

RECOMP II USERS' PROGRAM NO. 1094

PROGRAM TITLE: FAST SINE-COSINE SUBROUTINE (FLOATING POINT)

PROGRAM CLASSIFICATION: Subroutine

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PURPOSE: To provide a subroutine to compute both the sine and cosine of a radian argument with a small relative error. Speed and precision, rather than memory requirements, are the primary objectives of this subroutine.

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PROGRAM TITLE: FAST SINE-COSINE SUBROUTINE
(FLOATING POINT)

1. Purpose

To provide a subroutine to compute both the sine and cosine of a radian argument with a small relative error. Speed and precision, rather than memory requirements, are the primary objectives of this subroutine.

2. Restrictions

2.1 To obtain the sine and cosine to at least three decimal places of accuracy, the absolute size of the argument must be less than 10^8 radians. See 3.2 below. Can't be relocated into XX60 (XX any channel).

3. Method

3.1 The original angle is multiplied by a suitable multiplier ($4/\pi$ for arguments in radians) to transform the angle to units of half right-angles, or eighths of a circle. The integral part (except for the low order bit) is then removed, yielding an intermediate argument $0 \leq F < 2$. The final argument is then obtained by using either $\pm |F|$ or $\pm |2-F|$, whichever is smaller, with the sign chosen in accordance with the sign of the original angle. This final argument is then used in a continued fraction expansion for the sine; namely:

$$\text{sine } x = x \left\{ d_0 + x^2 \left[d_1 + \frac{e_1}{d_2 + x^2 + \frac{e_2}{d_3 + x^2}} \right] \right\}$$

Calling the value of the fraction above P, and $\sqrt{1 - P^2} = R$, the value of the sine and cosine are determined from the last 3 bits of the integer part of the argument expressed in half right-angle units, as follows:

<u>Last 3 Bits</u>		<u>Sin</u>	<u>Cos</u>
$X \geq 0$	$X < 0$		
000	111	P	R
111	000		
001	110	R	-P
010	101		
011	100	-P	-R
100	011		
101	010	-R	P
110	001		

3.2 Accuracy: The maximum relative error of the subroutine for either sine or cosine in the basic range $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ radians is less than 2×10^{-11} . For arguments outside of these limits, the absolute accuracy of either function is limited by the number of bits representing the fractional portion of the radian argument. In particular, the maximum absolute error of either sin or cos for radian arguments outside the basic range appears to be less than $3 \times 2^{E-38}$, where E is the binary exponent of the argument.

4. Usage

4.1 The argument must be a floating point number in the A and R registers.

4.2 Calling sequence

TRA 33-5129R

Normal return

Note
 \downarrow
 $L_0 + 0030.0$

The transfer may be in either half word.

4.3 Upon normal return, sine X is in the A and R registers, and also in 7774 and 7775; cosine X is in 7776 and 7777, all numbers being in floating point form.

4.4 Error return: None

4.5 The routine destroys the L and V loops

4.6 The routine is relocatable.

4.7 Options available: For arguments in degrees, change location

$$\left. \begin{array}{l} L_0 + 0124 \text{ to } +55\ 40551-02\ 66030 \\ L_0 + 0122 \text{ to } -00\ 00000\ -00\ 00021 \end{array} \right\} 1/45$$

This avoids the necessity for the programmer to convert an argument from degrees to radians before transferring to the sin/cos routine.

5. Coding Information

5.1 The routine occupies words 0000 to 0137₈ or 96 words.

5.2 Unused locations. A total of 26 locations are unused, as follows:

<u>Location (octal)</u>	<u>Number</u>
$L_0 + 0024 - 0025$	2
0042 - 0044	3
0046 - 0047	2
0062 - 0064	3
0067 - 0070	2
0072 - 0073	2
0102 - 0105	4
0107 - 0113	5
0123 - 0125	<u>3</u>
	26

5.3 Erasable locations. Location $L_0 + 0076$ is used for temporary storage.

5.4 Constants (all floating point)

	<u>Value</u>	<u>Location (octal)</u>
d_0	0.78539 81633 97	$L_0 + 0022$
d_1	-0.15789 09220 72	+ 0020
d_2	184.56259 0042	+ 0060
d_3	22.19509 18999	+ 0065
e_1	56.26206 97951	+ 0126
e_2	12090.47453 99	+ 0026
$4/\pi$	1.27323 95447 4	+ 0121
	1.	+ 0040
	2.	+ 0100

5.5 Execution time: 284 ms maximum

6. Checkout

The routine has been checked over a wide range of values, using both degrees and radians as arguments. Comparisons with standard tables revealed no errors in excess of those indicated in 3.2.

0000.0	+30.7774.0	FCA	0020.0	-50.3270.1	---
	+35.7776.0	FST		-41.5242.0	---
0001.0	+30.7772.0	FCA	0021.0	-00.0000.0	---
	+57.7770.0	TRA		-00.0001.0	---
0002.0	+34.7772.0	FCS	0022.0	+62.2077.0	---
	+35.7776.0	FST		+52.4207.0	---
0003.0	+30.7774.0	FCA	0023.0	+00.0000.0	---
	+57.7770.0	TRA		-00.0000.0	---
0004.0	+34.7774.0	FCS	0024.0	-00.0000.0	---
	+35.7776.0	FST		-00.0000.0	---
0005.0	+34.7772.0	FCS	0025.0	-00.0000.0	---
	+57.7770.0	TRA		-00.0000.0	---
0006.0	+30.7772.0	FCA	0026.0	+57.1647.1	---
	+35.7776.0	FST		-27.5561.1	---
0007.0	+34.7774.0	FCS	0027.0	+00.0000.0	---
	+57.7770.0	TRA		-00.0007.0	---
0010.0	+30.0126.0	FCA	0030.0	+15.0076.0	SAX
	+05.7776.0	FDV		+57.0045.0	TRA
0011.0	+04.0020.0	FAD	0031.0	+34.7774.0	FCS
	+07.7772.0	FMP		+06.7772.1	FSB
0012.0	+04.0022.0	FAD	0032.0	+04.7774.0	FAD
	+07.7774.0	FMP		+04.7772.1	FAD
0013.0	+35.7772.0	FST	0033.0	+50.7766.1	TZE
	+07.7772.0	FMP		+35.7774.0	FST
0014.0	+33.7767.0	EXT	0034.0	+43.0000.0	XAR
	+04.0040.0	FAD		+50.7767.0	TZE
0015.0	+35.7763.0	FST	0035.0	+51.7767.0	TMI
	+44.7763.0	FSQ		+43.0000.0	XAR
0016.0	+35.7774.0	FST	0036.0	+06.0100.0	FSB
	+57.7771.0	TRA		+35.7774.0	FST
0017.0	-77.7777.1	---	0037.0	+64.0130.0	CTL
	+77.7777.1	---		+57.7760.0	TRA

0040.0	+40.0000.0	---	0060.0	+56.1100.0	---
	-00.0000.0	---		-27.4651.1	---
0041.0	+00.0000.0	---	0061.0	+00.0000.0	---
	-00.0000.1	---		-00.0004.0	---
0042.0	-00.0000.0	---	0062.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0043.0	-00.0000.0	---	0063.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0044.0	-00.0000.0	---	0064.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0045.0	+64.0050.0	CTL	0065.0	+54.3076.0	---
	+57.7760.0	TRA		-61.2744.0	---
0046.0	-00.0000.0	---	0066.0	+00.0000.0	---
	-00.0000.0	---		-00.0002.1	---
0047.0	-00.0000.0	---	0067.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0050.0	+66.0070.0	CTV	0070.0	-00.0000.0	---
	+01.7771.0	ADD		-00.0000.0	---
0051.0	+60.7770.0	STO	0071.0	+35.7774.0	FST
	+00.7776.0	CLA		+57.0000.1	TRA
0052.0	+07.0121.0	FMP	0072.0	-00.0000.0	---
	+35.7772.0	FST		-00.0000.0	---
0053.0	+51.7764.1	TMI	0073.0	-00.0000.0	---
	+04.0115.0	FAD		-00.0000.0	---
0054.0	+57.7765.0	TRA	0074.0	+40.0000.0	---
	+04.0117.0	FAD		-00.0000.0	---
0055.0	+33.7777.0	EXT	0075.0	+00.0000.0	---
	+01.0106.0	ADD		-00.0023.1	---
0056.0	+33.0114.0	EXT	0076.0	-00.0000.0	---
	+60.7771.0	STO		-00.0000.0	---
0057.0	+64.0030.0	CTL	0077.0	+00.0000.0	---
	+57.7761.0	TRA		-00.0007.0	---

0100.0	+40.0000.0	---	0120.0	+00.0000.0	---
	-00.0000.0	---		-00.0023.0	---
0101.0	+00.0000.0	---	0121.0	+50.5746.0	---
	-00.0001.0	---		-15.5625.0	---
0102.0	-00.0000.0	---	0122.0	+00.0000.0	---
	-00.0000.0	---		-00.0000.1	---
0103.0	-00.0000.0	---	0123.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0104.0	-00.0000.0	---	0124.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0105.0	-00.0000.0	---	0125.0	-00.0000.0	---
	-00.0000.0	---		-00.0000.0	---
0106.0	+64.0000.0	CTL	0126.0	+70.2061.0	---
	+57.7761.0	TRA		+60.0616.0	---
0107.0	-00.0000.0	---	0127.0	+00.0000.0	---
	-00.0000.0	---		-00.0003.0	---
0110.0	-00.0000.0	---	0130.0	+30.7772.0	FCA
	-00.0000.0	---		+52.7762.0	TPL
0111.0	-00.0000.0	---	0131.0	+34.7774.0	FCS
	-00.0000.0	---		+60.7774.0	STO
0112.0	-00.0000.0	---	0132.0	+30.7774.0	FCA
	-00.0000.0	---		+07.7774.0	FMP
0113.0	-00.0000.0	---	0133.0	+35.7772.0	FST
	-00.0000.0	---		+04.0065.0	FAD
0114.0	+77.7777.1	---	0134.0	+35.7763.0	FST
	+77.7766.0	---		+30.0026.0	FCA
0115.0	+40.0000.0	---	0135.0	+05.7763.0	FDV
	-00.0000.0	---		+04.7772.0	FAD
0116.0	+00.0000.0	---	0136.0	+04.0060.0	FAD
	-00.0023.0	---		+35.7776.0	FST
0117.0	+60.0000.0	---	0137.0	+64.0010.0	CTL
	-00.0007.0	---		+57.7760.0	TRA