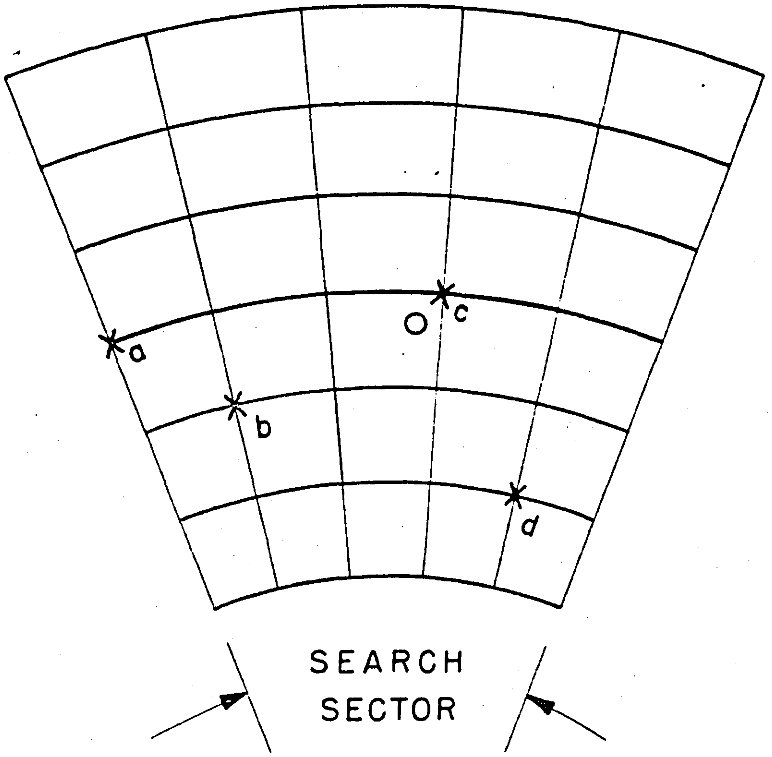
[illegible] and if the computer added one to this counter on the receipt of each new piece of incoming data, then the search sector for that aircraft might begin again when the time counter reached a value of, say, 725. If the time counter reached a value of about 775 without any data being found which corresponded with the tracked aircraft, then this would serve as an indication of a radar miss and appropriate action could be taken.

The exact details of the use and setting of the time counter will be discussed more fully in Section 3.2.6. For present purposes it is only necessary to note that a method is available for establishing a search sector for correlation and that the use of such a sector means that the computer need be concerned with a particular tracked aircraft only during a relatively short period of time during each antenna scan. The saving of time and resulting efficient use of the computer time because of this latter fact makes it possible to employ the computer for multiple aircraft tracking without any need of utilizing the internal storage of the computer as a buffer storage for incoming data; that is, since each piece of incoming data need not be compared with all aircraft but only with the few which it falls [illegible] a piece of incoming data can be handled and processed in less than [illegible] second.

3.2.4 Selecting the Best Piece of Data -- The Search Area

The previous section has discussed the establishment of a search sector within which all incoming pieces of data are to be inspected for correspondence to the predicted position of the tracked aircraft. As will be noted in Section 3.3, the predicted position of an aircraft will generally be given with greater precision than 1/2 mile; the radar data, on the other hand, describes the intersection points on a grid whose lines are about one mile apart. As an example of the situation which might be encountered, consider Figure 2. In this figure the circled dot represents the predicted position of the tracked aircraft and the crosses represents the positions of the incoming radar data in the search sector. An inspection of data a, b, c, and d should lead to the conclusion that the position labelled c is probably the quantized data most closely corresponding to the aircraft, that is, c is the "best" piece of data. This decision is based on the fact that c is the one which is closest to the predicted position of the aircraft.

It is desirable to discard a piece of data as soon as possible in view of a general need to economize on storage space. The computer could store all the pieces of incoming data falling within the search sector and then determine which one of these is the "best"; far more desirable is a means by which each piece of



A POSSIBLE ORIENTATION OF A PREDICTED POSITION
AND
INCOMING DATA IN A SEARCH SECTOR

FIG. 2

-50641-

data can be used and then discarded before the next piece is made available from the telephone line. Of similar importance is the need to be able to determine when the antenna has passed through the search sector of a tracked aircraft without finding a piece of data adequately corresponding to the predicted position.

As a convenience in explaining the method employed, let it be assumed that the predicted position of a tracked aircraft is $\bar{\bar{x}}_p$ and $\bar{\bar{y}}_p$. Let it further be assumed that the data falling within the search sector corresponds to positions:

x_a, y_a; x_b, y_b; x_c, y_c; x_d, y_d; etc.

The computer then considers a circle about the point $\bar{\bar{x}}_p$, $\bar{\bar{y}}_p$. This circle is called the search area.* The initial radius of this circle is about three miles, this size being chosen in consideration of possible errors in the predicted position, especially after initiation (see Section 3.3), and the effects of quantization. Let this initial radius be designated as r_0.

The action of the computer in selecting the "best" piece of data is as follows: when the data x_a, y_a is received, the quantity $(\bar{\bar{x}}_p - x_a)^2 + (\bar{\bar{y}}_p - y_a)^2$ is formed; this is denoted as r_a^2. The computer then checks to see if r_a^2 is less than r_0^2. If so, the computer selects x_a, y_a as the probable position of the aircraft and substitutes the value r_a^2 in the place of r_0^2; if $r_a^2 \geqq r_0^2$, the data is rejected. When the next piece of data -- x_b, y_b -- is received, $(\bar{\bar{x}}_p - x_b)^2 + (\bar{\bar{y}}_p - y_b)^2$ is formed. If this quantity, r_b^2, is found to be less than the previously selected minimum r^2 -- r_0^2 or r_a^2 -- then the piece of data is selected as the new minimum. Again, if r_b^2 is found to be equal or greater than the previously selected minimum, the data is rejected and no change is made in the minimum r^2. This process is continued for all pieces of data falling within the search sector; whenever a better piece of data is found, the size of the search area is reduced. When this procedure has been carried out for all pieces of data received within the search sector, the "best" piece of data-- the definition of "best" being <u>the closest</u> -- will have been chosen.

---

* The word <u>area</u> is used since in preliminary programming a square was used and other shaped areas were considered. Corresponding terminology for analog equipment is the <u>gate</u> or <u>box</u>.

It should be noted that if it is found that the final radius of the search area is still r_0 after the antenna has passed through the search sector, then it can be assumed that the radar missed the aircraft. The action in cases of a miss is further described in detail in Section 3.2.5 . If the aircraft was not missed, after the computer senses that it has passed through the search sector it proceeds with the smoothing and prediction action. (See Section 5.2)

At the present time, two different initial sizes of search area are used. The larger size is usually set at about four miles, the smaller at three miles. Immediately after initiation or on the succeeding scan after a miss, r_0 is set to the larger size. On any scan after the computer has successfully tracked the aircraft, the smaller value of r_0 is used.

The initial size of the search area is somewhat of a critical matter, and the values indicated above were selected only after a careful study of the effects of various values of r_0. In particular, it is desirable to select the r_0 small enough so that only the quantized data of the tracked aircraft falls within the area. If the r_0 were larger and several pieces of data fell within the area, then whenever the radar missed the tracked aircraft and yet reported other echoes (possibly other aircraft) nearby, the computer would not suspect a miss and would tend to commence tracking on the other pieces of data (See Section 5.2). On the other hand, it is necessary that the search area be large enough to enable the computer to track the aircraft despite any sudden maneuvers or turns by the aircraft being tracked. In more sophisticated versions of the tracking program which may be written in the near future, more than two different sizes of search area may be used and the search area might be varied in accordance with the previous history of the tracking of that aircraft. In addition, the size of the search area is likely to be changed from a circle to some other shape, possibly one which is extended in the direction of motion of the aircraft.

3.2.5 Initiation and Cessation of Tracking

For initiation of tracking, it is necessary to indicate to the computer which of the incoming data represents the present position of the aircraft which is to be taken under consideration. The method used at the present time for the selection of the aircraft to be tracked has been termed manual initiation or manual aquisition. Manual initiation requires the services of a human operator who must identify the aircraft to be tracked from a

PPI display, either the display which is generated by the computer or that which is made available by the AFCRC digital scope. Automatic initiation, on the other hand, is a method by which the computer would employ some criteria in selecting particular radar returns to be tracked. These criteria might be the existence of otherwise untracked returns within the area of radar coverage, might be related to the appearance of returns near a peripheral boundary about the radar set, or might be concerned with certain characteristics of aircraft returns such as their positions and movements. Except for one preliminary programming study of automatic initiation, the Bedford experiments have all employed manual acquistion.

As a means of implementing manual acquistion, use is made of a photoelectric device termed a light gun.* This device consists of a phototube with a small aperture which will receive light incident on the end of an attached narrow tube or barrel. The end of this barrel or tube is placed on the PPI display over the position of the selected aircraft. When the aircraft return is next displayed, the illumination from the spot on the face of the cathode ray tube is used to trigger the light gun and form a pulse. This pulse is immediately used to change the sign digit of one of the flip-flop registers (FFO) of test storage. The action is quite similar to the use of the timing channel in connection with the input of radar data. The sign digit of the flip-flop is normally a 1, indicating a negative content, and the pulse from the light gun is used to reset sign digit to a 0 which gives the register a positive content The computer, after displaying each piece of radar data by setting up the horizontal and vertical decoders to the proper x and y values, stores the x and y coordinates until it has had a chance to investigate the flip-flop register associated with the light gun. If the computer discovers that this flip-flop has a positive content, the necessary action for initiation is undertaken.

When performing an interception it is also necessary to specify to the computer whether the aircraft being initiated is the target or the interceptor. This is performed by adjusting the toggle switches associated with the light gun flip-flop register so that it (the register) will be reset to either of two different negative values by the pulse from the light gun. In this way the computer first investigates the flip-flop to see if it is negative and if so, a further investigation is made to see to which of the two negative values it has been set.

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* A more detailed discussion of the light gun is available in E-2024

In the interception program, the computer is always tracking, or attempting to track, two aircraft. When the program is first inserted into the computer and radar data supplied, it attempts to track two imaginary aircraft at the origin. Since no radar returns at the origin are ever received, the computer acts as if the radar is continually missing the two aircraft and it waits in expectation of a return from the imaginary aircraft. The search sector is set up, but no returns are ever found within the search area and inasmuch as the smoothing program starts with a zero prediction of velocity (see Section 3.3), the two aircraft are repredicted at each scan as being at the origin. In a similar manner, the computer will continue to track, or attempt to track, the last aircraft initiated as target and interceptor until the light gun is used to acquire a new target or a new interceptor.

An evaluation of the action of the program in tracking either the target or interceptor can be made by reference to the filtered (F scope) display on which only the tracked aircraft are displayed. As noted, the computer checks and investigates returns until the radar antenna has passed through the search sector of a tracked aircraft; at this point the size of the search area is investigated to determine whether the radar saw the aircraft. If so, a display of the position of the aircraft is made on the F scope; if the radar missed the aircraft or the computer is improperly tracking, no display is given on the F scope. By superimposing the two scope presentations and noting whether or not a double display (the tracked or filtered display appears slightly later in time) occurs, it can be seen whether the computer is tracking the aircraft. Other visual scope displays or presentations in flip-flop indicator lights could be made to note the successful tracking of either target or interceptor; at present such additional displays have not been needed.

In the multiple tracking program mentioned in Section 1.4 which tracks five aircraft but which carries out no interception computations, there is no need for designation as target or interceptor. The tracking section of this program actually has two modes of operation: tracking or non-tracking. Tracking of a target is accomplished by use of the light gun on the main (D scope) display; cessation of tracking on a target is performed by use of the light gun on the filtered (F scope) display. In this program, faulty tracking or "misses" are indicated by the illumination of two points on the F scope, one at the position at which the aircraft should be and the other at the origin.*

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* The display at the position of the aircraft is needed to implement cessation.

For purposes of keeping count of the number of aircraft tracked, a binary indication is shown in one set of flip-flop lights.

For both interception and multiple tracking programs, the action following initiation is quite similar and will be discussed more fully in succeeding sections. In brief, the time counters must be set (see Section 3.2.6), the initial x and y velocities set to zero (see Section 3.3), the large size of search area inserted, and the x and y positions of the aircraft must be transferred from the display section of the program to the set of registers allocated to that aircraft.

3.2.6 Setting of the Time Counter

The important values or counts associated with the time counter are shown in Figure 3. The values shown are the values used by the computer multiplied by 2^{15} (They would be stored in the computer as 100×2^{-15}, 550×2^{-15}, etc.)** Assume that the position of the aircraft is as shown with the cross.

Upon initiation the time counter is set to -650. This count will be increased by 1 each 1/50 second. The computer begins correlating the incoming data with the aircraft when the time counter gets to +1. Each time a better fit of data is found the time counter is reset to +100. In time the best "best" fit of data will be found and 100 counts later when the time counter reaches 200, the search sector is ended and the counter is reset to -650. No correlation is again attempted with that aircraft until the time counter again becomes positive. It should be noted that the resetting of the counter to +100 upon finding a better piece of data ensures that the search sector is centered on the aircraft regardless of its movements.

** The values presently used are much larger than is necessary; satisfactory operation can be achieved under present operation of the DRR with a sector of about 10 counts.

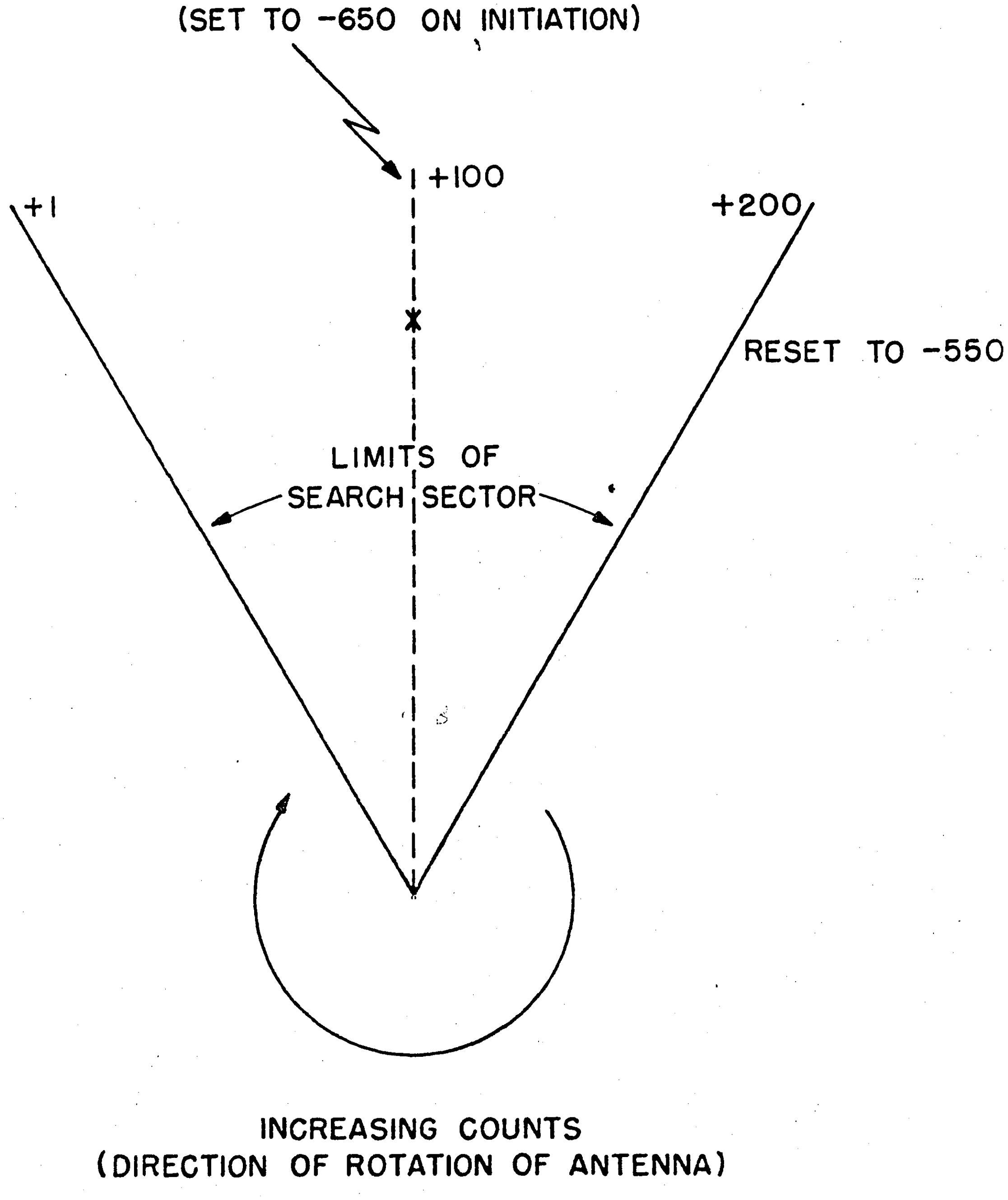

FIG. 3
TIME COUNTER VALUES

A-50586

3.3 Smoothing

3.3.1 Nature of the Problem and Its Restrictions

As noted, of paramount importance in the tracking and interception actions of the computer is the determination of the velocity of the aircraft under consideration. This velocity must be determined from the heavily quantized range and azimuth indications which, for each aircraft, are made available only once each 15 seconds.

For the initial phases of the Bedford experiments, the problem of velocity smoothing and tracking was first attacked in its simplest form and only aircraft flying straight paths, or paths without violent turns, were assumed. The importance of such a restriction is evident in view of the fact that it is possible for an aircraft to fly in a small circle in such a way that the quantization of the data makes it appear that the aircraft is stationary. Although the methods used were designed only for straight-line flight, they have been sufficiently successful in practive to permit tracking and smoothing for aircraft making 180° turns; work is now being carried out, however, on more advanced methods which will permit successful operation in all types of conditions.

In considering the problem and the desired results, it must be remembered that some methods of attack were precluded because of

a) limitations of internal storage space in the computer

and

b) requirements of computer time.

The storage restriction was by far the most stringent and dictated the use of a method which did not require the storage of a large number of previous pieces of radar data. Fortunately the method which was selected to minimize on storage turned out to be highly economical of operating time.

3.3.2 Effects of Quantization and Sampling

A convenient method of visualizing the effects of quantization and sampling (i.e., taking readings only at certain intervals of time) on the observed values of range and azimuth is shown in Figure 4. The flight of a constant-speed aircraft along a generalized coordinate X as a function of time is depicted by the heavy line, the slope of which is the X velocity of the aircraft. On top of this plot is superimposed a grid of which the vertical lines represent the sampling times -- i.e., the times of successive observations by the antenna* -- and the horizontal lines represent the quantized values of X at which the aircraft can be reported.

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* In general, as the aircraft changes its position the sampling period may increase or decrease slightly. This change, however, is negligible for aircraft at greater than a 20 mile range and flying at normal air speeds.

FIG. 4

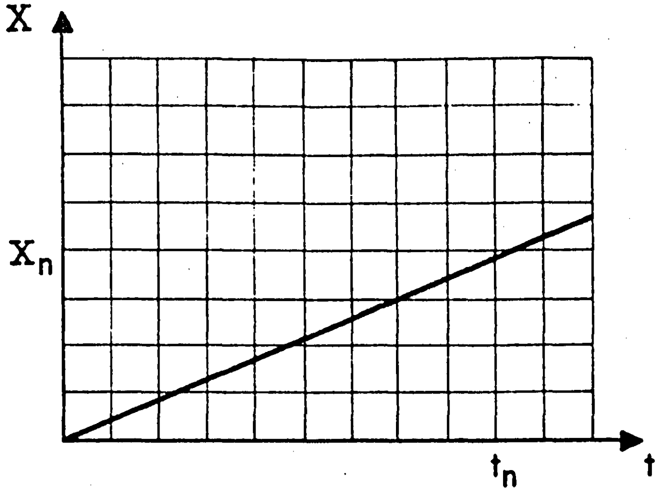


FIG. 5

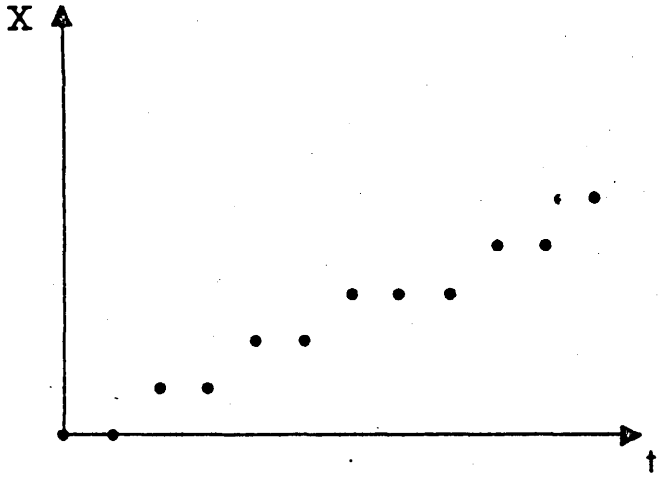
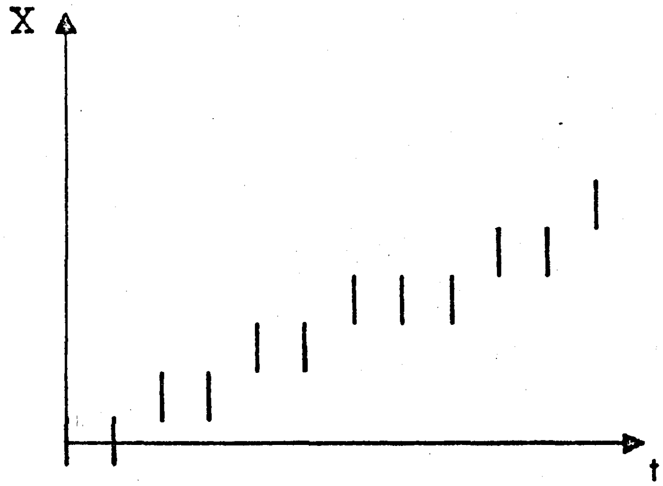


FIG. 6



QUANTIZATION OF DATA

Since the quantization is $\pm$ 1/2 mile, the horizontal lines have a one-mile separation. At any time, t_n, the aircraft would be reported as being at the quantized value, X_n; which is closest to the true position. Perfect quantization and error-free radar data are assumed.

If in Figure 4 the path of the aircraft were replaced by the values of X_n at times t_n, the plot of Figure 5 is obtained. Figure 5 is capable of further interpretation if through each quantized and sampled observation a line indicating the possible true positions of the aircraft is drawn. By doing this, a plot such as that in Figure 6 is obtained.

The true course of the aircraft is such that it intersects each of the vertical lines in Figure 6. In view of this fact, the path or the velocity might be determined by a method of curve-fitting. Such curve-fitting is not particularly easy to mechanize even for a straight line plot as in Figures 4, 5, and 6; it becomes considerably more complicated when there are curved flight paths and changes in the slopes on an X-t plot. It should also be noted that for a certain configuration and limited number of observations, it is possible to construct a number of different straight lines through the vertical lines of Figure 6, thereby permitting a spread in the possible velocities. Methods of curve-fitting were excluded from use in the smoothing program under the simplified conditions of the initial Bedford experiments due to the fact that these methods required an excessive amount of storage space.

3.3.3 Smoothing Equations

The method and equations to be described below have been modeled after the so-called "aided-tracking" method which has been used successfully in analog computers. In its full generality, however, the method is seen to be an application of a procedure of "successive approximations."

Let it be assumed that by some means the computer has a fairly good knowledge of the position, $\overline{x}_n$, of an aircraft at time, t_n, and that the computer has been able to secure a rough estimate of the aircraft's velocity*. Call this estimate $\dot{x}_n$.

---

* The notation $\overline{x}_n$ for position is used to represent a value which has more precision and more significant digits than a quantized radar reading.

From the two quantities $\bar{x}_n$ and $\mathring{x}_n$ it is possible to make a prediction of the position of the aircraft during the next scan of the antenna. The new position would be given as:

$$\bar{x}_{n+1} = \bar{x}_n + \mathring{x}_n \Delta t ,$$

where Δt is the period of the antenna rotation. If, on the next scan, it is observed that the aircraft is not at $\bar{x}_{n+1}$, but is displaced from that point by an amount greater than the possible error introduced by quantization, it would be logical to assume that either $\bar{x}_n$, or $\mathring{x}_n$, or both, were in error. It would then appear to be proper to increase or decrease the estimate of the aircraft's velocity by an amount dependent upon the difference of the observed and predicted values of position. A modification should also be made in $\bar{x}_n$ or $\bar{x}_{n+1}$ before the next extrapolation of position is attempted.

It is, of course, possible to accept the most recent observation as the best-estimate of the aircraft's position and use this in the place of $\bar{x}_{n+1}$. There are several reasons, however, why this might not be too satisfactory: first, the observed value is likely to be in error due to quantization; and second, the radar might miss the tracked aircraft but might pick up and report an aircraft flying nearby. In the latter case, the computer might be led to the erroneous conclusion that this observation corresponded to the tracked aircraft. As can be seen from the equations below, in this situation if the computer gives a certain weighting to the observation, it will tend to hesitate at "locking onto" this second aircraft and will tend to wait for the tracked aircraft to reappear. If, on the other hand, the tracked aircraft had made a turn and the radar observation, although appearing to be in error, actually corresponded to the aircraft which the computer had thought was travelling in a straight line, the effect of not completely accepting the new observation will only temporarily delay the computer in following the aircraft in the turn.

The form of the smoothing equations are as follows, with $\mathring{x}_n$ and $\mathring{x}_{n-1}$ being smoothed values of velocity, $\bar{x}_n$ and $\bar{x}_{n+1}$ being smoothed values of position, and x_n being the quantized observation at $t = n$.

$$D_x = x_n - \bar{x}_n \qquad \text{Equation 1}$$

$$\dot{x}_n = \dot{x}_{n-1} + g(D_x / \Delta t) \qquad \text{" \quad 2}$$

$$\bar{x}_{n+1} = \bar{x}_n + h(D_x) + \dot{x}_n \Delta t \qquad \text{" \quad 3}$$

The quantity D_x, the difference between the observed and predicted values of position, is a measure of the success of the tracking and smoothing operations. The expressions $g(\frac{D_x}{\Delta t})$ and $h(D_x)$ are the correction functions, and represent the corrections to be made in smoothed velocity and position for measured values of D_x.

The physical meaning of Equations 2 and 3 is quite simple: a new velocity is derived from the old velocity by adding a correction term which is a function, g, of the difference between the observed and smoothed position; a new smoothed or predicted position is obtained by adding a correction term h(D) to the present smoothed position and by adding in the expected amount that the aircraft will travel in the next scan.

In actual application, $\Delta t = 1$ (scan) and velocities are described in distance per scan. The above equations are actually used for both the x and y coordinates in the form:

$$\dot{x}_n = \dot{x}_{n-1} + g(D_x) \qquad \text{Equation 4}$$

$$\bar{x}_{n+1} = \bar{x}_n + h(D_x) + \dot{x}_n \qquad \text{" \quad 5}$$

$$\dot{y}_n = \dot{y}_{n-1} + g(D_y) \qquad \text{" \quad 6}$$

$$\bar{y}_{n+1} = \bar{y}_n + h(D_y) + \dot{y}_n \qquad \text{" \quad 7}$$

where

$$D_x = x_n - \bar{x}_n \qquad \text{" \quad 8}$$

$$D_y = y_n - \bar{y}_n \qquad \text{" \quad 9}$$

In actual operation, initial values of zero are used for $\dot{x}$ and $\dot{y}$, and the observed x and y values of position upon initiation are used for $\bar{x}_o$ and $\bar{y}_o$[*]. When the radar misses the tracked aircraft, D_x and D_y are assumed zero. The functions g and h are chosen so that $g(0)$ and $h(0)$ are also zero. Under such conditions the smoothing equations can be used even if a miss has occurred, and the computer will merely extrapolate the x and y positions forward with the most recently determined values of velocity.

---

[*] Actually, under certain conditions it might be possible to make a fairly good estimate of the magnitude of velocity of the aircraft, and in more restricted situations, it would be feasible to guess the initial heading of the aircraft. To provide for generality in the Bedford experiments, the initial guess of zero x and y velocity components is used.

3.3.1 The Correction Functions

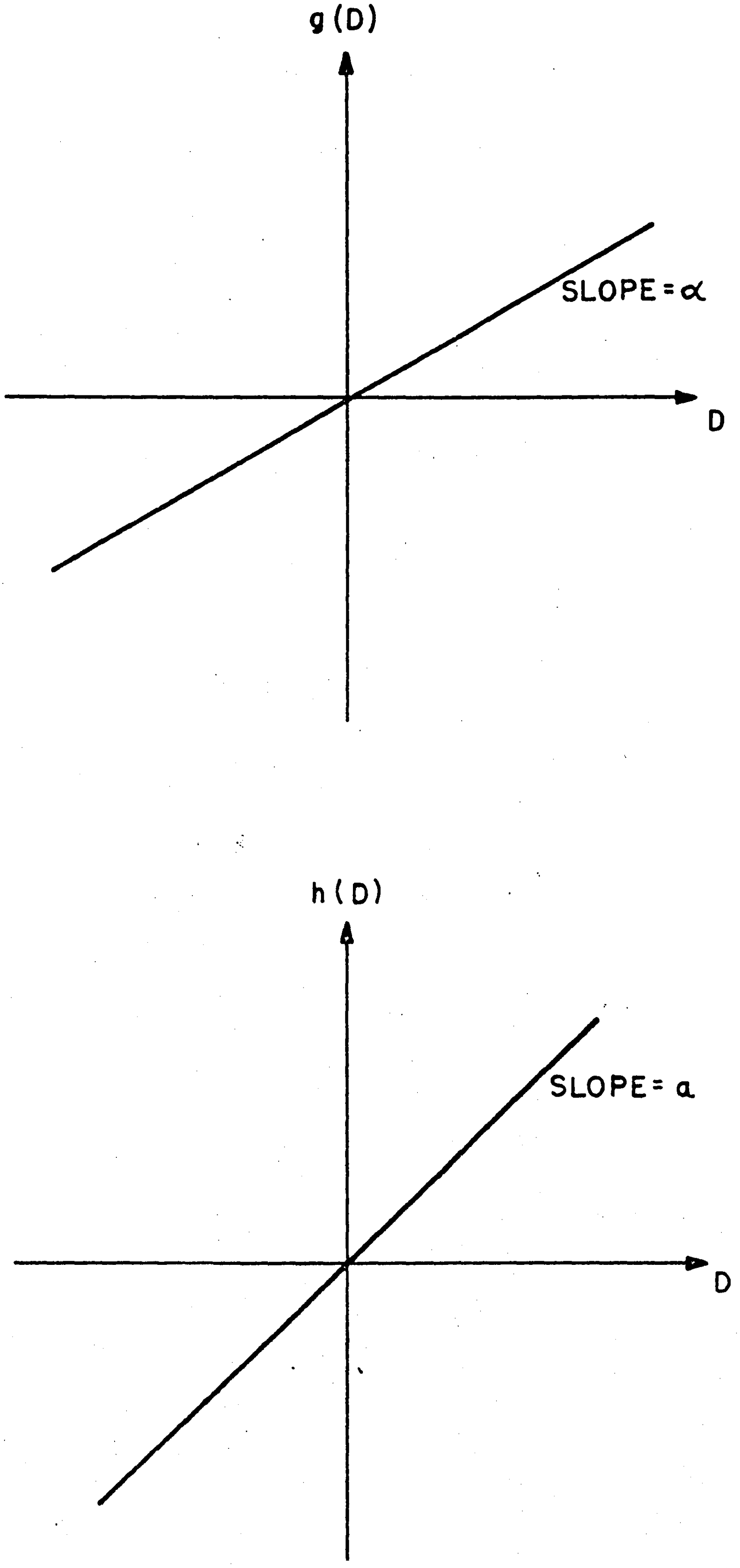
If the above smoothing equations are written for several consecutive values of n and then are combined, it can be shown that g and h represent weighting functions, and their values are of great importance in "weighting" past values of smoothed positions and velocity in the current estimates.

There are a number of forms that the g and h functions might reasonably take. If the corrections are to be linear (proportional), the curves of Figure 7 apply. This type of correction function is the one currently employed in the smoothing process, having been chosen for initial use because of the fact that it appeared to be satisfactory and was obtainable at a relatively low expense of storage space.

The most important choice which had to be made in respect to the use of this linear correction was that of the slopes, α and a, of the velocity and position correction curves. By making α and a equal to unity, one essentially makes a full correction, introducing the error in full as the correction. Such a choice of parameters would be satisfactory if there were no quantization in the radar data; with the quantization present, however, an α and a of unity produces large oscillations in smoothed velocity and position.

As noted above, zero is used as the best a priori guess of the x and y velocity components of a tracked aircraft. It is then desirable to proceed from this initial guess to the actual velocity components of the aircraft as rapidly as possible; once this value is reached, the effects of quantization must be minimized. These two requirements demand opposite extremes in the values of a and α ; values of a and α close to unity will permit a rapid approximation to the velocity (components), while low values of a and α are necessary if the smoothed velocity is to settle down to a steady value. (The entire problem has a close analogy to an electrical network response or filter problem. The requirement is for the design of a filter which in response to a sampled and quantized ramp function input of position will supply a velocity step-function output. The selection of a and α can be compared with the problem of selecting suitable values of parameters in an R, L, C circuit; depending upon the choices of parameters, underdamped, overdamped, or damped output may be obtained.)

Thus, in using the linear type of correction, it was first necessary to select values of a and α which gave a satisfactory compromise between the time that it took to proceed



LINEAR CORRECTION CURVES

FIG. 7

A-50645-1

from the initial estimate of zero to the true velocity of the aircraft and the resulting effects of quantization after the steady-state value had been reached. The smoothing equations with the linear type of correction are actually two linear difference equations, and as such a homogenous (transient) solution can be found rather easily by analytical means. The equations can also be solved for sampled but not quantized values of x_n. Both solutions depend heavily upon the values of a and α, and even without the effects of quantization it is difficult to choose values for a and α.

As an aid in the selection of a and α, the computer has been programmed to simulate qunatized data. This data is smoothed by the computer and the results are displayed along with the true values of position and velocity on an oscilloscope. Various values of a and α can be inserted manually, and a simulated flight test of about 100 miles length can be completed in less than a second. Pictures of the results for various combinations of a and α were taken and the pictures were compared in an attempt to select optimum values. The choice finally reached was that of $\alpha = 1/16$, $a = 5/16$. These values cause the velocity response to overshoot slightly; the velocity gets up to within about 10% of the true value in about 10 scans and remains within about ± 5% during the steady state.

As an aid in analysing the performance of the smoothing under actual data, a program has also been written which prints out, once each scan, the velocity and position information for a tracked aircraft. An example of this printed data is given in Table I. The study of a number of such printed results of smoothing actual radar data has shown a close similarity to the results obtained with simulated data.

3.3.5 Non-Linear Correction

As noted, the linear type of correction was initially considered and used because of its simplicity and economy. Experience of the past months and closer investigations of the smoothing problem have now indicated that great improvements can be made if non-linear correction were employed. It is also fairly obvious that the smoothing problem consists of two main parts: a) the transient state, existing after initiation or after an aircraft turns or changes velocity, and b) the steady-state situation when the aircraft is continuing with constant velocity components. Work is now progressing on the study and implementation of non-linear correction. It appears [redacted] 25, 1951 [illegible] pm

PRINTED RECORD OF TRACKING WHILE SCANNING

Note: Each line of entries correspond to one
rotation of the radar set.
Positive X direction to the EAST,
Positive Y direction to the NORTH.

SMOOTHED POSITIONS (Nautical Miles)		SMOOTHED VELOCITIES (Knots)		OBSERVED DEVIATION (Nautical Miles)		DATA CONSIDERED
X	Y	X	Y	X	Y	
[illegible]	012.4	-145.2	-027.1	-000.7	+000.5	1
[illegible]	[illegible]	-158.4	-018.6	+000.3	-000.4	1
[illegible]	[illegible]	-152.7	-026.2	-000.3	+000.4	1
[illegible]	[illegible]	158.4	-019.6	+000.3	+000.3	1
[illegible]	012.4	-151.6	-014.0	+000.0	+000.0	0
[illegible]	[illegible]	-151.8	-014.0	+000.5	+000.0	1
[illegible]	[illegible]	[illegible]	-013.1	+000.0	-000.1	1
[illegible]	[illegible]	[illegible]	-015.8	-000.3	-000.3	1
[illegible]	[illegible]	[illegible]	-021.5	+000.3	-000.1	1
[illegible]	-013.0	[illegible]	-023.4	-000.0	-000.2	1
[illegible]	[illegible]	[illegible]	-028.1	+000.0	+000.0	0
[illegible]	013.3	-142.4	-028.1	+000.1	-000.2	1
[illegible]	013.5	-140.5	-031.8	+000.2	-000.3	1
[illegible]	013.7	-144.3	-036.5	+000.0	+001.0	1
[illegible]	[illegible]	-143.4	-020.5	+000.0	-000.5	1
[illegible]	013.6	[illegible]	-030.0	+000.0	+000.0	0
[illegible]	014.0	-142.4	-030.0	-000.0	+000.6	1
[illegible]	013.7	-144.3	-018.6	+000.4	+000.5	1
[illegible]	013.6	-136.8	-010.2	-000.0	+000.1	1
[illegible]	013.6	-135.8	-007.4	-000.3	-000.1	2
[illegible]	-013.7	[illegible]	-009.3	+000.6	-001.2	1
[illegible]	[illegible]	-133.1	-029.0	-000.2	+000.2	2
[illegible]	-014.3	-137.7	-025.2	+000.6	-001.0	1
[illegible]	-014.7	-126.5	-040.2	+000.0	-000.7	1
-052.2	-015.2	-125.5	-052.4	-000.3	-000.5	1
-052.8	-015.6	-131.2	-061.8	+000.0	+000.0	0
-053.4	-016.0	-131.2	-061.8	+000.0	+000.0	0
[illegible]	-016.2	131.2	-061.8	+000.3	+000.0	1
[illegible]	-016.4	-124.6	-060.8	+000.0	+000.0	0
[illegible]	-016.7	124.6	-060.8	-000.5	-000.0	1
[illegible]	-017.0	-133.1	-061.8	+000.0	+000.0	0
-056.1	-017.2	-133.1	-061.8	-000.2	+000.1	1
-056.7	-017.4	-136.8	-059.0	+000.0	+000.0	0
[illegible]	-017.6	-136.8	-059.0	+000.0	+000.0	0

Table I Printed Record of Track-While-Scan

The above data was produced and printed out by the computer during an actual flight test on May 25, 1951. Each line of data corresponds to a scan of the antenna. The printed data represents a period of time after tracking had been initiated; for this reason the 3rd and 4th columns do not show the initial zero velocity estimates. The right-hand column represents the number of returns falling within the search area; a 0 indicates a radar miss.

that it is advantageous to make the correction non-linear with the size of the currently-observed D. As noted, at the present time x and y positions and velocity components are smoothed independently. A further step may be to employ a technique which effectively ties together the two velocity components, and the smoothing of velocity magnitude and heading could be attempted.

IV. INTERCEPTION

4.1 General Considerations and Assumptions

The second major function of the computer in the program at hand is to compute the heading instructions necessary to guide one aircraft -- the interceptor -- on an interception course with the second aircraft -- the target. As noted, these interception instructions are intended solely for the mid-course guidance and in the ultimate state are intended only to position the interceptor such that the final phases of the attack can be carried out under the direction of an airborne radar and fire control system. In the experiments, however, due to the lack of the appropriate equipment for directing the final phases, collision course heading instructions are given to the pilot until the aircraft paths cross.

The radar system at Bedford provides no height information and the experiments are necessarily of a two dimensional nature. For safety in the flight tests, the target and the interceptor fly at altitudes differing by 500 feet. Other restrictions are made necessary because of insufficient computer storage capacity. At the present time, no specific programming provisions are made to handle the effects of wind and although the computer tracks both aircraft and derives a smoothed velocity for each, the smoothed velocity of the interceptor is used only in the tracking and not in the interception computations. The velocity of the interceptor is pre-arranged with the pilot of that aircraft and its value is inserted manually into the machine through a flip-flop register. In order to compensate for some of the effects of the wind, the inserted velocity is usually altered to account for head-winds or tail-winds along the general interception course.

In dealing with the interception equations it is convenient to describe three types of interception courses. A <u>pursuit</u> course is that in which the interceptor is always directed towards the present position of the target. The determination of the instructions for such a course involves only a knowledge of the target's present position and not its velocity. The pursuit course is generally curved in nature. A <u>lead-pursuit</u> course is that in which the interceptor is directed toward some point on the extrapolated path of the target. Instructions for this course require a knowledge of the target's velocity and heading. The <u>collision course</u> is a full lead-pursuit course in which the interceptor is directed to cross the extrapolated path of the target at a point where it will collide with the target. A collision course requires a knowledge of the target's velocity and heading, and if the target is flying a straight path, a collision course entails a straight minimum-distance path for the interceptor.

4.2 Interception Equations

The following notation will be used in deriving the interception equations. The quantities defined are listed in Figure 8:

V_I = magnitude of velocity of interceptor

V_{Tx} = target's velocity along x coordinate

V_{Ty} = target's velocity along y coordinate

Δx = x coordinate difference in positions of target and interceptor

Δy = y coordinate difference in positions of target and interceptor

τ = time to interception

ψ = heading angle (with respect to positive x axis) to be given to the interceptor.

The equations defining the collision course are those which state that the aircraft and interceptor will be at the same point in space at the same time. For the two coordinates these equations are:

$$(V_I \cos \psi)\tau = \Delta x + V_{Tx} \cdot \tau \qquad \text{Equation 10}$$

$$(V_I \sin \psi)\tau = \Delta y + V_{Ty} \cdot \tau \qquad \text{Equation 11}$$

4.3 Solution of Equations

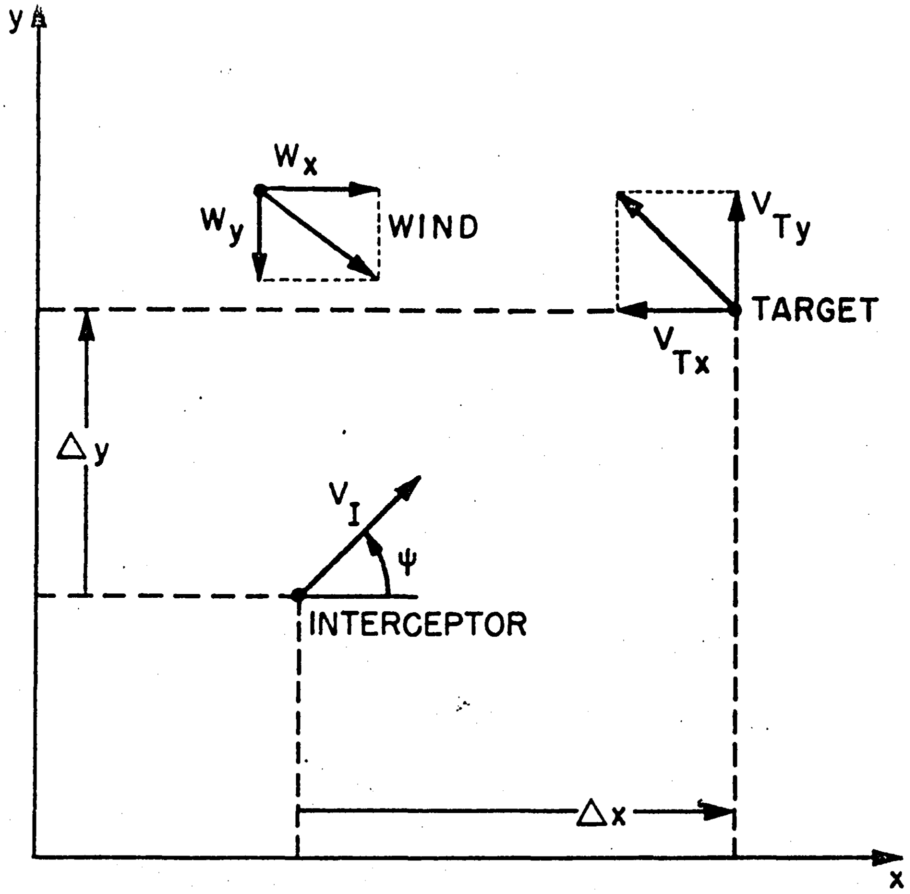
Although a number of methods for solving the above equations might be used, the particular requirements invoked by the computer are such that an iterative procedure is desirable. The following procedure has been found to be the most satisfactory under these conditions and is presently used:

If Equations 10 and 11 are divided by τ, squared, and added to each other:

$$V_I^2 \cos^2 \psi + V_I^2 \sin^2 \psi = \left(\frac{\Delta x}{\tau} + V_{Tx}\right)^2 + \left(\frac{\Delta y}{\tau} + V_{Ty}\right)^2$$

or

$$V_I^2 = \left(\frac{\Delta x}{\tau} + V_{Tx}\right)^2 + \left(\frac{\Delta y}{\tau} + V_{Ty}\right)^2 \qquad \text{Equation 12}$$



LEGEND:

V_I = AIRSPEED OF INTERCEPTOR

V_{Tx} = TARGET'S GROUND VELOCITY ALONG X COORDINATE

V_{Ty} = TARGET'S GROUND VELOCITY ALONG Y COORDINATE

W_x = WIND COMPONENT ALONG X COORDINATE

W_y = WIND COMPONENT ALONG Y COORDINATE

Δx = X COORDINATE DIFFERENCE OF TARGET & INTERCEPTOR

Δy = Y COORDINATE DIFFERENCE OF TARGET & INTERCEPTOR

τ = TIME TO INTERCEPTION

ψ = HEADING ANGLE (SHOWN WITH RESPECT TO POSITIVE X AXIS)

INTERCEPTION NOTATION

FIG. 34

If, as has been the case in the Bedford experiments, the interceptor's velocity is greater than that of the target, then for $\frac{1}{\tau} = 0$, Equation 12 will be an inequality with the left-hand side greater than the right. This inequality will hold as $\frac{1}{\tau}$ is increased, until $\frac{1}{\tau}$ reaches the proper values for the solution of the equation. The iterative method of solution, then, is to start with $\frac{1}{\tau} = 0$ and to increase $\frac{1}{\tau}$ by small increments until:

$$\left(\frac{\Delta x}{\tau} + V_{Tx}\right)^2 + \left(\frac{\Delta y}{\tau} + V_{Ty}\right)^2 - V_I^2 \geqslant 0$$

Actually, the procedure used corresponds to assuming at first that the time to interception is very large (infinite) and then decreasing this time until the proper value is found. It is more convenient in practice to increase $\frac{1}{\tau}$ in small increments rather than decreasing τ itself; in this way, the time estimate is quickly dropped from infinity to reasonable values and to values which have successively smaller increments.

After the proper value of $\frac{1}{\tau}$ is reached, Equation 11 is divided by Equation 10, giving:

$$\frac{\sin\psi}{\cos\psi} = \tan\psi = \frac{\Delta y/\tau + V_{Ty}}{\Delta x/\tau + V_{Tx}} \qquad \text{Equation 13}$$

Several methods are available for finding the inverse tangent. One which is quite convenient employs the approximation:

$$\psi = \tan^{-1}\frac{g}{h} = \frac{g}{4|g|}\left[1 - \frac{h(.6|g| + |h|)}{h^2 + 1.2|hg| + g^2}\right] \qquad \text{Equation 14}$$

This approximation permits the determination of ψ as a part of a revolution between -1/2 and +1/2. The maximum error is 1/1800 revolution or .2 degrees. Following the evaluation of ψ in this form, the angle is transformed to a clockwise sense relative to magnetic north and is then converted to a binary-coded form for display in a flip-flop.

V FLOW DIAGRAM

5.1 General Considerations

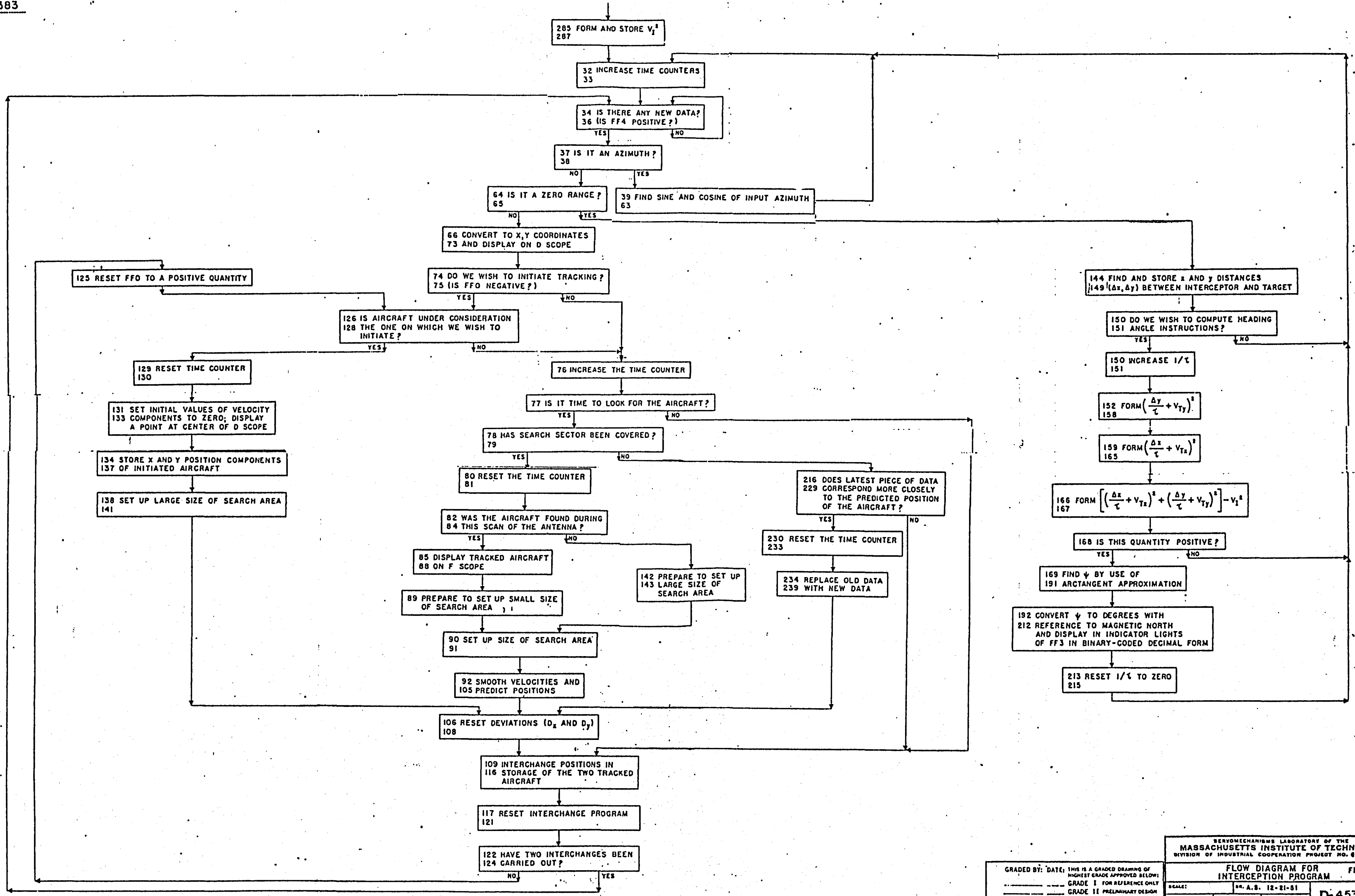
The general principles and the logic behind the track-while-scan function has been explained in Section III and the important factors regarding the interception computations were described in Section IV. This section describes how these various pieces go together to give the complete interception program.

The track-while-scan function must be carried out both for the target and the interceptor. To track one aircraft requires both the necessary program orders and a small number of storage registers containing data pertinent to the tracked aircraft. For tracking two aircraft, it is possible to use essentially the same program orders and a second set of data storage registers; it is necessary, however, to provide that proper steps be taken so that the program orders will operate with this second set of data. One method of handling such a problem would be to modify the pertinent addresses of the program -- that is, those orders whose addresses refer to the aircraft data. This method was rejected because of space considerations, and the method presently used does not alter the addresses in the program, but rather interchanges the positions of the two sets of data so that the program operates first upon one set, and after the interchange, on the second set. After two such interchanges each set of data is back in its initial position.

5.2 Explanation of Flow Diagram

The numbers beside the descriptive material on Figure 9 refer to the addresses of the orders used to perform each of the described functions. For ease in reference, the various parts of the flow diagram will be described by these register addresses. For example, the notation (285-287) refers to the "Form and Store V_I^2" at the top center of Figure 9.

After the program has been read into the computer, the first function carried out is that of extracting the manually-inserted velocity of the interceptor, V_I, from one of the FF's, squaring it to form V_I^2, and storing this for future use by the interception program. See (285-287). This section of the program is performed only once, this being immediately after the program has been read into the computer.



GRADED BY: DATE: THIS IS A GRADED DRAWING OF HIGHEST GRADE APPROVED BELOW:
GRADE I FOR REFERENCE ONLY
GRADE II PRELIMINARY DESIGN
GRADE III FINAL DESIGN

SERVOMECHANISMS LABORATORY OF THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DIVISION OF INDUSTRIAL COOPERATION PROJECT NO. 6883

FLOW DIAGRAM FOR
INTERCEPTION PROGRAM

FIG. 9

SCALE:
DR. A.S. 12-21-51
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The best place for investigating the general action of the computer is at (34-36). The three orders in these registers inspect the input FF for data. These three orders are continuously cycled through until data is found, that is, until FF4 has received some input data which makes its contents positive. When no data is found, the cycling action is indicated by the "no" output from (34-36).

When data is found it may either be an azimuth, a range, or a zero range. These three cases will now be discussed individually.

a) Azimuth

At (37-38) an inspection is made of the appropriate digit (digit position 1) to determine if the data is an azimuth. If so, the program proceeds to (39-63) where the sine and cosine of the azimuth are found. Following this, a return is made to (32-33) where the time counters for each of the two aircraft are increased by unity.

b) Zero Range

If the data is not an azimuth, it must be either a range or a zero range. Zero ranges are checked for at (64-65). If a zero range is discovered, the program proceeds to (144-149) at the right-hand side of Figure 9. This section of the program performs the interception computations and will be discussed below following a description of the tracking section.

c) Range

If a range is found at (64-65) this range is associated with the azimuth most recently received. A conversion is made to (x,y) coordinates and the (x,y) coordinate of the point is displayed on the D-scope.

Immediately after a range is received and a point displayed, a check is next made at (74-75) to determine if initiation of tracking on this displayed spot had been desired. If initiation had been desired, a pulse from the light-gun would have returned and set the contents of FFO to a negative value. Consider, for the present, that initiation was not desired; the program then proceeds from the "no" output at (74-75) to (76).

At (76) the time counter is increased by unity. At (77) a check is made to see if the new value of the time counter is positive or negative; if negative, it is not time to attempt any correlation (Section 3.2.6), and the computer proceeds to (109-116). At this point, the set of registers containing the target and interceptor are interchanged. Following this interchange, the interchange program is reset at (117-121). A check is then made to see if two interchanges have been carried out; if not, the computer returns via (125) and (126-128)--see explanation below--to (76) to perform the above-mentioned checks against the other aircraft. If at (122-124) the indication is that two interchanges have been made, the computer has tested the most recent piece of incoming data against both aircraft and the program should leave (122-124) on the "yes" output and return to (34-36).

After the first interchange has been carried out, the computer returns to (76) through (125) and (126-128). At (125) the light-gun flip-flop is reset to a positive quantity, so that another initiation can be detected at a future time and at (126-128) a check is made upon initiation. It should be noted that extreme care must be taken in order that initiation by the light-gun also properly identifies the aircraft as a target or interceptor; this action is undertaken at (126-128).

At some point within the scan of the radar, the computer, having come through (76), will find at (77) that the time counter is positive. The computer proceeds to (78 - 79) where it is first necessary to determine whether the search sector has just been completed -- that is, whether the time counter has a reading greater than 200. If the search sector has not been covered, the computer will proceed from the "no" output to (216-229). At this point a check is made to see if this most recent piece of incoming data corresponds any more closely to the predicted position of the tracked aircraft (aircraft or interceptor as the case may be) than a previously received piece of data (see Section 3.2.4). If this latest piece of data is not better, the computer proceeds from the output of (216 - 229) to (109 -116) for the interchange program. When a new piece of data does appear to be better than any previous piece, the time counter is reset to 100 at (230-233), the previously selected piece of radar data is discarded in favor of the most recent piece at (234-239), and the computer proceeds to the interchange (109-116), resetting I_x and D_y to their new value.

At some later time, upon passing through (76), (77), and (78-79) it will be found that the time counter has reached a value greater than 200. It should be noted that although the time counters are increased on each azimuth and zero range -- this increase being made at (32-33)-- the contents of the time counter are only investigated by that part of the program starting at (76). For this reason, it is possible for the time counter to have reached a value greater than 200 before the time counter is investigated at (78 - 79) and thus it must be properly reset at (80-81). A check is then made to see if the aircraft had been observed during the preceeding scan of the antenna. This check, at (82-84), is performed, see section 3.2.4 by an investigation of the sum of the squares of the (x,y) coordinate differences between the predicted (smoothed) aircraft position and the best observed radar position.

If the aircraft was missed during the preceeding scan of the radar, a larger size of search area is set up for the next scan through the combination of (142-143) and (90-91). If the aircraft was seen during the scan, a spot is displayed on the F-scope by the action of (85-88) and the small size of search area is established as a result of (89) and (90 - 91). Regardless of whether the aircraft was or was not seen during the scan, coordinate velocities are smoothed and positions are predicted at (92-105). As noted in Section 3.3, if the aircraft has been missed during the scan, the effect of the smoothing and prediction section is to retain the same coordinate velocities and use them to extrapolate position one more scan. At (106-108), the deviations (D_x D_y) are reset to zero in anticipation of possibly missing the aircraft during the next scan.

If initiation of tracking is desired, the computer will have been directed to (129 - 130). As noted above, this is done only after the proper comparison is made at (126 - 128) which provides the correct action depending upon the results of the interchange program. At (129-130) the time counter is reset to a negative value of -550×2^{-15}. At (131 - 133) the initial values of the velocity components are set to zero, and in the process of doing so, a spot is displayed at the center of the D-scope as an aid to the human observer. The x and y position components corresponding to the received piece of radar data are then stored as the initial position of the initiated aircraft. This is done at (134 - 137). At (138 - 141) the large size of the search area is set up and the program then proceeds to (106 - 108) where D_x and D_y are set to zero.

One step in the solution of the interception equations(i.e. an incremental increase of $1/\tau$) is performed each time the computer detects a zero range. After the detection of a zero range, the program proceeds from the "yes" output of (64-65) to (144-149). These orders store Δx and Δy, the coordinate distances between the target and the interceptor. At (150-151) a check is made to see if the human operator desires the interception equations to be solved; this is indicated by the algebraic sign of an externally reset flip-flop register. If the computations are to be made, $1/\tau$ is increased at (150-151), $(\Delta y/\tau + V_{Ty})^2$ is formed at (152-158), and $(\frac{\Delta x}{\tau} + V_{Tx})^2$ is formed at (159-165). The difference $\left[(\frac{\Delta x}{\tau} + V_{Tx})^2 + (\frac{\Delta y}{\tau} + V_{Ty})^2\right] - V_I^2$ is formed at (166-167), and the sign of the difference is checked at (168). If the computation was completed at this step, the computer proceeds on the "yes" output of (168) to (169-191) and (192-212). These two parts of the program find the heading angle by means of the arctangent approximation and then convert the angle to degrees measured with respect to magnetic north. The heading angle is finally converted to binary -- coded decimal form and displayed in the lights of FF3. After the angle has been set up in the flip-flop register, $1/\tau$ is reset to zero at (213-215) in preparation for the next computation. As shown, the computer always performs the interception computations provided that the flip-flop is set so as to give the proper result at (150-151); once the computation is performed, it will be repeated, giving the same result unless a scan has passed and the smoothed positions of the aircraft result in a different Δx or Δy at (144-149).

Signed David R. Israel
David R. Israel

Approved C. R. Wieser
C. R. Wieser

Figures Attached:

APPENDIX

a) Location of Flip-Flop Registers, "Universal" Constants, and Data Storage

0) +0

1) 1/4

8) FF3 angle display flip-flop

23) FF0 light gun flip-flop

28) FF4 data input flip-flop

29) FF1 $1/\tau$

30) FF2 V_I

31) $1/2 - 2^{-15}$

250), 260) time counter

251), 261) x smoothed position

252), 262) y smoothed position

253), 263) y velocity

254), 264) x velocity

255), 265) x difference between observed and predicted position, (D_x)

256), 266) y difference between observed and predicted position, (D_y)

257), 267) search area size - initial

258), 268) search area size - running

b) Complete Coded Program

```
32  ao  250     84  cp  142    136  ca  272    188  qe  275    240  ri    0
33  ao  260     85  ca  251    137  ts  252    189  cp  704*   241  -3/16
34  cs    0     86  qh    0    138  ca  243    190  cs  275    242  +7/24
35  qe   28     87  ca  252    139  ts  257    191  ts  275    243  ri   32
36  cp   34     88  qf  252    140  ts  258    192  ao    0**   244  ri   18
37  su   31     89  ca  244    141  sp  106    193  dv   29    245  ri    0
38  cp   64     90  ts  257    142  ca  243    194  sl    6    246  ri  650
39  su    1     91  ts  258    143  sp   90    195  ts   30    247  ri  100
40  sl    1     92  ca  255    144  cs  261    196  cs  242    248  ri  200
41  ts  272     93  mr  271    145  ad  251    197  ad  275    249  ri  650
42  cm  272     94  ad  254    146  ts  281    198  sa  106    250  ri    0
43  ts  273     95  ts  254    147  cs  262    199  mh  276    251  ri    0
44  mr  189     96  sa  255    148  ad  252    200  sr  302    252  ri    0
45  su  189     97  sa  251    149  ts  282    201  ts    8    253  ri    0
46  mh  273     98  ts  251    150  ao   29    202  sl    4    254  ri    0
47  sl    2     99  ca  256    151  cp   32    203  mh  277    255  ri    0
48  ad  279    100  mr  271    152  mh  282    204  ts  278    256  ri    0
49  mr  273    101  ad  253    153  sl    7    205  sl   15    257  ri    0
50  ad  270    102  ts  253    154  ad  253    206  mh  277    258  ri    0
51  mh  272    103  sa  256    155  sl    5    207  ts   29    259  ri    0
52  ts  273    104  sa  252    156  ts  275    208  ca  278    260  ri    0
53  cs  241    105  ts  252    157  mr  275    209  sl    5    261  ri    0
54  ts  241    106  ca    0    158  ts  283    210  ad   29    262  ri    0
55  cp   32    107  ts  255    159  ca   29    211  sl    1    263  ri    0
56  ca  273    108  ts  256    160  mh  281    212  td    8    264  ri    0
57  ts  240    109  ca  250    161  sl    7    213  ca    0    265  ri    0
58  ca  272    110  qe  260    162  ad  254    214  ts   29    266  ri    0
59  su    1    111  ts  250    163  sl    5    215  sp   32    267  ri    0
60  cp   62    112  ao  109    164  ts  278    216  cs  274    268  ri    0
61  ad  106    113  ao  110    165  mr  278    217  sa  251    269  ri    0
62  ad   31    114  ao  111    166  su  280    218  ts  269    270  +.3879
63  sp   41    115  su   91    167  ad  283    219  mr  269    271  +.3333
64  ad   31    116  cp  109    168  cp   32    220  ts  245    272  ri    0
65  cp  144    117  ca   32    169  cm  275    221  cs  272    273  ri    0
66  ts  272    118  td  109    170  mr  284    222  sa  252    274  ri    0
67  mh  240    119  td  111    171  mr  278    223  ts  259    275  ri    0
68  sl    5    120  ca   33    172  qe  283    224  mr  259    276  -.9000
69  qh  274    121  td  110    173  sr    2    225  sa  245    277  ri   10
70  cs  272    122  cs  241    174  ts  281    226  ts  245    278  ri    0
71  mh  273    123  ts  241    175  cm  278    227  ca  258    279  +.1206
72  sl    5    124  cp   34    176  mr  278    228  su  245    280  ri    0
73  qd  272    125  qe   23    177  sr    2    229  cp  109    281  ri    0
74  ca   23    126  mr  106    178  ad  283    230  ca  247    282  ri    0
75  cp  127    127  su  241    179  qe  283    231  ts  250    283  ri    0
76  ao  250    128  cp   76    180  ad  283    232  ca  245    284  +.1500
77  cp  109    129  cs  249    181  ts  278    233  ts  258    285  ca   30
78  su  248    130  ts  250    182  cm  278    234  cs  269    286  mr   30
79  cp  216    131  cs    0    183  ad  281    235  mr   86    287  ts  280
80  su  246    132  qh  253    184  qe  283    236  ts  255
81  ts  250    133  qd  254    185  dv  283    237  cs  259
82  ca  257    134  ca  274    186  sl   13    238  mr   86
83  su  258    135  ts  251    187  su    1    239  sp  108
```

*(+.8965)

**Orders 192-195 should be disregarded.