RECOMP II USERS' PROGRAM NO. 1100

PROGRAM TITLE: RECOMP II BESSEL FUNCTION OF THE SECOND
                          KIND, ORDER ONE SUBROUTINE (FLOATING
                          POINT, RELOCATABLE)

PROGRAM CLASSIFICATION: Subroutine

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PURPOSE: To compute the Bessel function of the second
                          kind, order one of a floating point argument,
                          found in the A and R registers.

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                 ORDER ONE SUBROUTINE (FLOATING POINT, RELOCATABLE)

1. PURPOSE

   1.1 To compute the Bessel function of the second kind, order one of a
   floating point argument, found in the A and R registers.

2. METHOD

   2.1 For $0 < X < 8$,

$$N_1(X) = \frac{2}{\pi} \left(\text{Gamma} + \text{Log}_e \frac{X}{2} \right) J_1(X) - \frac{2}{\pi X} - \frac{1}{\pi} \sum_{n=0}^{N} A_n ,$$

   where

$$A_n = (-1)^n \frac{(X/2)^{2n+1}}{n!(n+1)!} \left[2\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right) + \frac{1}{n+1} \right]$$

   and N is the smallest integer for which $A_N \leq 10^{-12}$.

   2.2 For $8 \leq x < \infty$

$$N_1(X) = \left[\frac{2}{\pi X}\right]^{1/2} \left\{ \sin\left(X - \frac{3\pi}{4}\right) \right.$$

$$\left. - \sum_{n=1}^{N} (-1)^n \left[B_{2n-1}\cos\left(X - \frac{3\pi}{4}\right) + B_{2n} \sin\left(X - \frac{3\pi}{4}\right)\right]\right\}$$

   where

$$B_i = \frac{1^2 \cdot 3^2 \cdot 5^2 \ldots (2_i-3)^2(2_i-1)(2_i+1)}{i! \, (8X)^i}$$

and where N is the first integer for which

$$B_{2N} > B_{2n-1} \quad \text{or} \quad B_{2N} < 10^{-12},$$

whichever occurs first.

3. RESTRICTIONS

    3.1 Range of Argument $X > 0$

    3.2 Contents of L and V loops are destroyed.

    3.3 The following subroutines should be in memory: sin-cos(AN-012.1) $\log_e$ (AN-037), J_1 (AN-056).

    3.4 The following modifications should be made in AN-056 so that it will use AN-012.1

| L(Origin plus 0011) | TRA 0000 | N∅P |
| | N∅P | N∅P |

4. USE

    4.1 Calling Sequence

α	FCA	ARG
	TRA	N_1
α+1	PZE	L(AN-012.1)
	PZE	L(AN-056)
α+2	PZE	L(AN-037)
	ERROR	RETURN
α+3	NORMAL	RETURN

    4.2 This routine occupies words 0000 thru 0167 or 120 full words.

    4.3 Minimum Accuracy: 8 decimal places. If speed is desired and several fewer accurate places are needed, it is suggested that 10^{-P} be entered into location 0162-3, where P is one more than the number of accurate decimal places required.

    4.4 After a normal exit the value of the function is found in the A and R registers. Error return is caused by negative argument.

5. CODING INFORMATION

5.1 Constants

5.1.1 Floating Point

1.0	in	0002
2.0	in	0004
8.0	in	0010
$2/\pi = 0.63661\ 9772$	in	0050
Gamma $= 0.57721\ 56649$	in	0056
$3\pi/4 =$	in	0164
3.0	in	0166
10^{-12}	in	0162

5.1.2 Fixed Point

1 at B39	in	0003
1 at B38	in	0005

5.2 Erasable Locations: 0006-7, 0054-5, 0136-7, 0156-7.

5.3 Master Tape includes:

1) Basic routine, 0000-0167

2) Relocation matrix 3777-4167

3) AN-004 Relocation routine, 7724-7757.

N_1 → Set Up Exit Addresses

$X : 0$ — $\leq$ → ERROR EXIT

$X : 8$ — < 8 → $SUM = \frac{2}{\pi}(Gamma + Log_e \frac{X}{2})J_1(X) - \frac{2}{\pi X}$

> 8

$n = 1$
$SUM = 0$

$$SUM = \left[\frac{2}{\pi X}\right]^{1/2} \left[\sin(X - \frac{3\pi}{4})\right]$$

$$C_n = B_{2n-1} \cos(X - \frac{3\pi}{4}) + B_{2n} \sin(X - \frac{3\pi}{4})$$

$$SUM = SUM - (-1)^n C_n \left[\frac{2}{\pi X}\right]^{1/2}$$

$\frac{B_{2n}}{B_{2n-1}} : 1$ — > 1

< 1

$B_{2n} : 10^{-12}$ — $> 10^{-12}$

$< 10^{-12}$

$n = n+1$

NORMAL EXIT

$n = 0$

Compute A_n

$SUM = SUM + A_n$

$A_n : 10^{-12}$ — $> 10^{-12}$

$< 10^{-12}$

$n = n+1$

NORMAL EXIT