RECOMP II USERS' PROGRAM NO. 1099

PROGRAM TITLE:	RECOMP II BESSEL FUNCTION OF THE SECOND KIND, ORDER ZERO SUBROUTINE (FLOATING POINT, RELOCATABLE)
PROGRAM CLASSIFICATION:	Subroutine
AUTHOR:	Philip Sheldon Sylvania Electronic Systems A Division of Sylvania Electric Products, Inc. Waltham, Massachusetts
PURPOSE:	To compute the Bessel function of the second kind, order zero of a floating point argument, found in the A and R registers.
DATE:	10 October 1961

Published by

RECOMP Users' Library

at

AUTONETICS INDUSTRIAL PRODUCTS

A DIVISION OF NORTH AMERICAN AVIATION, INC.
3400 E. 70th Street, Long Beach 5, Calif.

PROGRAM TITLE: RECOMP II BESSEL FUNCTION OF THE SECOND KIND, ORDER ZERO SUBROUTINE (FLOATING POINT, RELOCATABLE)

1. PURPOSE

1.1 To compute the Bessel function of the second kind, order zero of a floating point argument, found in the A and R registers.

2. METHOD

2.1 For $0 < X < 8$ the following formula is used:

$$N_o(X) = \frac{2}{\pi}\left(\text{Gamma} + \text{Log}_e \frac{X}{2}\right) J_o(X) + \sum_{n=1}^{N} A_n,$$

where

$$A_n = (-1)^n \frac{2}{\pi} \frac{(X/2)^{2n}}{(n!)(n!)} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right)$$

and N is the smallest integer for which $A_N \leq 10^{-12}$.

2.2 For $8 \leq X < \infty$

$$N_o(X) = \left[\frac{2}{\pi X}\right]^{1/2} \sum_{n=0}^{N} (-1)^n \left[B_{2n} \sin\left(X - \frac{\pi}{4}\right) - B_{2n+1} \cos\left(X - \frac{\pi}{4}\right)\right]$$

where

$$B_i = \frac{1^2 3^2 \ldots (2_i - 1)^2}{i!(8X)^i}$$

and where N is the first integer for which

$$B_{2N+1} > B_{2N} \quad \text{or} \quad B_{2N+1} < 10^{-12},$$

whichever happens first.

3. RESTRICTIONS

3.1 Range of argument $X > 0$

3.2 Contents of L and V loops are destroyed.

3.3 The following subroutines must be in memory: sin-cos (AN-012.1), $\log_e$ (AN-037), J_o(AN-055).

3.4 The following modifications should be made in AN-055 so that it will use AN-012.1.

L (origin + 0052)	TRA	0000	FCA	7762
	NØP		NØP	

4. USE

4.1 Calling Sequence

α	FCA	ARG
	TRA	N_o
$\alpha + 1$	PZE	L(AN-012.1)
	PZE	L(AN-055)
$\alpha + 2$	PZE	L(AN-037)
	ERRØR	RETURN
$\alpha + 3$	NØRMAL	RETURN

4.2 The routine occupies words 0000 thru 0163_8 or 116 full words.

4.3 Minimum accuracy: 8 decimal places. When speed is desired and several fewer accurate places are needed, it is suggested that 10^{-P} be entered into location 0162-3, where P is one greater than the number of accurate decimal places required.

4.4 After a normal exit the value of the function is found in the A and R registers. Error return is caused by negative argument.

5. CODING INFORMATION

5.1 Constants

5.1.1 Floating Point

1.0	in	0002
2.0	in	0004
8.0	in	0010
$2/\pi$ = 0.63661 9772	in	0050
Gamma = 0.57721 56649	in	0056
$\pi/4$ = 0.78539 81634	in	0102
10^{-12}	in	0162

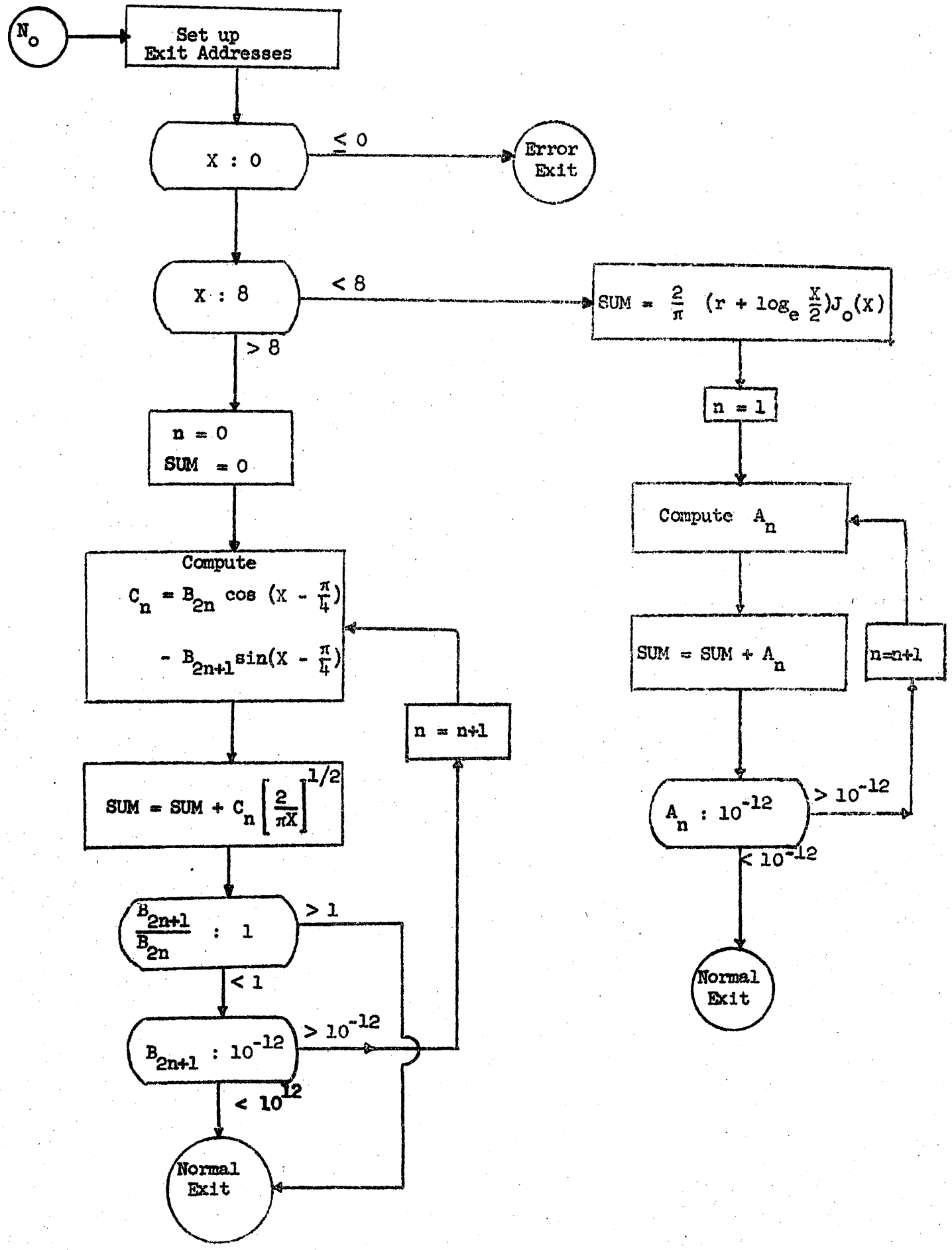
5.1.2 Fixed Point

1 at B 39	in	0003
1 at B 38	in	0005

5.2 Erasable locations: 0006-7, 0054-5, 0066-7, 0076-7, 0100-1, 0136-7, 0156-7.

5.3 Master Tape includes:

1) Basic routine, 0000-0163.
2) Relocation matrix, 4000-4163.
3) AN-004 Relocation routine, 7724-7757.


N o
Set up Exit Addresses
X : 0
≤ 0
Error Exit
X : 8
< 8
> 8
SUM = 2/π (r + log e X/2)J o(X)
n = 0
SUM = 0
n = 1
Compute
C n = B 2n cos (X - π/4)
- B 2n+1 sin(X - π/4)
Compute A n
SUM = SUM + A n
n=n+1
n = n+1
SUM = SUM + C n [2/πX]^1/2
A n : 10^-12
> 10^-12
< 10^-12
B 2n+1 / B 2n : 1
> 1
< 1
B 2n+1 : 10^-12
> 10^-12
< 10^12
Normal Exit
Normal Exit