

Java Source and Bytecode Formalizations in Isabelle: μ Java

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Contents

1	Preface	5
1.1	Introduction	5
1.2	Theory Dependencies	7
2	Java Source Language	9
2.1	Some Auxiliary Definitions	10
2.2	Java types	11
2.3	Class Declarations and Programs	12
2.4	Relations between Java Types	13
2.5	Java Values	16
2.6	Program State	17
2.7	Expressions and Statements	20
2.8	System Classes	21
2.9	Well-formedness of Java programs	22
2.10	Well-typedness Constraints	28
2.11	Operational Evaluation (big step) Semantics	32
2.12	Conformity Relations for Type Soundness Proof	37
2.13	Type Safety Proof	42
2.14	Example MicroJava Program	45
2.15	Example for generating executable code from Java semantics	51
3	Java Virtual Machine	55
3.1	State of the JVM	56
3.2	Instructions of the JVM	57
3.3	JVM Instruction Semantics	58
3.4	Exception handling in the JVM	61
3.5	Program Execution in the JVM	62
3.6	Example for generating executable code from JVM semantics	63
3.7	A Defensive JVM	66
4	Bytecode Verifier	69
4.1	Semilattices	70
4.2	The Error Type	75
4.3	Fixed Length Lists	80
4.4	Typing and Dataflow Analysis Framework	86
4.5	Products as Semilattices	87
4.6	More on Semilattices	89
4.7	More about Options	91

4.8	The Lightweight Bytecode Verifier	93
4.9	Correctness of the LBV	98
4.10	Completeness of the LBV	100
4.11	Lifting the Typing Framework to <code>err</code> , <code>app</code> , and <code>eff</code>	103
4.12	The Java Type System as Semilattice	105
4.13	The JVM Type System as Semilattice	107
4.14	Effect of Instructions on the State Type	112
4.15	Monotonicity of <code>eff</code> and <code>app</code>	119
4.16	The Bytecode Verifier	120
4.17	The Typing Framework for the JVM	122
4.18	LBV for the JVM	125
4.19	BV Type Safety Invariant	128
4.20	BV Type Safety Proof	133
4.21	Welltyped Programs produce no Type Errors	140
4.22	Kildall's Algorithm	144
4.23	Kildall for the JVM	147
4.24	Example Welltypings	153

Chapter 1

Preface

1.1 Introduction

This document contains the automatically generated listings of the Isabelle sources for μ Java. μ Java is a reduced model of JavaCard, dedicated to the study of the interaction of the source language, byte code, the byte code verifier and the compiler. In order to make the Isabelle sources more accessible, this introduction provides a brief survey of the main concepts of μ Java.

The μ Java **source language** (see Chapter 2) only comprises a part of the original JavaCard language. It models features such as:

- The basic “primitive types” of Java
- Object orientation, in particular classes, and relevant relations on classes (subclass, widening)
- Methods and method signatures
- Inheritance and overriding of methods, dynamic binding
- Representatives of “relevant” expressions and statements
- Generation and propagation of system exceptions

However, the following features are missing in μ Java wrt. JavaCard:

- Some primitive types (**byte**, **short**)
- Interfaces and related concepts, arrays
- Most numeric operations, syntactic variants of statements (**do-loop**, **for-loop**)
- Complex block structure, method bodies with multiple returns
- Abrupt termination (**break**, **continue**)
- Class and method modifiers (such as **static** and **public/private** access modifiers)
- User-defined exception classes and an explicit **throw**-statement. Exceptions cannot be caught.

- A “definite assignment” check

In addition, features are missing that are not part of the JavaCard language, such as multithreading and garbage collection. No attempt has been made to model peculiarities of JavaCard such as the applet firewall or the transaction mechanism.

For a more complete Isabelle model of JavaCard, the reader should consult the Bali formalization (<http://isabelle.in.tum.de/verificard/Bali/document.pdf>), which models most of the source language features of JavaCard, however without describing the bytecode level.

The central topics of the source language formalization are:

- Description of the structure of the “runtime environment”, in particular structure of classes and the program state
- Definition of syntax, typing rules and operational semantics of statements and expressions
- Definition of “conformity” (characterizing type safety) and a type safety proof

The μ Java **virtual machine** (see Chapter 3) corresponds rather directly to the source level, in the sense that the same data types are supported and bytecode instructions required for emulating the source level operations are provided. Again, only one representative of different variants of instructions has been selected; for example, there is only one comparison operator. The formalization of the bytecode level is purely descriptive (“no theorems”) and rather brief as compared to the source level; all questions related to type systems for and type correctness of bytecode are dealt with in chapter on bytecode verification.

The problem of **bytecode verification** (see Chapter 4) is dealt with in several stages:

- First, the notion of “method type” is introduced, which corresponds to the notion of “type” on the source level.
- Well-typedness of instructions wrt. a method type is defined (see Section 4.16). Roughly speaking, determining well-typedness is *type checking*.
- It is shown that bytecode that is well-typed in this sense can be safely executed – a type soundness proof on the bytecode level (Section 4.20).
- Given raw bytecode, one of the purposes of bytecode verification is to determine a method type that is well-typed according to the above definition. Roughly speaking, this is *type inference*. The Isabelle formalization presents bytecode verification as an instance of an abstract dataflow algorithm (Kildall’s algorithm, see Sections 4.22 to 4.23).

Bytecode verification in μ Java so far takes into account:

- Operations and branching instructions
- Exceptions

Initialization during object creation is not accounted for in the present document (see the formalization in <http://isabelle.in.tum.de/verificard/obj-init/document.pdf>), neither is the `jsr` instruction.

1.2 Theory Dependencies

Figure 1.1 shows the dependencies between the Isabelle theories in the following sections.

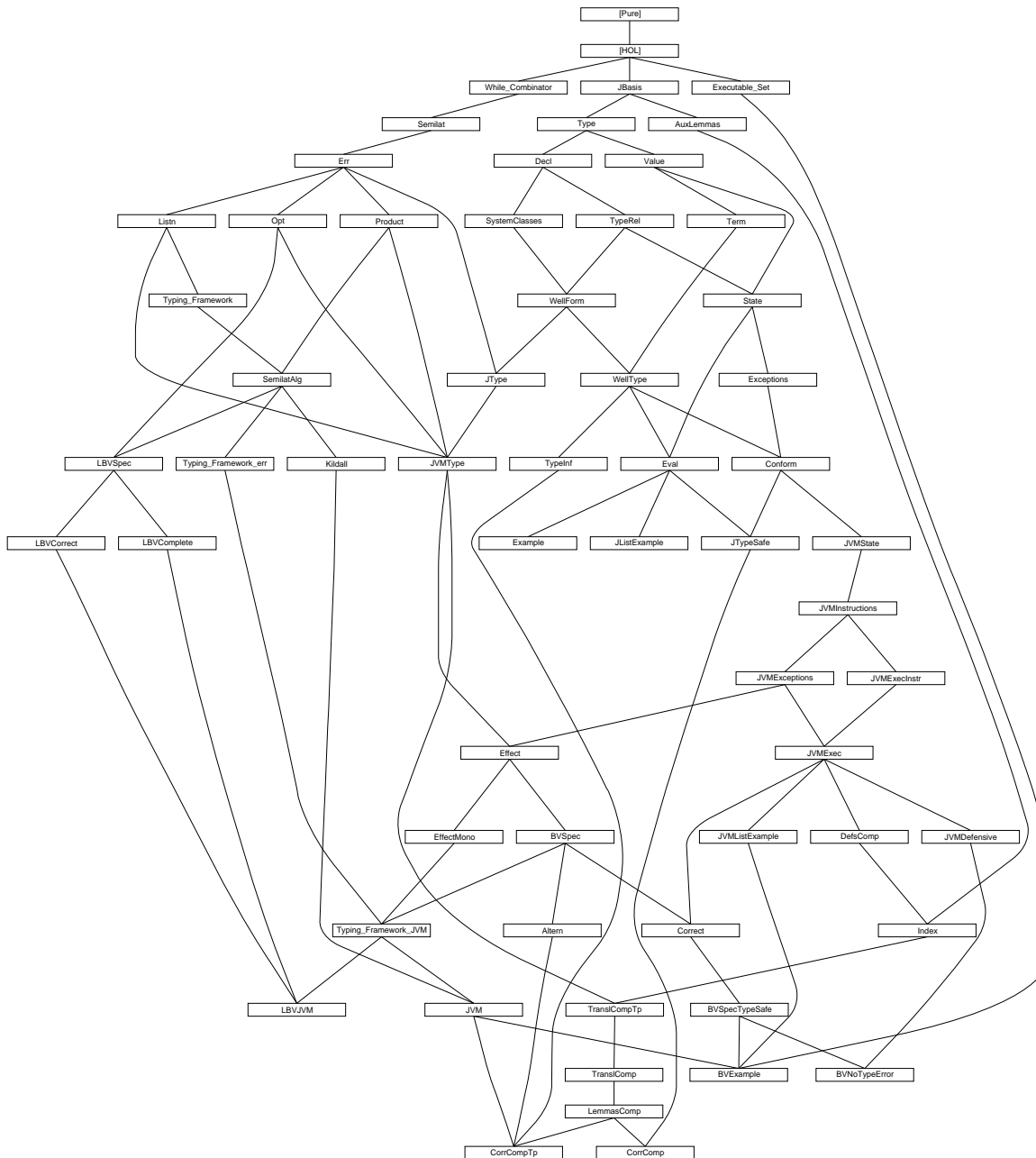


Figure 1.1: Theory Dependency Graph

Chapter 2

Java Source Language

2.1 Some Auxiliary Definitions

theory *JBasis* imports *Main* begin

lemmas [simp] = Let_def

2.1.1 unique

constdefs

unique :: "('a × 'b) list => bool"
 "unique == distinct ∘ map fst"

lemma fst_in_set_lemma [rule_format (no_asm)]:
 "(x, y) : set xys --> x : fst ` set xys"
 <proof>

lemma unique_Nil [simp]: "unique []"
 <proof>

lemma unique_Cons [simp]: "unique ((x,y)#l) = (unique l & (!y. (x,y) ~: set l))"
 <proof>

lemma unique_append [rule_format (no_asm)]: "unique l' ==> unique l -->
 (! (x,y):set l. ! (x',y'):set l'. x' ~ x) --> unique (l @ l')"
 <proof>

lemma unique_map_inj [rule_format (no_asm)]:
 "unique l --> inj f --> unique (map (%(k,x). (f k, g k x)) l)"
 <proof>

2.1.2 More about Maps

lemma map_of_SomeI [rule_format (no_asm)]:
 "unique l --> (k, x) : set l --> map_of l k = Some x"
 <proof>

lemma Ball_set_table':
 "(∀ (x,y) ∈ set l. P x y) --> (∀ x. ∀ y. map_of l x = Some y --> P x y)"
 <proof>

lemmas Ball_set_table = Ball_set_table' [THEN mp]

lemma table_of_remap_SomeD [rule_format (no_asm)]:
 "map_of (map (λ((k,k'),x). (k,(k',x))) t) k = Some (k',x) -->
 map_of t (k, k') = Some x"
 <proof>

end

2.2 Java types

theory Type imports JBasis begin

typeddecl *cnam*

— exceptions

datatype

xcpt
= *NullPointer*
| *ClassCast*
| *OutOfMemory*

— class names

datatype *cname*

= *Object*
| *Xcpt xcpt*
| *Cname cnam*

typeddecl *vnam* — variable or field name

typeddecl *mname* — method name

— names for *This* pointer and local/field variables

datatype *vname*

= *This*
| *VName vnam*

— primitive type, cf. 4.2

datatype *prim_ty*

= *Void* — 'result type' of void methods
| *Boolean*
| *Integer*

— reference type, cf. 4.3

datatype *ref_ty*

= *NullT* — null type, cf. 4.1
| *ClassT cname* — class type

— any type, cf. 4.1

datatype *ty*

= *PrimT prim_ty* — primitive type
| *RefT ref_ty* — reference type

syntax

NT :: "*ty*"
Class :: "*cname* => *ty*"

translations

"*NT*" == "*RefT NullT*"
"*Class C*" == "*RefT (ClassT C)*"

end

2.3 Class Declarations and Programs

theory *Decl* imports *Type* begin

types

fdecl = "*vname* \times *ty*" — field declaration, cf. 8.3 (, 9.3)

sig = "*mname* \times *ty list*" — signature of a method, cf. 8.4.2

'*c mdecl* = "*sig* \times *ty* \times '*c*" — method declaration in a class

'*c "class"* = "*cname* \times *fdecl list* \times '*c mdecl list*"
— class = superclass, fields, methods

'*c cdecl* = "*cname* \times '*c class*" — class declaration, cf. 8.1

'*c prog* = "'*c cdecl list*" — program

translations

"*fdecl*" <= (type) "*vname* \times *ty*"

"*sig*" <= (type) "*mname* \times *ty list*"

"*mdecl c*" <= (type) "*sig* \times *ty* \times *c*"

"*class c*" <= (type) "*cname* \times *fdecl list* \times (*c mdecl*) *list*"

"*cdecl c*" <= (type) "*cname* \times (*c class*)"

"*prog c*" <= (type) "(*c cdecl*) *list*"

constdefs

"*class*" :: "'*c prog* => (*cname* \rightarrow '*c class*)"

"*class* \equiv *map_of*"

is_class :: "'*c prog* => *cname* => bool"

"*is_class G C* \equiv *class G C* \neq None"

lemma *finite_is_class*: "finite {*C. is_class G C*}"
<proof>

consts

is_type :: "'*c prog* => *ty* => bool"

primrec

"*is_type G (PrimT pt)* = True"

"*is_type G (RefT t)* = (case *t* of NullT => True | ClassT *C* => *is_class G C*)"

end

2.4 Relations between Java Types

theory TypeRel imports Decl begin

— direct subclass, cf. 8.1.3

inductive

subcls1 :: "'c prog => [cname, cname] => bool" ("_ ⊢ _ <C1 _" [71,71,71] 70)
for G :: "'c prog"

where

subcls1I: "⌊class G C = Some (D,rest); C ≠ Object⌋ ⇒ G ⊢ C <C1 D"

abbreviation

subcls :: "'c prog => [cname, cname] => bool" ("_ ⊢ _ ≤C _" [71,71,71] 70)
where "G ⊢ C ≤C D ≡ (subcls1 G) ^** C D"

lemma subcls1D:

"G ⊢ C <C1 D ⇒ C ≠ Object ∧ (∃ fs ms. class G C = Some (D,fs,ms))"

⟨proof⟩

lemma subcls1_def2:

"subcls1 G = (λC D. (C, D) ∈
(SIGMA C: {C. is_class G C} . {D. C ≠ Object ∧ fst (the (class G C)) = D}))"

⟨proof⟩

lemma finite_subcls1: "finite {(C, D). subcls1 G C D}"

⟨proof⟩

lemma subcls_is_class: "(subcls1 G) ^++ C D ==> is_class G C"

⟨proof⟩

lemma subcls_is_class2 [rule_format (no_asm)]:

"G ⊢ C ≤C D ⇒ is_class G D → is_class G C"

⟨proof⟩

constdefs

class_rec :: "'c prog ⇒ cname ⇒ 'a ⇒
(cname ⇒ fdecl list ⇒ 'c mdecl list ⇒ 'a ⇒ 'a) ⇒ 'a"
"class_rec G == wfrec {(C, D). (subcls1 G) ^--1 C D}
(λr C t f. case class G C of
None ⇒ arbitrary
| Some (D,fs,ms) ⇒
f C fs ms (if C = Object then t else r D t f))"

lemma class_rec_lemma: "wfP ((subcls1 G) ^--1) ⇒ class G C = Some (D,fs,ms) ⇒

class_rec G C t f = f C fs ms (if C=Object then t else class_rec G D t f)"

⟨proof⟩

definition

"wf_class G = wfP ((subcls1 G) ^--1)"

lemma class_rec_func [code func]:

"class_rec G C t f = (if wf_class G then
(case class G C
of None ⇒ arbitrary

```

      | Some (D, fs, ms) => f C fs ms (if C = Object then t else class_rec G D t f))
    else class_rec G C t f)"
⟨proof⟩

```

consts

```

method :: "'c prog × cname => ( sig  → cname × ty × 'c )"
field  :: "'c prog × cname => ( vname → cname × ty      )"
fields :: "'c prog × cname => ((vname × cname) × ty) list"

```

— methods of a class, with inheritance, overriding and hiding, cf. 8.4.6

```

defs method_def: "method ≡ λ(G,C). class_rec G C empty (λC fs ms ts.
      ts ++ map_of (map (λ(s,m). (s,(C,m))) ms))"

```

```

lemma method_rec_lemma: "[|class G C = Some (D,fs,ms); wfP ((subcls1 G)^--1)|] ==>
  method (G,C) = (if C = Object then empty else method (G,D)) ++
  map_of (map (λ(s,m). (s,(C,m))) ms)"
⟨proof⟩

```

```

defs fields_def: "fields ≡ λ(G,C). class_rec G C []      (λC fs ms ts.
      map (λ(fn,ft). ((fn,C),ft)) fs @ ts)"

```

```

lemma fields_rec_lemma: "[|class G C = Some (D,fs,ms); wfP ((subcls1 G)^--1)|] ==>
  fields (G,C) =
  map (λ(fn,ft). ((fn,C),ft)) fs @ (if C = Object then [] else fields (G,D))"
⟨proof⟩

```

```

defs field_def: "field == map_of o (map (λ((fn,fd),ft). (fn,(fd,ft)))) o fields"

```

lemma field_fields:

```

"field (G,C) fn = Some (fd, fT) ==> map_of (fields (G,C)) (fn, fd) = Some fT"
⟨proof⟩

```

inductive

```

widen  :: "'c prog => [ty    , ty    ] => bool" ("_ ⊢ _ ≤ _"    [71,71,71] 70)
for G :: "'c prog"

```

where

```

  refl [intro!, simp]:      "G ⊢      T ≤ T"    — identity conv., cf. 5.1.1
| subcls      : "G ⊢ C ≤ C D ==> G ⊢ Class C ≤ Class D"
| null [intro!]:      "G ⊢      NT ≤ RefT R"

```

lemmas refl = HOL.refl

— casting conversion, cf. 5.5 / 5.1.5

— left out casts on primitive types

inductive

```

cast  :: "'c prog => [ty    , ty    ] => bool" ("_ ⊢ _ ≤? _"    [71,71,71] 70)
for G :: "'c prog"

```

where

```

  widen: "G ⊢ C ≤ D ==> G ⊢ C ≤? D"
| subcls: "G ⊢ D ≤ C C ==> G ⊢ Class C ≤? Class D"

```

lemma widen_PrimT_RefT [iff]: "(G ⊢ PrimT pT ≤ RefT rT) = False"

⟨proof⟩

lemma widen_RefT: " $G \vdash \text{RefT } R \preceq T \implies \exists t. T = \text{RefT } t$ "
 $\langle \text{proof} \rangle$

lemma widen_RefT2: " $G \vdash S \preceq \text{RefT } R \implies \exists t. S = \text{RefT } t$ "
 $\langle \text{proof} \rangle$

lemma widen_Class: " $G \vdash \text{Class } C \preceq T \implies \exists D. T = \text{Class } D$ "
 $\langle \text{proof} \rangle$

lemma widen_Class_NullT [iff]: " $(G \vdash \text{Class } C \preceq NT) = \text{False}$ "
 $\langle \text{proof} \rangle$

lemma widen_Class_Class [iff]: " $(G \vdash \text{Class } C \preceq \text{Class } D) = (G \vdash C \preceq C \ D)$ "
 $\langle \text{proof} \rangle$

lemma widen_NT_Class [simp]: " $G \vdash T \preceq NT \implies G \vdash T \preceq \text{Class } D$ "
 $\langle \text{proof} \rangle$

lemma cast_PrimT_RefT [iff]: " $(G \vdash \text{PrimT } pT \preceq ? \text{RefT } rT) = \text{False}$ "
 $\langle \text{proof} \rangle$

lemma cast_RefT: " $G \vdash C \preceq ? \text{Class } D \implies \exists rT. C = \text{RefT } rT$ "
 $\langle \text{proof} \rangle$

theorem widen_trans[trans]: " $\llbracket G \vdash S \preceq U; G \vdash U \preceq T \rrbracket \implies G \vdash S \preceq T$ "
 $\langle \text{proof} \rangle$

end

2.5 Java Values

theory *Value* **imports** *Type* **begin**

typedec *loc* — locations, i.e. abstract references on objects

datatype *loc*

 = *XcptRef* *xcpt* — special locations for pre-allocated system exceptions
 / *Loc* *loc* — usual locations (references on objects)

datatype *val*

 = *Unit* — dummy result value of void methods
 / *Null* — null reference
 / *Bool* *bool* — Boolean value
 / *Intg* *int* — integer value, name *Intg* instead of *Int* because of clash with *HOL/Set.thy*
 / *Addr* *loc* — addresses, i.e. locations of objects

consts

the_Bool :: "*val* => *bool*"
 the_Intg :: "*val* => *int*"
 the_Addr :: "*val* => *loc*"

primrec

 "*the_Bool* (*Bool* *b*) = *b*"

primrec

 "*the_Intg* (*Intg* *i*) = *i*"

primrec

 "*the_Addr* (*Addr* *a*) = *a*"

consts

defpval :: "*prim_ty* => *val*" — default value for primitive types
 default_val :: "*ty* => *val*" — default value for all types

primrec

 "*defpval* *Void* = *Unit*"
 "*defpval* *Boolean* = *Bool False*"
 "*defpval* *Integer* = *Intg 0*"

primrec

 "*default_val* (*PrimT* *pt*) = *defpval* *pt*"
 "*default_val* (*RefT* *r*) = *Null*"

end

2.6 Program State

theory State imports TypeRel Value begin

types

`fields' = "(vname × cname → val)"` — field name, defining class, value

`obj = "cname × fields'"` — class instance with class name and fields

constdefs

`obj_ty :: "obj => ty"`

`"obj_ty obj == Class (fst obj)"`

`init_vars :: "('a × ty) list => ('a → val)"`

`"init_vars == map_of o map (λ(n,T). (n,default_val T))"`

types `ahheap = "loc → obj"` — "heap" used in a translation below

`locals = "vname → val"` — simple state, i.e. variable contents

`state = "ahheap × locals"` — heap, local parameter including This

`xstate = "val option × state"` — state including exception information

syntax

`heap :: "state => ahheap"`

`locals :: "state => locals"`

`Norm :: "state => xstate"`

`abrupt :: "xstate => val option"`

`store :: "xstate => state"`

`lookup_obj :: "state => val => obj"`

translations

`"heap" => "fst"`

`"locals" => "snd"`

`"Norm s" == "(None,s)"`

`"abrupt" => "fst"`

`"store" => "snd"`

`"lookup_obj s a'" == "the (heap s (the_Addr a'))"`

constdefs

`raise_if :: "bool ⇒ xcpt ⇒ val option ⇒ val option"`

`"raise_if b x xo ≡ if b ∧ (xo = None) then Some (Addr (XcptRef x)) else xo"`

`new_Addr :: "ahheap => loc × val option"`

`"new_Addr h ≡ SOME (a,x). (h a = None ∧ x = None) | x = Some (Addr (XcptRef OutOfMemory))"`

`np :: "val => val option => val option"`

`"np v == raise_if (v = Null) NullPointer"`

`c_hupd :: "ahheap => xstate => xstate"`

`"c_hupd h' == λ(xo,(h,l)). if xo = None then (None,(h',l)) else (xo,(h,l))"`

`cast_ok :: "'c prog => cname => ahheap => val => bool"`

"cast_ok G C h v == v = Null \vee G \vdash obj_ty (the (h (the_Addr v))) \preceq Class C"

lemma obj_ty_def2 [simp]: "obj_ty (C,fs) = Class C"
 <proof>

lemma new_AddrD: "new_Addr hp = (ref, xcp) \implies
 hp ref = None \wedge xcp = None \vee xcp = Some (Addr (XcptRef OutOfMemory))"
 <proof>

lemma raise_if_True [simp]: "raise_if True x y \neq None"
 <proof>

lemma raise_if_False [simp]: "raise_if False x y = y"
 <proof>

lemma raise_if_Some [simp]: "raise_if c x (Some y) \neq None"
 <proof>

lemma raise_if_Some2 [simp]:
 "raise_if c z (if x = None then Some y else x) \neq None"
 <proof>

lemma raise_if_SomeD [rule_format (no_asm)]:
 "raise_if c x y = Some z \longrightarrow c \wedge Some z = Some (Addr (XcptRef x)) | y = Some z"
 <proof>

lemma raise_if_NoneD [rule_format (no_asm)]:
 "raise_if c x y = None \longrightarrow \neg c \wedge y = None"
 <proof>

lemma np_NoneD [rule_format (no_asm)]:
 "np a' x' = None \longrightarrow x' = None \wedge a' \neq Null"
 <proof>

lemma np_None [rule_format (no_asm), simp]: "a' \neq Null \longrightarrow np a' x' = x'"
 <proof>

lemma np_Some [simp]: "np a' (Some xc) = Some xc"
 <proof>

lemma np_Null [simp]: "np Null None = Some (Addr (XcptRef NullPointer))"
 <proof>

lemma np_Addr [simp]: "np (Addr a) None = None"
 <proof>

lemma np_raise_if [simp]: "(np Null (raise_if c xc None)) =
 Some (Addr (XcptRef (if c then xc else NullPointer)))"
 <proof>

lemma c_hupd_fst [simp]: "fst (c_hupd h (x, s)) = x"
 <proof>

end

2.7 Expressions and Statements

theory *Term* imports *Value* begin

datatype *binop* = *Eq* | *Add* — function codes for binary operation

datatype *expr*

- = *NewC* *cname* — class instance creation
- | *Cast* *cname* *expr* — type cast
- | *Lit* *val* — literal value, also references
- | *BinOp* *binop* *expr* *expr* — binary operation
- | *LAcc* *vname* — local (incl. parameter) access
- | *LAss* *vname* *expr* ("*_* := *_*" [90,90]90) — local assign
- | *FAcc* *cname* *expr* *vname* ("{*_*}_ . *_*" [10,90,99]90) — field access
- | *FAss* *cname* *expr* *vname* *expr* ("{*_*}_ . *_* := *_*" [10,90,99,90]90) — field ass.
- | *Call* *cname* *expr* *mname* *"ty list"* *"expr list"* ("*_* . *_* ' ({*_*}_ ')" [10,90,99,10,10] 90) — method call

datatype *stmt*

- = *Skip* — empty statement
- | *Expr* *expr* — expression statement
- | *Comp* *stmt* *stmt* ("*_* ; *_*" [61,60]60)
- | *Cond* *expr* *stmt* *stmt* ("*If* ' (*_*) *_* *Else* *_* " [80,79,79]70)
- | *Loop* *expr* *stmt* ("*While* ' (*_*) *_* " [80,79]70)

end

2.8 System Classes

theory *SystemClasses* **imports** *Decl* **begin**

This theory provides definitions for the *Object* class, and the system exceptions.

constdefs

```
ObjectC :: "'c cdecl"
"ObjectC ≡ (Object, (arbitrary, [], []))"
```

```
NullPointerC :: "'c cdecl"
"NullPointerC ≡ (Xcpt NullPointer, (Object, [], []))"
```

```
ClassCastC :: "'c cdecl"
"ClassCastC ≡ (Xcpt ClassCast, (Object, [], []))"
```

```
OutOfMemoryC :: "'c cdecl"
"OutOfMemoryC ≡ (Xcpt OutOfMemory, (Object, [], []))"
```

```
SystemClasses :: "'c cdecl list"
"SystemClasses ≡ [ObjectC, NullPointerC, ClassCastC, OutOfMemoryC]"
```

end

2.9 Well-formedness of Java programs

theory *WellForm* imports *TypeRel* *SystemClasses* begin

for static checks on expressions and statements, see *WellType*.

improvements over Java Specification 1.0 (cf. 8.4.6.3, 8.4.6.4, 9.4.1):

- a method implementing or overwriting another method may have a result type that widens to the result type of the other method (instead of identical type)

simplifications:

- for uniformity, *Object* is assumed to be declared like any other class

```
types 'c wf_mb = "'c prog => cname => 'c mdecl => bool"
```

```
constdefs
```

```
wf_syscls :: "'c prog => bool"
"wf_syscls G == let cs = set G in Object ∈ fst ' cs ∧ (∀x. Xcpt x ∈ fst ' cs)"
```

```
wf_fdecl :: "'c prog => fdecl => bool"
"wf_fdecl G == λ(fn,ft). is_type G ft"
```

```
wf_mhead :: "'c prog => sig => ty => bool"
"wf_mhead G == λ(mn,pTs) rT. (∀T∈set pTs. is_type G T) ∧ is_type G rT"
```

```
ws_cdecl :: "'c prog => 'c cdecl => bool"
"ws_cdecl G ==
  λ(C, (D,fs,ms)).
    (∀f∈set fs. wf_fdecl G f) ∧ unique fs ∧
    (∀(sig,rT,mb)∈set ms. wf_mhead G sig rT) ∧ unique ms ∧
    (C ≠ Object → is_class G D ∧ ¬G⊢D⊆C C)"
```

```
ws_prog :: "'c prog => bool"
"ws_prog G ==
  wf_syscls G ∧ (∀c∈set G. ws_cdecl G c) ∧ unique G"
```

```
wf_mrT :: "'c prog => 'c cdecl => bool"
"wf_mrT G ==
  λ(C, (D,fs,ms)).
    (C ≠ Object → (∀(sig,rT,b)∈set ms. ∀D' rT' b'.
      method(G,D) sig = Some(D',rT',b') → G⊢rT⊆rT'))"
```

```
wf_cdecl_mdecl :: "'c wf_mb => 'c prog => 'c cdecl => bool"
"wf_cdecl_mdecl wf_mb G ==
  λ(C, (D,fs,ms)). (∀m∈set ms. wf_mb G C m)"
```

```
wf_prog :: "'c wf_mb => 'c prog => bool"
"wf_prog wf_mb G ==
  ws_prog G ∧ (∀c∈set G. wf_mrT G c ∧ wf_cdecl_mdecl wf_mb G c)"
```

```

wf_mdecl :: "'c wf_mb => 'c wf_mb"
"wf_mdecl wf_mb G C ==  $\lambda(\text{sig}, rT, mb). \text{wf\_mhead } G \text{ sig } rT \wedge \text{wf\_mb } G \text{ C } (\text{sig}, rT, mb)"$ 

wf_cdecl :: "'c wf_mb => 'c prog => 'c cdecl => bool"
"wf_cdecl wf_mb G ==
   $\lambda(C, (D, fs, ms)).$ 
   $(\forall f \in \text{set } fs. \text{wf\_fdecl } G \quad f) \wedge \text{unique } fs \wedge$ 
   $(\forall m \in \text{set } ms. \text{wf\_mdecl } wf\_mb \text{ G C } m) \wedge \text{unique } ms \wedge$ 
   $(C \neq \text{Object} \longrightarrow \text{is\_class } G \text{ D} \wedge \neg G \vdash D \preceq C \wedge$ 
     $(\forall (\text{sig}, rT, b) \in \text{set } ms. \forall D' \text{ rT}' \text{ b}'.$ 
       $\text{method}(G, D) \text{ sig} = \text{Some}(D', rT', b') \longrightarrow G \vdash rT \preceq rT'))"$ 

lemma wf_cdecl_mrT_cdecl_mdecl:
  "(wf_cdecl wf_mb G c) = (ws_cdecl G c  $\wedge$  wf_mrT G c  $\wedge$  wf_cdecl_mdecl wf_mb G c)"
<proof>

lemma wf_cdecl_ws_cdecl [intro]: "wf_cdecl wf_mb G cd  $\implies$  ws_cdecl G cd"
<proof>

lemma wf_prog_ws_prog [intro]: "wf_prog wf_mb G  $\implies$  ws_prog G"
<proof>

lemma wf_prog_wf_mdecl:
  "[ wf_prog wf_mb G; (C, S, fs, mdecls)  $\in$  set G; ((mn, pTs), rT, code)  $\in$  set mdecls ]
   $\implies \text{wf\_mdecl } wf\_mb \text{ G C } ((mn, pTs), rT, code)"$ 
<proof>

lemma class_wf:
  "[| class G C = Some c; wf_prog wf_mb G |]
   $\implies \text{wf\_cdecl } wf\_mb \text{ G } (C, c) \wedge \text{wf\_mrT } G \text{ (C, c)}$ 
<proof>

lemma class_wf_struct:
  "[| class G C = Some c; ws_prog G |]
   $\implies \text{ws\_cdecl } G \text{ (C, c)}$ 
<proof>

lemma class_Object [simp]:
  "ws_prog G  $\implies \exists X \text{ fs } ms. \text{class } G \text{ Object} = \text{Some } (X, fs, ms)"$ 
<proof>

lemma class_Object_syscls [simp]:
  "wf_syscls G  $\implies \text{unique } G \implies \exists X \text{ fs } ms. \text{class } G \text{ Object} = \text{Some } (X, fs, ms)"$ 
<proof>

lemma is_class_Object [simp]: "ws_prog G  $\implies \text{is\_class } G \text{ Object}"$ 
<proof>

lemma is_class_xcpt [simp]: "ws_prog G  $\implies \text{is\_class } G \text{ (Xcpt } x)"$ 
<proof>

lemma subcls1_wfD: "[| G  $\vdash C \prec C1D$ ; ws_prog G |]  $\implies D \neq C \wedge \neg (\text{subcls1 } G)^{++} D \text{ C}"$ 
<proof>

```

lemma *wf_cdecl_supD*:

"!!r. $\llbracket \text{ws_cdecl } G \text{ (C,D,r)}; C \neq \text{Object} \rrbracket \implies \text{is_class } G \text{ D}$ "
 $\langle \text{proof} \rangle$

lemma *subcls_asym*: " $\llbracket \text{ws_prog } G; (\text{subcls1 } G)^{++} C \text{ D} \rrbracket \implies \neg (\text{subcls1 } G)^{++} D \text{ C}$ "

$\langle \text{proof} \rangle$

lemma *subcls_irrefl*: " $\llbracket \text{ws_prog } G; (\text{subcls1 } G)^{++} C \text{ D} \rrbracket \implies C \neq D$ "

$\langle \text{proof} \rangle$

lemma *acyclic_subcls1*: " $\text{ws_prog } G \implies \text{acyclicP } (\text{subcls1 } G)$ "

$\langle \text{proof} \rangle$

lemma *wf_subcls1*: " $\text{ws_prog } G \implies \text{wfP } ((\text{subcls1 } G)^{--1})$ "

$\langle \text{proof} \rangle$

lemma *subcls_induct*:

" $\llbracket \text{wf_prog } \text{wf_mb } G; !!C. \forall D. (\text{subcls1 } G)^{++} C \text{ D} \dashrightarrow P \text{ D} \implies P \text{ C} \rrbracket \implies P \text{ C}$ "
 (is " $?A \implies \text{PROP } ?P \implies _$ ")

$\langle \text{proof} \rangle$

lemma *subcls1_induct*:

" $\llbracket \text{is_class } G \text{ C}; \text{wf_prog } \text{wf_mb } G; P \text{ Object};$
 $!!C \text{ D fs ms. } \llbracket C \neq \text{Object}; \text{is_class } G \text{ C}; \text{class } G \text{ C} = \text{Some } (D, \text{fs}, \text{ms}) \wedge$
 $\text{wf_cdecl } \text{wf_mb } G \text{ (C,D,fs,ms)} \wedge G \vdash C \prec C1 \text{ D} \wedge \text{is_class } G \text{ D} \wedge P \text{ D} \rrbracket \implies P \text{ C}$
 $\rrbracket \implies P \text{ C}$ "
 (is " $?A \implies ?B \implies ?C \implies \text{PROP } ?P \implies _$ ")

$\langle \text{proof} \rangle$

lemma *subcls_induct_struct*:

" $\llbracket \text{ws_prog } G; !!C. \forall D. (\text{subcls1 } G)^{++} C \text{ D} \dashrightarrow P \text{ D} \implies P \text{ C} \rrbracket \implies P \text{ C}$ "
 (is " $?A \implies \text{PROP } ?P \implies _$ ")

$\langle \text{proof} \rangle$

lemma *subcls1_induct_struct*:

" $\llbracket \text{is_class } G \text{ C}; \text{ws_prog } G; P \text{ Object};$
 $!!C \text{ D fs ms. } \llbracket C \neq \text{Object}; \text{is_class } G \text{ C}; \text{class } G \text{ C} = \text{Some } (D, \text{fs}, \text{ms}) \wedge$
 $\text{ws_cdecl } G \text{ (C,D,fs,ms)} \wedge G \vdash C \prec C1 \text{ D} \wedge \text{is_class } G \text{ D} \wedge P \text{ D} \rrbracket \implies P \text{ C}$
 $\rrbracket \implies P \text{ C}$ "
 (is " $?A \implies ?B \implies ?C \implies \text{PROP } ?P \implies _$ ")

$\langle \text{proof} \rangle$

lemmas *method_rec* = *wf_subcls1* [THEN [2] *method_rec_lemma*]

lemmas *fields_rec* = *wf_subcls1* [THEN [2] *fields_rec_lemma*]

lemma *field_rec*: " $\llbracket \text{class } G \text{ C} = \text{Some } (D, \text{fs}, \text{ms}); \text{ws_prog } G \rrbracket$
 $\implies \text{field } (G, C) =$

(if $C = \text{Object}$ then empty else $\text{field } (G, D)$) ++
 $\text{map_of } (\text{map } (\lambda(s, f). (s, C, f)) \text{ fs})$ "

$\langle \text{proof} \rangle$

lemma *method_Object* [simp]:

"method (G, Object) sig = Some (D, mh, code) \implies ws_prog G \implies D = Object"
 <proof>

lemma *fields_Object* [simp]: "[(vn, C), T] \in set (fields (G, Object)); ws_prog G]
 \implies C = Object"
 <proof>

lemma *subcls_C_Object*: "[is_class G C; ws_prog G] \implies G \vdash C \preceq C Object"
 <proof>

lemma *is_type_rTI*: "wf_mhead G sig rT \implies is_type G rT"
 <proof>

lemma *widen_fields_defpl'*: "[is_class G C; ws_prog G] \implies
 $\forall ((fn, fd), fT) \in \text{set (fields (G, C))}. G \vdash C \preceq_C fd$ "
 <proof>

lemma *widen_fields_defpl*:
 "[((fn, fd), fT) \in set (fields (G, C)); ws_prog G; is_class G C] \implies
 G \vdash C \preceq C fd"
 <proof>

lemma *unique_fields*:
 "[is_class G C; ws_prog G] \implies unique (fields (G, C))"
 <proof>

lemma *fields_mono_lemma* [rule_format (no_asm)]:
 "[ws_prog G; (subcls1 G)^** C' C] \implies
 x \in set (fields (G, C)) \rightarrow x \in set (fields (G, C'))"
 <proof>

lemma *fields_mono*:
 "[map_of (fields (G, C)) fn = Some f; G \vdash D \preceq C C; is_class G D; ws_prog G]
 \implies map_of (fields (G, D)) fn = Some f"
 <proof>

lemma *widen_cfs_fields*:
 "[field (G, C) fn = Some (fd, fT); G \vdash D \preceq C C; ws_prog G] \implies
 map_of (fields (G, D)) (fn, fd) = Some fT"
 <proof>

lemma *method_wf_mdecl* [rule_format (no_asm)]:
 "wf_prog wf_mb G \implies is_class G C \implies
 method (G, C) sig = Some (md, mh, m)
 \rightarrow G \vdash C \preceq C md \wedge wf_mdecl wf_mb G md (sig, (mh, m))"
 <proof>

lemma *method_wf_mhead* [rule_format (no_asm)]:
 "ws_prog G \implies is_class G C \implies
 method (G, C) sig = Some (md, rT, mb)
 \rightarrow G \vdash C \preceq C md \wedge wf_mhead G sig rT"
 <proof>

$\langle proof \rangle$

```
lemma subcls_widen_methd [rule_format (no_asm)]:
  "[| G ⊢ T' ≤ C T; wf_prog wf_mb G |] ==>
    ∀ D rT b. method (G,T) sig = Some (D,rT ,b) -->
      (∃ D' rT' b'. method (G,T') sig = Some (D',rT',b') ∧ G ⊢ D' ≤ C D ∧ G ⊢ rT' ≤ rT)"
  <proof>
```

```
lemma subtype_widen_methd:
  "[| G ⊢ C ≤ C D; wf_prog wf_mb G;
    method (G,D) sig = Some (md, rT, b) |]
    ==> ∃ md' rT' b'. method (G,C) sig = Some (md',rT',b') ∧ G ⊢ rT' ≤ rT"
  <proof>
```

```
lemma method_in_md [rule_format (no_asm)]:
  "ws_prog G ==> is_class G C ==> ∀ D. method (G,C) sig = Some (D,mh,code)
    --> is_class G D ∧ method (G,D) sig = Some (D,mh,code)"
  <proof>
```

```
lemma method_in_md_struct [rule_format (no_asm)]:
  "ws_prog G ==> is_class G C ==> ∀ D. method (G,C) sig = Some (D,mh,code)
    --> is_class G D ∧ method (G,D) sig = Some (D,mh,code)"
  <proof>
```

```
lemma fields_in_fd [rule_format (no_asm)]: "[| wf_prog wf_mb G; is_class G C |]
  ==> ∀ vn D T. (((vn,D),T) ∈ set (fields (G,C)))
    → (is_class G D ∧ ((vn,D),T) ∈ set (fields (G,D))))"
  <proof>
```

```
lemma field_in_fd [rule_format (no_asm)]: "[| wf_prog wf_mb G; is_class G C |]
  ==> ∀ vn D T. (field (G,C) vn = Some (D,T))
    → is_class G D ∧ field (G,D) vn = Some (D,T))"
  <proof>
```

```
lemma widen_methd:
  "[| method (G,C) sig = Some (md,rT,b); wf_prog wf_mb G; G ⊢ T'' ≤ C C |]
    ==> ∃ md' rT' b'. method (G,T'') sig = Some (md',rT',b') ∧ G ⊢ rT' ≤ rT"
  <proof>
```

```
lemma widen_field: "[| (field (G,C) fn) = Some (fd, fT); wf_prog wf_mb G; is_class G C
|]
  ==> G ⊢ C ≤ C fd"
  <proof>
```

```
lemma Call_lemma:
  "[| method (G,C) sig = Some (md,rT,b); G ⊢ T'' ≤ C C; wf_prog wf_mb G;
    class G C = Some y |] ==> ∃ T' rT' b. method (G,T'') sig = Some (T',rT',b) ∧
    G ⊢ rT' ≤ rT ∧ G ⊢ T'' ≤ C T' ∧ wf_mhead G sig rT' ∧ wf_mb G T' (sig,rT',b)"
  <proof>
```

lemma *fields_is_type_lemma* [rule_format (no_asm)]:

"[|is_class G C; ws_prog G|] ==>
 $\forall f \in \text{set } (\text{fields } (G,C)). \text{is_type } G \text{ (snd } f)$ "
 <proof>

lemma *fields_is_type*:

"[|map_of (fields (G,C)) fn = Some f; ws_prog G; is_class G C|] ==>
 is_type G f"
 <proof>

lemma *field_is_type*: "[| ws_prog G; is_class G C; field (G, C) fn = Some (fd, fT) |]
 $\implies \text{is_type } G \text{ fT}$ "

<proof>

lemma *methd*:

"[| ws_prog G; (C,S,fs,mdecls) \in set G; (sig,rT,code) \in set mdecls |]
 $\implies \text{method } (G,C) \text{ sig} = \text{Some}(C,rT,code) \wedge \text{is_class } G \text{ C}$ "
 <proof>

lemma *wf_mb'E*:

"[| wf_prog wf_mb G; $\bigwedge C \ S \ fs \ ms \ m. [(C,S,fs,ms) \in \text{set } G; m \in \text{set } ms] \implies \text{wf_mb}' \ G \ C \ m$ |]
 $\implies \text{wf_prog } \text{wf_mb}' \ G$ "
 <proof>

lemma *fst_mono*: " $A \subseteq B \implies \text{fst } 'A \subseteq \text{fst } 'B$ " <proof>

lemma *wf_syscls*:

"set SystemClasses \subseteq set G $\implies \text{wf_syscls } G$ "
 <proof>

end

2.10 Well-typedness Constraints

theory *WellType* **imports** *Term WellForm* **begin**

the formulation of well-typedness of method calls given below (as well as the Java Specification 1.0) is a little too restrictive: Is does not allow methods of class *Object* to be called upon references of interface type.

simplifications:

- the type rules include all static checks on expressions and statements, e.g. definedness of names (of parameters, locals, fields, methods)

local variables, including method parameters and *This*:

types

```
lenv    = "vname  $\rightarrow$  ty"
'c env  = "'c prog  $\times$  lenv"
```

syntax

```
prg      :: "'c env  $\Rightarrow$  'c prog"
localT :: "'c env  $\Rightarrow$  (vname  $\rightarrow$  ty)"
```

translations

```
"prg"       $\Rightarrow$  "fst"
"localT"     $\Rightarrow$  "snd"
```

consts

```
more_spec :: "'c prog  $\Rightarrow$  (ty  $\times$  'x)  $\times$  ty list  $\Rightarrow$ 
              (ty  $\times$  'x)  $\times$  ty list  $\Rightarrow$  bool"
appl_methds :: "'c prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  ((ty  $\times$  ty)  $\times$  ty list) set"
max_spec :: "'c prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  ((ty  $\times$  ty)  $\times$  ty list) set"
```

defs

```
more_spec_def: "more_spec G ==  $\lambda((d,h),pTs).$   $\lambda((d',h'),pTs').$   $G \vdash d \preceq d' \wedge$ 
              list_all2 ( $\lambda T T'. G \vdash T \preceq T'$ ) pTs pTs'"
```

— applicable methods, cf. 15.11.2.1

```
appl_methds_def: "appl_methds G C ==  $\lambda(mn, pTs).$ 
              {((Class md,rT),pTs') | md rT mb pTs'.
              method (G,C) (mn, pTs') = Some (md,rT,mb)  $\wedge$ 
              list_all2 ( $\lambda T T'. G \vdash T \preceq T'$ ) pTs pTs'}
```

— maximally specific methods, cf. 15.11.2.2

```
max_spec_def: "max_spec G C sig == {m. m  $\in$  appl_methds G C sig  $\wedge$ 
              ( $\forall m' \in$  appl_methds G C sig.
              more_spec G m' m  $\rightarrow$  m' = m)}
```

lemma *max_spec2appl_meths*:

```
"x  $\in$  max_spec G C sig  $\Rightarrow$  x  $\in$  appl_methds G C sig"
```

<proof>

lemma *appl_methsD*:

```

"((md,rT),pTs') ∈ appl_methds G C (mn, pTs) ==>
  ∃ D b. md = Class D ∧ method (G,C) (mn, pTs') = Some (D,rT,b)
  ∧ list_all2 (λT T'. G ⊢ T ≤ T') pTs pTs'"
⟨proof⟩

```

```

lemmas max_spec2mheads = insertI1 [THEN [2] equalityD2 [THEN subsetD],
  THEN max_spec2appl_meths, THEN appl_methsD]

```

consts

```

typeof :: "(loc => ty option) => val => ty option"

```

primrec

```

"typeof dt Unit      = Some (PrimT Void)"
"typeof dt Null      = Some NT"
"typeof dt (Bool b)  = Some (PrimT Boolean)"
"typeof dt (Intg i)  = Some (PrimT Integer)"
"typeof dt (Addr a)  = dt a"

```

```

lemma is_type_typeof [rule_format (no_asm), simp]:
  "(∀ a. v ≠ Addr a) --> (∃ T. typeof t v = Some T ∧ is_type G T)"
⟨proof⟩

```

```

lemma typeof_empty_is_type [rule_format (no_asm)]:
  "typeof (λ a. None) v = Some T ⟶ is_type G T"
⟨proof⟩

```

```

lemma typeof_default_val: "∃ T. (typeof dt (default_val ty) = Some T) ∧ G ⊢ T ≤ ty"
⟨proof⟩

```

types

```

java_mb = "vname list × (vname × ty) list × stmt × expr"
— method body with parameter names, local variables, block, result expression.
— local variables might include This, which is hidden anyway

```

inductive

```

ty_expr :: "'c env => expr => ty => bool" ("_ ⊢ _ :: _" [51, 51, 51] 50)
and ty_exprs :: "'c env => expr list => ty list => bool" ("_ ⊢ _ [::] _" [51, 51, 51]
50)

```

```

and wt_stmt :: "'c env => stmt => bool" ("_ ⊢ _ √" [51, 51] 50)

```

where

```

NewC: "[| is_class (prg E) C |] ==>
  E ⊢ NewC C :: Class C" — cf. 15.8

```

— cf. 15.15

```

| Cast: "[| E ⊢ e :: C; is_class (prg E) D;
  prg E ⊢ C ≤? Class D |] ==>
  E ⊢ Cast D e :: Class D"

```

— cf. 15.7.1

```

| Lit: "[| typeof (λ v. None) x = Some T |] ==>
  E ⊢ Lit x :: T"

```

```

— cf. 15.13.1
/ LAcc: "[| localT E v = Some T; is_type (prg E) T |] ==>
  E⊢LAcc v::T"

/ BinOp: "[| E⊢e1::T;
  E⊢e2::T;
  if bop = Eq then T' = PrimT Boolean
  else T' = T ∧ T = PrimT Integer |] ==>
  E⊢BinOp bop e1 e2::T'"

— cf. 15.25, 15.25.1
/ LAss: "[| v ~= This;
  E⊢LAcc v::T;
  E⊢e::T';
  prg E⊢T' ⋖ T |] ==>
  E⊢v::=e::T'"

— cf. 15.10.1
/ FAcc: "[| E⊢a::Class C;
  field (prg E,C) fn = Some (fd,fT) |] ==>
  E⊢{fd}a..fn::fT"

— cf. 15.25, 15.25.1
/ FAss: "[| E⊢{fd}a..fn::T;
  E⊢v      ::T';
  prg E⊢T' ⋖ T |] ==>
  E⊢{fd}a..fn:=v::T'"

— cf. 15.11.1, 15.11.2, 15.11.3
/ Call: "[| E⊢a::Class C;
  E⊢ps[::]pTs;
  max_spec (prg E) C (mn, pTs) = {(md,rT),pTs'} |] ==>
  E⊢{C}a..mn({pTs'}ps)::rT"

— well-typed expression lists

— cf. 15.11.???
/ Nil: "E⊢[][::][]"

— cf. 15.11.???
/ Cons: "[| E⊢e::T;
  E⊢es[::]Ts |] ==>
  E⊢e#es[::]T#Ts"

— well-typed statements

/ Skip: "E⊢Skip√"

/ Expr: "[| E⊢e::T |] ==>
  E⊢Expr e√"

/ Comp: "[| E⊢s1√;
```

$$E \vdash s2 \sqrt{\quad} \mid \Rightarrow \\ E \vdash s1;; s2 \sqrt{\quad}$$

— cf. 14.8

```
| Cond: "[| E ⊢ e :: PrimT Boolean;
           E ⊢ s1 √;
           E ⊢ s2 √ |] ==>
           E ⊢ If(e) s1 Else s2 √"
```

— cf. 14.10

```
| Loop: "[| E ⊢ e :: PrimT Boolean;
            E ⊢ s √ |] ==>
            E ⊢ While(e) s √"
```

constdefs

```
wf_java_mdecl :: "'c prog => cname => java_mb mdecl => bool"
"wf_java_mdecl G C == λ((mn,pTs),rT,(pns,lvars,blk,res)).
  length pTs = length pns ∧
  distinct pns ∧
  unique lvars ∧
  This ∉ set pns ∧ This ∉ set (map fst lvars) ∧
  (∀pn∈set pns. map_of lvars pn = None) ∧
  (∀(vn,T)∈set lvars. is_type G T) &
  (let E = (G,map_of lvars(pns[↦]pTs)(This↦Class C)) in
   E ⊢ blk √ ∧ (∃T. E ⊢ res :: T ∧ G ⊢ T ≤ rT))"
```

syntax

```
wf_java_prog :: "'c prog => bool"
```

translations

```
"wf_java_prog" == "wf_prog wf_java_mdecl"
```

```
lemma wf_java_prog_wf_java_mdecl: "[|
  wf_java_prog G; (C, D, fds, mths) ∈ set G; jmdcl ∈ set mths |]
  ==> wf_java_mdecl G C jmdcl"
<proof>
```

```
lemma wt_is_type: "(E ⊢ e :: T → ws_prog (prg E) → is_type (prg E) T) ∧
  (E ⊢ es[::]Ts → ws_prog (prg E) → Ball (set Ts) (is_type (prg E))) ∧
  (E ⊢ c √ → True)"
<proof>
```

```
lemmas ty_expr_is_type = wt_is_type [THEN conjunct1, THEN mp, rule_format]
```

```
lemma expr_class_is_class: "
  [ws_prog (prg E); E ⊢ e :: Class C] ==> is_class (prg E) C"
<proof>
```

end

2.11 Operational Evaluation (big step) Semantics

theory Eval imports State WellType begin

— Auxiliary notions

constdefs

```
fits      :: "java_mb prog  $\Rightarrow$  state  $\Rightarrow$  val  $\Rightarrow$  ty  $\Rightarrow$  bool" ("_,_  $\vdash_{\text{fits}}$  _" [61,61,61,61] 60)
"G,s  $\vdash a'$  fits T  $\equiv$  case T of PrimT T'  $\Rightarrow$  False | RefT T'  $\Rightarrow$  a'=Null  $\vee$  G  $\vdash_{\text{obj\_ty}}$ (lookup_obj
s a')  $\preceq$  T"
```

constdefs

```
catch :: "java_mb prog  $\Rightarrow$  xstate  $\Rightarrow$  cname  $\Rightarrow$  bool" ("_,_  $\vdash_{\text{catch}}$  _" [61,61,61] 60)
"G,s  $\vdash_{\text{catch}}$  C  $\equiv$  case abrupt s of None  $\Rightarrow$  False | Some a  $\Rightarrow$  G,store s  $\vdash$  a fits Class C"
```

constdefs

```
lupd      :: "vname  $\Rightarrow$  val  $\Rightarrow$  state  $\Rightarrow$  state" ("lupd'(_  $\mapsto$  _)" [10,10] 1000)
"lupd vn v  $\equiv$   $\lambda$  (hp,loc). (hp, (loc(vn  $\mapsto$  v)))"
```

constdefs

```
new_xcpt_var :: "vname  $\Rightarrow$  xstate  $\Rightarrow$  xstate"
"new_xcpt_var vn  $\equiv$   $\lambda$ (x,s). Norm (lupd(vn  $\mapsto$  the x) s)"
```

— Evaluation relations

inductive

```
eval :: "[java_mb prog, xstate, expr, val, xstate]  $\Rightarrow$  bool "
(" _  $\vdash$  _  $\rightarrow$  _" [51,82,60,82,82] 81)
and evals :: "[java_mb prog, xstate, expr list,
               val list, xstate]  $\Rightarrow$  bool "
(" _  $\vdash$  _  $\rightarrow$  _" [51,82,60,51,82] 81)
and exec :: "[java_mb prog, xstate, stmt, xstate]  $\Rightarrow$  bool "
(" _  $\vdash$  _  $\rightarrow$  _" [51,82,60,82] 81)
for G :: "java_mb prog"
where
```

— evaluation of expressions

XcptE: "G \vdash (Some xc,s) \rightarrow arbitrary \rightarrow (Some xc,s)" — cf. 15.5

— cf. 15.8.1

```
| NewC: "[| h = heap s; (a,x) = new_Addr h;
           h' = h(a  $\mapsto$  (C, init_vars (fields (G,C)))) |]  $\Rightarrow$ 
           G  $\vdash_{\text{Norm}}$  s  $\rightarrow$  NewC C  $\rightarrow$  Addr a  $\rightarrow$  c_hupd h' (x,s)"
```

— cf. 15.15

```
| Cast: "[| G  $\vdash_{\text{Norm}}$  s0  $\rightarrow$  v  $\rightarrow$  (x1,s1);
           x2 = raise_if ( $\neg$  cast_ok G C (heap s1) v) ClassCast x1 |]  $\Rightarrow$ 
           G  $\vdash_{\text{Norm}}$  s0  $\rightarrow$  Cast C e  $\rightarrow$  v  $\rightarrow$  (x2,s1)"
```



```

— cf. 15.7.1
| Lit: "G⊢Norm s -Lit v>v-> Norm s"

| BinOp: "[| G⊢Norm s -e1>v1-> s1;
              G⊢s1      -e2>v2-> s2;
              v = (case bop of Eq => Bool (v1 = v2)
                    | Add => Intg (the_Intg v1 + the_Intg v2)) |] ==>
              G⊢Norm s -BinOp bop e1 e2>v-> s2"

— cf. 15.13.1, 15.2
| LAcc: "G⊢Norm s -LAcc v>the (locals s v)-> Norm s"

— cf. 15.25.1
| LAss: "[| G⊢Norm s -e>v-> (x, (h, l));
              l' = (if x = None then l(va↦v) else l) |] ==>
              G⊢Norm s -va::e>v-> (x, (h, l'))"

— cf. 15.10.1, 15.2
| FAcc: "[| G⊢Norm s0 -e>a'-> (x1, s1);
              v = the (snd (the (heap s1 (the_Addr a')))) (fn, T)) |] ==>
              G⊢Norm s0 -{T}e..fn>v-> (np a' x1, s1)"

— cf. 15.25.1
| FAss: "[| G⊢      Norm s0 -e1>a'-> (x1, s1); a = the_Addr a';
              G⊢(np a' x1, s1) -e2>v -> (x2, s2);
              h = heap s2; (c, fs) = the (h a);
              h' = h(a↦(c, (fs((fn, T)↦v)))) |] ==>
              G⊢Norm s0 -{T}e1..fn:=e2>v-> c_hupd h' (x2, s2)"

— cf. 15.11.4.1, 15.11.4.2, 15.11.4.4, 15.11.4.5, 14.15
| Call: "[| G⊢Norm s0 -e>a'-> s1; a = the_Addr a';
              G⊢s1 -ps[>]pvs-> (x, (h, l)); dynT = fst (the (h a));
              (md, rT, pns, lvars, blk, res) = the (method (G, dynT) (mn, pTs));
              G⊢(np a' x, (h, (init_vars lvars) (pns[>]pvs) (This↦a'))) -blk-> s3;
              G⊢ s3 -res>v -> (x4, s4) |] ==>
              G⊢Norm s0 -{C}e..mn({pTs}ps)>v-> (x4, (heap s4, l))"

— evaluation of expression lists

— cf. 15.5
| XcptEs: "G⊢(Some xc, s) -e[>]arbitrary-> (Some xc, s)"

— cf. 15.11.???
| Nil: "G⊢Norm s0 -[] [>] []-> Norm s0"

— cf. 15.6.4
| Cons: "[| G⊢Norm s0 -e > v -> s1;
              G⊢      s1 -es[>]vs-> s2 |] ==>
              G⊢Norm s0 -e#es[>]v#vs-> s2"

— execution of statements

```

```

— cf. 14.1
| XcptS: "G⊢ (Some xc, s) -c-> (Some xc, s)"

— cf. 14.5
| Skip: "G⊢ Norm s -Skip-> Norm s"

— cf. 14.7
| Expr: "[| G⊢ Norm s0 -e>v-> s1 |] ==>
          G⊢ Norm s0 -Expr e-> s1"

— cf. 14.2
| Comp: "[| G⊢ Norm s0 -c1-> s1;
            G⊢      s1 -c2-> s2 |] ==>
          G⊢ Norm s0 -c1;; c2-> s2"

— cf. 14.8.2
| Cond: "[| G⊢ Norm s0 -e>v-> s1;
            G⊢ s1 -(if the_Bool v then c1 else c2)-> s2 |] ==>
          G⊢ Norm s0 -If(e) c1 Else c2-> s2"

— cf. 14.10, 14.10.1
| LoopF: "[| G⊢ Norm s0 -e>v-> s1; ¬the_Bool v |] ==>
          G⊢ Norm s0 -While(e) c-> s1"
| LoopT: "[| G⊢ Norm s0 -e>v-> s1; the_Bool v;
            G⊢ s1 -c-> s2; G⊢ s2 -While(e) c-> s3 |] ==>
          G⊢ Norm s0 -While(e) c-> s3"

lemma eval_evals_exec_induct = eval_evals_exec.induct [split_format (complete)]

lemma NewCI: "[| new_Addr (heap s) = (a, x);
                s' = c_hupd (heap s (a ↦ (C, init_vars (fields (G, C))))) (x, s) |] ==>
                G⊢ Norm s -NewC C>Addr a-> s'"
⟨proof⟩

lemma eval_evals_exec_no_xcpt:
  "!!s s'. (G⊢ (x, s) -e > v -> (x', s') --> x'=None --> x=None) ∧
            (G⊢ (x, s) -es[>]vs-> (x', s') --> x'=None --> x=None) ∧
            (G⊢ (x, s) -c -> (x', s') --> x'=None --> x=None)"
⟨proof⟩

lemma eval_no_xcpt: "G⊢ (x, s) -e>v-> (None, s') ==> x=None"
⟨proof⟩

lemma evals_no_xcpt: "G⊢ (x, s) -e[>]v-> (None, s') ==> x=None"
⟨proof⟩

lemma exec_no_xcpt: "G ⊢ (x, s) -c-> (None, s')
  ==> x = None"
⟨proof⟩

lemma eval_evals_exec_xcpt:
  "!!s s'. (G⊢ (x, s) -e > v -> (x', s') --> x=Some xc --> x'=Some xc ∧ s'=s) ∧
            (G⊢ (x, s) -es[>]vs-> (x', s') --> x=Some xc --> x'=Some xc ∧ s'=s) ∧
            (G⊢ (x, s) -c -> (x', s') --> x=Some xc --> x'=Some xc ∧ s'=s)"

```

```

      (G ⊢ (x, s) -es[>-] vs-→ (x', s') --> x = Some xc --> x' = Some xc ∧ s' = s) ∧
      (G ⊢ (x, s) -c      -> (x', s') --> x = Some xc --> x' = Some xc ∧ s' = s) "
⟨proof⟩

```

```

lemma eval_xcpt: "G ⊢ (Some xc, s) -e>-v-→ (x', s') ==> x' = Some xc ∧ s' = s"
⟨proof⟩

```

```

lemma exec_xcpt: "G ⊢ (Some xc, s) -s0-→ (x', s') ==> x' = Some xc ∧ s' = s"
⟨proof⟩

```

end

theory Exceptions **imports** State **begin**

a new, blank object with default values in all fields:

```

constdefs
  blank :: "'c prog ⇒ cname ⇒ obj"
  "blank G C ≡ (C, init_vars (fields(G, C)))"

  start_heap :: "'c prog ⇒ aheap"
  "start_heap G ≡ empty (XcptRef NullPointer ↦ blank G (Xcpt NullPointer))
                    (XcptRef ClassCast ↦ blank G (Xcpt ClassCast))
                    (XcptRef OutOfMemory ↦ blank G (Xcpt OutOfMemory))"

```

```

consts
  cname_of :: "ahheap ⇒ val ⇒ cname"

```

```

translations
  "cname_of hp v" == "fst (the (hp (the_Addr v)))"

```

```

constdefs
  preallocated :: "ahheap ⇒ bool"
  "preallocated hp ≡ ∀ x. ∃ fs. hp (XcptRef x) = Some (Xcpt x, fs)"

```

```

lemma preallocatedD:
  "preallocated hp ⇒ ∃ fs. hp (XcptRef x) = Some (Xcpt x, fs)"
⟨proof⟩

```

```

lemma preallocatedE [elim?]:
  "preallocated hp ⇒ (∧ fs. hp (XcptRef x) = Some (Xcpt x, fs) ⇒ P hp) ⇒ P hp"
⟨proof⟩

```

```

lemma cname_of_xcp:
  "raise_if b x None = Some xcp ⇒ preallocated hp
   ⇒ cname_of (hp::ahheap) xcp = Xcpt x"
⟨proof⟩

```

```

lemma preallocated_start:
  "preallocated (start_heap G)"
⟨proof⟩

```

36

end

2.12 Conformity Relations for Type Soundness Proof

theory Conform imports State WellType Exceptions begin

types 'c env' = "'c prog \times (vname \rightarrow ty)" — same as env of WellType.thy

constdefs

```

hext :: "aheap  $\Rightarrow$  aheap  $\Rightarrow$  bool" ("_  $\leq$ |" [51,51] 50)
"h $\leq$ |" h' ==  $\forall a C fs. h a = \text{Some}(C, fs) \rightarrow (\exists fs'. h' a = \text{Some}(C, fs'))$ "

conf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  val  $\Rightarrow$  ty  $\Rightarrow$  bool"
      ("_,_ |- _  $:: \leq$ |" [51,51,51,51] 50)
"G,h|-v $:: \leq T$  ==  $\exists T'. \text{typeof}(\text{option\_map } \text{obj\_ty } o h) v = \text{Some } T' \wedge G \vdash T' \preceq T$ "

lconf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  ('a  $\rightarrow$  val)  $\Rightarrow$  ('a  $\rightarrow$  ty)  $\Rightarrow$  bool"
      ("_,_ |- _  $:: \leq$ |" [51,51,51,51] 50)
"G,h|-vs $:: \leq Ts$  ==  $\forall n T. Ts n = \text{Some } T \rightarrow (\exists v. vs n = \text{Some } v \wedge G,h|-v $:: \leq T$ )$ "

oconf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  obj  $\Rightarrow$  bool" ("_,_ |- _ [ok]" [51,51,51] 50)
"G,h|-obj [ok] == G,h|-snd obj $:: \leq$ map_of (fields (G,fst obj))"

hconf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  bool" ("_ |-h _ [ok]" [51,51] 50)
"G|-h h [ok] ==  $\forall a \text{ obj}. h a = \text{Some } \text{obj} \rightarrow G,h|-obj [ok]$ "

xconf :: "aheap  $\Rightarrow$  val option  $\Rightarrow$  bool"
"xconf hp vo == preallocated hp  $\wedge$  ( $\forall v. (vo = \text{Some } v) \rightarrow (\exists xc. v = (\text{Addr } (\text{XcptRef } xc))))$ "

conforms :: "xstate  $\Rightarrow$  java_mb env'  $\Rightarrow$  bool" ("_  $:: \leq$ |" [51,51] 50)
"s $:: \leq E$  == prg E|-h heap (store s) [ok]  $\wedge$ 
      prg E,heap (store s)|-locals (store s) $:: \leq$ localT E  $\wedge$ 
      xconf (heap (store s)) (abrupt s)"

syntax (xsymbols)
hext      :: "aheap  $\Rightarrow$  aheap  $\Rightarrow$  bool"
          ("_  $\leq$ |" [51,51] 50)

conf      :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  val  $\Rightarrow$  ty  $\Rightarrow$  bool"
          ("_,_  $\vdash$  _  $:: \leq$ |" [51,51,51,51] 50)

lconf     :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  ('a  $\rightarrow$  val)  $\Rightarrow$  ('a  $\rightarrow$  ty)  $\Rightarrow$  bool"
          ("_,_  $\vdash$  _  $:: \leq$ |" [51,51,51,51] 50)

oconf     :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  obj  $\Rightarrow$  bool"
          ("_,_  $\vdash$  _  $\sqrt{\phantom{x}}$ " [51,51,51] 50)

hconf     :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  bool"
          ("_  $\vdash$  h _  $\sqrt{\phantom{x}}$ " [51,51] 50)

conforms  :: "state  $\Rightarrow$  java_mb env'  $\Rightarrow$  bool"
          ("_  $:: \leq$ |" [51,51] 50)

```

2.12.1 hext

lemma hextI:

" $\forall a \ C \ fs \ . \ h \ a = \text{Some } (C, fs) \ \rightarrow$
 $(\exists fs' . \ h' \ a = \text{Some } (C, fs')) \ \Rightarrow \ h \leq h'$ "

$\langle \text{proof} \rangle$

lemma hext_objD: " $[h \leq h'; \ h \ a = \text{Some } (C, fs) \] \ \Rightarrow \ \exists fs' . \ h' \ a = \text{Some } (C, fs')$ "

$\langle \text{proof} \rangle$

lemma hext_refl [simp]: " $h \leq h$ "

$\langle \text{proof} \rangle$

lemma hext_new [simp]: " $h \ a = \text{None} \ \Rightarrow \ h \leq h(a \mapsto x)$ "

$\langle \text{proof} \rangle$

lemma hext_trans: " $[h \leq h'; \ h' \leq h''] \ \Rightarrow \ h \leq h''$ "

$\langle \text{proof} \rangle$

lemma hext_upd_obj: " $h \ a = \text{Some } (C, fs) \ \Rightarrow \ h \leq h(a \mapsto (C, fs'))$ "

$\langle \text{proof} \rangle$

2.12.2 conf

lemma conf_Null [simp]: " $G, h \vdash \text{Null} :: \preceq T = G \vdash \text{RefT Null} T \preceq T$ "

$\langle \text{proof} \rangle$

lemma conf_litval [rule_format (no_asm), simp]:

" $\text{typeof } (\lambda v. \text{None}) \ v = \text{Some } T \ \rightarrow \ G, h \vdash v :: \preceq T$ "

$\langle \text{proof} \rangle$

lemma conf_AddrI: " $[h \ a = \text{Some } \text{obj}; \ G \vdash \text{obj_ty } \text{obj} \preceq T] \ \Rightarrow \ G, h \vdash \text{Addr } a :: \preceq T$ "

$\langle \text{proof} \rangle$

lemma conf_obj_AddrI: " $[h \ a = \text{Some } (C, fs); \ G \vdash C \preceq C \ D] \ \Rightarrow \ G, h \vdash \text{Addr } a :: \preceq \text{Class } D$ "

$\langle \text{proof} \rangle$

lemma defval_conf [rule_format (no_asm)]:

" $\text{is_type } G \ T \ \rightarrow \ G, h \vdash \text{default_val } T :: \preceq T$ "

$\langle \text{proof} \rangle$

lemma conf_upd_obj:

" $h \ a = \text{Some } (C, fs) \ \Rightarrow \ (G, h(a \mapsto (C, fs'))) \vdash x :: \preceq T = (G, h) \vdash x :: \preceq T$ "

$\langle \text{proof} \rangle$

lemma conf_widen [rule_format (no_asm)]:

" $\text{wf_prog } \text{wf_mb } G \ \Rightarrow \ G, h \vdash x :: \preceq T \ \rightarrow \ G \vdash T \preceq T' \ \rightarrow \ G, h \vdash x :: \preceq T'$ "

$\langle \text{proof} \rangle$

lemma conf_hext [rule_format (no_asm)]: " $h \leq h' \ \Rightarrow \ G, h \vdash v :: \preceq T \ \rightarrow \ G, h' \vdash v :: \preceq T$ "

$\langle \text{proof} \rangle$

lemma new_locD: " $[h \ a = \text{None}; \ G, h \vdash \text{Addr } t :: \preceq T] \ \Rightarrow \ t \neq a$ "

$\langle \text{proof} \rangle$

```

lemma conf_RefTD [rule_format (no_asm)]:
  "G,h⊢a'::⊆RefT T --> a' = Null |
  (∃ a obj T'. a' = Addr a ∧ h a = Some obj ∧ obj_ty obj = T' ∧ G⊢T'⊆RefT T)"
<proof>

```

```

lemma conf_NullTD: "G,h⊢a'::⊆RefT NullT ==> a' = Null"
<proof>

```

```

lemma non_npD: "[|a' ≠ Null; G,h⊢a'::⊆RefT t|] ==>
  ∃ a C fs. a' = Addr a ∧ h a = Some (C,fs) ∧ G⊢Class C⊆RefT t"
<proof>

```

```

lemma non_np_objD: "!!G. [|a' ≠ Null; G,h⊢a'::⊆ Class C|] ==>
  (∃ a C' fs. a' = Addr a ∧ h a = Some (C',fs) ∧ G⊢C'⊆C C)"
<proof>

```

```

lemma non_np_objD' [rule_format (no_asm)]:
  "a' ≠ Null ==> wf_prog wf_mb G ==> G,h⊢a'::⊆RefT t -->
  (∃ a C fs. a' = Addr a ∧ h a = Some (C,fs) ∧ G⊢Class C⊆RefT t)"
<proof>

```

```

lemma conf_list_gext_widen [rule_format (no_asm)]:
  "wf_prog wf_mb G ==> ∀Ts Ts'. list_all2 (conf G h) vs Ts -->
  list_all2 (λT T'. G⊢T⊆T') Ts Ts' --> list_all2 (conf G h) vs Ts'"
<proof>

```

2.12.3 lconf

```

lemma lconfD: "[| G,h⊢vs[:⊆]Ts; Ts n = Some T |] ==> G,h⊢(the (vs n))::⊆T"
<proof>

```

```

lemma lconf_hext [elim]: "[| G,h⊢l[:⊆]L; h≤|h' |] ==> G,h'⊢l[:⊆]L"
<proof>

```

```

lemma lconf_upd: "!!X. [| G,h⊢l[:⊆]lT;
  G,h⊢v::⊆T; lT va = Some T |] ==> G,h⊢l(va↦v)[:⊆]lT"
<proof>

```

```

lemma lconf_init_vars_lemma [rule_format (no_asm)]:
  "∀x. P x --> R (dv x) x ==> (∀x. map_of fs f = Some x --> P x) -->
  (∀T. map_of fs f = Some T -->
  (∃ v. map_of (map (λ(f,ft). (f, dv ft)) fs) f = Some v ∧ R v T))"
<proof>

```

```

lemma lconf_init_vars [intro!]:
  "∀n. ∀T. map_of fs n = Some T --> is_type G T ==> G,h⊢init_vars fs[:⊆]map_of fs"
<proof>

```

```

lemma lconf_ext: "[|G,s⊢l[:⊆]L; G,s⊢v::⊆T|] ==> G,s⊢l(vn↦v)[:⊆]L(vn↦T)"
<proof>

```

```

lemma lconf_ext_list [rule_format (no_asm)]:
  "G,h⊢l[:⊆]L ==> ∀vs Ts. distinct vns --> length Ts = length vns -->
  list_all2 (λv T. G,h⊢v::⊆T) vs Ts --> G,h⊢l(vns[↦]vs)[:⊆]L(vns[↦]Ts)"

```

<proof>

lemma *lconf_restr*: " $\llbracket 1T \text{ vn} = \text{None}; G, h \vdash 1 \text{ } [:: \preceq] 1T(\text{vn} \mapsto T) \rrbracket \implies G, h \vdash 1 \text{ } [:: \preceq] 1T$ "
<proof>

2.12.4 oconf

lemma *oconf_hext*: " $G, h \vdash \text{obj} \sqrt{} \implies h \leq |h' \implies G, h' \vdash \text{obj} \sqrt{}$ "
<proof>

lemma *oconf_obj*: " $G, h \vdash (C, fs) \sqrt{} =$
 $(\forall T f. \text{map_of}(\text{fields } (G, C)) f = \text{Some } T \rightarrow (\exists v. fs f = \text{Some } v \wedge G, h \vdash v :: \preceq T))$ "
<proof>

lemmas *oconf_objD* = *oconf_obj* [THEN *iffD1*, THEN *spec*, THEN *spec*, THEN *mp*]

2.12.5 hconf

lemma *hconfD*: " $[|G \vdash h \sqrt{}; h \text{ a} = \text{Some obj}|] \implies G, h \vdash \text{obj} \sqrt{}$ "
<proof>

lemma *hconfI*: " $\forall a \text{ obj}. h \text{ a} = \text{Some obj} \rightarrow G, h \vdash \text{obj} \sqrt{} \implies G \vdash h \sqrt{}$ "
<proof>

2.12.6 xconf

lemma *xconf_raise_if*: " $xconf h x \implies xconf h (\text{raise_if } b \text{ xcn } x)$ "
<proof>

2.12.7 conforms

lemma *conforms_heapD*: " $(x, (h, l)) :: \preceq (G, lT) \implies G \vdash h \sqrt{}$ "
<proof>

lemma *conforms_localD*: " $(x, (h, l)) :: \preceq (G, lT) \implies G, h \vdash l [:: \preceq] lT$ "
<proof>

lemma *conforms_xcptD*: " $(x, (h, l)) :: \preceq (G, lT) \implies xconf h x$ "
<proof>

lemma *conformsI*: " $[|G \vdash h \sqrt{}; G, h \vdash l [:: \preceq] lT; xconf h x|] \implies (x, (h, l)) :: \preceq (G, lT)$ "
<proof>

lemma *conforms_restr*: " $\llbracket 1T \text{ vn} = \text{None}; s :: \preceq (G, lT(\text{vn} \mapsto T)) \rrbracket \implies s :: \preceq (G, lT)$ "
<proof>

lemma *conforms_xcpt_change*: " $\llbracket (x, (h, l)) :: \preceq (G, lT); xconf h x \rightarrow xconf h x' \rrbracket \implies$
 $(x', (h, l)) :: \preceq (G, lT)$ "
<proof>

lemma *preallocated_hext*: " $\llbracket \text{preallocated } h; h \leq |h' \rrbracket \implies \text{preallocated } h'$ "
<proof>

lemma *xconf_hext*: " $\llbracket xconf h \text{ vo}; h \leq |h' \rrbracket \implies xconf h' \text{ vo}$ "

$\langle proof \rangle$

lemma *conforms_hext*: " $[| (x, (h, l)) :: \preceq (G, lT); h \leq |h'|; G \vdash h' \checkmark |]$
 $\impl (x, (h', l)) :: \preceq (G, lT)$ "

$\langle proof \rangle$

lemma *conforms_upd_obj*:

" $[| (x, (h, l)) :: \preceq (G, lT); G, h(a \mapsto obj) \vdash obj \checkmark; h \leq |h(a \mapsto obj)|]$
 $\impl (x, (h(a \mapsto obj), l)) :: \preceq (G, lT)$ "

$\langle proof \rangle$

lemma *conforms_upd_local*:

" $[| (x, (h, l)) :: \preceq (G, lT); G, h \vdash v :: \preceq T; lT \text{ va} = \text{Some } T |]$
 $\impl (x, (h, l(va \mapsto v))) :: \preceq (G, lT)$ "

$\langle proof \rangle$

end

2.13 Type Safety Proof

theory JTypeSafe imports Eval Conform begin

declare split_beta [simp]

lemma NewC_conforms:

```
"[| h a = None; (x, (h, l)) ::  $\preceq$  (G, lT); wf_prog wf_mb G; is_class G C |] ==>
  (x, (h(a  $\mapsto$  (C, (init_vars (fields (G, C))))), l)) ::  $\preceq$  (G, lT)"
<proof>
```

lemma Cast_conf:

```
"[| wf_prog wf_mb G; G, h  $\vdash$  v ::  $\preceq$  CC; G  $\vdash$  CC  $\preceq$ ? Class D; cast_ok G D h v |]
  ==> G, h  $\vdash$  v ::  $\preceq$  Class D"
<proof>
```

lemma FAcc_type_sound:

```
"[| wf_prog wf_mb G; field (G, C) fn = Some (fd, ft); (x, (h, l)) ::  $\preceq$  (G, lT);
  x' = None --> G, h  $\vdash$  a' ::  $\preceq$  Class C; np a' x' = None |] ==>
  G, h  $\vdash$  the (snd (the (h (the_Addr a')))) (fn, fd) ::  $\preceq$  ft"
<proof>
```

lemma FAss_type_sound:

```
"[| wf_prog wf_mb G; a = the_Addr a'; (c, fs) = the (h a);
  (G, lT)  $\vdash$  v :: T'; G  $\vdash$  T'  $\preceq$  ft;
  (G, lT)  $\vdash$  aa :: Class C;
  field (G, C) fn = Some (fd, ft); h''  $\leq$  |h';
  x' = None --> G, h'  $\vdash$  a' ::  $\preceq$  Class C; h'  $\leq$  |h;
  Norm (h, l) ::  $\preceq$  (G, lT); G, h  $\vdash$  x ::  $\preceq$  T'; np a' x' = None |] ==>
  h''  $\leq$  |h(a  $\mapsto$  (c, (fs((fn, fd)  $\mapsto$  x))))  $\wedge$ 
  Norm(h(a  $\mapsto$  (c, (fs((fn, fd)  $\mapsto$  x))), l) ::  $\preceq$  (G, lT)  $\wedge$ 
  G, h(a  $\mapsto$  (c, (fs((fn, fd)  $\mapsto$  x))))  $\vdash$  x ::  $\preceq$  T'"
<proof>
```

```
lemma Call_lemma2: "[| wf_prog wf_mb G; list_all2 (conf G h) pvs pTs;
  list_all2 ( $\lambda$ T T'. G  $\vdash$  T  $\preceq$  T') pTs pTs'; wf_mhead G (mn, pTs') rT;
  length pTs' = length pns; distinct pns;
  Ball (set lvars) (split ( $\lambda$ vn. is_type G))
  |] ==> G, h  $\vdash$  init_vars lvars(pns[ $\mapsto$ ]pvs) [::  $\preceq$ ] map_of lvars(pns[ $\mapsto$ ]pTs'"
<proof>
```

lemma Call_type_sound:

```
"[| wf_java_prog G; a'  $\neq$  Null; Norm (h, l) ::  $\preceq$  (G, lT); class G C = Some y;
  max_spec G C (mn, pTs_a) = {(mda, rTa), pTs'}; xc  $\leq$  |xh; xh  $\leq$  |h;
  list_all2 (conf G h) pvs pTs_a;
  (md, rT, pns, lvars, blk, res) =
    the (method (G, fst (the (h (the_Addr a')))) (mn, pTs'));
   $\forall$  lT. (np a' None, h, init_vars lvars(pns[ $\mapsto$ ]pvs)(This  $\mapsto$  a')) ::  $\preceq$  (G, lT) -->
  (G, lT)  $\vdash$  blk  $\surd$  --> h  $\leq$  |xi  $\wedge$  (xcptb, xi, xl) ::  $\preceq$  (G, lT);
   $\forall$  lT. (xcptb, xi, xl) ::  $\preceq$  (G, lT) --> ( $\forall$  lT. (G, lT)  $\vdash$  res :: T -->
```

```

      xi ≤ |h' ∧ (x', h', xj) :: ≤(G, lT) ∧ (x' = None --> G, h' ⊢ v :: ≤T));
    G, xh ⊢ a' :: ≤ Class C
  |] ==>
    xc ≤ |h' ∧ (x', (h', l)) :: ≤(G, lT) ∧ (x' = None --> G, h' ⊢ v :: ≤rTa)"
  <proof>

```

```

declare split_if [split del]
declare fun_upd_apply [simp del]
declare fun_upd_same [simp]
declare wf_prog_ws_prog [simp]

```

<ML>

```
declare [[unify_search_bound = 40, unify_trace_bound = 40]]
```

theorem eval_evals_exec_type_sound:

```

"wf_java_prog G ==>
  (G ⊢ (x, (h, l)) -e >v -> (x', (h', l'))) -->
    (∀ lT. (x, (h, l)) :: ≤(G, lT) --> (∀ T. (G, lT) ⊢ e :: T -->
      h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT) ∧ (x' = None --> G, h' ⊢ v :: ≤ T)))) ∧
  (G ⊢ (x, (h, l)) -es[>]vs-> (x', (h', l'))) -->
    (∀ lT. (x, (h, l)) :: ≤(G, lT) --> (∀ Ts. (G, lT) ⊢ es[::]Ts -->
      h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT) ∧ (x' = None --> list_all2 (λv T. G, h' ⊢ v :: ≤T) vs
      Ts)))) ∧
  (G ⊢ (x, (h, l)) -c -> (x', (h', l'))) -->
    (∀ lT. (x, (h, l)) :: ≤(G, lT) --> (G, lT) ⊢ c √ -->
      h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT)))"
  <proof>

```

```
declare [[unify_search_bound = 20, unify_trace_bound = 20]]
```

lemma eval_type_sound: "!!E s s'.

```

  [| wf_java_prog G; G ⊢ (x, s) -e>v -> (x', s'); (x, s) :: ≤E; E ⊢ e :: T; G = prg E |]
  ==> (x', s') :: ≤E ∧ (x' = None --> G, heap s' ⊢ v :: ≤T) ∧ heap s ≤ | heap s'"
  <proof>

```

lemma evals_type_sound: "!!E s s'.

```

  [| wf_java_prog G; G ⊢ (x, s) -es[>]vs-> (x', s'); (x, s) :: ≤E; E ⊢ es[::]Ts; G = prg E |]

  ==> (x', s') :: ≤E ∧ (x' = None --> (list_all2 (λv T. G, heap s' ⊢ v :: ≤T) vs Ts)) ∧ heap
  s ≤ | heap s'"
  <proof>

```

lemma exec_type_sound: "!!E s s'.

```

  [| wf_java_prog G; G ⊢ (x, s) -s0-> (x', s'); (x, s) :: ≤E; E ⊢ s0 √; G = prg E |]
  ==> (x', s') :: ≤E ∧ heap s ≤ | heap s'"
  <proof>

```

theorem all_methods_understood:

```

"/[G=prg E; wf_java_prog G; G⊢(x,s) -e>a'-> Norm s'; a' ≠ Null;
  (x,s)::⊑E; E⊢e::Class C; method (G,C) sig ≠ None/] ==>
  method (G,fst (the (heap s' (the_Addr a')))) sig ≠ None"
⟨proof⟩

declare split_beta [simp del]
declare fun_upd_apply [simp]
declare wf_prog_ws_prog [simp del]

end

```

2.14 Example MicroJava Program

theory Example imports SystemClasses Eval begin

The following example MicroJava program includes: class declarations with inheritance, hiding of fields, and overriding of methods (with refined result type), instance creation, local assignment, sequential composition, method call with dynamic binding, literal values, expression statement, local access, type cast, field assignment (in part), skip.

```

class Base {
  boolean vee;
  Base foo(Base x) {return x;}
}

class Ext extends Base {
  int vee;
  Ext foo(Base x) {((Ext)x).vee=1; return null;}
}

class Example {
  public static void main (String args[]) {
    Base e=new Ext();
    e.foo(null);
  }
}

datatype cnam' = Base' | Ext'
datatype vnam' = vee' | x' | e'

consts
  cnam' :: "cnam' => cname"
  vnam' :: "vnam' => vname"

— cnam' and vnam' are intended to be isomorphic to cnam and vnam
axioms
  inj_cnam': "(cnam' x = cnam' y) = (x = y)"
  inj_vnam': "(vnam' x = vnam' y) = (x = y)"

  surj_cnam': "∃ m. n = cnam' m"
  surj_vnam': "∃ m. n = vnam' m"

declare inj_cnam' [simp] inj_vnam' [simp]

syntax
  Base :: cname
  Ext  :: cname
  vee  :: vname
  x    :: vname
  e    :: vname

translations
  "Base" == "cnam' Base'"

```

```

"Ext" == "cnam' Ext'"
"vee" == "VName (vnam' vee')"
"x" == "VName (vnam' x')"
"e" == "VName (vnam' e')"

```

axioms

```

Base_not_Object: "Base  $\neq$  Object"
Ext_not_Object: "Ext  $\neq$  Object"
Base_not_Xcpt: "Base  $\neq$  Xcpt z"
Ext_not_Xcpt: "Ext  $\neq$  Xcpt z"
e_not_This: "e  $\neq$  This"

```

```

declare Base_not_Object [simp] Ext_not_Object [simp]
declare Base_not_Xcpt [simp] Ext_not_Xcpt [simp]
declare e_not_This [simp]
declare Base_not_Object [symmetric, simp]
declare Ext_not_Object [symmetric, simp]
declare Base_not_Xcpt [symmetric, simp]
declare Ext_not_Xcpt [symmetric, simp]

```

consts

```

foo_Base:: java_mb
foo_Ext :: java_mb
BaseC :: "java_mb cdecl"
ExtC :: "java_mb cdecl"
test :: stmt
foo :: mname
a :: loc
b :: loc

```

defs

```

foo_Base_def:"foo_Base == ([x],[],Skip,LAcc x)"
BaseC_def:"BaseC == (Base, (Object,
    [(vee, PrimT Boolean)],
    [((foo,[Class Base]),Class Base,foo_Base)]))"
foo_Ext_def:"foo_Ext == ([x],[],Expr( {Ext}Cast Ext
    (LAcc x)..vee:=Lit (Intg Numeral1)),
    Lit Null)"
ExtC_def: "ExtC == (Ext, (Base ,
    [(vee, PrimT Integer)],
    [((foo,[Class Base]),Class Ext,foo_Ext)]))"

test_def:"test == Expr(e::=NewC Ext);;
    Expr({Base}LAcc e..foo({[Class Base]}[Lit Null]))"

```

abbreviation

```

NP :: xcpt where
  "NP == NullPointer"

```

abbreviation

```

tprg :: "java_mb prog" where
  "tprg == [ObjectC, BaseC, ExtC, ClassCastC, NullPointerC, OutOfMemoryC]"

```

abbreviation

```
obj1 :: obj where
"obj1 == (Ext, empty((vee, Base)↦Bool False) ((vee, Ext )↦Intg 0))"
```

```
abbreviation "s0 == Norm    (empty, empty)"
```

```
abbreviation "s1 == Norm    (empty(a↦obj1),empty(e↦Addr a))"
```

```
abbreviation "s2 == Norm    (empty(a↦obj1),empty(x↦Null)(This↦Addr a))"
```

```
abbreviation "s3 == (Some NP, empty(a↦obj1),empty(e↦Addr a))"
```

```
lemmas map_of_Cons = map_of.simps(2)
```

```
lemma map_of_Cons1 [simp]: "map_of ((aa,bb)#ps) aa = Some bb"
```

```
<proof>
```

```
lemma map_of_Cons2 [simp]: "aa≠k ==> map_of ((k,bb)#ps) aa = map_of ps aa"
```

```
<proof>
```

```
declare map_of_Cons [simp del] — sic!
```

```
lemma class_tprg_Object [simp]: "class tprg Object = Some (arbitrary, [], [])"
```

```
<proof>
```

```
lemma class_tprg_NP [simp]: "class tprg (Xcpt NP) = Some (Object, [], [])"
```

```
<proof>
```

```
lemma class_tprg_OM [simp]: "class tprg (Xcpt OutOfMemory) = Some (Object, [], [])"
```

```
<proof>
```

```
lemma class_tprg_CC [simp]: "class tprg (Xcpt ClassCast) = Some (Object, [], [])"
```

```
<proof>
```

```
lemma class_tprg_Base [simp]:
```

```
"class tprg Base = Some (Object,
  [(vee, PrimT Boolean)],
  [(foo, [Class Base]), Class Base, foo_Base])"
```

```
<proof>
```

```
lemma class_tprg_Ext [simp]:
```

```
"class tprg Ext = Some (Base,
  [(vee, PrimT Integer)],
  [(foo, [Class Base]), Class Ext, foo_Ext])"
```

```
<proof>
```

```
lemma not_Object_subcls [elim!]: "(subcls1 tprg)^++ Object C ==> R"
```

```
<proof>
```

```
lemma subcls_ObjectD [dest!]: "tprg⊢Object≤C C ==> C = Object"
```

```
<proof>
```

```
lemma not_Base_subcls_Ext [elim!]: "(subcls1 tprg)^++ Base Ext ==> R"
```

```
<proof>
```

```
lemma class_tprgD:
```

```
"class tprg C = Some z ==> C=Object ∨ C=Base ∨ C=Ext ∨ C=Xcpt NP ∨ C=Xcpt ClassCast
∨ C=Xcpt OutOfMemory"
```

```
<proof>
```

```

lemma not_class_subcls_class [elim!]: "(subcls1 tprg)^++ C C ==> R"
<proof>

lemma unique_classes: "unique tprg"
<proof>

lemmas subcls_direct = subcls1I [THEN r_into_rtranclp [where r="subcls1 G"], standard]

lemma Ext_subcls_Base [simp]: "tprg ⊢ Ext ≤C Base"
<proof>

lemma Ext_widen_Base [simp]: "tprg ⊢ Class Ext ≤ Class Base"
<proof>

declare ty_expr_ty_exprs_wt_stmt.intros [intro!]

lemma acyclic_subcls1': "acyclicP (subcls1 tprg)"
<proof>

lemmas wf_subcls1' = acyclic_subcls1' [THEN finite_subcls1 [THEN finite_acyclic_wf_converse
[to_pred]]]

lemmas fields_rec' = wf_subcls1' [THEN [2] fields_rec_lemma]

lemma fields_Object [simp]: "fields (tprg, Object) = []"
<proof>

declare is_class_def [simp]

lemma fields_Base [simp]: "fields (tprg, Base) = [((vee, Base), PrimT Boolean)]"
<proof>

lemma fields_Ext [simp]:
  "fields (tprg, Ext) = [((vee, Ext), PrimT Integer)] @ fields (tprg, Base)"
<proof>

lemmas method_rec' = wf_subcls1' [THEN [2] method_rec_lemma]

lemma method_Object [simp]: "method (tprg, Object) = map_of []"
<proof>

lemma method_Base [simp]: "method (tprg, Base) = map_of
  [((foo, [Class Base]), Base, (Class Base, foo_Base))]"
<proof>

lemma method_Ext [simp]: "method (tprg, Ext) = (method (tprg, Base) ++ map_of
  [((foo, [Class Base]), Ext, (Class Ext, foo_Ext))])"
<proof>

lemma wf_foo_Base:
  "wf_mdecl wf_java_mdecl tprg Base ((foo, [Class Base]), (Class Base, foo_Base))"
<proof>

```



```
lemma wf_foo_Ext:
  "wf_mdecl wf_java_mdecl tprg Ext ((foo, [Class Base]), (Class Ext, foo_Ext))"
  <proof>
```

```
lemma wf_ObjectC:
  "ws_cdecl tprg ObjectC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg ObjectC ∧ wf_mrT tprg ObjectC"
  <proof>
```

```
lemma wf_NP:
  "ws_cdecl tprg NullPointerC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg NullPointerC ∧ wf_mrT tprg NullPointerC"
  <proof>
```

```
lemma wf_OM:
  "ws_cdecl tprg OutOfMemoryC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg OutOfMemoryC ∧ wf_mrT tprg OutOfMemoryC"
  <proof>
```

```
lemma wf_CC:
  "ws_cdecl tprg ClassCastC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg ClassCastC ∧ wf_mrT tprg ClassCastC"
  <proof>
```

```
lemma wf_BaseC:
  "ws_cdecl tprg BaseC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg BaseC ∧ wf_mrT tprg BaseC"
  <proof>
```

```
lemma wf_ExtC:
  "ws_cdecl tprg ExtC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg ExtC ∧ wf_mrT tprg ExtC"
  <proof>
```

```
lemma [simp]: "fst ObjectC = Object" <proof>
```

```
lemma wf_tprg:
  "wf_prog wf_java_mdecl tprg"
  <proof>
```

```
lemma appl_methds_foo_Base:
  "appl_methds tprg Base (foo, [NT]) =
   {((Class Base, Class Base), [Class Base])}"
  <proof>
```

```
lemma max_spec_foo_Base: "max_spec tprg Base (foo, [NT]) =
  {((Class Base, Class Base), [Class Base])}"
  <proof>
```

<ML>

```
lemma wt_test: "(tprg, empty(e ↦ Class Base)) ⊢
  Expr(e ::= NewC Ext);; Expr({Base}LAcc e..foo({?pTs'}[Lit Null])) √"
```

$\langle proof \rangle$

$\langle ML \rangle$

```

declare split_if [split del]
declare init_vars_def [simp] c_hupd_def [simp] cast_ok_def [simp]
lemma exec_test:
  " [/new_Addr (heap (snd s0)) = (a, None)] ==>
    tprg ⊢ s0 -test-> ?s"
 $\langle proof \rangle$ 

end

```

2.15 Example for generating executable code from Java semantics

theory JListExample imports Eval SystemClasses begin

$\langle ML \rangle$

consts

```
list_name :: cname
append_name :: mname
val_nam :: vnam
next_nam :: vnam
l_nam :: vnam
l1_nam :: vnam
l2_nam :: vnam
l3_nam :: vnam
l4_nam :: vnam
```

constdefs

```
val_name :: vname
"val_name == VName val_nam"

next_name :: vname
"next_name == VName next_nam"

l_name :: vname
"l_name == VName l_nam"

l1_name :: vname
"l1_name == VName l1_nam"

l2_name :: vname
"l2_name == VName l2_nam"

l3_name :: vname
"l3_name == VName l3_nam"

l4_name :: vname
"l4_name == VName l4_nam"

list_class :: "java_mb class"
"list_class ==
  (Object,
   [(val_name, PrimT Integer), (next_name, RefT (ClassT list_name))],
   [(append_name, [RefT (ClassT list_name)]), PrimT Void,
    ([l_name], [],
     If(BinOp Eq ({list_name}(LAcc This)..next_name) (Lit Null))
       Expr ({list_name}(LAcc This)..next_name:=LAcc l_name)
     Else
       Expr ({list_name}({list_name}(LAcc This)..next_name)..
        append_name({[RefT (ClassT list_name)]}[LAcc l_name])),
    Lit Unit))])"

example_prg :: "java_mb prog"
```

```

"example_prg == [ObjectC, (list_name, list_class)]"

types_code
  cname ("string")
  vnam ("string")
  mname ("string")
  loc' ("int")

consts_code
  "new_Addr" ("⟨module⟩new'_addr {x. case x of None => True | Some y => False *} / {x
None *} {x Loc *}")
  attach {x
fun new_addr p none loc hp =
  let fun nr i = if p (hp (loc i)) then (loc i, none) else nr (i+1);
  in nr 0 end;
x}

  "arbitrary" ("(raise Match)")
  "arbitrary :: val" ("{* Unit *}")
  "arbitrary :: cname" ("")

  "Object" ("Object")
  "list_name" ("list")
  "append_name" ("append")
  "val_nam" ("val")
  "next_nam" ("next")
  "l_nam" ("l")
  "l1_nam" ("l1")
  "l2_nam" ("l2")
  "l3_nam" ("l3")
  "l4_nam" ("l4")

code_module J
contains
  test = "example_prg ⊢ Norm (empty, empty)
    -(Expr (l1_name := NewC list_name));;
    Expr ({list_name}(LAcc l1_name)..val_name := Lit (Intg 1));;
    Expr (l2_name := NewC list_name);;
    Expr ({list_name}(LAcc l2_name)..val_name := Lit (Intg 2));;
    Expr (l3_name := NewC list_name);;
    Expr ({list_name}(LAcc l3_name)..val_name := Lit (Intg 3));;
    Expr (l4_name := NewC list_name);;
    Expr ({list_name}(LAcc l4_name)..val_name := Lit (Intg 4));;
    Expr ({list_name}(LAcc l1_name)..
      append_name({[RefT (ClassT list_name)] [LAcc l2_name]}));;
    Expr ({list_name}(LAcc l1_name)..
      append_name({[RefT (ClassT list_name)] [LAcc l3_name]}));;
    Expr ({list_name}(LAcc l1_name)..
      append_name({[RefT (ClassT list_name)] [LAcc l4_name]})) -> _"

```

2.15.1 Big step execution

⟨ML⟩

end

Chapter 3

Java Virtual Machine

3.1 State of the JVM

```
theory JVMState
imports "../J/Conform"
begin
```

3.1.1 Frame Stack

```
types
  opstack   = "val list"
  locvars   = "val list"
  p_count   = nat

  frame = "opstack ×
           locvars ×
           cname ×
           sig ×
           p_count"

  — operand stack
  — local variables (including this pointer and method parameters)
  — name of class where current method is defined
  — method name + parameter types
  — program counter within frame
```

3.1.2 Exceptions

```
constdefs
  raise_system_xcpt :: "bool ⇒ xcpt ⇒ val option"
  "raise_system_xcpt b x ≡ raise_if b x None"
```

3.1.3 Runtime State

```
types
  jvm_state = "val option × aheap × frame list" — exception flag, heap, frames
```

3.1.4 Lemmas

```
lemma new_Addr_OutOfMemory:
  "snd (new_Addr hp) = Some xcp ⇒ xcp = Addr (XcptRef OutOfMemory)"
  ⟨proof⟩

end
```


3.2 Instructions of the JVM

theory *JVMInstructions* imports *JVMState* begin

datatype

```

instr = Load nat           — load from local variable
      | Store nat          — store into local variable
      | LitPush val        — push a literal (constant)
      | New cname          — create object
      | Getfield vname cname — Fetch field from object
      | Putfield vname cname — Set field in object
      | Checkcast cname    — Check whether object is of given type
      | Invoke cname mname "(ty list)" — inv. instance meth of an object
      | Return             — return from method
      | Pop                — pop top element from opstack
      | Dup                — duplicate top element of opstack
      | Dup_x1             — duplicate top element and push 2 down
      | Dup_x2             — duplicate top element and push 3 down
      | Swap              — swap top and next to top element
      | IAdd               — integer addition
      | Goto int           — goto relative address
      | Ifcmpeq int        — branch if int/ref comparison succeeds
      | Throw              — throw top of stack as exception

```

types

```

bytecode = "instr list"
exception_entry = "p_count × p_count × p_count × cname"
                — start-pc, end-pc, handler-pc, exception type
exception_table = "exception_entry list"
jvm_method = "nat × nat × bytecode × exception_table"
            — max stacksize, size of register set, instruction sequence, handler table
jvm_prog = "jvm_method prog"

```

end

3.3 JVM Instruction Semantics

theory *JVMExecInstr* imports *JVMInstructions* *JVMState* begin

consts

```
exec_instr :: "[instr, jvm_prog, aheap, opstack, locvars,
               cname, sig, p_count, frame list] => jvm_state"
```

primrec

```
"exec_instr (Load idx) G hp stk vars Cl sig pc frs =
  (None, hp, ((vars ! idx) # stk, vars, Cl, sig, pc+1)#frs)"
```

```
"exec_instr (Store idx) G hp stk vars Cl sig pc frs =
  (None, hp, (tl stk, vars[idx:=hd stk], Cl, sig, pc+1)#frs)"
```

```
"exec_instr (LitPush v) G hp stk vars Cl sig pc frs =
  (None, hp, (v # stk, vars, Cl, sig, pc+1)#frs)"
```

```
"exec_instr (New C) G hp stk vars Cl sig pc frs =
  (let (oref,xp') = new_Addr hp;
   fs = init_vars (fields(G,C));
   hp' = if xp'=None then hp(oref ↦ (C,fs)) else hp;
   pc' = if xp'=None then pc+1 else pc
   in
   (xp', hp', (Addr oref#stk, vars, Cl, sig, pc')#frs))"
```

```
"exec_instr (Getfield F C) G hp stk vars Cl sig pc frs =
  (let oref = hd stk;
   xp' = raise_system_xcpt (oref=None) NullPointer;
   (oc,fs) = the(hp(the_Addr oref));
   pc' = if xp'=None then pc+1 else pc
   in
   (xp', hp, (the(fs(F,C))#(tl stk), vars, Cl, sig, pc')#frs))"
```

```
"exec_instr (Putfield F C) G hp stk vars Cl sig pc frs =
  (let (fval,oref)= (hd stk, hd(tl stk));
   xp' = raise_system_xcpt (oref=None) NullPointer;
   a = the_Addr oref;
   (oc,fs) = the(hp a);
   hp' = if xp'=None then hp(a ↦ (oc, fs((F,C) ↦ fval))) else hp;
   pc' = if xp'=None then pc+1 else pc
   in
   (xp', hp', (tl (tl stk), vars, Cl, sig, pc')#frs))"
```

```
"exec_instr (Checkcast C) G hp stk vars Cl sig pc frs =
  (let oref = hd stk;
   xp' = raise_system_xcpt (¬ cast_ok G C hp oref) ClassCast;
   stk' = if xp'=None then stk else tl stk;
   pc' = if xp'=None then pc+1 else pc
   in
   (xp', hp, (stk', vars, Cl, sig, pc')#frs))"
```

```
"exec_instr (Invoke C mn ps) G hp stk vars Cl sig pc frs =
  (let n = length ps;
```

```

    argsoref = take (n+1) stk;
    oref = last argsoref;
    xp' = raise_system_xcpt (oref=None) NullPointer;
    dynT = fst(the(hp(the_Addr oref)));
    (dc,mh,mxs,mxl,c)= the (method (G,dynT) (mn,ps));
    frs' = if xp'=None then
        [([],rev argsoref@replicate mxl arbitrary,dc,(mn,ps),0)]
        else []
in
    (xp', hp, frs'@(stk, vars, Cl, sig, pc)#frs))"
— Because exception handling needs the pc of the Invoke instruction,
— Invoke doesn't change stk and pc yet (Return does that).

"exec_instr Return G hp stk0 vars Cl sig0 pc frs =
  (if frs=[] then
    (None, hp, [])
  else
    let val = hd stk0; (stk,loc,C,sig,pc) = hd frs;
        (mn,pt) = sig0; n = length pt
    in
      (None, hp, (val#(drop (n+1) stk),loc,C,sig,pc+1)#tl frs))"
— Return drops arguments from the caller's stack and increases
— the program counter in the caller

"exec_instr Pop G hp stk vars Cl sig pc frs =
  (None, hp, (tl stk, vars, Cl, sig, pc+1)#frs)"

"exec_instr Dup G hp stk vars Cl sig pc frs =
  (None, hp, (hd stk # stk, vars, Cl, sig, pc+1)#frs)"

"exec_instr Dup_x1 G hp stk vars Cl sig pc frs =
  (None, hp, (hd stk # hd (tl stk) # hd stk # (tl (tl stk)),
    vars, Cl, sig, pc+1)#frs)"

"exec_instr Dup_x2 G hp stk vars Cl sig pc frs =
  (None, hp,
    (hd stk # hd (tl stk) # (hd (tl (tl stk))) # hd stk # (tl (tl (tl stk)))),
    vars, Cl, sig, pc+1)#frs)"

"exec_instr Swap G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk,hd (tl stk))
  in
    (None, hp, (val2#val1#(tl (tl stk)), vars, Cl, sig, pc+1)#frs))"

"exec_instr IAdd G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk,hd (tl stk))
  in
    (None, hp, (Intg ((the_Intg val1)+(the_Intg val2))#(tl (tl stk)),
    vars, Cl, sig, pc+1)#frs))"

"exec_instr Ifcmpeq i G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk, hd (tl stk));
    pc' = if val1 = val2 then nat(int pc+i) else pc+1
  in

```

```

      (None, hp, (tl (tl stk), vars, Cl, sig, pc')#frs))"

"exec_instr (Goto i) G hp stk vars Cl sig pc frs =
  (None, hp, (stk, vars, Cl, sig, nat(int pc+i))#frs)"

"exec_instr Throw G hp stk vars Cl sig pc frs =
  (let xcpt = raise_system_xcpt (hd stk = Null) NullPointer;
    xcpt' = if xcpt = None then Some (hd stk) else xcpt
   in
    (xcpt', hp, (stk, vars, Cl, sig, pc)#frs))"

end

```

3.4 Exception handling in the JVM

theory JVMExceptions imports JVMInstructions begin

constdefs

```
match_exception_entry :: "jvm_prog  $\Rightarrow$  cname  $\Rightarrow$  p_count  $\Rightarrow$  exception_entry  $\Rightarrow$  bool"
"match_exception_entry G cn pc ee ==
  let (start_pc, end_pc, handler_pc, catch_type) = ee in
  start_pc <= pc  $\wedge$  pc < end_pc  $\wedge$  G  $\vdash$  cn  $\preceq_C$  catch_type"
```

consts

```
match_exception_table :: "jvm_prog  $\Rightarrow$  cname  $\Rightarrow$  p_count  $\Rightarrow$  exception_table
 $\Rightarrow$  p_count option"
```

primrec

```
"match_exception_table G cn pc [] = None"
"match_exception_table G cn pc (e#es) = (if match_exception_entry G cn pc e
  then Some (fst (snd (snd e)))
  else match_exception_table G cn pc es)"
```

consts

```
ex_table_of :: "jvm_method  $\Rightarrow$  exception_table"
```

translations

```
"ex_table_of m" == "snd (snd (snd m))"
```

consts

```
find_handler :: "jvm_prog  $\Rightarrow$  val option  $\Rightarrow$  aheap  $\Rightarrow$  frame list  $\Rightarrow$  jvm_state"
```

primrec

```
"find_handler G xcpt hp [] = (xcpt, hp, [])"
"find_handler G xcpt hp (fr#frs) =
  (case xcpt of
    None  $\Rightarrow$  (None, hp, fr#frs)
  | Some xc  $\Rightarrow$ 
    let (stk, loc, C, sig, pc) = fr in
    (case match_exception_table G (cname_of hp xc) pc
      (ex_table_of (snd(snd(the(method (G,C) sig))))) of
      None  $\Rightarrow$  find_handler G (Some xc) hp frs
    | Some handler_pc  $\Rightarrow$  (None, hp, ([xc], loc, C, sig, handler_pc)#frs)))"
```

System exceptions are allocated in all heaps:

Only program counters that are mentioned in the exception table can be returned by *match_exception_table*:

lemma match_exception_table_in_et:

```
"match_exception_table G C pc et = Some pc'  $\implies \exists e \in \text{set et. pc'} = \text{fst (snd (snd e))}"
\langle proof \rangle$ 
```

end

3.5 Program Execution in the JVM

theory *JVMExec* imports *JVMExecInstr* *JVMExceptions* begin

consts

exec :: "*jvm_prog* \times *jvm_state* \Rightarrow *jvm_state* option"

— *exec* is not recursive. *recdef* is just used for pattern matching

recdef *exec* "{}"

"*exec* (*G*, *xp*, *hp*, []) = None"

"*exec* (*G*, None, *hp*, (*stk*,*loc*,*C*,*sig*,*pc*)#*frs*) =
(let
 i = *fst*(*snd*(*snd*(*snd*(*snd*(*the*(*method* (*G*,*C*) *sig*)))))) ! *pc*;
 (*xcpt*', *hp*', *frs*') = *exec_instr* *i* *G* *hp* *stk* *loc* *C* *sig* *pc* *frs*
 in Some (*find_handler* *G* *xcpt*' *hp*' *frs*'))"

"*exec* (*G*, Some *xp*, *hp*, *frs*) = None"

constdefs

exec_all :: "[*jvm_prog*,*jvm_state*,*jvm_state*] \Rightarrow bool"
 (" _ |- _ -jvm-> _" [61,61,61]60)
"*G* |- *s* -jvm-> *t* == (*s*,*t*) \in {(*s*,*t*). *exec*(*G*,*s*) = Some *t*}^*"

syntax (*xsymbols*)

exec_all :: "[*jvm_prog*,*jvm_state*,*jvm_state*] \Rightarrow bool"
 (" _ \vdash _ -jvm \rightarrow _" [61,61,61]60)

The start configuration of the JVM: in the start heap, we call a method *m* of class *C* in program *G*. The *this* pointer of the frame is set to *Null* to simulate a static method invocation.

constdefs

start_state :: "*jvm_prog* \Rightarrow *cname* \Rightarrow *mname* \Rightarrow *jvm_state*"
"*start_state* *G* *C* *m* \equiv
let (*C*',*rT*,*mxs*,*mxl*,*i*,*et*) = *the* (*method* (*G*,*C*) (*m*,[])) in
 (None, *start_heap* *G*, [([]), Null # replicate *mxl* arbitrary, *C*, (*m*,[]), 0]))"

end

3.6 Example for generating executable code from JVM semantics

```
theory JVMListExample imports "../J/SystemClasses" JVMExec begin
```

```
consts
```

```
list_nam :: cnam
test_nam :: cnam
append_name :: mname
makelist_name :: mname
val_nam :: vnam
next_nam :: vnam
```

```
constdefs
```

```
list_name :: cname
"list_name == Cname list_nam"

test_name :: cname
"test_name == Cname test_nam"

val_name :: vname
"val_name == VName val_nam"

next_name :: vname
"next_name == VName next_nam"
```

```
append_ins :: bytecode
```

```
"append_ins ==
  [Load 0,
   Getfield next_name list_name,
   Dup,
   LitPush Null,
   Ifcmpeq 4,
   Load 1,
   Invoke list_name append_name [Class list_name],
   Return,
   Pop,
   Load 0,
   Load 1,
   Putfield next_name list_name,
   LitPush Unit,
   Return]"
```

```
list_class :: "jvm_method class"
```

```
"list_class ==
  (Object,
   [(val_name, PrimT Integer), (next_name, Class list_name)],
   [(append_name, [Class list_name]), PrimT Void,
    (3, 0, append_ins, [(1,2,8,Xcpt NullPointer))]))"
```

```
make_list_ins :: bytecode
```

```
"make_list_ins ==
  [New list_name,
   Dup,
```

```

    Store 0,
    LitPush (Intg 1),
    Putfield val_name list_name,
    New list_name,
    Dup,
    Store 1,
    LitPush (Intg 2),
    Putfield val_name list_name,
    New list_name,
    Dup,
    Store 2,
    LitPush (Intg 3),
    Putfield val_name list_name,
    Load 0,
    Load 1,
    Invoke list_name append_name [Class list_name],
    Pop,
    Load 0,
    Load 2,
    Invoke list_name append_name [Class list_name],
    Return]"

test_class :: "jvm_method class"
"test_class ==
  (Object, [],
   [((makelist_name, []), PrimT Void, (3, 2, make_list_ins, []))])"

E :: jvm_prog
"E == SystemClasses @ [(list_name, list_class), (test_name, test_class)]"

types_code
  cnam ("string")
  vnam ("string")
  mname ("string")
  loc' ("int")

consts_code
  "new_Addr" ("⟨module⟩new'_addr {x. case x of None => True | Some y => False *}/ {x
None *}/ {x Loc *}")
  attach {x
fun new_addr p none loc hp =
  let fun nr i = if p (hp (loc i)) then (loc i, none) else nr (i+1);
  in nr 0 end;
x}

  "arbitrary" ("(raise Match)")
  "arbitrary :: val" ("{* Unit *}")
  "arbitrary :: cname" ("{* Object *}")

  "list_nam" ("list")
  "test_nam" ("test")
  "append_name" ("append")
  "makelist_name" ("makelist")

```



```
"val_nam" ("val")  
"next_nam" ("next")
```

definition

```
"test = exec (E, start_state E test_name makelist_name)"
```

3.6.1 Single step execution

code_module *JVM*

contains

```
exec = exec  
test = test
```

$\langle ML \rangle$

end

3.7 A Defensive JVM

theory *JVMDefensive* imports *JVMExec* begin

Extend the state space by one element indicating a type error (or other abnormal termination)

datatype 'a type_error = *TypeError* | *Normal* 'a

syntax "fifth" :: "'a × 'b × 'c × 'd × 'e × 'f ⇒ 'e"

translations

"fifth x" == "fst(snd(snd(snd(snd x))))"

consts *isAddr* :: "val ⇒ bool"

recdef *isAddr* "{}"

"isAddr (Addr loc) = True"

"isAddr v = False"

consts *isIntg* :: "val ⇒ bool"

recdef *isIntg* "{}"

"isIntg (Intg i) = True"

"isIntg v = False"

constdefs

isRef :: "val ⇒ bool"

"isRef v ≡ v = Null ∨ isAddr v"

consts

check_instr :: "[instr, jvm_prog, aheap, opstack, locvars,
cname, sig, p_count, nat, frame list] ⇒ bool"

primrec

"check_instr (Load idx) G hp stk vars C sig pc mxs frs =
(idx < length vars ∧ size stk < mxs)"

"check_instr (Store idx) G hp stk vars Cl sig pc mxs frs =
(0 < length stk ∧ idx < length vars)"

"check_instr (LitPush v) G hp stk vars Cl sig pc mxs frs =
(¬isAddr v ∧ size stk < mxs)"

"check_instr (New C) G hp stk vars Cl sig pc mxs frs =
(is_class G C ∧ size stk < mxs)"

"check_instr (Getfield F C) G hp stk vars Cl sig pc mxs frs =
(0 < length stk ∧ is_class G C ∧ field (G,C) F ≠ None ∧
let (C', T) = the (field (G,C) F); ref = hd stk in
C' = C ∧ isRef ref ∧ (ref ≠ Null →
hp (the_Addr ref) ≠ None ∧
let (D, vs) = the (hp (the_Addr ref)) in
G ⊢ D ≤_C C ∧ vs (F,C) ≠ None ∧ G, hp ⊢ the (vs (F,C)) :: ≤ T)))"

"check_instr (Putfield F C) G hp stk vars Cl sig pc mxs frs =
(1 < length stk ∧ is_class G C ∧ field (G,C) F ≠ None ∧

```

(let (C', T) = the (field (G,C) F); v = hd stk; ref = hd (tl stk) in
  C' = C  $\wedge$  isRef ref  $\wedge$  (ref  $\neq$  Null  $\longrightarrow$ 
    hp (the_Addr ref)  $\neq$  None  $\wedge$ 
    (let (D,vs) = the (hp (the_Addr ref)) in
      G  $\vdash$  D  $\preceq_C$  C  $\wedge$  G, hp  $\vdash$  v ::  $\preceq$  T))))"

```

```

"check_instr (Checkcast C) G hp stk vars Cl sig pc mxs frs =
(0 < length stk  $\wedge$  is_class G C  $\wedge$  isRef (hd stk))"

```

```

"check_instr (Invoke C mn ps) G hp stk vars Cl sig pc mxs frs =
(length ps < length stk  $\wedge$ 
(let n = length ps; v = stk!n in
isRef v  $\wedge$  (v  $\neq$  Null  $\longrightarrow$ 
  hp (the_Addr v)  $\neq$  None  $\wedge$ 
  method (G, cname_of hp v) (mn, ps)  $\neq$  None  $\wedge$ 
  list_all2 ( $\lambda$  v T. G, hp  $\vdash$  v ::  $\preceq$  T) (rev (take n stk)) ps))))"

```

```

"check_instr Return G hp stk0 vars Cl sig0 pc mxs frs =
(0 < length stk0  $\wedge$  (0 < length frs  $\longrightarrow$ 
  method (G, Cl) sig0  $\neq$  None  $\wedge$ 
  (let v = hd stk0; (C, rT, body) = the (method (G, Cl) sig0) in
    Cl = C  $\wedge$  G, hp  $\vdash$  v ::  $\preceq$  rT))))"

```

```

"check_instr Pop G hp stk vars Cl sig pc mxs frs =
(0 < length stk)"

```

```

"check_instr Dup G hp stk vars Cl sig pc mxs frs =
(0 < length stk  $\wedge$  size stk < mxs)"

```

```

"check_instr Dup_x1 G hp stk vars Cl sig pc mxs frs =
(1 < length stk  $\wedge$  size stk < mxs)"

```

```

"check_instr Dup_x2 G hp stk vars Cl sig pc mxs frs =
(2 < length stk  $\wedge$  size stk < mxs)"

```

```

"check_instr Swap G hp stk vars Cl sig pc mxs frs =
(1 < length stk)"

```

```

"check_instr IAdd G hp stk vars Cl sig pc mxs frs =
(1 < length stk  $\wedge$  isIntg (hd stk)  $\wedge$  isIntg (hd (tl stk)))"

```

```

"check_instr (Ifcmpeq b) G hp stk vars Cl sig pc mxs frs =
(1 < length stk  $\wedge$  0  $\leq$  int pc+b)"

```

```

"check_instr (Goto b) G hp stk vars Cl sig pc mxs frs =
(0  $\leq$  int pc+b)"

```

```

"check_instr Throw G hp stk vars Cl sig pc mxs frs =
(0 < length stk  $\wedge$  isRef (hd stk))"

```

constdefs

```

check :: "jvm_prog  $\Rightarrow$  jvm_state  $\Rightarrow$  bool"
"check G s  $\equiv$  let (xcpt, hp, frs) = s in
  (case frs of []  $\Rightarrow$  True | (stk, loc, C, sig, pc)#frs'  $\Rightarrow$ 

```

```

      (let (C',rt,mxs,mxl,ins,et) = the (method (G,C) sig); i = ins!pc in
        pc < size ins ∧
        check_instr i G hp stk loc C sig pc mxs frs'))"

exec_d :: "jvm_prog ⇒ jvm_state type_error ⇒ jvm_state option type_error"
"exec_d G s ≡ case s of
  TypeError ⇒ TypeError
  | Normal s' ⇒ if check G s' then Normal (exec (G, s')) else TypeError"

consts
  "exec_all_d" :: "jvm_prog ⇒ jvm_state type_error ⇒ jvm_state type_error ⇒ bool"
  ("_ |- _ -jvmd-> _" [61,61,61]60)

syntax (xsymbols)
  "exec_all_d" :: "jvm_prog ⇒ jvm_state type_error ⇒ jvm_state type_error ⇒ bool"
  ("_ ⊢ _ -jvmd→ _" [61,61,61]60)

defs
  exec_all_d_def:
  "G ⊢ s -jvmd→ t ≡
    (s,t) ∈ ({(s,t). exec_d G s = TypeError ∧ t = TypeError} ∪
      {(s,t). ∃ t'. exec_d G s = Normal (Some t') ∧ t = Normal t'})*"

declare split_paired_All [simp del]
declare split_paired_Ex [simp del]

lemma [dest!]:
  "(if P then A else B) ≠ B ⇒ P"
  ⟨proof⟩

lemma exec_d_no_errorI [intro]:
  "check G s ⇒ exec_d G (Normal s) ≠ TypeError"
  ⟨proof⟩

theorem no_type_error_commutates:
  "exec_d G (Normal s) ≠ TypeError ⇒
    exec_d G (Normal s) = Normal (exec (G, s))"
  ⟨proof⟩

lemma defensive_imp_aggressive:
  "G ⊢ (Normal s) -jvmd→ (Normal t) ⇒ G ⊢ s -jvm→ t"
  ⟨proof⟩

end

```

Chapter 4

Bytecode Verifier

4.1 Semilattices

```

theory Semilat
imports Main While_Combinator
begin

types 'a ord    = "'a  $\Rightarrow$  'a  $\Rightarrow$  bool"
      'a binop  = "'a  $\Rightarrow$  'a  $\Rightarrow$  'a"
      'a sl     = "'a set * 'a ord * 'a binop"

consts
  "@lesub"    :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool" ("(_ /<=_ __ _)" [50, 1000, 51] 50)
  "@lesssub"  :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool" ("(_ /<'__ _)" [50, 1000, 51] 50)
defs
  lesub_def:   "x <=_r y == r x y"
  lesssub_def: "x <_r y == x <=_r y & x ~= y"

syntax (xsymbols)
  "@lesub"    :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool" ("(_ / $\leq$  _)" [50, 1000, 51] 50)

consts
  "@plussub"  :: "'a  $\Rightarrow$  ('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b  $\Rightarrow$  'c" ("(_ /+ '__ _)" [65, 1000, 66] 65)
defs
  plussub_def: "x +_f y == f x y"

syntax (xsymbols)
  "@plussub"  :: "'a  $\Rightarrow$  ('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b  $\Rightarrow$  'c" ("(_ /+_ _)" [65, 1000, 66] 65)

syntax (xsymbols)
  "@plussub"  :: "'a  $\Rightarrow$  ('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b  $\Rightarrow$  'c" ("(_ / $\sqcup$  _)" [65, 1000, 66] 65)

constdefs
  order :: "'a ord  $\Rightarrow$  bool"
  "order r == (!x. x <=_r x) &
    (!x y. x <=_r y & y <=_r x  $\longrightarrow$  x=y) &
    (!x y z. x <=_r y & y <=_r z  $\longrightarrow$  x <=_r z)"

  acc :: "'a ord  $\Rightarrow$  bool"
  "acc r == wfP ( $\lambda$ y x. x <_r y)"

  top :: "'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool"
  "top r T == !x. x <=_r T"

  closed :: "'a set  $\Rightarrow$  'a binop  $\Rightarrow$  bool"
  "closed A f == !x:A. !y:A. x +_f y : A"

  semilat :: "'a sl  $\Rightarrow$  bool"
  "semilat ==  $\%$ (A,r,f). order r & closed A f &
    (!x:A. !y:A. x <=_r x +_f y) &
    (!x:A. !y:A. y <=_r x +_f y) &
    (!x:A. !y:A. !z:A. x <=_r z & y <=_r z  $\longrightarrow$  x +_f y <=_r z)"

  is_ub :: "'a ord  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  'a  $\Rightarrow$  bool"

```

```

"is_ub r x y u == r x u & r y u"

is_lub :: "'a ord ⇒ 'a ⇒ 'a ⇒ 'a ⇒ bool"
"is_lub r x y u == is_ub r x y u & (!z. is_ub r x y z ⟶ r u z)"

some_lub :: "'a ord ⇒ 'a ⇒ 'a ⇒ 'a"
"some_lub r x y == SOME z. is_lub r x y z"

locale (open) semilat =
  fixes A :: "'a set"
  and r :: "'a ord"
  and f :: "'a binop"
  assumes semilat: "semilat(A,r,f)"

lemma order_refl [simp, intro]:
  "order r ⟹ x <=_r x"
  ⟨proof⟩

lemma order_antisym:
  "[ order r; x <=_r y; y <=_r x ] ⟹ x = y"
  ⟨proof⟩

lemma order_trans:
  "[ order r; x <=_r y; y <=_r z ] ⟹ x <=_r z"
  ⟨proof⟩

lemma order_less_irrefl [intro, simp]:
  "order r ⟹ ~ x <_r x"
  ⟨proof⟩

lemma order_less_trans:
  "[ order r; x <_r y; y <_r z ] ⟹ x <_r z"
  ⟨proof⟩

lemma topD [simp, intro]:
  "top r T ⟹ x <=_r T"
  ⟨proof⟩

lemma top_le_conv [simp]:
  "[ order r; top r T ] ⟹ (T <=_r x) = (x = T)"
  ⟨proof⟩

lemma semilat_Def:
  "semilat(A,r,f) == order r & closed A f &
    (!x:A. !y:A. x <=_r x +_f y) &
    (!x:A. !y:A. y <=_r x +_f y) &
    (!x:A. !y:A. !z:A. x <=_r z & y <=_r z ⟶ x +_f y <=_r z)"
  ⟨proof⟩

lemma (in semilat) orderI [simp, intro]:
  "order r"
  ⟨proof⟩

lemma (in semilat) closedI [simp, intro]:

```

"closed A f"
 ⟨proof⟩

lemma closedD:
 "⟦ closed A f; x:A; y:A ⟧ \implies x +_f y : A"
 ⟨proof⟩

lemma closed_UNIV [simp]: "closed UNIV f"
 ⟨proof⟩

lemma (in semilat) closed_f [simp, intro]:
 "⟦ x:A; y:A ⟧ \implies x +_f y : A"
 ⟨proof⟩

lemma (in semilat) refl_r [intro, simp]:
 "x <=_r x"
 ⟨proof⟩

lemma (in semilat) antisym_r [intro?]:
 "⟦ x <=_r y; y <=_r x ⟧ \implies x = y"
 ⟨proof⟩

lemma (in semilat) trans_r [trans, intro?]:
 "⟦ x <=_r y; y <=_r z ⟧ \implies x <=_r z"
 ⟨proof⟩

lemma (in semilat) ub1 [simp, intro?]:
 "⟦ x:A; y:A ⟧ \implies x <=_r x +_f y"
 ⟨proof⟩

lemma (in semilat) ub2 [simp, intro?]:
 "⟦ x:A; y:A ⟧ \implies y <=_r x +_f y"
 ⟨proof⟩

lemma (in semilat) lub [simp, intro?]:
 "⟦ x <=_r z; y <=_r z; x:A; y:A; z:A ⟧ \implies x +_f y <=_r z"
 ⟨proof⟩

lemma (in semilat) plus_le_conv [simp]:
 "⟦ x:A; y:A; z:A ⟧ \implies (x +_f y <=_r z) = (x <=_r z & y <=_r z)"
 ⟨proof⟩

lemma (in semilat) le_iff_plus_unchanged:
 "⟦ x:A; y:A ⟧ \implies (x <=_r y) = (x +_f y = y)"
 ⟨proof⟩

lemma (in semilat) le_iff_plus_unchanged2:
 "⟦ x:A; y:A ⟧ \implies (x <=_r y) = (y +_f x = y)"
 ⟨proof⟩


```
lemma (in semilat) plus_assoc [simp]:
  assumes a: "a ∈ A" and b: "b ∈ A" and c: "c ∈ A"
  shows "a +_f (b +_f c) = a +_f b +_f c"
<proof>
```

```
lemma (in semilat) plus_com_lemma:
  "[a ∈ A; b ∈ A] ⇒ a +_f b ≤_r b +_f a"
<proof>
```

```
lemma (in semilat) plus_commutative:
  "[a ∈ A; b ∈ A] ⇒ a +_f b = b +_f a"
<proof>
```

```
lemma is_lubD:
  "is_lub r x y u ⇒ is_ub r x y u & (!z. is_ub r x y z → r u z)"
<proof>
```

```
lemma is_ubI:
  "[r x u; r y u] ⇒ is_ub r x y u"
<proof>
```

```
lemma is_ubD:
  "is_ub r x y u ⇒ r x u & r y u"
<proof>
```

```
lemma is_lub_bigger1 [iff]:
  "is_lub (r^**) x y y = r^** x y"
<proof>
```

```
lemma is_lub_bigger2 [iff]:
  "is_lub (r^**) x y x = r^** y x"
<proof>
```

```
lemma extend_lub:
  "[single_valuedP r; is_lub (r^**) x y u; r x' x]
  ⇒ EX v. is_lub (r^**) x' y v"
<proof>
```

```
lemma single_valued_has_lubs [rule_format]:
  "[single_valuedP r; r^** x u] ⇒ (!y. r^** y u →
  (EX z. is_lub (r^**) x y z))"
<proof>
```

```
lemma some_lub_conv:
  "[acyclicP r; is_lub (r^**) x y u] ⇒ some_lub (r^**) x y = u"
<proof>
```

```
lemma is_lub_some_lub:
  "[single_valuedP r; acyclicP r; r^** x u; r^** y u]
  ⇒ is_lub (r^**) x y (some_lub (r^**) x y)"
<proof>
```

4.1.1 An executable lub-finder

constdefs

```
exec_lub :: "('a  $\Rightarrow$  'a  $\Rightarrow$  bool)  $\Rightarrow$  ('a  $\Rightarrow$  'a)  $\Rightarrow$  'a binop"
"exec_lub r f x y == while ( $\lambda z. \neg r^{**} x z$ ) f y"
```

lemma acyclic_single_valued_finite:

```
"[[acyclicP r; single_valuedP r; r^{**} x y ]]
 $\implies$  finite ({(x, y). r x y}  $\cap$  {a. r^{**} x a}  $\times$  {b. r^{**} b y})]"
<proof>
```

lemma exec_lub_conv:

```
"[[ acyclicP r; !x y. r x y  $\longrightarrow$  f x = y; is_lub (r^{**}) x y u ]]  $\implies$ 
exec_lub r f x y = u"
<proof>
```

lemma is_lub_exec_lub:

```
"[[ single_valuedP r; acyclicP r; r^{**} x u; r^{**} y u; !x y. r x y  $\longrightarrow$  f x = y ]]
 $\implies$  is_lub (r^{**}) x y (exec_lub r f x y)"
<proof>
```

end

4.2 The Error Type

```

theory Err imports Semilat begin

datatype 'a err = Err | OK 'a

types 'a ebinop = "'a ⇒ 'a ⇒ 'a err"
      'a esl =    "'a set * 'a ord * 'a ebinop"

consts
  ok_val :: "'a err ⇒ 'a"
primrec
  "ok_val (OK x) = x"

constdefs
  lift :: "('a ⇒ 'b err) ⇒ ('a err ⇒ 'b err)"
  "lift f e == case e of Err ⇒ Err | OK x ⇒ f x"

  lift2 :: "('a ⇒ 'b ⇒ 'c err) ⇒ 'a err ⇒ 'b err ⇒ 'c err"
  "lift2 f e1 e2 ==
    case e1 of Err ⇒ Err
              | OK x ⇒ (case e2 of Err ⇒ Err | OK y ⇒ f x y)"

  le :: "'a ord ⇒ 'a err ord"
  "le r e1 e2 ==
    case e2 of Err ⇒ True |
              OK y ⇒ (case e1 of Err ⇒ False | OK x ⇒ x <=_r y)"

  sup :: "('a ⇒ 'b ⇒ 'c) ⇒ ('a err ⇒ 'b err ⇒ 'c err)"
  "sup f == lift2(%x y. OK(x +_f y))"

  err :: "'a set ⇒ 'a err set"
  "err A == insert Err {x . ? y:A. x = OK y}"

  esl :: "'a sl ⇒ 'a esl"
  "esl == %(A,r,f). (A,r, %x y. OK(f x y))"

  sl :: "'a esl ⇒ 'a err sl"
  "sl == %(A,r,f). (err A, le r, lift2 f)"

syntax
  err_semilat :: "'a esl ⇒ bool"
translations
  "err_semilat L" == "semilat(Err.sl L)"

consts
  strict :: "('a ⇒ 'b err) ⇒ ('a err ⇒ 'b err)"
primrec
  "strict f Err = Err"
  "strict f (OK x) = f x"

lemma strict_Some [simp]:
  "(strict f x = OK y) = (∃ z. x = OK z ∧ f z = OK y)"

```

$\langle proof \rangle$

lemma not_Err_eq:

" $(x \neq Err) = (\exists a. x = OK\ a)$ "

$\langle proof \rangle$

lemma not_OK_eq:

" $(\forall y. x \neq OK\ y) = (x = Err)$ "

$\langle proof \rangle$

lemma unfold_le_sub_err:

" $e1 \leq (le\ r)\ e2 \implies le\ r\ e1\ e2$ "

$\langle proof \rangle$

lemma le_err_refl:

" $!x. x \leq_r x \implies e \leq (Err.le\ r)\ e$ "

$\langle proof \rangle$

lemma le_err_trans [rule_format]:

" $order\ r \implies e1 \leq (le\ r)\ e2 \longrightarrow e2 \leq (le\ r)\ e3 \longrightarrow e1 \leq (le\ r)\ e3$ "

$\langle proof \rangle$

lemma le_err_antisym [rule_format]:

" $order\ r \implies e1 \leq (le\ r)\ e2 \longrightarrow e2 \leq (le\ r)\ e1 \longrightarrow e1=e2$ "

$\langle proof \rangle$

lemma OK_le_err_OK:

" $(OK\ x \leq (le\ r)\ OK\ y) = (x \leq_r y)$ "

$\langle proof \rangle$

lemma order_le_err [iff]:

" $order(le\ r) = order\ r$ "

$\langle proof \rangle$

lemma le_Err [iff]: " $e \leq (le\ r)\ Err$ "

$\langle proof \rangle$

lemma Err_le_conv [iff]:

" $Err \leq (le\ r)\ e \iff (e = Err)$ "

$\langle proof \rangle$

lemma le_OK_conv [iff]:

" $e \leq (le\ r)\ OK\ x \iff (\exists y. e = OK\ y \ \&\ y \leq_r x)$ "

$\langle proof \rangle$

lemma OK_le_conv:

" $OK\ x \leq (le\ r)\ e \iff (e = Err \mid (\exists y. e = OK\ y \ \&\ x \leq_r y))$ "

$\langle proof \rangle$

lemma top_Err [iff]: " $top\ (le\ r)\ Err$ "

$\langle proof \rangle$

lemma OK_less_conv [rule_format, iff]:

" $OK\ x < (le\ r)\ e \iff (e = Err \mid (\exists y. e = OK\ y \ \&\ x <_r y))$ "

$\langle proof \rangle$

lemma not_Err_less [rule_format, iff]:

" $\sim(\text{Err} \leq (\text{le } r) \ x)$ "

$\langle proof \rangle$

lemma semilat_errI [intro]: **includes** semilat

shows "semilat(err A, Err.le r, lift2($\%x \ y. \text{OK}(f \ x \ y)$))"

$\langle proof \rangle$

lemma err_semilat_eslI_aux:

includes semilat **shows** "err_semilat(esl(A,r,f))"

$\langle proof \rangle$

lemma err_semilat_eslI [intro, simp]:

" $\bigwedge L. \text{semilat } L \implies \text{err_semilat}(\text{esl } L)$ "

$\langle proof \rangle$

lemma acc_err [simp, intro!]: " $\text{acc } r \implies \text{acc}(\text{le } r)$ "

$\langle proof \rangle$

lemma Err_in_err [iff]: " $\text{Err} : \text{err } A$ "

$\langle proof \rangle$

lemma Ok_in_err [iff]: " $(\text{OK } x : \text{err } A) = (x:A)$ "

$\langle proof \rangle$

4.2.1 lift

lemma lift_in_errI:

" $\llbracket e : \text{err } S; !x:S. e = \text{OK } x \longrightarrow f \ x : \text{err } S \rrbracket \implies \text{lift } f \ e : \text{err } S$ "

$\langle proof \rangle$

lemma Err_lift2 [simp]:

" $\text{Err } +_-(\text{lift2 } f) \ x = \text{Err}$ "

$\langle proof \rangle$

lemma lift2_Err [simp]:

" $x +_-(\text{lift2 } f) \ \text{Err} = \text{Err}$ "

$\langle proof \rangle$

lemma OK_lift2_OK [simp]:

" $\text{OK } x +_-(\text{lift2 } f) \ \text{OK } y = x +_f y$ "

$\langle proof \rangle$

4.2.2 sup

lemma Err_sup_Err [simp]:

" $\text{Err } +_-(\text{Err.sup } f) \ x = \text{Err}$ "

$\langle proof \rangle$

lemma Err_sup_Err2 [simp]:

" $x +_-(\text{Err.sup } f) \ \text{Err} = \text{Err}$ "

$\langle proof \rangle$

lemma *Err_sup_OK [simp]:*
 "OK x +_(Err.sup f) OK y = OK(x +_f y)"
 <proof>

lemma *Err_sup_eq_OK_conv [iff]:*
 "(Err.sup f ex ey = OK z) = (? x y. ex = OK x & ey = OK y & f x y = z)"
 <proof>

lemma *Err_sup_eq_Err [iff]:*
 "(Err.sup f ex ey = Err) = (ex=Err | ey=Err)"
 <proof>

4.2.3 semilat (err A) (le r) f

lemma *semilat_le_err_Err_plus [simp]:*
 "[[x: err A; semilat(err A, le r, f)]] \implies Err +_f x = Err"
 <proof>

lemma *semilat_le_err_plus_Err [simp]:*
 "[[x: err A; semilat(err A, le r, f)]] \implies x +_f Err = Err"
 <proof>

lemma *semilat_le_err_OK1:*
 "[[x:A; y:A; semilat(err A, le r, f); OK x +_f OK y = OK z]]
 \implies x <=_r z"
 <proof>

lemma *semilat_le_err_OK2:*
 "[[x:A; y:A; semilat(err A, le r, f); OK x +_f OK y = OK z]]
 \implies y <=_r z"
 <proof>

lemma *eq_order_le:*
 "[[x=y; order r] \implies x <=_r y"
 <proof>

lemma *OK_plus_OK_eq_Err_conv [simp]:*
 assumes "x:A" and "y:A" and "semilat(err A, le r, fe)"
 shows "((OK x) +_fe (OK y) = Err) = (~(? z:A. x <=_r z & y <=_r z))"
 <proof>

4.2.4 semilat (err(Union AS))

lemma *all_bex_swap_lemma [iff]:*
 "(!x. (? y:A. x = f y) \longrightarrow P x) = (!y:A. P(f y))"
 <proof>

lemma *closed_err_Union_lift2I:*
 "[[!A:AS. closed (err A) (lift2 f); AS ~={};
 !A:AS. !B:AS. A~B \longrightarrow (!a:A. !b:B. a +_f b = Err)]]
 \implies closed (err(Union AS)) (lift2 f)"
 <proof>

If $AS = \{\}$ the thm collapses to $Semilat.order\ r \wedge closed\ \{Err\}\ f \wedge Err \sqcup_f Err = Err$ which

may not hold

lemma `err_semilat_UnionI`:

```
"[ !A:AS. err_semilat(A, r, f); AS ~= {};  
  !A:AS.!B:AS. A~=B  $\longrightarrow$  (!a:A.!b:B. ~ a <=_r b & a +_f b = Err) ]  
   $\implies$  err_semilat(Union AS, r, f)"  
<proof>
```

end

4.3 Fixed Length Lists

theory *Listn* imports *Err* begin

constdefs

```
list :: "nat  $\Rightarrow$  'a set  $\Rightarrow$  'a list set"
"list n A == {xs. length xs = n & set xs  $\leq$  A}"
```

```
le :: "'a ord  $\Rightarrow$  ('a list)ord"
"le r == list_all2 (%x y. x  $\leq_r$  y)"
```

```
syntax "@lesublist" :: "'a list  $\Rightarrow$  'a ord  $\Rightarrow$  'a list  $\Rightarrow$  bool"
  ("(_ / $\leq$ [_] _)" [50, 0, 51] 50)
syntax "@lesssublist" :: "'a list  $\Rightarrow$  'a ord  $\Rightarrow$  'a list  $\Rightarrow$  bool"
  ("(_ / $<$ [_] _)" [50, 0, 51] 50)
```

translations

```
"x  $\leq$ [r] y" == "x  $\leq$ _(Listn.le r) y"
"x  $<$ [r] y" == "x  $<$ _(Listn.le r) y"
```

constdefs

```
map2 :: "('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'a list  $\Rightarrow$  'b list  $\Rightarrow$  'c list"
"map2 f == (%xs ys. map (split f) (zip xs ys))"
```

```
syntax "@plussublist" :: "'a list  $\Rightarrow$  ('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b list  $\Rightarrow$  'c list"
  ("(_ / $+$ [_] _)" [65, 0, 66] 65)
```

translations "x $+$ [f] y" == "x $+$ _(map2 f) y"

consts coalesce :: "'a err list \Rightarrow 'a list err"

primrec

```
"coalesce [] = OK[]"
"coalesce (ex#exs) = Err.sup (op #) ex (coalesce exs)"
```

constdefs

```
sl :: "nat  $\Rightarrow$  'a sl  $\Rightarrow$  'a list sl"
"sl n == %(A,r,f). (list n A, le r, map2 f)"
```

```
sup :: "('a  $\Rightarrow$  'b  $\Rightarrow$  'c err)  $\Rightarrow$  'a list  $\Rightarrow$  'b list  $\Rightarrow$  'c list err"
"sup f == %xs ys. if size xs = size ys then coalesce(xs  $+$ [f] ys) else Err"
```

```
upto_esl :: "nat  $\Rightarrow$  'a esl  $\Rightarrow$  'a list esl"
"upto_esl m == %(A,r,f). (Union{list n A |n. n  $\leq$  m}, le r, sup f)"
```

lemmas [simp] = set_update_subsetI

lemma unfold_lesub_list:

```
"xs  $\leq$ [r] ys == Listn.le r xs ys"
<proof>
```

lemma Nil_le_conv [iff]:

```
"([ ]  $\leq$ [r] ys) = (ys = [ ])"
<proof>
```

lemma Cons_notle_Nil [iff]:

"~ x#xs <=[r] []"
 <proof>

lemma Cons_le_Cons [iff]:
 "x#xs <=[r] y#ys = (x <=_r y & xs <=[r] ys)"
 <proof>

lemma Cons_less_Conss [simp]:
 "order r \implies
 x#xs <_(Listn.le r) y#ys =
 (x <_r y & xs <=[r] ys | x = y & xs <_(Listn.le r) ys)"
 <proof>

lemma list_update_le_cong:
 "[i < size xs; xs <=[r] ys; x <=_r y] \implies xs[i:=x] <=[r] ys[i:=y]"
 <proof>

lemma le_listD:
 "[xs <=[r] ys; p < size xs] \implies xs!p <=_r ys!p"
 <proof>

lemma le_list_refl:
 "!x. x <=_r x \implies xs <=[r] xs"
 <proof>

lemma le_list_trans:
 "[order r; xs <=[r] ys; ys <=[r] zs] \implies xs <=[r] zs"
 <proof>

lemma le_list_antisym:
 "[order r; xs <=[r] ys; ys <=[r] xs] \implies xs = ys"
 <proof>

lemma order_listI [simp, intro!]:
 "order r \implies order(Listn.le r)"
 <proof>

lemma lesub_list_impl_same_size [simp]:
 "xs <=[r] ys \implies size ys = size xs"
 <proof>

lemma lesssub_list_impl_same_size:
 "xs <_(Listn.le r) ys \implies size ys = size xs"
 <proof>

lemma le_list_appendI:
 "[b c d. a <=[r] b \implies c <=[r] d \implies a@c <=[r] b@d]"
 <proof>

lemma le_listI:
 "length a = length b \implies ($\bigwedge n. n < \text{length } a \implies a!n <=_r b!n$) \implies a <=[r] b"

$\langle proof \rangle$

lemma *listI*:

" $\llbracket \text{length } xs = n; \text{ set } xs \leq A \rrbracket \implies xs : \text{list } n \ A$ "

$\langle proof \rangle$

lemma *listE_length [simp]*:

" $xs : \text{list } n \ A \implies \text{length } xs = n$ "

$\langle proof \rangle$

lemma *less_lengthI*:

" $\llbracket xs : \text{list } n \ A; p < n \rrbracket \implies p < \text{length } xs$ "

$\langle proof \rangle$

lemma *listE_set [simp]*:

" $xs : \text{list } n \ A \implies \text{set } xs \leq A$ "

$\langle proof \rangle$

lemma *list_0 [simp]*:

" $\text{list } 0 \ A = \{[]\}$ "

$\langle proof \rangle$

lemma *in_list_Suc_iff*:

" $(xs : \text{list } (\text{Suc } n) \ A) = (\exists y \in A. \exists ys \in \text{list } n \ A. xs = y \# ys)$ "

$\langle proof \rangle$

lemma *Cons_in_list_Suc [iff]*:

" $(x \# xs : \text{list } (\text{Suc } n) \ A) = (x \in A \ \& \ xs : \text{list } n \ A)$ "

$\langle proof \rangle$

lemma *list_not_empty*:

" $\exists a. a \in A \implies \exists xs. xs : \text{list } n \ A$ "

$\langle proof \rangle$

lemma *nth_in [rule_format, simp]*:

" $!i \ n. \text{length } xs = n \longrightarrow \text{set } xs \leq A \longrightarrow i < n \longrightarrow (xs!i) : A$ "

$\langle proof \rangle$

lemma *listE_nth_in*:

" $\llbracket xs : \text{list } n \ A; i < n \rrbracket \implies (xs!i) : A$ "

$\langle proof \rangle$

lemma *listn_Cons_Suc [elim!]*:

" $l \# xs \in \text{list } n \ A \implies (\bigwedge n'. n = \text{Suc } n' \implies l \in A \implies xs \in \text{list } n' \ A \implies P) \implies P$ "

$\langle proof \rangle$

lemma *listn_appendE [elim!]*:

" $a @ b \in \text{list } n \ A \implies (\bigwedge n1 \ n2. n = n1 + n2 \implies a \in \text{list } n1 \ A \implies b \in \text{list } n2 \ A \implies P) \implies$

P "

$\langle proof \rangle$

```

lemma listt_update_in_list [simp, intro!]:
  "[[ xs : list n A; x ∈ A ]] ⇒ xs[i := x] : list n A"
  <proof>

lemma plus_list_Nil [simp]:
  "[[] +[f] xs = []"
  <proof>

lemma plus_list_Cons [simp]:
  "(x#xs) +[f] ys = (case ys of [] ⇒ [] | y#ys ⇒ (x +_f y)#(xs +[f] ys))"
  <proof>

lemma length_plus_list [rule_format, simp]:
  "!ys. length(xs +[f] ys) = min(length xs) (length ys)"
  <proof>

lemma nth_plus_list [rule_format, simp]:
  "!xs ys i. length xs = n → length ys = n → i < n →
    (xs +[f] ys)!i = (xs!i) +_f (ys!i)"
  <proof>

lemma (in semilat) plus_list_ub1 [rule_format]:
  "[[ set xs ≤ A; set ys ≤ A; size xs = size ys ]]
   ⇒ xs ≤[r] xs +[f] ys"
  <proof>

lemma (in semilat) plus_list_ub2:
  "[[set xs ≤ A; set ys ≤ A; size xs = size ys ]]
   ⇒ ys ≤[r] xs +[f] ys"
  <proof>

lemma (in semilat) plus_list_lub [rule_format]:
shows "!xs ys zs. set xs ≤ A → set ys ≤ A → set zs ≤ A
  → size xs = n & size ys = n →
  xs ≤[r] zs & ys ≤[r] zs → xs +[f] ys ≤[r] zs"
  <proof>

lemma (in semilat) list_update_incr [rule_format]:
  "x ∈ A ⇒ set xs ≤ A →
  (!i. i < size xs → xs ≤[r] xs[i := x +_f xs!i])"
  <proof>

lemma acc_le_listI [intro!]:
  "[[ order r; acc r ]] ⇒ acc(Listn.le r)"
  <proof>

lemma closed_listI:
  "closed S f ⇒ closed (list n S) (map2 f)"
  <proof>

lemma Listn_sl_aux:
includes semilat shows "semilat (Listn.sl n (A,r,f))"

```

<proof>

lemma Listn_sl: " $\bigwedge L. \text{semilat } L \implies \text{semilat } (\text{Listn.sl } n \ L)$ "

<proof>

lemma coalesce_in_err_list [rule_format]:

" $!x\text{es}. x\text{es} : \text{list } n \ (\text{err } A) \longrightarrow \text{coalesce } x\text{es} : \text{err}(\text{list } n \ A)$ "

<proof>

lemma lem: " $\bigwedge x \text{ xs}. x \ +_{\text{op } \#} \text{ xs} = x\#\text{xs}$ "

<proof>

lemma coalesce_eq_OK1_D [rule_format]:

" $\text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f) \implies$
 $!x\text{s}. x\text{s} : \text{list } n \ A \longrightarrow (!y\text{s}. y\text{s} : \text{list } n \ A \longrightarrow$
 $(!z\text{s}. \text{coalesce } (x\text{s} \ +[f] \ y\text{s}) = \text{OK } z\text{s} \longrightarrow x\text{s} \leq_{[r]} z\text{s}))$ "

<proof>

lemma coalesce_eq_OK2_D [rule_format]:

" $\text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f) \implies$
 $!x\text{s}. x\text{s} : \text{list } n \ A \longrightarrow (!y\text{s}. y\text{s} : \text{list } n \ A \longrightarrow$
 $(!z\text{s}. \text{coalesce } (x\text{s} \ +[f] \ y\text{s}) = \text{OK } z\text{s} \longrightarrow y\text{s} \leq_{[r]} z\text{s}))$ "

<proof>

lemma lift2_le_ub:

" $[\text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f); x \in A; y \in A; x \ +_f \ y = \text{OK } z;$
 $u \in A; x \leq_{\text{err}} u; y \leq_{\text{err}} u] \implies z \leq_{\text{err}} u$ "

<proof>

lemma coalesce_eq_OK_ub_D [rule_format]:

" $\text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f) \implies$
 $!x\text{s}. x\text{s} : \text{list } n \ A \longrightarrow (!y\text{s}. y\text{s} : \text{list } n \ A \longrightarrow$
 $(!z\text{s } u\text{s}. \text{coalesce } (x\text{s} \ +[f] \ y\text{s}) = \text{OK } z\text{s} \ \& \ x\text{s} \leq_{[r]} u\text{s} \ \& \ y\text{s} \leq_{[r]} u\text{s}$
 $\ \& \ u\text{s} : \text{list } n \ A \longrightarrow z\text{s} \leq_{[r]} u\text{s}))$ "

<proof>

lemma lift2_eq_ErrD:

" $[x \ +_f \ y = \text{Err}; \text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f); x \in A; y \in A]$
 $\implies \sim(\exists u \in A. x \leq_{\text{err}} u \ \& \ y \leq_{\text{err}} u)$ "

<proof>

lemma coalesce_eq_Err_D [rule_format]:

" $[\text{semilat}(\text{err } A, \text{Err.le } r, \text{lift2 } f)]$
 $\implies !x\text{s}. x\text{s} \in \text{list } n \ A \longrightarrow (!y\text{s}. y\text{s} \in \text{list } n \ A \longrightarrow$
 $\text{coalesce } (x\text{s} \ +[f] \ y\text{s}) = \text{Err} \longrightarrow$
 $\sim(\exists z\text{s} \in \text{list } n \ A. x\text{s} \leq_{[r]} z\text{s} \ \& \ y\text{s} \leq_{[r]} z\text{s}))$ "

<proof>

lemma closed_err_lift2_conv:

" $\text{closed } (\text{err } A) \ (\text{lift2 } f) = (\forall x \in A. \forall y \in A. x \ +_f \ y : \text{err } A)$ "

<proof>

lemma closed_map2_list [rule_format]:

```

"closed (err A) (lift2 f)  $\implies$ 
 $\forall xs. xs : list\ n\ A \longrightarrow (\forall ys. ys : list\ n\ A \longrightarrow$ 
  map2 f xs ys : list n (err A))"
<proof>

```

```

lemma closed_lift2_sup:
  "closed (err A) (lift2 f)  $\implies$ 
  closed (err (list n A)) (lift2 (sup f))"
  <proof>

```

```

lemma err_semilat_sup:
  "err_semilat (A,r,f)  $\implies$ 
  err_semilat (list n A, Listn.le r, sup f)"
  <proof>

```

```

lemma err_semilat_upto_esl:
  " $\bigwedge L. err\_semilat\ L \implies err\_semilat(upto\_esl\ m\ L)$ "
  <proof>

```

```

end

```

4.4 Typing and Dataflow Analysis Framework

theory *Typing_Framework* imports *Listn* begin

The relationship between dataflow analysis and a welltyped-instruction predicate.

types

```
's step_type = "nat  $\Rightarrow$  's  $\Rightarrow$  (nat  $\times$  's) list"
```

constdefs

```
stable :: "'s ord  $\Rightarrow$  's step_type  $\Rightarrow$  's list  $\Rightarrow$  nat  $\Rightarrow$  bool"
"stable r step ss p == !(q,s'):set(step p (ss!p)). s' <=_r ss!q"
```

```
stables :: "'s ord  $\Rightarrow$  's step_type  $\Rightarrow$  's list  $\Rightarrow$  bool"
"stables r step ss == !p<size ss. stable r step ss p"
```

```
wt_step ::
"'s ord  $\Rightarrow$  's  $\Rightarrow$  's step_type  $\Rightarrow$  's list  $\Rightarrow$  bool"
"wt_step r T step ts ==
!p<size(ts). ts!p ~= T & stable r step ts p"
```

```
is_bcv :: "'s ord  $\Rightarrow$  's  $\Rightarrow$  's step_type
 $\Rightarrow$  nat  $\Rightarrow$  's set  $\Rightarrow$  ('s list  $\Rightarrow$  's list)  $\Rightarrow$  bool"
"is_bcv r T step n A bcv == !ss : list n A.
(!p<n. (bcv ss)!p ~= T) =
(? ts: list n A. ss <=[r] ts & wt_step r T step ts)"
```

end

4.5 Products as Semilattices

theory Product imports Err begin

constdefs

```

le :: "'a ord  $\Rightarrow$  'b ord  $\Rightarrow$  ('a * 'b) ord"
"le rA rB == %(a,b) (a',b'). a <=_rA a' & b <=_rB b'"

sup :: "'a ebinop  $\Rightarrow$  'b ebinop  $\Rightarrow$  ('a * 'b) ebinop"
"sup f g == %(a1,b1)(a2,b2). Err.sup Pair (a1+_f a2) (b1+_g b2)"

esl :: "'a esl  $\Rightarrow$  'b esl  $\Rightarrow$  ('a * 'b) esl"
"esl == %(A,rA,fA) (B,rB,fB). (A <*> B, le rA rB, sup fA fB)"

syntax "@lesubprod" :: "'a*'b  $\Rightarrow$  'a ord  $\Rightarrow$  'b ord  $\Rightarrow$  'b  $\Rightarrow$  bool"
  ("(_ /<='(_,_') _)" [50, 0, 0, 51] 50)
translations "p <=(rA,rB) q" == "p <=_ (Product.le rA rB) q"

```

lemma unfold_lesub_prod:

```

  "p <=(rA,rB) q == le rA rB p q"
  <proof>

```

lemma le_prod_Pair_conv [iff]:

```

  "((a1,b1) <=(rA,rB) (a2,b2)) = (a1 <=_rA a2 & b1 <=_rB b2)"
  <proof>

```

lemma less_prod_Pair_conv:

```

  "((a1,b1) <_ (Product.le rA rB) (a2,b2)) =
   (a1 <_rA a2 & b1 <_rB b2 | a1 <=_rA a2 & b1 <_rB b2)"
  <proof>

```

lemma order_le_prod [iff]:

```

  "order(Product.le rA rB) = (order rA & order rB)"
  <proof>

```

lemma acc_le_prodI [intro!]:

```

  "[ acc rA; acc rB ]  $\Longrightarrow$  acc(Product.le rA rB)"
  <proof>

```

lemma closed_lift2_sup:

```

  "[ closed (err A) (lift2 f); closed (err B) (lift2 g) ]  $\Longrightarrow$ 
   closed (err(A<*>B)) (lift2(sup f g))"
  <proof>

```

lemma unfold_plussub_lift2:

```

  "e1 +_(lift2 f) e2 == lift2 f e1 e2"
  <proof>

```

lemma plus_eq_Err_conv [simp]:

```

  assumes "x:A" and "y:A"
  and "semilat(err A, Err.le r, lift2 f)"

```

```

shows "(x +_f y = Err) = (~(? z:A. x <=_r z & y <=_r z))"
<proof>

```

```

lemma err_semilat_Product_esl:
  "∧L1 L2. [ err_semilat L1; err_semilat L2 ] ⇒ err_semilat(Product.esl L1 L2)"
<proof>

```

```

end

```


4.6 More on Semilattices

theory SemilatAlg imports Typing_Framework Product begin

constdefs

```
lesubstep_type :: "(nat × 's) list ⇒ 's ord ⇒ (nat × 's) list ⇒ bool"
  ("(_ /<=_|_ | _)" [50, 0, 51] 50)
"x <=_|r| y ≡ ∀ (p,s) ∈ set x. ∃ s'. (p,s') ∈ set y ∧ s <=_r s'"
```

consts

```
"@plusplussub" :: "'a list ⇒ ('a ⇒ 'a ⇒ 'a) ⇒ 'a ⇒ 'a" ("(_ /++'__ _)" [65, 1000, 66] 65)
```

primrec

```
"[] ++_f y = y"
"(x#xs) ++_f y = xs ++_f (x +_f y)"
```

constdefs

```
bounded :: "'s step_type ⇒ nat ⇒ bool"
"bounded step n == !p<n. !s. !(q,t):set(step p s). q<n"

pres_type :: "'s step_type ⇒ nat ⇒ 's set ⇒ bool"
"pres_type step n A == ∀ s∈A. ∀ p<n. ∀ (q,s')∈set (step p s). s' ∈ A"

mono :: "'s ord ⇒ 's step_type ⇒ nat ⇒ 's set ⇒ bool"
"mono r step n A ==
  ∀ s p t. s ∈ A ∧ p < n ∧ s <=_r t ⟶ step p s <=_|r| step p t"
```

lemma pres_typeD:

```
"[ pres_type step n A; s∈A; p<n; (q,s')∈set (step p s) ] ⟹ s' ∈ A"
⟨proof⟩
```

lemma monoD:

```
"[ mono r step n A; p < n; s∈A; s <=_r t ] ⟹ step p s <=_|r| step p t"
⟨proof⟩
```

lemma boundedD:

```
"[ bounded step n; p < n; (q,t) : set (step p xs) ] ⟹ q < n"
⟨proof⟩
```

lemma lesubstep_type_refl [simp, intro]:

```
"(∧ x. x <=_r x) ⟹ x <=_|r| x"
⟨proof⟩
```

lemma lesub_step_typeD:

```
"a <=_|r| b ⟹ (x,y) ∈ set a ⟹ ∃ y'. (x, y') ∈ set b ∧ y <=_r y'"
⟨proof⟩
```

lemma list_update_le_listI [rule_format]:

```
"set xs <= A ⟶ set ys <= A ⟶ xs <=[r] ys ⟶ p < size xs ⟶
  x <=_r ys!p ⟶ semilat(A,r,f) ⟶ x∈A ⟶
  xs[p := x +_f xs!p] <=[r] ys"
```

$\langle proof \rangle$

lemma *plusplus_closed*: *includes semilat* **shows**

" $\bigwedge y. \llbracket \text{set } x \subseteq A; y \in A \rrbracket \implies x ++_f y \in A$ "

$\langle proof \rangle$

lemma (in *semilat*) *pp_ub2*:

" $\bigwedge y. \llbracket \text{set } x \subseteq A; y \in A \rrbracket \implies y \leq_r x ++_f y$ "

$\langle proof \rangle$

lemma (in *semilat*) *pp_ub1*:

shows " $\bigwedge y. \llbracket \text{set } ls \subseteq A; y \in A; x \in \text{set } ls \rrbracket \implies x \leq_r ls ++_f y$ "

$\langle proof \rangle$

lemma (in *semilat*) *pp_lub*:

assumes *z*: " $z \in A$ "

shows

" $\bigwedge y. y \in A \implies \text{set } xs \subseteq A \implies \forall x \in \text{set } xs. x \leq_r z \implies y \leq_r z \implies xs ++_f y \leq_r z$ "

$\langle proof \rangle$

lemma *ub1'*: *includes semilat*

shows " $\llbracket \forall (p, s) \in \text{set } S. s \in A; y \in A; (a, b) \in \text{set } S \rrbracket$

$\implies b \leq_r \text{map snd } [(p', t') \leftarrow S. p' = a] ++_f y$ "

$\langle proof \rangle$

lemma *plusplus_empty*:

" $\forall s'. (q, s') \in \text{set } S \longrightarrow s' ++_f ss ! q = ss ! q \implies$

$(\text{map snd } [(p', t') \leftarrow S. p' = q] ++_f ss ! q) = ss ! q$ "

$\langle proof \rangle$

end

4.7 More about Options

theory Opt imports Err begin

constdefs

```
le :: "'a ord  $\Rightarrow$  'a option ord"
"le r o1 o2 == case o2 of None  $\Rightarrow$  o1=None |
                    Some y  $\Rightarrow$  (case o1 of None  $\Rightarrow$  True
                               | Some x  $\Rightarrow$  x <=_r y)"
```

```
opt :: "'a set  $\Rightarrow$  'a option set"
"opt A == insert None {x . ? y:A. x = Some y}"
```

```
sup :: "'a ebinop  $\Rightarrow$  'a option ebinop"
"sup f o1 o2 ==
  case o1 of None  $\Rightarrow$  OK o2 | Some x  $\Rightarrow$  (case o2 of None  $\Rightarrow$  OK o1
    | Some y  $\Rightarrow$  (case f x y of Err  $\Rightarrow$  Err | OK z  $\Rightarrow$  OK (Some z)))"
```

```
esl :: "'a esl  $\Rightarrow$  'a option esl"
"esl == %(A,r,f). (opt A, le r, sup f)"
```

```
lemma unfold_le_opt:
  "o1 <=_ (le r) o2 =
    (case o2 of None  $\Rightarrow$  o1=None |
      Some y  $\Rightarrow$  (case o1 of None  $\Rightarrow$  True | Some x  $\Rightarrow$  x <=_r y))"
<proof>
```

```
lemma le_opt_refl:
  "order r  $\Longrightarrow$  o1 <=_ (le r) o1"
<proof>
```

```
lemma le_opt_trans [rule_format]:
  "order r  $\Longrightarrow$ 
    o1 <=_ (le r) o2  $\longrightarrow$  o2 <=_ (le r) o3  $\longrightarrow$  o1 <=_ (le r) o3"
<proof>
```

```
lemma le_opt_antisym [rule_format]:
  "order r  $\Longrightarrow$  o1 <=_ (le r) o2  $\longrightarrow$  o2 <=_ (le r) o1  $\longrightarrow$  o1=o2"
<proof>
```

```
lemma order_le_opt [intro!,simp]:
  "order r  $\Longrightarrow$  order (le r)"
<proof>
```

```
lemma None_bot [iff]:
  "None <=_ (le r) ox"
<proof>
```

```
lemma Some_le [iff]:
  "(Some x <=_ (le r) ox) = (? y. ox = Some y & x <=_r y)"
<proof>
```

```
lemma le_None [iff]:
  "(ox <=_ (le r) None) = (ox = None)"
```

$\langle proof \rangle$

lemma *OK_None_bot [iff]:*
 "OK None \leq (Err.le (le r)) x"
 $\langle proof \rangle$

lemma *sup_None1 [iff]:*
 "x $+$ (sup f) None = OK x"
 $\langle proof \rangle$

lemma *sup_None2 [iff]:*
 "None $+$ (sup f) x = OK x"
 $\langle proof \rangle$

lemma *None_in_opt [iff]:*
 "None : opt A"
 $\langle proof \rangle$

lemma *Some_in_opt [iff]:*
 "(Some x : opt A) = (x:A)"
 $\langle proof \rangle$

lemma *semilat_opt [intro, simp]:*
 " $\bigwedge L. \text{err_semilat } L \implies \text{err_semilat } (\text{Opt.esl } L)$ "
 $\langle proof \rangle$

lemma *top_le_opt_Some [iff]:*
 "top (le r) (Some T) = top r T"
 $\langle proof \rangle$

lemma *Top_le_conv:*
 " $\llbracket \text{order } r; \text{top } r \text{ } T \rrbracket \implies (T \leq_r x) = (x = T)$ "
 $\langle proof \rangle$

lemma *acc_le_optI [intro!]:*
 "acc r \implies acc(le r)"
 $\langle proof \rangle$

lemma *option_map_in_optionI:*
 " $\llbracket \text{ox} : \text{opt } S; !x:S. \text{ox} = \text{Some } x \longrightarrow f \text{ } x : S \rrbracket$
 $\implies \text{option_map } f \text{ } \text{ox} : \text{opt } S$ "
 $\langle proof \rangle$

end

4.8 The Lightweight Bytecode Verifier

```
theory LBVSPEC imports SemilatAlg Opt begin
```

```
types
```

```
  's certificate = "'s list"
```

```
consts
```

```
merge :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ nat ⇒ (nat × 's) list ⇒ 's
⇒ 's"
```

```
primrec
```

```
"merge cert f r T pc []      x = x"
"merge cert f r T pc (s#ss) x = merge cert f r T pc ss (let (pc',s') = s in
                        if pc'=pc+1 then s' +_f x
                        else if s' <=_r (cert!pc') then x
                        else T)"
```

```
constdefs
```

```
wtl_inst :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒
              's step_type ⇒ nat ⇒ 's ⇒ 's"
"wtl_inst cert f r T step pc s ≡ merge cert f r T pc (step pc s) (cert!(pc+1))"
```

```
wtl_cert :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ 's ⇒
              's step_type ⇒ nat ⇒ 's ⇒ 's"
```

```
"wtl_cert cert f r T B step pc s ≡
  if cert!pc = B then
    wtl_inst cert f r T step pc s
  else
    if s <=_r (cert!pc) then wtl_inst cert f r T step pc (cert!pc) else T"
```

```
consts
```

```
wtl_inst_list :: "'a list ⇒ 's certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ 's ⇒
                  's step_type ⇒ nat ⇒ 's ⇒ 's"
```

```
primrec
```

```
"wtl_inst_list []      cert f r T B step pc s = s"
"wtl_inst_list (i#is) cert f r T B step pc s =
  (let s' = wtl_cert cert f r T B step pc s in
   if s' = T ∨ s = T then T else wtl_inst_list is cert f r T B step (pc+1) s')"
```

```
constdefs
```

```
cert_ok :: "'s certificate ⇒ nat ⇒ 's ⇒ 's ⇒ 's set ⇒ bool"
"cert_ok cert n T B A ≡ (∀i < n. cert!i ∈ A ∧ cert!i ≠ T) ∧ (cert!n = B)"
```

```
constdefs
```

```
bottom :: "'a ord ⇒ 'a ⇒ bool"
"bottom r B ≡ ∀x. B <=_r x"
```

```
locale (open) lbv = semilat +
```

```
  fixes T :: "'a" ("⊤")
```

```
  fixes B :: "'a" ("⊥")
```

```
  fixes step :: "'a step_type"
```

```
  assumes top: "top r ⊤"
```

```
  assumes T_A: "⊤ ∈ A"
```

```

assumes bot: "bottom  $r \perp$ "
assumes B_A: " $\perp \in A$ "

fixes merge :: "'a certificate  $\Rightarrow$  nat  $\Rightarrow$  (nat  $\times$  'a) list  $\Rightarrow$  'a  $\Rightarrow$  'a"
defines mrg_def: "merge cert  $\equiv$  LBVSPEC.merge cert f r  $\top$ "

fixes wti :: "'a certificate  $\Rightarrow$  nat  $\Rightarrow$  'a  $\Rightarrow$  'a"
defines wti_def: "wti cert  $\equiv$  wtl_inst cert f r  $\top$  step"

fixes wtc :: "'a certificate  $\Rightarrow$  nat  $\Rightarrow$  'a  $\Rightarrow$  'a"
defines wtc_def: "wtc cert  $\equiv$  wtl_cert cert f r  $\top \perp$  step"

fixes wtl :: "'b list  $\Rightarrow$  'a certificate  $\Rightarrow$  nat  $\Rightarrow$  'a  $\Rightarrow$  'a"
defines wtl_def: "wtl ins cert  $\equiv$  wtl_inst_list ins cert f r  $\top \perp$  step"

lemma (in lbv) wti:
  "wti c pc s  $\equiv$  merge c pc (step pc s) (c!(pc+1))"
  <proof>

lemma (in lbv) wtc:
  "wtc c pc s  $\equiv$  if c!pc =  $\perp$  then wti c pc s else if s  $\leq_r$  c!pc then wti c pc (c!pc)
  else  $\top$ "
  <proof>

lemma cert_okD1 [intro?]:
  "cert_ok c n T B A  $\implies$  pc < n  $\implies$  c!pc  $\in$  A"
  <proof>

lemma cert_okD2 [intro?]:
  "cert_ok c n T B A  $\implies$  c!n = B"
  <proof>

lemma cert_okD3 [intro?]:
  "cert_ok c n T B A  $\implies$  B  $\in$  A  $\implies$  pc < n  $\implies$  c!Suc pc  $\in$  A"
  <proof>

lemma cert_okD4 [intro?]:
  "cert_ok c n T B A  $\implies$  pc < n  $\implies$  c!pc  $\neq$  T"
  <proof>

declare Let_def [simp]

4.8.1 more semilattice lemmas

lemma (in lbv) sup_top [simp, elim]:
  assumes x: "x  $\in$  A"
  shows "x ++_f  $\top$  =  $\top$ "
  <proof>

lemma (in lbv) plusplussup_top [simp, elim]:
  "set xs  $\subseteq$  A  $\implies$  xs ++_f  $\top$  =  $\top$ "
  <proof>

```

```

lemma (in semilat) pp_ub1':
  assumes S: "snd'set S  $\subseteq$  A"
  assumes y: "y  $\in$  A" and ab: "(a, b)  $\in$  set S"
  shows "b  $\leq_r$  map snd [(p', t')  $\leftarrow$  S . p' = a] ++_f y"
<proof>

```

```

lemma (in lbv) bottom_le [simp, intro]:
  " $\perp \leq_r x$ "
<proof>

```

```

lemma (in lbv) le_bottom [simp]:
  " $x \leq_r \perp = (x = \perp)$ "
<proof>

```

4.8.2 merge

```

lemma (in lbv) merge_Nil [simp]:
  "merge c pc [] x = x" <proof>

```

```

lemma (in lbv) merge_Cons [simp]:
  "merge c pc (l#ls) x = merge c pc ls (if fst l=pc+1 then snd l ++_f x
                                         else if snd l  $\leq_r$  (c!fst l) then x
                                         else  $\top$ )"
<proof>

```

```

lemma (in lbv) merge_Err [simp]:
  "snd'set ss  $\subseteq$  A  $\implies$  merge c pc ss  $\top = \top$ "
<proof>

```

```

lemma (in lbv) merge_not_top:
  " $\bigwedge x. \text{snd'set } ss \subseteq A \implies \text{merge c pc ss } x \neq \top \implies$ 
 $\forall (pc', s') \in \text{set } ss. (pc' \neq pc+1 \longrightarrow s' \leq_r (c!pc'))$ "
  (is " $\bigwedge x. ?\text{set } ss \implies ?\text{merge } ss \ x \implies ?P \ ss$ ")
<proof>

```

```

lemma (in lbv) merge_def:
  shows
    " $\bigwedge x. x \in A \implies \text{snd'set } ss \subseteq A \implies$ 
    merge c pc ss x =
    (if  $\forall (pc', s') \in \text{set } ss. pc' \neq pc+1 \longrightarrow s' \leq_r c!pc'$  then
      map snd [(p', t')  $\leftarrow$  ss. p'=pc+1] ++_f x
    else  $\top$ )"
  (is " $\bigwedge x. \_ \implies \_ \implies ?\text{merge } ss \ x = ?\text{if } ss \ x$  is " $\bigwedge x. \_ \implies \_ \implies ?P \ ss \ x$ ")
<proof>

```

```

lemma (in lbv) merge_not_top_s:
  assumes x: "x  $\in$  A" and ss: "snd'set ss  $\subseteq$  A"
  assumes m: "merge c pc ss x  $\neq \top$ "
  shows "merge c pc ss x = (map snd [(p', t')  $\leftarrow$  ss. p'=pc+1] ++_f x)"
<proof>

```

4.8.3 wtl-inst-list

lemmas [iff] = not_Err_eq

lemma (in lbv) wtl_Nil [simp]: "wtl [] c pc s = s"
 ⟨proof⟩

lemma (in lbv) wtl_Cons [simp]:
 "wtl (i#is) c pc s =
 (let s' = wtc c pc s in if s' = \top \vee s = \top then \top else wtl is c (pc+1) s')"
 ⟨proof⟩

lemma (in lbv) wtl_Cons_not_top:
 "wtl (i#is) c pc s \neq \top =
 (wtc c pc s \neq \top \wedge s \neq \top \wedge wtl is c (pc+1) (wtc c pc s) \neq \top)"
 ⟨proof⟩

lemma (in lbv) wtl_top [simp]: "wtl ls c pc \top = \top "
 ⟨proof⟩

lemma (in lbv) wtl_not_top:
 "wtl ls c pc s \neq \top \implies s \neq \top "
 ⟨proof⟩

lemma (in lbv) wtl_append [simp]:
 " \bigwedge pc s. wtl (a@b) c pc s = wtl b c (pc+length a) (wtl a c pc s)"
 ⟨proof⟩

lemma (in lbv) wtl_take:
 "wtl is c pc s \neq \top \implies wtl (take pc' is) c pc s \neq \top "
 (is "?wtl is \neq _ \implies _")
 ⟨proof⟩

lemma take_Suc:
 " $\forall n. n < \text{length } l \longrightarrow \text{take } (\text{Suc } n) l = (\text{take } n l) @ [l!n]$ " (is "?P l")
 ⟨proof⟩

lemma (in lbv) wtl_Suc:
 assumes suc: "pc+1 < length is"
 assumes wtl: "wtl (take pc is) c 0 s \neq \top "
 shows "wtl (take (pc+1) is) c 0 s = wtc c pc (wtl (take pc is) c 0 s)"
 ⟨proof⟩

lemma (in lbv) wtl_all:
 assumes all: "wtl is c 0 s \neq \top " (is "?wtl is \neq _")
 assumes pc: "pc < length is"
 shows "wtc c pc (wtl (take pc is) c 0 s) \neq \top "
 ⟨proof⟩

4.8.4 preserves-type

lemma (in lbv) merge_pres:
 assumes s0: "snd'set ss \subseteq A" and x: "x \in A"
 shows "merge c pc ss x \in A"
 ⟨proof⟩


```
lemma pres_typeD2:
  "pres_type step n A  $\implies$  s  $\in$  A  $\implies$  p < n  $\implies$  snd' set (step p s)  $\subseteq$  A"
  <proof>
```

```
lemma (in lbv) wti_pres [intro?]:
  assumes pres: "pres_type step n A"
  assumes cert: "c!(pc+1)  $\in$  A"
  assumes s_pc: "s  $\in$  A" "pc < n"
  shows "wti c pc s  $\in$  A"
  <proof>
```

```
lemma (in lbv) wtc_pres:
  assumes pres: "pres_type step n A"
  assumes cert: "c!pc  $\in$  A" and cert': "c!(pc+1)  $\in$  A"
  assumes s: "s  $\in$  A" and pc: "pc < n"
  shows "wtc c pc s  $\in$  A"
  <proof>
```

```
lemma (in lbv) wtl_pres:
  assumes pres: "pres_type step (length is) A"
  assumes cert: "cert_ok c (length is)  $\top \perp$  A"
  assumes s: "s  $\in$  A"
  assumes all: "wtl is c 0 s  $\neq \top$ "
  shows "pc < length is  $\implies$  wtl (take pc is) c 0 s  $\in$  A"
  (is "?len pc  $\implies$  ?wtl pc  $\in$  A")
  <proof>
```

```
end
```

4.9 Correctness of the LBV

theory *LBVCorrect* **imports** *LBVSpec Typing_Framework* **begin**

```

locale (open) lbvs = lbv +
  fixes s0 :: 'a ("s0")
  fixes c :: "'a list"
  fixes ins :: "'b list"
  fixes phi :: "'a list" (" $\varphi$ ")
  defines phi_def:
    " $\varphi \equiv \text{map } (\lambda pc. \text{ if } c!pc = \perp \text{ then wtl (take pc ins) c 0 s0 \text{ else } c!pc)$ "
    "[0..length ins]"

```

```

assumes bounded: "bounded step (length ins)"
assumes cert: "cert_ok c (length ins)  $\top \perp A$ "
assumes pres: "pres_type step (length ins) A"

```

```

lemma (in lbvs) phi_None [intro?]:
  " $\llbracket pc < \text{length ins}; c!pc = \perp \rrbracket \implies \varphi ! pc = \text{wtl (take pc ins) c 0 s0}$ "
  <proof>

```

```

lemma (in lbvs) phi_Some [intro?]:
  " $\llbracket pc < \text{length ins}; c!pc \neq \perp \rrbracket \implies \varphi ! pc = c ! pc$ "
  <proof>

```

```

lemma (in lbvs) phi_len [simp]:
  "length  $\varphi$  = length ins"
  <proof>

```

```

lemma (in lbvs) wtl_suc_pc:
  assumes all: "wtl ins c 0 s0  $\neq \top$ "
  assumes pc: "pc+1 < length ins"
  shows "wtl (take (pc+1) ins) c 0 s0  $\leq_r \varphi!(pc+1)$ "
  <proof>

```

```

lemma (in lbvs) wtl_stable:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "
  assumes s0: "s0  $\in A$ "
  assumes pc: "pc < length ins"
  shows "stable r step  $\varphi$  pc"
  <proof>

```

```

lemma (in lbvs) phi_not_top:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "
  assumes pc: "pc < length ins"
  shows " $\varphi!pc \neq \top$ "
  <proof>

```

```

lemma (in lbvs) phi_in_A:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "

```

```

    assumes s0: "s0 ∈ A"
    shows "φ ∈ list (length ins) A"
  ⟨proof⟩

```

```

lemma (in lbvs) phi0:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes 0: "0 < length ins"
  shows "s0 ≤r φ!0"
  ⟨proof⟩

```

```

theorem (in lbvs) wtl_sound:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes s0: "s0 ∈ A"
  shows "∃ ts. wt_step r ⊤ step ts"
  ⟨proof⟩

```

```

theorem (in lbvs) wtl_sound_strong:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes s0: "s0 ∈ A"
  assumes nz: "0 < length ins"
  shows "∃ ts ∈ list (length ins) A. wt_step r ⊤ step ts ∧ s0 ≤r ts!0"
  ⟨proof⟩

```

```

end

```

4.10 Completeness of the LBV

```

theory LBVComplete
imports LBVSpec Typing_Framework
begin

constdefs
  is_target :: "[ 's step_type, 's list, nat ]  $\Rightarrow$  bool"
  "is_target step phi pc'  $\equiv$ 
     $\exists pc\ s'.\ pc' \neq pc+1 \wedge pc < \text{length } \phi \wedge (pc', s') \in \text{set } (\text{step } pc\ (\phi!pc))"$ 

  make_cert :: "[ 's step_type, 's list, 's ]  $\Rightarrow$  's certificate"
  "make_cert step phi B  $\equiv$ 
    map ( $\lambda pc.$  if is_target step phi pc then phi!pc else B) [0.. $\text{length } \phi$ ] @ [B]"

lemma [code]:
  "is_target step phi pc' =
    list_ex ( $\lambda pc.$  pc'  $\neq$  pc+1  $\wedge$  pc' mem (map fst (step pc (phi!pc)))) [0.. $\text{length } \phi$ ]"
  <proof>

locale (open) lbvc = lbv +
  fixes phi :: "'a list" ("φ")
  fixes c    :: "'a list"
  defines cert_def: "c  $\equiv$  make_cert step φ ⊥"

  assumes mono: "mono r step (length φ) A"
  assumes pres: "pres_type step (length φ) A"
  assumes phi:  " $\forall pc < \text{length } \phi. \phi!pc \in A \wedge \phi!pc \neq \top$ "
  assumes bounded: "bounded step (length φ)"

  assumes B_neq_T: "⊥  $\neq$  ⊤"

lemma (in lbvc) cert: "cert_ok c (length φ) ⊤ ⊥ A"
  <proof>

lemmas [simp del] = split_paired_Ex

lemma (in lbvc) cert_target [intro?]:
  "[ (pc', s')  $\in$  set (step pc (φ!pc));
    pc'  $\neq$  pc+1; pc < length φ; pc' < length φ ]
 $\implies$  c!pc' = φ!pc"
  <proof>

lemma (in lbvc) cert_approx [intro?]:
  "[ pc < length φ; c!pc  $\neq$  ⊥ ]
 $\implies$  c!pc = φ!pc"
  <proof>

lemma (in lbv) le_top [simp, intro]:

```

"x <=_r \top "
 <proof>

lemma (in lbv) merge_mono:
 assumes less: "ss2 <=|r| ss1"
 assumes x: "x \in A"
 assumes ss1: "snd'set ss1 \subseteq A"
 assumes ss2: "snd'set ss2 \subseteq A"
 shows "merge c pc ss2 x <=_r merge c pc ss1 x" (is "?s2 <=_r ?s1")
 <proof>

lemma (in lbvc) wti_mono:
 assumes less: "s2 <=_r s1"
 assumes pc: "pc < length φ "
 assumes s1: "s1 \in A"
 assumes s2: "s2 \in A"
 shows "wti c pc s2 <=_r wti c pc s1" (is "?s2' <=_r ?s1'")
 <proof>

lemma (in lbvc) wtc_mono:
 assumes less: "s2 <=_r s1"
 assumes pc: "pc < length φ "
 assumes s1: "s1 \in A"
 assumes s2: "s2 \in A"
 shows "wtc c pc s2 <=_r wtc c pc s1" (is "?s2' <=_r ?s1'")
 <proof>

lemma (in lbv) top_le_conv [simp]:
 " \top <=_r x = (x = \top)"
 <proof>

lemma (in lbv) neq_top [simp, elim]:
 "[x <=_r y; y \neq \top] \implies x \neq \top "
 <proof>

lemma (in lbvc) stable_wti:
 assumes stable: "stable r step φ pc"
 assumes pc: "pc < length φ "
 shows "wti c pc (φ !pc) \neq \top "
 <proof>

lemma (in lbvc) wti_less:
 assumes stable: "stable r step φ pc"
 assumes suc_pc: "Suc pc < length φ "
 shows "wti c pc (φ !pc) <=_r φ !Suc pc" (is "?wti <=_r _")
 <proof>

lemma (in lbvc) stable_wtc:
 assumes stable: "stable r step phi pc"
 assumes pc: "pc < length φ "

shows "wtc c pc ($\varphi!pc$) $\neq \top$ "
 <proof>

lemma (in lbvc) wtc_less:
 assumes stable: "stable r step φ pc"
 assumes suc_pc: "Suc pc < length φ "
 shows "wtc c pc ($\varphi!pc$) $\leq_r \varphi!Suc\ pc$ " (is "?wtc \leq_r _")
 <proof>

lemma (in lbvc) wt_step_wtl_lemma:
 assumes wt_step: "wt_step r \top step φ "
 shows " $\bigwedge pc\ s. pc + length\ ls = length\ \varphi \implies s \leq_r \varphi!pc \implies s \in A \implies s \neq \top \implies$
 wtl ls c pc s $\neq \top$ "
 (is " $\bigwedge pc\ s. _ \implies _ \implies _ \implies _ \implies ?wtl\ ls\ pc\ s \neq _$ ")
 <proof>

theorem (in lbvc) wtl_complete:
 assumes wt: "wt_step r \top step φ "
 and s: " $s \leq_r \varphi!0$ " " $s \in A$ " " $s \neq \top$ "
 and len: "length ins = length phi"
 shows "wtl ins c 0 s $\neq \top$ "
 <proof>

end

4.11 Lifting the Typing Framework to err, app, and eff

```
theory Typing_Framework_err imports Typing_Framework SemilatAlg begin
```

```
constdefs
```

```
wt_err_step :: "'s ord  $\Rightarrow$  's err step_type  $\Rightarrow$  's err list  $\Rightarrow$  bool"
"wt_err_step r step ts  $\equiv$  wt_step (Err.le r) Err step ts"
```

```
wt_app_eff :: "'s ord  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  's step_type  $\Rightarrow$  's list  $\Rightarrow$  bool"
"wt_app_eff r app step ts  $\equiv$ 
 $\forall p < \text{size } ts. \text{app } p (ts!p) \wedge (\forall (q,t) \in \text{set } (\text{step } p (ts!p)). t \leq_r ts!q)$ "
```

```
map_snd :: "('b  $\Rightarrow$  'c)  $\Rightarrow$  ('a  $\times$  'b) list  $\Rightarrow$  ('a  $\times$  'c) list"
"map_snd f  $\equiv$  map ( $\lambda(x,y). (x, f y)$ )"
```

```
error :: "nat  $\Rightarrow$  (nat  $\times$  'a err) list"
"error n  $\equiv$  map ( $\lambda x. (x, \text{Err})$ ) [0.. $n$ ]"
```

```
err_step :: "nat  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  's step_type  $\Rightarrow$  's err step_type"
"err_step n app step p t  $\equiv$ 
  case t of
    Err  $\Rightarrow$  error n
  | OK t'  $\Rightarrow$  if app p t' then map_snd OK (step p t') else error n"
```

```
app_mono :: "'s ord  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  nat  $\Rightarrow$  's set  $\Rightarrow$  bool"
"app_mono r app n A  $\equiv$ 
 $\forall s p t. s \in A \wedge p < n \wedge s \leq_r t \longrightarrow \text{app } p t \longrightarrow \text{app } p s$ "
```

```
lemmas err_step_defs = err_step_def map_snd_def error_def
```

```
lemma bounded_err_stepD:
  "bounded (err_step n app step) n  $\implies$ 
 $p < n \implies \text{app } p a \implies (q,b) \in \text{set } (\text{step } p a) \implies$ 
 $q < n$ "
  <proof>
```

```
lemma in_map_sndD: "(a,b)  $\in$  set (map_snd f xs)  $\implies \exists b'. (a,b') \in$  set xs"
  <proof>
```

```
lemma bounded_err_stepI:
  " $\forall p. p < n \longrightarrow (\forall s. \text{app } p s \longrightarrow (\forall (q,s') \in \text{set } (\text{step } p s). q < n))$ 
 $\implies \text{bounded } (\text{err_step } n \text{ app step}) n$ "
  <proof>
```

```
lemma bounded_lift:
  "bounded step n  $\implies \text{bounded } (\text{err_step } n \text{ app step}) n$ "
  <proof>
```

```

lemma le_list_map_OK [simp]:
  "∧b. map OK a <=[Err.le r] map OK b = (a <=[r] b)"
  ⟨proof⟩

```

```

lemma map_snd_lessI:
  "x <=|r| y ⇒ map_snd OK x <=|Err.le r| map_snd OK y"
  ⟨proof⟩

```

```

lemma mono_lift:
  "order r ⇒ app_mono r app n A ⇒ bounded (err_step n app step) n ⇒
  ∀s p t. s ∈ A ∧ p < n ∧ s <=_r t ⇒ app p t ⇒ step p s <=|r| step p t ⇒
  mono (Err.le r) (err_step n app step) n (err A)"
  ⟨proof⟩

```

```

lemma in_errorD:
  "(x,y) ∈ set (error n) ⇒ y = Err"
  ⟨proof⟩

```

```

lemma pres_type_lift:
  "∀s∈A. ∀p. p < n ⇒ app p s ⇒ (∀(q, s')∈set (step p s). s' ∈ A)
  ⇒ pres_type (err_step n app step) n (err A)"
  ⟨proof⟩

```

There used to be a condition here that each instruction must have a successor. This is not needed any more, because the definition of `error` trivially ensures that there is a successor for the critical case where `app` does not hold.

```

lemma wt_err_imp_wt_app_eff:
  assumes wt: "wt_err_step r (err_step (size ts) app step) ts"
  assumes b: "bounded (err_step (size ts) app step) (size ts)"
  shows "wt_app_eff r app step (map ok_val ts)"
  ⟨proof⟩

```

```

lemma wt_app_eff_imp_wt_err:
  assumes app_eff: "wt_app_eff r app step ts"
  assumes bounded: "bounded (err_step (size ts) app step) (size ts)"
  shows "wt_err_step r (err_step (size ts) app step) (map OK ts)"
  ⟨proof⟩

```

end

4.12 The Java Type System as Semilattice

theory JType imports "../J/WellForm" Err begin

constdefs

```
super :: "'a prog ⇒ cname ⇒ cname"
"super G C == fst (the (class G C))"
```

lemma superI:

```
"G ⊢ C <C1 D ⇒ super G C = D"
⟨proof⟩
```

constdefs

```
is_ref :: "ty ⇒ bool"
"is_ref T == case T of PrimT t ⇒ False | RefT r ⇒ True"

sup :: "'c prog ⇒ ty ⇒ ty ⇒ ty err"
"sup G T1 T2 ==
case T1 of PrimT P1 ⇒ (case T2 of PrimT P2 ⇒
  (if P1 = P2 then OK (PrimT P1) else Err) | RefT R ⇒ Err)
| RefT R1 ⇒ (case T2 of PrimT P ⇒ Err | RefT R2 ⇒
  (case R1 of NullT ⇒ (case R2 of NullT ⇒ OK NT | ClassT C ⇒ OK (Class C))
  | ClassT C ⇒ (case R2 of NullT ⇒ OK (Class C)
  | ClassT D ⇒ OK (Class (exec_lub (subcls1 G) (super G) C D))))))"

subtype :: "'c prog ⇒ ty ⇒ ty ⇒ bool"
"subtype G T1 T2 == G ⊢ T1 ≤ T2"

is_ty :: "'c prog ⇒ ty ⇒ bool"
"is_ty G T == case T of PrimT P ⇒ True | RefT R ⇒
  (case R of NullT ⇒ True | ClassT C ⇒ (subcls1 G)^** C Object)"
```

translations

```
"types G" == "Collect (is_type G)"
```

constdefs

```
esl :: "'c prog ⇒ ty esl"
"esl G == (types G, subtype G, sup G)"
```

lemma PrimT_PrimT: "(G ⊢ xb ≤ PrimT p) = (xb = PrimT p)"
 ⟨proof⟩

lemma PrimT_PrimT2: "(G ⊢ PrimT p ≤ xb) = (xb = PrimT p)"
 ⟨proof⟩

lemma is_tyI:

```
"[ is_type G T; ws_prog G ] ⇒ is_ty G T"
⟨proof⟩
```

lemma is_type_conv:

```
"ws_prog G ⇒ is_type G T = is_ty G T"
⟨proof⟩
```

```

lemma order_widen:
  "acyclicP (subcls1 G)  $\implies$  order (subtype G)"
  <proof>

lemma wf_converse_subcls1_impl_acc_subtype:
  "wfP ((subcls1 G)--1)  $\implies$  acc (subtype G)"
  <proof>

lemma closed_err_types:
  "[[ ws_prog G; single_valuedP (subcls1 G); acyclicP (subcls1 G) ] ]
   $\implies$  closed (err (types G)) (lift2 (sup G))"
  <proof>

lemma sup_subtype_greater:
  "[[ ws_prog G; single_valuedP (subcls1 G); acyclicP (subcls1 G);
    is_type G t1; is_type G t2; sup G t1 t2 = OK s ] ]
   $\implies$  subtype G t1 s  $\wedge$  subtype G t2 s"
  <proof>

lemma sup_subtype_smallest:
  "[[ ws_prog G; single_valuedP (subcls1 G); acyclicP (subcls1 G);
    is_type G a; is_type G b; is_type G c;
    subtype G a c; subtype G b c; sup G a b = OK d ] ]
   $\implies$  subtype G d c"
  <proof>

lemma sup_exists:
  "[[ subtype G a c; subtype G b c; sup G a b = Err ] ]  $\implies$  False"
  <proof>

lemma err_semilat_JType_esl_lemma:
  "[[ ws_prog G; single_valuedP (subcls1 G); acyclicP (subcls1 G) ] ]
   $\implies$  err_semilat (esl G)"
  <proof>

lemma single_valued_subcls1:
  "ws_prog G  $\implies$  single_valuedP (subcls1 G)"
  <proof>

theorem err_semilat_JType_esl:
  "ws_prog G  $\implies$  err_semilat (esl G)"
  <proof>

end

```

4.13 The JVM Type System as Semilattice

theory JVMType imports Opt Product Listn JType begin

types

```
locvars_type = "ty err list"
opstack_type = "ty list"
state_type   = "opstack_type × locvars_type"
state        = "state_type option err"    — for Kildall
method_type  = "state_type option list"   — for BVSpec
class_type   = "sig ⇒ method_type"
prog_type    = "cname ⇒ class_type"
```

constdefs

```
stk_esl :: "'c prog ⇒ nat ⇒ ty list esl"
"stk_esl S maxs == upto_esl maxs (JType.esl S)"

reg_sl :: "'c prog ⇒ nat ⇒ ty err list sl"
"reg_sl S maxr == Listn.sl maxr (Err.sl (JType.esl S))"

sl :: "'c prog ⇒ nat ⇒ nat ⇒ state sl"
"sl S maxs maxr ==
Err.sl(Opt.esl(Product.esl (stk_esl S maxs) (Err.esl(reg_sl S maxr))))"
```

constdefs

```
states :: "'c prog ⇒ nat ⇒ nat ⇒ state set"
"states S maxs maxr == fst(sl S maxs maxr)"

le :: "'c prog ⇒ nat ⇒ nat ⇒ state ord"
"le S maxs maxr == fst(snd(sl S maxs maxr))"

sup :: "'c prog ⇒ nat ⇒ nat ⇒ state binop"
"sup S maxs maxr == snd(snd(sl S maxs maxr))"
```

constdefs

```
sup_ty_opt :: "[code prog, ty err, ty err] ⇒ bool"
(" _ |- _ <=o _" [71,71] 70)
"sup_ty_opt G == Err.le (subtype G)"

sup_loc :: "[code prog, locvars_type, locvars_type] ⇒ bool"
(" _ |- _ <=l _" [71,71] 70)
"sup_loc G == Listn.le (sup_ty_opt G)"

sup_state :: "[code prog, state_type, state_type] ⇒ bool"
(" _ |- _ <=s _" [71,71] 70)
"sup_state G == Product.le (Listn.le (subtype G)) (sup_loc G)"

sup_state_opt :: "[code prog, state_type option, state_type option] ⇒ bool"
(" _ |- _ <= ' _" [71,71] 70)
"sup_state_opt G == Opt.le (sup_state G)"
```

```

syntax (xsymbols)
  sup_ty_opt    :: "[code prog,ty err,ty err] ⇒ bool"
                  ("_ ⊢ _ <=o _" [71,71] 70)
  sup_loc       :: "[code prog,locvars_type,locvars_type] ⇒ bool"
                  ("_ ⊢ _ <=l _" [71,71] 70)
  sup_state     :: "[code prog,state_type,state_type] ⇒ bool"
                  ("_ ⊢ _ <=s _" [71,71] 70)
  sup_state_opt :: "[code prog,state_type option,state_type option] ⇒ bool"
                  ("_ ⊢ _ <=' _" [71,71] 70)

lemma JVM_states_unfold:
  "states S maxs maxr == err(opt((Union {list n (types S) |n. n <= maxs}) <*>
    list maxr (err(types S)))))"
  <proof>

lemma JVM_le_unfold:
  "le S m n ==
    Err.le (Opt.le (Product.le (Listn.le (subtype S)) (Listn.le (Err.le (subtype S)))))"
  <proof>

lemma JVM_le_convert:
  "le G m n (OK t1) (OK t2) = G ⊢ t1 <=' t2"
  <proof>

lemma JVM_le_Err_conv:
  "le G m n = Err.le (sup_state_opt G)"
  <proof>

lemma zip_map [rule_format]:
  "∀a. length a = length b ⟶
    zip (map f a) (map g b) = map (λ(x,y). (f x, g y)) (zip a b)"
  <proof>

lemma [simp]: "Err.le r (OK a) (OK b) = r a b"
  <proof>

lemma stk_convert:
  "Listn.le (subtype G) a b = G ⊢ map OK a <=l map OK b"
  <proof>

lemma sup_state_conv:
  "(G ⊢ s1 <=s s2) ==
    (G ⊢ map OK (fst s1) <=l map OK (fst s2)) ∧ (G ⊢ snd s1 <=l snd s2)"
  <proof>

lemma subtype_refl [simp]:
  "subtype G t t"
  <proof>

theorem sup_ty_opt_refl [simp]:

```

" $G \vdash t \leq_o t$ "
 $\langle proof \rangle$

lemma `le_list_refl2 [simp]:`
 " $(\bigwedge xs. r \ xs \ xs) \implies Listn.le \ r \ xs \ xs$ "
 $\langle proof \rangle$

theorem `sup_loc_refl [simp]:`
 " $G \vdash t \leq_l t$ "
 $\langle proof \rangle$

theorem `sup_state_refl [simp]:`
 " $G \vdash s \leq_s s$ "
 $\langle proof \rangle$

theorem `sup_state_opt_refl [simp]:`
 " $G \vdash s \leq' s$ "
 $\langle proof \rangle$

theorem `anyConvErr [simp]:`
 " $(G \vdash Err \leq_o any) = (any = Err)$ "
 $\langle proof \rangle$

theorem `OKanyConvOK [simp]:`
 " $(G \vdash (OK \ ty') \leq_o (OK \ ty)) = (G \vdash ty' \preceq ty)$ "
 $\langle proof \rangle$

theorem `sup_ty_opt_OK:`
 " $G \vdash a \leq_o (OK \ b) \implies \exists x. a = OK \ x$ "
 $\langle proof \rangle$

lemma `widen_PrimT_conv1 [simp]:`
 " $\llbracket G \vdash S \preceq T; S = PrimT \ x \rrbracket \implies T = PrimT \ x$ "
 $\langle proof \rangle$

theorem `sup_PTS_eq:`
 " $(G \vdash OK \ (PrimT \ p) \leq_o X) = (X=Err \vee X = OK \ (PrimT \ p))$ "
 $\langle proof \rangle$

theorem `sup_loc_Nil [iff]:`
 " $(G \vdash [] \leq_l XT) = (XT=[])$ "
 $\langle proof \rangle$

theorem `sup_loc_Cons [iff]:`
 " $(G \vdash (Y\#YT) \leq_l XT) = (\exists X \ XT'. XT=X\#XT' \wedge (G \vdash Y \leq_o X) \wedge (G \vdash YT \leq_l XT'))$ "
 $\langle proof \rangle$

theorem `sup_loc_Cons2:`
 " $(G \vdash YT \leq_l (X\#XT)) = (\exists Y \ YT'. YT=Y\#YT' \wedge (G \vdash Y \leq_o X) \wedge (G \vdash YT' \leq_l XT))$ "
 $\langle proof \rangle$

lemma `sup_state_Cons:`
 " $(G \vdash (x\#xt, a) \leq_s (y\#yt, b)) =$

$((G \vdash x \preceq y) \wedge (G \vdash (xt, a) \leq_s (yt, b)))$
 $\langle proof \rangle$

theorem *sup_loc_length*:

" $G \vdash a \leq_l b \implies \text{length } a = \text{length } b$ "
 $\langle proof \rangle$

theorem *sup_loc_nth*:

" $\llbracket G \vdash a \leq_l b; n < \text{length } a \rrbracket \implies G \vdash (a!n) \leq_o (b!n)$ "
 $\langle proof \rangle$

theorem *all_nth_sup_loc*:

" $\forall b. \text{length } a = \text{length } b \longrightarrow (\forall n. n < \text{length } a \longrightarrow (G \vdash (a!n) \leq_o (b!n)))$
 $\longrightarrow (G \vdash a \leq_l b)"$ (is "?P a")
 $\langle proof \rangle$

theorem *sup_loc_append*:

" $\text{length } a = \text{length } b \implies$
 $(G \vdash (a@x) \leq_l (b@y)) = ((G \vdash a \leq_l b) \wedge (G \vdash x \leq_l y))"$
 $\langle proof \rangle$

theorem *sup_loc_rev [simp]*:

" $(G \vdash (\text{rev } a) \leq_l \text{rev } b) = (G \vdash a \leq_l b)"$
 $\langle proof \rangle$

theorem *sup_loc_update [rule_format]*:

" $\forall n y. (G \vdash a \leq_o b) \longrightarrow n < \text{length } y \longrightarrow (G \vdash x \leq_l y) \longrightarrow$
 $(G \vdash x[n := a] \leq_l y[n := b])"$ (is "?P x")
 $\langle proof \rangle$

theorem *sup_state_length [simp]*:

" $G \vdash s2 \leq_s s1 \implies$
 $\text{length } (\text{fst } s2) = \text{length } (\text{fst } s1) \wedge \text{length } (\text{snd } s2) = \text{length } (\text{snd } s1)"$
 $\langle proof \rangle$

theorem *sup_state_append_snd*:

" $\text{length } a = \text{length } b \implies$
 $(G \vdash (i, a@x) \leq_s (j, b@y)) = ((G \vdash (i, a) \leq_s (j, b)) \wedge (G \vdash (i, x) \leq_s (j, y)))"$
 $\langle proof \rangle$

theorem *sup_state_append_fst*:

" $\text{length } a = \text{length } b \implies$
 $(G \vdash (a@x, i) \leq_s (b@y, j)) = ((G \vdash (a, i) \leq_s (b, j)) \wedge (G \vdash (x, i) \leq_s (y, j)))"$
 $\langle proof \rangle$

theorem *sup_state_Cons1*:

" $(G \vdash (x\#xt, a) \leq_s (yt, b)) =$
 $(\exists y yt'. yt = y\#yt' \wedge (G \vdash x \preceq y) \wedge (G \vdash (xt, a) \leq_s (yt', b)))"$
 $\langle proof \rangle$

theorem *sup_state_Cons2*:

"($G \vdash (xt, a) \leq_s (y\#yt, b)$) =
 $(\exists x \, xt'. \, xt=x\#xt' \wedge (G \vdash x \preceq y) \wedge (G \vdash (xt', a) \leq_s (yt, b)))$ "
 ⟨proof⟩

theorem *sup_state_ignore_fst*:

" $G \vdash (a, x) \leq_s (b, y) \implies G \vdash (c, x) \leq_s (c, y)$ "
 ⟨proof⟩

theorem *sup_state_rev_fst*:

"($G \vdash (\text{rev } a, x) \leq_s (\text{rev } b, y)$) = ($G \vdash (a, x) \leq_s (b, y)$)"
 ⟨proof⟩

lemma *sup_state_opt_None_any [iff]*:

"($G \vdash \text{None} \leq' \text{any}$) = True"
 ⟨proof⟩

lemma *sup_state_opt_any_None [iff]*:

"($G \vdash \text{any} \leq' \text{None}$) = ($\text{any} = \text{None}$)"
 ⟨proof⟩

lemma *sup_state_opt_Some_Some [iff]*:

"($G \vdash (\text{Some } a) \leq' (\text{Some } b)$) = ($G \vdash a \leq_s b$)"
 ⟨proof⟩

lemma *sup_state_opt_any_Some [iff]*:

"($G \vdash (\text{Some } a) \leq' \text{any}$) = ($\exists b. \, \text{any} = \text{Some } b \wedge G \vdash a \leq_s b$)"
 ⟨proof⟩

lemma *sup_state_opt_Some_any*:

"($G \vdash \text{any} \leq' (\text{Some } b)$) = ($\text{any} = \text{None} \vee (\exists a. \, \text{any} = \text{Some } a \wedge G \vdash a \leq_s b)$)"
 ⟨proof⟩

theorem *sup_ty_opt_trans [trans]*:

"($\llbracket G \vdash a \leq_o b; G \vdash b \leq_o c \rrbracket \implies G \vdash a \leq_o c$)"
 ⟨proof⟩

theorem *sup_loc_trans [trans]*:

"($\llbracket G \vdash a \leq_l b; G \vdash b \leq_l c \rrbracket \implies G \vdash a \leq_l c$)"
 ⟨proof⟩

theorem *sup_state_trans [trans]*:

"($\llbracket G \vdash a \leq_s b; G \vdash b \leq_s c \rrbracket \implies G \vdash a \leq_s c$)"
 ⟨proof⟩

theorem *sup_state_opt_trans [trans]*:

"($\llbracket G \vdash a \leq' b; G \vdash b \leq' c \rrbracket \implies G \vdash a \leq' c$)"
 ⟨proof⟩

end

4.14 Effect of Instructions on the State Type

```

theory Effect
imports JVMType "../JVM/JVMExceptions"
begin

types
  succ_type = "(p_count × state_type option) list"

```

Program counter of successor instructions:

```

consts
  succs :: "instr ⇒ p_count ⇒ p_count list"
primrec
  "succs (Load idx) pc          = [pc+1]"
  "succs (Store idx) pc         = [pc+1]"
  "succs (LitPush v) pc         = [pc+1]"
  "succs (Getfield F C) pc      = [pc+1]"
  "succs (Putfield F C) pc      = [pc+1]"
  "succs (New C) pc             = [pc+1]"
  "succs (Checkcast C) pc       = [pc+1]"
  "succs Pop pc                 = [pc+1]"
  "succs Dup pc                 = [pc+1]"
  "succs Dup_x1 pc              = [pc+1]"
  "succs Dup_x2 pc              = [pc+1]"
  "succs Swap pc                = [pc+1]"
  "succs IAdd pc                = [pc+1]"
  "succs (Ifcmpeq b) pc         = [pc+1, nat (int pc + b)]"
  "succs (Goto b) pc            = [nat (int pc + b)]"
  "succs Return pc              = [pc]"
  "succs (Invoke C mn fpTs) pc = [pc+1]"
  "succs Throw pc               = [pc]"

```

Effect of instruction on the state type:

```

consts
  eff' :: "instr × jvm_prog × state_type ⇒ state_type"

recdef eff' "{}"
  "eff' (Load idx, G, (ST, LT))          = (ok_val (LT ! idx) # ST, LT)"
  "eff' (Store idx, G, (ts#ST, LT))       = (ST, LT[idx:= OK ts])"
  "eff' (LitPush v, G, (ST, LT))          = (the (typeof (λv. None) v) # ST, LT)"
  "eff' (Getfield F C, G, (oT#ST, LT))    = (snd (the (field (G,C) F)) # ST, LT)"
  "eff' (Putfield F C, G, (vT#oT#ST, LT)) = (ST,LT)"
  "eff' (New C, G, (ST,LT))               = (Class C # ST, LT)"
  "eff' (Checkcast C, G, (RefT rt#ST,LT)) = (Class C # ST,LT)"
  "eff' (Pop, G, (ts#ST,LT))              = (ST,LT)"
  "eff' (Dup, G, (ts#ST,LT))              = (ts#ts#ST,LT)"
  "eff' (Dup_x1, G, (ts1#ts2#ST,LT))      = (ts1#ts2#ts1#ST,LT)"
  "eff' (Dup_x2, G, (ts1#ts2#ts3#ST,LT))  = (ts1#ts2#ts3#ts1#ST,LT)"
  "eff' (Swap, G, (ts1#ts2#ST,LT))        = (ts2#ts1#ST,LT)"
  "eff' (IAdd, G, (PrimT Integer#PrimT Integer#ST,LT))
                                         = (PrimT Integer#ST,LT)"
  "eff' (Ifcmpeq b, G, (ts1#ts2#ST,LT))   = (ST,LT)"
  "eff' (Goto b, G, s)                    = s"

```


— Return has no successor instruction in the same method

```
"eff' (Return, G, s) = s"
```

— Throw always terminates abruptly

```
"eff' (Throw, G, s) = s"
```

```
"eff' (Invoke C mn fpTs, G, (ST,LT)) = (let ST' = drop (length fpTs) ST
  in (fst (snd (the (method (G,C) (mn,fpTs))))#(tl ST'),LT))"
```

consts

```
match_any :: "jvm_prog ⇒ p_count ⇒ exception_table ⇒ cname list"
```

primrec

```
"match_any G pc [] = []"
```

```
"match_any G pc (e#es) = (let (start_pc, end_pc, handler_pc, catch_type) = e;
  es' = match_any G pc es
  in
  if start_pc <= pc ∧ pc < end_pc then catch_type#es' else es')"
```

consts

```
match :: "jvm_prog ⇒ xcpt ⇒ p_count ⇒ exception_table ⇒ cname list"
```

primrec

```
"match G X pc [] = []"
```

```
"match G X pc (e#es) =
  (if match_exception_entry G (Xcpt X) pc e then [Xcpt X] else match G X pc es)"
```

lemma match_some_entry:

```
"match G X pc et = (if ∃e ∈ set et. match_exception_entry G (Xcpt X) pc e then [Xcpt
X] else [])"
⟨proof⟩
```

consts

```
xcpt_names :: "instr × jvm_prog × p_count × exception_table ⇒ cname list"
```

recdef xcpt_names "{}

```
"xcpt_names (Getfield F C, G, pc, et) = match G NullPointer pc et"
"xcpt_names (Putfield F C, G, pc, et) = match G NullPointer pc et"
"xcpt_names (New C, G, pc, et) = match G OutOfMemory pc et"
"xcpt_names (Checkcast C, G, pc, et) = match G ClassCast pc et"
"xcpt_names (Throw, G, pc, et) = match_any G pc et"
"xcpt_names (Invoke C m p, G, pc, et) = match_any G pc et"
"xcpt_names (i, G, pc, et) = []"
```

constdefs

```
xcpt_eff :: "instr ⇒ jvm_prog ⇒ p_count ⇒ state_type option ⇒ exception_table ⇒
succ_type"
```

```
"xcpt_eff i G pc s et ==
```

```
  map (λC. (the (match_exception_table G C pc et), case s of None ⇒ None | Some s' ⇒
Some ([Class C], snd s'))))
  (xcpt_names (i,G,pc,et))"
```

```
norm_eff :: "instr ⇒ jvm_prog ⇒ state_type option ⇒ state_type option"
```

```
"norm_eff i G == option_map (λs. eff' (i,G,s))"
```

```
eff :: "instr ⇒ jvm_prog ⇒ p_count ⇒ exception_table ⇒ state_type option ⇒ succ_type"
```

```
"eff i G pc et s == (map (λpc'. (pc',norm_eff i G s)) (succs i pc)) @ (xcpt_eff i G
```

```
pc s et)"
```

```
constdefs
```

```
  isPrimT :: "ty  $\Rightarrow$  bool"
  "isPrimT T == case T of PrimT T'  $\Rightarrow$  True | RefT T'  $\Rightarrow$  False"
```

```
  isRefT :: "ty  $\Rightarrow$  bool"
  "isRefT T == case T of PrimT T'  $\Rightarrow$  False | RefT T'  $\Rightarrow$  True"
```

```
lemma isPrimT [simp]:
```

```
  "isPrimT T = ( $\exists T'$ . T = PrimT T')"  $\langle$ proof $\rangle$ 
```

```
lemma isRefT [simp]:
```

```
  "isRefT T = ( $\exists T'$ . T = RefT T')"  $\langle$ proof $\rangle$ 
```

```
lemma "list_all2 P a b  $\implies \forall (x,y) \in \text{set } (\text{zip } a \text{ } b). P \ x \ y"$ 
 $\langle$ proof $\rangle$ 
```

Conditions under which eff is applicable:

```
consts
```

```
app' :: "instr  $\times$  jvm_prog  $\times$  p_count  $\times$  nat  $\times$  ty  $\times$  state_type  $\Rightarrow$  bool"
```

```
recdef app' "{}"
```

```
"app' (Load idx, G, pc, maxs, rT, s) =
  (idx < length (snd s)  $\wedge$  (snd s) ! idx  $\neq$  Err  $\wedge$  length (fst s) < maxs)"
"app' (Store idx, G, pc, maxs, rT, (ts#ST, LT)) =
  (idx < length LT)"
"app' (LitPush v, G, pc, maxs, rT, s) =
  (length (fst s) < maxs  $\wedge$  typeof ( $\lambda t.$  None) v  $\neq$  None)"
"app' (Getfield F C, G, pc, maxs, rT, (oT#ST, LT)) =
  (is_class G C  $\wedge$  field (G,C) F  $\neq$  None  $\wedge$  fst (the (field (G,C) F)) = C  $\wedge$ 
  G  $\vdash$  oT  $\preceq$  (Class C))"
"app' (Putfield F C, G, pc, maxs, rT, (vT#oT#ST, LT)) =
  (is_class G C  $\wedge$  field (G,C) F  $\neq$  None  $\wedge$  fst (the (field (G,C) F)) = C  $\wedge$ 
  G  $\vdash$  oT  $\preceq$  (Class C)  $\wedge$  G  $\vdash$  vT  $\preceq$  (snd (the (field (G,C) F))))"
"app' (New C, G, pc, maxs, rT, s) =
  (is_class G C  $\wedge$  length (fst s) < maxs)"
"app' (Checkcast C, G, pc, maxs, rT, (RefT rt#ST,LT)) =
  (is_class G C)"
"app' (Pop, G, pc, maxs, rT, (ts#ST,LT)) =
  True"
"app' (Dup, G, pc, maxs, rT, (ts#ST,LT)) =
  (1+length ST < maxs)"
"app' (Dup_x1, G, pc, maxs, rT, (ts1#ts2#ST,LT)) =
  (2+length ST < maxs)"
"app' (Dup_x2, G, pc, maxs, rT, (ts1#ts2#ts3#ST,LT)) =
  (3+length ST < maxs)"
"app' (Swap, G, pc, maxs, rT, (ts1#ts2#ST,LT)) =
  True"
"app' (IAdd, G, pc, maxs, rT, (PrimT Integer#PrimT Integer#ST,LT)) =
  True"
"app' (Ifcmpeq b, G, pc, maxs, rT, (ts#ts'#ST,LT)) =
  (0  $\leq$  int pc + b  $\wedge$  (isPrimT ts  $\wedge$  ts' = ts  $\vee$  isRefT ts  $\wedge$  isRefT ts'))"
```

```

"app' (Goto b, G, pc, maxs, rT, s) =
  (0 ≤ int pc + b)"
"app' (Return, G, pc, maxs, rT, (T#ST,LT)) =
  (G ⊢ T ≤ rT)"
"app' (Throw, G, pc, maxs, rT, (T#ST,LT)) =
  isRefT T"
"app' (Invoke C mn fpTs, G, pc, maxs, rT, s) =
  (length fpTs < length (fst s) ∧
   (let apTs = rev (take (length fpTs) (fst s));
      X      = hd (drop (length fpTs) (fst s))
   in
    G ⊢ X ≤ Class C ∧ is_class G C ∧ method (G,C) (mn,fpTs) ≠ None ∧
    list_all2 (λx y. G ⊢ x ≤ y) apTs fpTs)))

"app' (i,G, pc,maxs,rT,s) = False"

constdefs
  xcpt_app :: "instr ⇒ jvm_prog ⇒ nat ⇒ exception_table ⇒ bool"
  "xcpt_app i G pc et ≡ ∀C∈set(xcpt_names (i,G,pc,et)). is_class G C"

  app :: "instr ⇒ jvm_prog ⇒ nat ⇒ ty ⇒ nat ⇒ exception_table ⇒ state_type option
⇒ bool"
  "app i G maxs rT pc et s == case s of None ⇒ True | Some t ⇒ app' (i,G,pc,maxs,rT,t)
  ∧ xcpt_app i G pc et"

lemma match_any_match_table:
  "C ∈ set (match_any G pc et) ⇒ match_exception_table G C pc et ≠ None"
  ⟨proof⟩

lemma match_X_match_table:
  "C ∈ set (match G X pc et) ⇒ match_exception_table G C pc et ≠ None"
  ⟨proof⟩

lemma xcpt_names_in_et:
  "C ∈ set (xcpt_names (i,G,pc,et)) ⇒
  ∃e ∈ set et. the (match_exception_table G C pc et) = fst (snd (snd e))"
  ⟨proof⟩

lemma 1: "2 < length a ⇒ (∃ l l' l'' ls. a = l#l'#l''#ls)"
  ⟨proof⟩

lemma 2: "¬(2 < length a) ⇒ a = [] ∨ (∃ l. a = [l]) ∨ (∃ l l'. a = [l,l'])"
  ⟨proof⟩

lemmas [simp] = app_def xcpt_app_def

simp rules for app

lemma appNone[simp]: "app i G maxs rT pc et None = True" ⟨proof⟩

lemma appLoad[simp]:

```

```

"(app (Load idx) G maxs rT pc et (Some s)) = ( $\exists$  ST LT. s = (ST,LT)  $\wedge$  idx < length LT  $\wedge$ 
LT!idx  $\neq$  Err  $\wedge$  length ST < maxs)"
  <proof>

```

```

lemma appStore[simp]:

```

```

"(app (Store idx) G maxs rT pc et (Some s)) = ( $\exists$  ts ST LT. s = (ts#ST,LT)  $\wedge$  idx < length
LT)"
  <proof>

```

```

lemma appLitPush[simp]:

```

```

"(app (LitPush v) G maxs rT pc et (Some s)) = ( $\exists$  ST LT. s = (ST,LT)  $\wedge$  length ST < maxs
 $\wedge$  typeof ( $\lambda$ v. None) v  $\neq$  None)"
  <proof>

```

```

lemma appGetField[simp]:

```

```

"(app (Getfield F C) G maxs rT pc et (Some s)) =
( $\exists$  oT vT ST LT. s = (oT#ST, LT)  $\wedge$  is_class G C  $\wedge$ 
  field (G,C) F = Some (C,vT)  $\wedge$  G  $\vdash$  oT  $\preceq$  (Class C)  $\wedge$  ( $\forall$  x  $\in$  set (match G NullPointer
pc et). is_class G x))"
  <proof>

```

```

lemma appPutField[simp]:

```

```

"(app (Putfield F C) G maxs rT pc et (Some s)) =
( $\exists$  vT vT' oT ST LT. s = (vT#oT#ST, LT)  $\wedge$  is_class G C  $\wedge$ 
  field (G,C) F = Some (C, vT')  $\wedge$  G  $\vdash$  oT  $\preceq$  (Class C)  $\wedge$  G  $\vdash$  vT  $\preceq$  vT'  $\wedge$ 
  ( $\forall$  x  $\in$  set (match G NullPointer pc et). is_class G x))"
  <proof>

```

```

lemma appNew[simp]:

```

```

"(app (New C) G maxs rT pc et (Some s)) =
( $\exists$  ST LT. s=(ST,LT)  $\wedge$  is_class G C  $\wedge$  length ST < maxs  $\wedge$ 
  ( $\forall$  x  $\in$  set (match G OutOfMemory pc et). is_class G x))"
  <proof>

```

```

lemma appCheckcast[simp]:

```

```

"(app (Checkcast C) G maxs rT pc et (Some s)) =
( $\exists$  rT ST LT. s = (RefT rT#ST,LT)  $\wedge$  is_class G C  $\wedge$ 
  ( $\forall$  x  $\in$  set (match G ClassCast pc et). is_class G x))"
  <proof>

```

```

lemma appPop[simp]:

```

```

"(app Pop G maxs rT pc et (Some s)) = ( $\exists$  ts ST LT. s = (ts#ST,LT))"
  <proof>

```

```

lemma appDup[simp]:

```

```

"(app Dup G maxs rT pc et (Some s)) = ( $\exists$  ts ST LT. s = (ts#ST,LT)  $\wedge$  1+length ST < maxs)"
  <proof>

```

```

lemma appDup_x1[simp]:

```

```

"(app Dup_x1 G maxs rT pc et (Some s)) = ( $\exists$  ts1 ts2 ST LT. s = (ts1#ts2#ST,LT)  $\wedge$  2+length
ST < maxs)"

```

$\langle proof \rangle$

lemma appDup_x2[simp]:

"(app Dup_x2 G maxs rT pc et (Some s)) = (\exists ts1 ts2 ts3 ST LT. s = (ts1#ts2#ts3#ST,LT))
 \wedge 3+length ST < maxs)"
 $\langle proof \rangle$

lemma appSwap[simp]:

"app Swap G maxs rT pc et (Some s) = (\exists ts1 ts2 ST LT. s = (ts1#ts2#ST,LT))"
 $\langle proof \rangle$

lemma appIAdd[simp]:

"app IAdd G maxs rT pc et (Some s) = (\exists ST LT. s = (PrimT Integer#PrimT Integer#ST,LT))"
 (is "?app s = ?P s")
 $\langle proof \rangle$

lemma appIfcmpeq[simp]:

"app (Ifcmpeq b) G maxs rT pc et (Some s) =
 (\exists ts1 ts2 ST LT. s = (ts1#ts2#ST,LT) \wedge 0 \leq int pc + b \wedge
 ($(\exists$ p. ts1 = PrimT p \wedge ts2 = PrimT p) \vee (\exists r r'. ts1 = RefT r \wedge ts2 = RefT r')))"
 $\langle proof \rangle$

lemma appReturn[simp]:

"app Return G maxs rT pc et (Some s) = (\exists T ST LT. s = (T#ST,LT) \wedge (G \vdash T \preceq rT))"
 $\langle proof \rangle$

lemma appGoto[simp]:

"app (Goto b) G maxs rT pc et (Some s) = (0 \leq int pc + b)"
 $\langle proof \rangle$

lemma appThrow[simp]:

"app Throw G maxs rT pc et (Some s) =
 (\exists T ST LT r. s=(T#ST,LT) \wedge T = RefT r \wedge (\forall C \in set (match_any G pc et). is_class G C))"
 $\langle proof \rangle$

lemma appInvoke[simp]:

"app (Invoke C mn fpTs) G maxs rT pc et (Some s) = (\exists apTs X ST LT mD' rT' b'.
 s = ((rev apTs) @ (X # ST), LT) \wedge length apTs = length fpTs \wedge is_class G C \wedge
 G \vdash X \preceq Class C \wedge (\forall (aT,fT) \in set(zip apTs fpTs). G \vdash aT \preceq fT) \wedge
 method (G,C) (mn,fpTs) = Some (mD', rT', b') \wedge
 (\forall C \in set (match_any G pc et). is_class G C))" (is "?app s = ?P s")
 $\langle proof \rangle$

lemma effNone:

"(pc', s') \in set (eff i G pc et None) \implies s' = None"
 $\langle proof \rangle$

lemma xcpt_app_lemma [code]:

```

"xcpt_app i G pc et = list_all (is_class G) (xcpt_names (i, G, pc, et))"
⟨proof⟩

lemmas [simp del] = app_def xcpt_app_def

end

```

4.15 Monotonicity of eff and app

theory EffectMono imports Effect begin

lemma PrimT_PrimT: "(G ⊢ xb ≤ PrimT p) = (xb = PrimT p)"
 <proof>

lemma sup_loc_some [rule_format]:
 "∀ y n. (G ⊢ b ≤_l y) ⟶ n < length y ⟶ y!n = OK t ⟶
 (∃ t. b!n = OK t ∧ (G ⊢ (b!n) ≤_o (y!n)))" (is "?P b")
 <proof>

lemma all_widen_is_sup_loc:
 "∀ b. length a = length b ⟶
 (∀ (x, y) ∈ set (zip a b). G ⊢ x ≤ y) = (G ⊢ (map OK a) ≤_l (map OK b))"
 (is "∀ b. length a = length b ⟶ ?Q a b" is "?P a")
 <proof>

lemma append_length_n [rule_format]:
 "∀ n. n ≤ length x ⟶ (∃ a b. x = a@b ∧ length a = n)" (is "?P x")
 <proof>

lemma rev_append_cons:
 "n < length x ⟹ ∃ a b c. x = (rev a) @ b # c ∧ length a = n"
 <proof>

lemma sup_loc_length_map:
 "G ⊢ map f a ≤_l map g b ⟹ length a = length b"
 <proof>

lemmas [iff] = not_Err_eq

lemma app_mono:
 "[[G ⊢ s ≤' s'; app i G m rT pc et s'] ⟹ app i G m rT pc et s]"
 <proof>

lemmas [simp del] = split_paired_Ex

lemma eff'_mono:
 "[[app i G m rT pc et (Some s2); G ⊢ s1 ≤_s s2] ⟹
 G ⊢ eff' (i, G, s1) ≤_s eff' (i, G, s2)]"
 <proof>

lemmas [iff del] = not_Err_eq

end

4.16 The Bytecode Verifier

theory *BVSpec* imports *Effect* begin

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

constdefs

— The program counter will always be inside the method:

```
check_bounded :: "instr list  $\Rightarrow$  exception_table  $\Rightarrow$  bool"
"check_bounded ins et  $\equiv$ 
( $\forall pc < \text{length ins. } \forall pc' \in \text{set (succs (ins!pc) pc). } pc' < \text{length ins}) \wedge$ 
( $\forall e \in \text{set et. } \text{fst (snd (snd e))} < \text{length ins})"$ 
```

— The method type only contains declared classes:

```
check_types :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  JVMType.state list  $\Rightarrow$  bool"
"check_types G mxs mxr phi  $\equiv \text{set phi} \subseteq \text{states } G \text{ mxs mxr}"$ 
```

— An instruction is welltyped if it is applicable and its effect

— is compatible with the type at all successor instructions:

```
wt_instr :: "[instr, jvm_prog, ty, method_type, nat, p_count,
             exception_table, p_count]  $\Rightarrow$  bool"
"wt_instr i G rT phi mxs max_pc et pc  $\equiv$ 
app i G mxs rT pc et (phi!pc)  $\wedge$ 
( $\forall (pc', s') \in \text{set (eff i G pc et (phi!pc)). } pc' < \text{max\_pc} \wedge G \vdash s' \leq' \text{phi!pc'})"$ 
```

— The type at $pc=0$ conforms to the method calling convention:

```
wt_start :: "[jvm_prog, cname, ty list, nat, method_type]  $\Rightarrow$  bool"
"wt_start G C pTs mxl phi ==
G  $\vdash \text{Some } ([], (\text{OK } (\text{Class } C))\#((\text{map OK pTs}))@(\text{replicate mxl Err})) \leq' \text{phi!0}"$ 
```

— A method is welltyped if the body is not empty, if execution does not

— leave the body, if the method type covers all instructions and mentions

— declared classes only, if the method calling convention is respected, and

— if all instructions are welltyped.

```
wt_method :: "[jvm_prog, cname, ty list, ty, nat, nat, instr list,
              exception_table, method_type]  $\Rightarrow$  bool"
"wt_method G C pTs rT mxs mxl ins et phi  $\equiv$ 
let max_pc = length ins in
0 < max_pc  $\wedge$ 
length phi = length ins  $\wedge$ 
check_bounded ins et  $\wedge$ 
check_types G mxs (1+length pTs+mxl) (map OK phi)  $\wedge$ 
wt_start G C pTs mxl phi  $\wedge$ 
( $\forall pc. pc < \text{max\_pc} \longrightarrow \text{wt\_instr (ins!pc) } G \text{ rT phi mxs max\_pc et pc})"$ 
```

— A program is welltyped if it is wellformed and all methods are welltyped

```
wt_jvm_prog :: "[jvm_prog, prog_type]  $\Rightarrow$  bool"
"wt_jvm_prog G phi ==
wf_prog ( $\lambda G \ C \ (\text{sig}, rT, (\text{maxs}, \text{maxl}, b, \text{et})).$ 
wt_method G C (snd sig) rT maxs maxl b et (phi C sig)) G"
```

lemma *check_boundedD*:

```
" $\llbracket \text{check\_bounded ins et; } pc < \text{length ins;}$ 
```



```

    (pc', s') ∈ set (eff (ins!pc) G pc et s)  ]  ⇒
pc' < length ins"
⟨proof⟩

```

```

lemma wt_jvm_progD:
  "wt_jvm_prog G phi ⇒ (∃ wt. wf_prog wt G)"
⟨proof⟩

```

```

lemma wt_jvm_prog_impl_wt_instr:
  "[ wt_jvm_prog G phi; is_class G C;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et); pc < length ins ]
  ⇒ wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc"
⟨proof⟩

```

We could leave out the check $pc' < max_pc$ in the definition of wt_instr in the context of wt_method .

```

lemma wt_instr_def2:
  "[ wt_jvm_prog G Phi; is_class G C;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et); pc < length ins;
    i = ins!pc; phi = Phi C sig; max_pc = length ins ]
  ⇒ wt_instr i G rT phi maxs max_pc et pc =
    (app i G maxs rT pc et (phi!pc) ∧
    (∀ (pc', s') ∈ set (eff i G pc et (phi!pc)). G ⊢ s' <= phi!pc'))"
⟨proof⟩

```

```

lemma wt_jvm_prog_impl_wt_start:
  "[ wt_jvm_prog G phi; is_class G C;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et) ] ⇒
  0 < (length ins) ∧ wt_start G C (snd sig) maxl (phi C sig)"
⟨proof⟩

```

end

4.17 The Typing Framework for the JVM

theory *Typing_Framework_JVM* imports *Typing_Framework_err* *JVMType* *EffectMono* *BVSpec* begin

constdefs

```
exec :: "jvm_prog ⇒ nat ⇒ ty ⇒ exception_table ⇒ instr list ⇒ JVMType.state step_type"
"exec G maxs rT et bs ==
err_step (size bs) (λpc. app (bs!pc) G maxs rT pc et) (λpc. eff (bs!pc) G pc et)"
```

constdefs

```
opt_states :: "'c prog ⇒ nat ⇒ nat ⇒ (ty list × ty err list) option set"
"opt_states G maxs maxr ≡ opt (⋃ {list n (types G) | n. n ≤ maxs} × list maxr (err
(types G)))"
```

4.17.1 Executability of *check_bounded*

consts

```
list_all'_rec :: "('a ⇒ nat ⇒ bool) ⇒ nat ⇒ 'a list ⇒ bool"
```

primrec

```
"list_all'_rec P n [] = True"
"list_all'_rec P n (x#xs) = (P x n ∧ list_all'_rec P (Suc n) xs)"
```

constdefs

```
list_all' :: "('a ⇒ nat ⇒ bool) ⇒ 'a list ⇒ bool"
"list_all' P xs ≡ list_all'_rec P 0 xs"
```

lemma *list_all'_rec*:

```
"⋀n. list_all'_rec P n xs = (∀p < size xs. P (xs!p) (p+n))"
⟨proof⟩
```

lemma *list_all' [iff]*:

```
"list_all' P xs = (∀n < size xs. P (xs!n) n)"
⟨proof⟩
```

lemma *[code]*:

```
"check_bounded ins et =
(list_all' (λi pc. list_all (λpc'. pc' < length ins) (succs i pc)) ins ∧
list_all (λe. fst (snd (snd e)) < length ins) et)"
⟨proof⟩
```

4.17.2 Connecting JVM and Framework

lemma *check_bounded_is_bounded*:

```
"check_bounded ins et ⇒ bounded (λpc. eff (ins!pc) G pc et) (length ins)"
⟨proof⟩
```

lemma *special_ex_swap_lemma [iff]*:

```
"(? X. (? n. X = A n & P n) & Q X) = (? n. Q(A n) & P n)"
⟨proof⟩
```

lemmas *[iff del]* = *not_None_eq*

theorem *exec_pres_type*:

```
"wf_prog wf_mb S ⇒"
```

```

pres_type (exec S maxs rT et bs) (size bs) (states S maxs maxr)"
⟨proof⟩

lemmas [iff] = not_None_eq

lemma sup_state_opt_unfold:
  "sup_state_opt G ≡ Opt.le (Product.le (Listn.le (subtype G)) (Listn.le (Err.le (subtype G))))"
  ⟨proof⟩

lemma app_mono:
  "app_mono (sup_state_opt G) (λpc. app (bs!pc) G maxs rT pc et) (length bs) (opt_states G maxs maxr)"
  ⟨proof⟩

lemma list_appendI:
  "⟦a ∈ list x A; b ∈ list y A⟧ ⇒ a @ b ∈ list (x+y) A"
  ⟨proof⟩

lemma list_map [simp]:
  "(map f xs ∈ list (length xs) A) = (f ` set xs ⊆ A)"
  ⟨proof⟩

lemma [iff]:
  "(OK ` A ⊆ err B) = (A ⊆ B)"
  ⟨proof⟩

lemma [intro]:
  "x ∈ A ⇒ replicate n x ∈ list n A"
  ⟨proof⟩

lemma lesubstep_type_simple:
  "a <=[Product.le (op =) r] b ⇒ a <=|r| b"
  ⟨proof⟩

lemma eff_mono:
  "⟦p < length bs; s <=_(sup_state_opt G) t; app (bs!p) G maxs rT pc et t⟧
  ⇒ eff (bs!p) G p et s <=|sup_state_opt G| eff (bs!p) G p et t"
  ⟨proof⟩

lemma order_sup_state_opt:
  "ws_prog G ⇒ order (sup_state_opt G)"
  ⟨proof⟩

theorem exec_mono:
  "ws_prog G ⇒ bounded (exec G maxs rT et bs) (size bs) ⇒
  mono (JVMType.le G maxs maxr) (exec G maxs rT et bs) (size bs) (states G maxs maxr)"
  ⟨proof⟩

theorem semilat_JVM_sII:

```

```

"ws_prog G  $\impl$  semilat (JVMTType.sl G maxs maxr)"
<proof>

lemma sl_triple_conv:
  "JVMTType.sl G maxs maxr ==
    (states G maxs maxr, JVMTType.le G maxs maxr, JVMTType.sup G maxs maxr)"
  <proof>

lemma map_id [rule_format]:
  "( $\forall n < \text{length } xs. f (g (xs!n)) = xs!n$ )  $\longrightarrow$  map f (map g xs) = xs"
  <proof>

lemma is_type_pTs:
  "[[ wf_prog wf_mb G; (C,S,fs,mdecls)  $\in$  set G; ((mn,pTs),rT,code)  $\in$  set mdecls ]]
 $\impl$  set pTs  $\subseteq$  types G"
  <proof>

lemma jvm_prog_lift:
  assumes wf:
    "wf_prog ( $\lambda G C bd. P G C bd$ ) G"

  assumes rule:
    " $\bigwedge wf\_mb C mn pTs C rT maxs maxl b et bd.$ 
      wf_prog wf_mb G  $\impl$ 
      method (G,C) (mn,pTs) = Some (C,rT,maxs,maxl,b,et)  $\impl$ 
      is_class G C  $\impl$ 
      set pTs  $\subseteq$  types G  $\impl$ 
      bd = ((mn,pTs),rT,maxs,maxl,b,et)  $\impl$ 
      P G C bd  $\impl$ 
      Q G C bd"

  shows
    "wf_prog ( $\lambda G C bd. Q G C bd$ ) G"
  <proof>

end

```

4.18 LBV for the JVM

```

theory LBVJVM
imports LBVCorrect LBVComplete Typing_Framework_JVM
begin

types prog_cert = "cname  $\Rightarrow$  sig  $\Rightarrow$  JVMType.state list"

constdefs
  check_cert :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  JVMType.state list  $\Rightarrow$  bool"
  "check_cert G mxs mxr n cert  $\equiv$  check_types G mxs mxr cert  $\wedge$  length cert = n+1  $\wedge$ 
    ( $\forall i < n. \text{cert!}i \neq \text{Err}$ )  $\wedge$  cert!n = OK None"

  lbvjvm :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  ty  $\Rightarrow$  exception_table  $\Rightarrow$ 
    JVMType.state list  $\Rightarrow$  instr list  $\Rightarrow$  JVMType.state  $\Rightarrow$  JVMType.state"
  "lbvjvm G maxs maxr rT et cert bs  $\equiv$ 
    wtl_inst_list bs cert (JVMType.sup G maxs maxr) (JVMType.le G maxs maxr) Err (OK None)
  (exec G maxs rT et bs) 0"

  wt_lbv :: "jvm_prog  $\Rightarrow$  cname  $\Rightarrow$  ty list  $\Rightarrow$  ty  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$ 
    exception_table  $\Rightarrow$  JVMType.state list  $\Rightarrow$  instr list  $\Rightarrow$  bool"
  "wt_lbv G C pTs rT mxs mxl et cert ins  $\equiv$ 
    check_bounded ins et  $\wedge$ 
    check_cert G mxs (1+size pTs+mxl) (length ins) cert  $\wedge$ 
    0 < size ins  $\wedge$ 
    (let start = Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err));
      result = lbvjvm G mxs (1+size pTs+mxl) rT et cert ins (OK start)
    in result  $\neq$  Err)"

  wt_jvm_prog_lbv :: "jvm_prog  $\Rightarrow$  prog_cert  $\Rightarrow$  bool"
  "wt_jvm_prog_lbv G cert  $\equiv$ 
    wf_prog ( $\lambda G C (\text{sig}, rT, (\text{maxs}, \text{maxl}, b, \text{et})) . \text{wt\_lbv } G C (\text{snd sig}) rT \text{maxs maxl et (cert } C \text{ sig) } b) G$ )"

  mk_cert :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  ty  $\Rightarrow$  exception_table  $\Rightarrow$  instr list
     $\Rightarrow$  method_type  $\Rightarrow$  JVMType.state list"
  "mk_cert G maxs rT et bs phi  $\equiv$  make_cert (exec G maxs rT et bs) (map OK phi) (OK None)"

  prg_cert :: "jvm_prog  $\Rightarrow$  prog_type  $\Rightarrow$  prog_cert"
  "prg_cert G phi C sig  $\equiv$  let (C, rT, (maxs, maxl, ins, et)) = the (method (G, C) sig) in
    mk_cert G maxs rT et ins (phi C sig)"

lemma wt_method_def2:
  fixes pTs and mxl and G and mxs and rT and et and bs and phi
  defines [simp]: "mxr  $\equiv$  1 + length pTs + mxl"
  defines [simp]: "r  $\equiv$  sup_state_opt G"
  defines [simp]: "app0  $\equiv$   $\lambda pc. \text{app } (bs!pc) G \text{mxs rT pc et}$ "
  defines [simp]: "step0  $\equiv$   $\lambda pc. \text{eff } (bs!pc) G \text{pc et}$ "

shows
  "wt_method G C pTs rT mxs mxl bs et phi =
    (bs  $\neq$  []  $\wedge$ 
     length phi = length bs  $\wedge$ 

```

```

    check_bounded bs et ∧
    check_types G mxs mxr (map OK phi) ∧
    wt_start G C pTs mxl phi ∧
    wt_app_eff r app0 step0 phi)"
  ⟨proof⟩

```

lemma check_certD:

```

  "check_cert G mxs mxr n cert ⇒ cert_ok cert n Err (OK None) (states G mxs mxr)"
  ⟨proof⟩

```

lemma wt_lbv_wt_step:

```

  assumes wf: "wf_prog wf_mb G"
  assumes lbv: "wt_lbv G C pTs rT mxs mxl et cert ins"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  defines [simp]: "mxr ≡ 1+length pTs+mxl"

```

```

  shows "∃ ts ∈ list (size ins) (states G mxs mxr).
        wt_step (JVMType.le G mxs mxr) Err (exec G mxs rT et ins) ts
        ∧ OK (Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err))) ≤_(JVMType.le
G mxs mxr) ts!0"
  ⟨proof⟩

```

lemma wt_lbv_wt_method:

```

  assumes wf: "wf_prog wf_mb G"
  assumes lbv: "wt_lbv G C pTs rT mxs mxl et cert ins"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  shows "∃ phi. wt_method G C pTs rT mxs mxl ins et phi"
  ⟨proof⟩

```

lemma wt_method_wt_lbv:

```

  assumes wf: "wf_prog wf_mb G"
  assumes wt: "wt_method G C pTs rT mxs mxl ins et phi"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  defines [simp]: "cert ≡ mk_cert G mxs rT et ins phi"

```

```

  shows "wt_lbv G C pTs rT mxs mxl et cert ins"
  ⟨proof⟩

```

theorem jvm_lbv_correct:

```

  "wt_jvm_prog_lbv G Cert ⇒ ∃ Phi. wt_jvm_prog G Phi"
  ⟨proof⟩

```

theorem jvm_lbv_complete:

```
"wt_jvm_prog G Phi  $\impl$  wt_jvm_prog_lbv G (prg_cert G Phi)"  
   $\langle proof \rangle$   
end
```

4.19 BV Type Safety Invariant

theory *Correct* **imports** *BVSpec* *"../JVM/JVMExec"* **begin**

constdefs

```

approx_val :: "[jvm_prog, aheap, val, ty err]  $\Rightarrow$  bool"
"approx_val G h v any == case any of Err  $\Rightarrow$  True | OK T  $\Rightarrow$  G, h  $\vdash$  v ::  $\preceq$ T"

approx_loc :: "[jvm_prog, aheap, val list, locvars_type]  $\Rightarrow$  bool"
"approx_loc G hp loc LT == list_all2 (approx_val G hp) loc LT"

approx_stk :: "[jvm_prog, aheap, opstack, opstack_type]  $\Rightarrow$  bool"
"approx_stk G hp stk ST == approx_loc G hp stk (map OK ST)"

correct_frame :: "[jvm_prog, aheap, state_type, nat, bytecode]  $\Rightarrow$  frame  $\Rightarrow$  bool"
"correct_frame G hp ==  $\lambda$ (ST, LT) maxl ins (stk, loc, C, sig, pc).
    approx_stk G hp stk ST  $\wedge$  approx_loc G hp loc LT  $\wedge$ 
    pc < length ins  $\wedge$  length loc = length(snd sig) + maxl + 1"

```

consts

```

correct_frames :: "[jvm_prog, aheap, prog_type, ty, sig, frame list]  $\Rightarrow$  bool"

```

primrec

```

"correct_frames G hp phi rT0 sig0 [] = True"

"correct_frames G hp phi rT0 sig0 (f#frs) =
  (let (stk, loc, C, sig, pc) = f in
   ( $\exists$  ST LT rT maxs maxl ins et.
    phi C sig ! pc = Some (ST, LT)  $\wedge$  is_class G C  $\wedge$ 
    method (G, C) sig = Some(C, rT, (maxs, maxl, ins, et))  $\wedge$ 
    ( $\exists$  C' mn pTs. ins ! pc = (Invoke C' mn pTs)  $\wedge$ 
     (mn, pTs) = sig0  $\wedge$ 
     ( $\exists$  apTs D ST' LT'.
      (phi C sig) ! pc = Some ((rev apTs) @ (Class D) # ST', LT')  $\wedge$ 
      length apTs = length pTs  $\wedge$ 
      ( $\exists$  D' rT' maxs' maxl' ins' et'.
       method (G, D) sig0 = Some(D', rT', (maxs', maxl', ins', et'))  $\wedge$ 
       G  $\vdash$  rT0  $\preceq$  rT'))  $\wedge$ 
     correct_frame G hp (ST, LT) maxl ins f  $\wedge$ 
     correct_frames G hp phi rT sig frs))))"

```

constdefs

```

correct_state :: "[jvm_prog, prog_type, jvm_state]  $\Rightarrow$  bool"
("_,_ |-JVM _ [ok]" [51,51] 50)

```

"correct_state G phi == λ (xp, hp, frs).

case xp of

None \Rightarrow (case frs of

[] \Rightarrow True

| (f#fs) \Rightarrow G \vdash h hp \surd \wedge preallocated hp \wedge

(let (stk, loc, C, sig, pc) = f

in

\exists rT maxs maxl ins et s.

is_class G C \wedge


```

method (G,C) sig = Some(C,rT,(maxs,maxl,ins,et)) ∧
  phi C sig ! pc = Some s ∧
  correct_frame G hp s maxl ins f ∧
  correct_frames G hp phi rT sig fs))
| Some x ⇒ frs = []"

```

```

syntax (xsymbols)
correct_state :: "[jvm_prog,prog_type,jvm_state] ⇒ bool"
                ("_,_ ⊢ JVM _ √" [51,51] 50)

```

```

lemma sup_ty_opt_OK:
  "(G ⊢ X <=o (OK T')) = (∃ T. X = OK T ∧ G ⊢ T ≤ T')"
  <proof>

```

4.19.1 approx-val

```

lemma approx_val_Err [simp,intro!]:
  "approx_val G hp x Err"
  <proof>

```

```

lemma approx_val_OK [iff]:
  "approx_val G hp x (OK T) = (G, hp ⊢ x :: ≤ T)"
  <proof>

```

```

lemma approx_val_Null [simp,intro!]:
  "approx_val G hp Null (OK (RefT x))"
  <proof>

```

```

lemma approx_val_sup_heap:
  "[ approx_val G hp v T; hp ≤ hp' ] ⇒ approx_val G hp' v T"
  <proof>

```

```

lemma approx_val_heap_update:
  "[ hp a = Some obj'; G, hp ⊢ v :: ≤ T; obj_ty obj = obj_ty obj' ]
  ⇒ G, hp(a ↦ obj) ⊢ v :: ≤ T"
  <proof>

```

```

lemma approx_val_widen:
  "[ approx_val G hp v T; G ⊢ T <=o T'; wf_prog wt G ]
  ⇒ approx_val G hp v T'"
  <proof>

```

4.19.2 approx-loc

```

lemma approx_loc_Nil [simp,intro!]:
  "approx_loc G hp [] []"
  <proof>

```

```

lemma approx_loc_Cons [iff]:
  "approx_loc G hp (l#ls) (L#LT) =
  (approx_val G hp l L ∧ approx_loc G hp ls LT)"
  <proof>

```

lemma approx_loc_nth:

"[[approx_loc G hp loc LT; n < length LT]]
 \implies approx_val G hp (loc!n) (LT!n)"
 <proof>

lemma approx_loc_imp_approx_val_sup:

"[[approx_loc G hp loc LT; n < length LT; LT ! n = OK T; G \vdash T \preceq T'; wf_prog wt G]]
 \implies G, hp \vdash (loc!n) :: \preceq T'"
 <proof>

lemma approx_loc_conv_all_nth:

"approx_loc G hp loc LT =
 (length loc = length LT \wedge ($\forall n < \text{length loc. approx_val G hp (loc!n) (LT!n)$))"
 <proof>

lemma approx_loc_sup_heap:

"[[approx_loc G hp loc LT; hp \leq hp']]
 \implies approx_loc G hp' loc LT"
 <proof>

lemma approx_loc_widen:

"[[approx_loc G hp loc LT; G \vdash LT \leq LT'; wf_prog wt G]]
 \implies approx_loc G hp loc LT'"
 <proof>

lemma loc_widen_Err [dest]:

" $\bigwedge XT. G \vdash \text{replicate } n \text{ Err} \leq XT \implies XT = \text{replicate } n \text{ Err}$ "
 <proof>

lemma approx_loc_Err [iff]:

"approx_loc G hp (replicate n v) (replicate n Err)"
 <proof>

lemma approx_loc_subst:

"[[approx_loc G hp loc LT; approx_val G hp x X]]
 \implies approx_loc G hp (loc[idx:=x]) (LT[idx:=X])"
 <proof>

lemma approx_loc_append:

"length l1=length L1 \implies
 approx_loc G hp (l1@l2) (L1@L2) =
 (approx_loc G hp l1 L1 \wedge approx_loc G hp l2 L2)"
 <proof>

4.19.3 approx-stk

lemma approx_stk_rev_lem:

"approx_stk G hp (rev s) (rev t) = approx_stk G hp s t"
 <proof>

lemma approx_stk_rev:

"approx_stk G hp (rev s) t = approx_stk G hp s (rev t)"
 <proof>

lemma approx_stk_sup_heap:

" $\llbracket \text{approx_stk } G \text{ hp stk } ST; \text{hp} \leq / \text{hp}' \rrbracket \implies \text{approx_stk } G \text{ hp}' \text{ stk } ST$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_widen:

" $\llbracket \text{approx_stk } G \text{ hp stk } ST; G \vdash \text{map OK } ST \leq \text{map OK } ST'; \text{wf_prog wt } G \rrbracket$
 $\implies \text{approx_stk } G \text{ hp stk } ST'$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_Nil [iff]:

" $\text{approx_stk } G \text{ hp } [] []$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_Cons [iff]:

" $\text{approx_stk } G \text{ hp } (x\#\text{stk}) (S\#ST) =$
 $(\text{approx_val } G \text{ hp } x (\text{OK } S) \wedge \text{approx_stk } G \text{ hp stk } ST)$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_Cons_lemma [iff]:

" $\text{approx_stk } G \text{ hp stk } (S\#ST') =$
 $(\exists s \text{ stk}'. \text{stk} = s\#\text{stk}' \wedge \text{approx_val } G \text{ hp } s (\text{OK } S) \wedge \text{approx_stk } G \text{ hp stk}' ST')$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_append:

" $\text{approx_stk } G \text{ hp stk } (S@S') \implies$
 $(\exists s \text{ stk}'. \text{stk} = s@\text{stk}' \wedge \text{length } s = \text{length } S \wedge \text{length } \text{stk}' = \text{length } S' \wedge$
 $\text{approx_stk } G \text{ hp } s S \wedge \text{approx_stk } G \text{ hp stk}' S')$ "
 $\langle \text{proof} \rangle$

lemma approx_stk_all_widen:

" $\llbracket \text{approx_stk } G \text{ hp stk } ST; \forall (x, y) \in \text{set } (\text{zip } ST \text{ } ST'). G \vdash x \preceq y; \text{length } ST = \text{length } ST'; \text{wf_prog wt } G \rrbracket$
 $\implies \text{approx_stk } G \text{ hp stk } ST'$ "
 $\langle \text{proof} \rangle$

4.19.4 oconf

lemma oconf_field_update:

" $\llbracket \text{map_of } (\text{fields } (G, \text{oT})) \text{ FD} = \text{Some } T; G, \text{hp} \vdash v :: \preceq T; G, \text{hp} \vdash (\text{oT}, \text{fs}) \checkmark \rrbracket$
 $\implies G, \text{hp} \vdash (\text{oT}, \text{fs}(\text{FD} \mapsto v)) \checkmark$ "
 $\langle \text{proof} \rangle$

lemma oconf_newref:

" $\llbracket \text{hp } \text{oref} = \text{None}; G, \text{hp} \vdash \text{obj } \checkmark; G, \text{hp} \vdash \text{obj}' \checkmark \rrbracket \implies G, \text{hp}(\text{oref} \mapsto \text{obj}') \vdash \text{obj } \checkmark$ "
 $\langle \text{proof} \rangle$

lemma oconf_heap_update:

" $\llbracket \text{hp } a = \text{Some } \text{obj}'; \text{obj_ty } \text{obj}' = \text{obj_ty } \text{obj}''; G, \text{hp} \vdash \text{obj } \checkmark \rrbracket$
 $\implies G, \text{hp}(a \mapsto \text{obj}'') \vdash \text{obj } \checkmark$ "
 $\langle \text{proof} \rangle$

4.19.5 hconf

lemma hconf_newref:

" $\llbracket \text{hp } \text{oref} = \text{None}; G \vdash \text{h } \text{hp} \checkmark; G, \text{hp} \vdash \text{obj} \checkmark \rrbracket \implies G \vdash \text{h } \text{hp}(\text{oref} \mapsto \text{obj}) \checkmark$ "
 $\langle \text{proof} \rangle$

lemma hconf_field_update:

" $\llbracket \text{map_of } (\text{fields } (G, \text{oT})) X = \text{Some } T; \text{hp } a = \text{Some}(\text{oT}, \text{fs});$
 $G, \text{hp} \vdash v :: \preceq T; G \vdash \text{h } \text{hp} \checkmark \rrbracket$
 $\implies G \vdash \text{h } \text{hp}(a \mapsto (\text{oT}, \text{fs}(X \mapsto v))) \checkmark$ "
 $\langle \text{proof} \rangle$

4.19.6 preallocated

lemma preallocated_field_update:

" $\llbracket \text{map_of } (\text{fields } (G, \text{oT})) X = \text{Some } T; \text{hp } a = \text{Some}(\text{oT}, \text{fs});$
 $G \vdash \text{h } \text{hp} \checkmark; \text{preallocated } \text{hp} \rrbracket$
 $\implies \text{preallocated } (\text{hp}(a \mapsto (\text{oT}, \text{fs}(X \mapsto v))))$ "
 $\langle \text{proof} \rangle$

lemma

assumes none: " $\text{hp } \text{oref} = \text{None}$ " **and** alloc: " $\text{preallocated } \text{hp}$ "
shows preallocated_newref: " $\text{preallocated } (\text{hp}(\text{oref} \mapsto \text{obj}))$ "
 $\langle \text{proof} \rangle$

4.19.7 correct-frames

lemmas [simp del] = fun_upd_apply

lemma correct_frames_field_update [rule_format]:

" $\forall rT \ C \ \text{sig}.$
 $\text{correct_frames } G \ \text{hp} \ \text{phi} \ rT \ \text{sig} \ \text{frs} \longrightarrow$
 $\text{hp } a = \text{Some } (C, \text{fs}) \longrightarrow$
 $\text{map_of } (\text{fields } (G, C)) \ \text{fl} = \text{Some } \text{fd} \longrightarrow$
 $G, \text{hp} \vdash v :: \preceq \text{fd}$
 $\longrightarrow \text{correct_frames } G \ (\text{hp}(a \mapsto (C, \text{fs}(\text{fl} \mapsto v)))) \ \text{phi} \ rT \ \text{sig} \ \text{frs}$ "
 $\langle \text{proof} \rangle$

lemma correct_frames_newref [rule_format]:

" $\forall rT \ C \ \text{sig}.$
 $\text{hp } x = \text{None} \longrightarrow$
 $\text{correct_frames } G \ \text{hp} \ \text{phi} \ rT \ \text{sig} \ \text{frs} \longrightarrow$
 $\text{correct_frames } G \ (\text{hp}(x \mapsto \text{obj})) \ \text{phi} \ rT \ \text{sig} \ \text{frs}$ "
 $\langle \text{proof} \rangle$

end

4.20 BV Type Safety Proof

theory BVSpecTypeSafe imports Correct begin

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

4.20.1 Preliminaries

Simp and intro setup for the type safety proof:

```
lemmas defs1 = sup_state_conv correct_state_def correct_frame_def
            wt_instr_def eff_def norm_eff_def
```

```
lemmas widen_rules[intro] = approx_val_widen approx_loc_widen approx_stk_widen
```

```
lemmas [simp del] = split_paired_All
```

If we have a welltyped program and a conforming state, we can directly infer that the current instruction is well typed:

```
lemma wt_jvm_prog_impl_wt_instr_cor:
  "[[ wt_jvm_prog G phi; method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
    G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) √ ] ]
  ⇒ wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc"
  <proof>
```

4.20.2 Exception Handling

Exceptions don't touch anything except the stack:

```
lemma exec_instr_xcpt:
  "(fst (exec_instr i G hp stk vars Cl sig pc frs) = Some xcp)
  = (∃ stk'. exec_instr i G hp stk vars Cl sig pc frs =
    (Some xcp, hp, (stk', vars, Cl, sig, pc)#frs))"
  <proof>
```

Relates *match_any* from the Bytecode Verifier with *match_exception_table* from the operational semantics:

```
lemma in_match_any:
  "match_exception_table G xcpt pc et = Some pc' ⇒
  ∃ C. C ∈ set (match_any G pc et) ∧ G ⊢ xcpt ≤C C ∧
    match_exception_table G C pc et = Some pc'"
  (is "PROP ?P et" is "?match et ⇒ ?match_any et")
  <proof>
```

```
lemma match_et_imp_match:
  "match_exception_table G (Xcpt X) pc et = Some handler
  ⇒ match G X pc et = [Xcpt X]"
  <proof>
```

We can prove separately that the recursive search for exception handlers (*find_handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occurred conforms.

lemma *uncaught_xcpt_correct*:

```
" $\wedge f. \llbracket \text{wt\_jvm\_prog } G \text{ phi}; \text{ xcp} = \text{Addr } \text{adr}; \text{ hp } \text{adr} = \text{Some } T; \\ G, \text{phi} \vdash \text{JVM } (\text{None}, \text{hp}, f\#\text{frs}) \checkmark \rrbracket \\ \implies G, \text{phi} \vdash \text{JVM } (\text{find\_handler } G (\text{Some } \text{xcp}) \text{ hp } \text{frs}) \checkmark "$ "
(is " $\wedge f. \llbracket ?\text{wt}; ?\text{adr}; ?\text{hp}; ?\text{correct } (\text{None}, \text{hp}, f\#\text{frs}) \rrbracket \implies ?\text{correct } (? \text{find } \text{frs}) "$ ")
<proof>
```

declare *raise_if_def* [simp]

The requirement of lemma *uncaught_xcpt_correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

lemma *exec_instr_xcpt_hp*:

```
" $\llbracket \text{fst } (\text{exec\_instr } (\text{ins!pc}) G \text{ hp } \text{stk } \text{vars } C \text{ sig } \text{pc } \text{frs}) = \text{Some } \text{xcp}; \\ \text{wt\_instr } (\text{ins!pc}) G \text{ rT } (\text{phi } C \text{ sig}) \text{ maxs } (\text{length } \text{ins}) \text{ et } \text{pc}; \\ G, \text{phi} \vdash \text{JVM } (\text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc})\#\text{frs}) \checkmark \rrbracket \\ \implies \exists \text{adr } T. \text{ xcp} = \text{Addr } \text{adr} \wedge \text{ hp } \text{adr} = \text{Some } T "$ "
(is " $\llbracket ?\text{xcp}; ?\text{wt}; ?\text{correct} \rrbracket \implies ?\text{thesis} "$ ")
<proof>
```

lemma *cname_of_xcp* [intro]:

```
" $\llbracket \text{preallocated } \text{hp}; \text{ xcp} = \text{Addr } (\text{XcptRef } x) \rrbracket \implies \text{cname\_of } \text{hp } \text{xcp} = \text{Xcpt } x "$ "
<proof>
```

Finally we can state that, whenever an exception occurs, the resulting next state always conforms:

lemma *xcpt_correct*:

```
" $\llbracket \text{wt\_jvm\_prog } G \text{ phi}; \\ \text{method } (G, C) \text{ sig} = \text{Some } (C, \text{rT}, \text{maxs}, \text{maxl}, \text{ins}, \text{et}); \\ \text{wt\_instr } (\text{ins!pc}) G \text{ rT } (\text{phi } C \text{ sig}) \text{ maxs } (\text{length } \text{ins}) \text{ et } \text{pc}; \\ \text{fst } (\text{exec\_instr } (\text{ins!pc}) G \text{ hp } \text{stk } \text{loc } C \text{ sig } \text{pc } \text{frs}) = \text{Some } \text{xcp}; \\ \text{Some } \text{state}' = \text{exec } (G, \text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc})\#\text{frs}); \\ G, \text{phi} \vdash \text{JVM } (\text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc})\#\text{frs}) \checkmark \rrbracket \\ \implies G, \text{phi} \vdash \text{JVM } \text{state}' \checkmark "$ "
<proof>
```

4.20.3 Single Instructions

In this section we look at each single (welltyped) instruction, and prove that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume, that on exception occurs for this (single step) execution.

lemmas [iff] = *not_Err_eq*

lemma *Load_correct*:

```
" $\llbracket \text{wf\_prog } \text{wt } G; \\ \text{method } (G, C) \text{ sig} = \text{Some } (C, \text{rT}, \text{maxs}, \text{maxl}, \text{ins}, \text{et}); \\ \text{ins!pc} = \text{Load } \text{idx}; \\ \text{wt\_instr } (\text{ins!pc}) G \text{ rT } (\text{phi } C \text{ sig}) \text{ maxs } (\text{length } \text{ins}) \text{ et } \text{pc}; \\ \text{Some } \text{state}' = \text{exec } (G, \text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc})\#\text{frs}); \\ G, \text{phi} \vdash \text{JVM } (\text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc})\#\text{frs}) \checkmark \rrbracket \\ \implies G, \text{phi} \vdash \text{JVM } \text{state}' \checkmark "$ "
<proof>
```

lemma Store_correct:

```
"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = Store idx;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemma LitPush_correct:

```
"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = LitPush v;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemma Cast_conf2:

```
"[[ wf_prog ok G; G,h⊢v::⊆RefT rt; cast_ok G C h v;
  G⊢Class C⊆T; is_class G C ]
⇒ G,h⊢v::⊆T"
⟨proof⟩
```

lemmas defs2 = defs1 raise_system_xcpt_def

lemma Checkcast_correct:

```
"[[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = Checkcast D;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemma Getfield_correct:

```
"[[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = Getfield F D;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemma Putfield_correct:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = Putfield F D;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemma New_correct:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = New X;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemmas [simp del] = split_paired_Ex

lemma Invoke_correct:

```
"[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Invoke C' mn pTs;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemmas [simp del] = map_append

lemma Return_correct:

```
"[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Return;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

lemmas [simp] = map_append

lemma Goto_correct:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
```



```

ins ! pc = Goto branch;
wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

lemma Ifcmpeq_correct:

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Ifcmpeq branch;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

lemma Pop_correct:

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Pop;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

lemma Dup_correct:

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

lemma Dup_x1_correct:

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup_x1;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

lemma Dup_x2_correct:

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup_x2;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;

```

```

  G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓ ]
⇒ G,phi ⊢ JVM state' ✓"
⟨proof⟩

```

lemma Swap_correct:

```

" [ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Swap;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓ ]
⇒ G,phi ⊢ JVM state' ✓"
⟨proof⟩

```

lemma IAdd_correct:

```

" [ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = IAdd;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓ ]
⇒ G,phi ⊢ JVM state' ✓"
⟨proof⟩

```

lemma Throw_correct:

```

" [ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Throw;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢ JVM state' ✓"
⟨proof⟩

```

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in well-typed programs, a conforming state is transformed into another conforming state when one instruction is executed.

theorem instr_correct:

```

" [ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
  G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓ ]
⇒ G,phi ⊢ JVM state' ✓"
⟨proof⟩

```

4.20.4 Main

lemma correct_state_impl_Some_method:

```

"G,phi ⊢ JVM (None, hp, (stk,loc,C,sig,pc)#frs) ✓
⇒ ∃ meth. method (G,C) sig = Some(C,meth)"
⟨proof⟩

```

lemma *BV_correct_1* [rule_format]:

" $\llbracket \text{wt_jvm_prog } G \text{ phi}; G, \text{phi} \vdash_{\text{JVM}} \text{state} \sqrt{} \rrbracket$
 $\implies \text{exec } (G, \text{state}) = \text{Some state}' \longrightarrow G, \text{phi} \vdash_{\text{JVM}} \text{state}' \sqrt{}$ "
 <proof>

lemma *L0*:

" $\llbracket \text{xp} = \text{None}; \text{frs} \neq [] \rrbracket \implies (\exists \text{state}'. \text{exec } (G, \text{xp}, \text{hp}, \text{frs}) = \text{Some state}')$ "
 <proof>

lemma *L1*:

" $\llbracket \text{wt_jvm_prog } G \text{ phi}; G, \text{phi} \vdash_{\text{JVM}} (\text{xp}, \text{hp}, \text{frs}) \sqrt{}; \text{xp} = \text{None}; \text{frs} \neq [] \rrbracket$
 $\implies \exists \text{state}'. \text{exec } (G, \text{xp}, \text{hp}, \text{frs}) = \text{Some state}' \wedge G, \text{phi} \vdash_{\text{JVM}} \text{state}' \sqrt{}$ "
 <proof>

theorem *BV_correct* [rule_format]:

" $\llbracket \text{wt_jvm_prog } G \text{ phi}; G \vdash s \text{ -jvm} \rightarrow t \rrbracket \implies G, \text{phi} \vdash_{\text{JVM}} s \sqrt{} \longrightarrow G, \text{phi} \vdash_{\text{JVM}} t \sqrt{}$ "
 <proof>

theorem *BV_correct_implies_approx*:

" $\llbracket \text{wt_jvm_prog } G \text{ phi};$
 $G \vdash s0 \text{ -jvm} \rightarrow (\text{None}, \text{hp}, (\text{stk}, \text{loc}, C, \text{sig}, \text{pc}) \# \text{frs}); G, \text{phi} \vdash_{\text{JVM}} s0 \sqrt{} \rrbracket$
 $\implies \text{approx_stk } G \text{ hp stk } (\text{fst } (\text{the } (\text{phi } C \text{ sig } ! \text{pc}))) \wedge$
 $\text{approx_loc } G \text{ hp loc } (\text{snd } (\text{the } (\text{phi } C \text{ sig } ! \text{pc})))$ "
 <proof>

lemma

fixes $G :: \text{jvm_prog } (" \Gamma ")$
 assumes $\text{wf}: " \text{wf_prog wf_mb } \Gamma "$
 shows $\text{hconf_start}: " \Gamma \vdash h (\text{start_heap } \Gamma) \sqrt{} "$
 <proof>

lemma

fixes $G :: \text{jvm_prog } (" \Gamma ")$ and $\text{Phi} :: \text{prog_type } (" \Phi ")$
 shows *BV_correct_initial*:
 $" \text{wt_jvm_prog } \Gamma \Phi \implies \text{is_class } \Gamma C \implies \text{method } (\Gamma, C) (m, []) = \text{Some } (C, b) "$
 $\implies \Gamma, \Phi \vdash_{\text{JVM}} \text{start_state } G C m \sqrt{} "$
 <proof>

theorem

fixes $G :: \text{jvm_prog } (" \Gamma ")$ and $\text{Phi} :: \text{prog_type } (" \Phi ")$
 assumes *welltyped*: $" \text{wt_jvm_prog } \Gamma \Phi "$ and
 $\text{main_method}: " \text{is_class } \Gamma C " " \text{method } (\Gamma, C) (m, []) = \text{Some } (C, b) "$
 shows *typesafe*:
 $" G \vdash \text{start_state } \Gamma C m \text{ -jvm} \rightarrow s \implies \Gamma, \Phi \vdash_{\text{JVM}} s \sqrt{} "$
 <proof>

end

4.21 Welltyped Programs produce no Type Errors

theory BVNoTypeError imports "../JVM/JVMDefensive" BVSPECTypeSafe begin

Some simple lemmas about the type testing functions of the defensive JVM:

```
lemma typeof_NoneD [simp,dest]:
  "typeof ( $\lambda v. \text{None}$ )  $v = \text{Some } x \implies \neg \text{isAddr } v$ "
  <proof>
```

```
lemma isRef_def2:
  "isRef  $v = (v = \text{Null} \vee (\exists \text{loc}. v = \text{Addr } \text{loc}))$ "
  <proof>
```

```
lemma app'Store[simp]:
  "app' (Store  $\text{idx}, G, pc, \text{maxs}, rT, (ST, LT)) = (\exists T \text{ ST}'. ST = T\#ST' \wedge \text{idx} < \text{length } LT)$ "
  <proof>
```

```
lemma app'GetField[simp]:
  "app' (Getfield  $F \ C, G, pc, \text{maxs}, rT, (ST, LT)) =$ 
  ( $\exists oT \ vT \ ST'. ST = oT\#ST' \wedge \text{is\_class } G \ C \wedge$ 
  field ( $G, C$ )  $F = \text{Some } (C, vT) \wedge G \vdash oT \preceq \text{Class } C$ )"
  <proof>
```

```
lemma app'PutField[simp]:
  "app' (Putfield  $F \ C, G, pc, \text{maxs}, rT, (ST, LT)) =$ 
  ( $\exists vT \ vT' \ oT \ ST'. ST = vT\#oT\#ST' \wedge \text{is\_class } G \ C \wedge$ 
  field ( $G, C$ )  $F = \text{Some } (C, vT') \wedge$ 
   $G \vdash oT \preceq \text{Class } C \wedge G \vdash vT \preceq vT')$ "
  <proof>
```

```
lemma app'Checkcast[simp]:
  "app' (Checkcast  $C, G, pc, \text{maxs}, rT, (ST, LT)) =$ 
  ( $\exists rT \ ST'. ST = \text{RefT } rT\#ST' \wedge \text{is\_class } G \ C$ )"
  <proof>
```

```
lemma app'Pop[simp]:
  "app' (Pop,  $G, pc, \text{maxs}, rT, (ST, LT)) = (\exists T \ ST'. ST = T\#ST')$ "
  <proof>
```

```
lemma app'Dup[simp]:
  "app' (Dup,  $G, pc, \text{maxs}, rT, (ST, LT)) =$ 
  ( $\exists T \ ST'. ST = T\#ST' \wedge \text{length } ST < \text{maxs}$ )"
  <proof>
```

```
lemma app'Dup_x1[simp]:
  "app' (Dup_x1,  $G, pc, \text{maxs}, rT, (ST, LT)) =$ 
  ( $\exists T1 \ T2 \ ST'. ST = T1\#T2\#ST' \wedge \text{length } ST < \text{maxs}$ )"
  <proof>
```

```
lemma app'Dup_x2[simp]:
```

```
"app' (Dup_x2, G, pc, maxs, rT, (ST,LT)) =
(∃ T1 T2 T3 ST'. ST = T1#T2#T3#ST' ∧ length ST < maxs)"
⟨proof⟩
```

```
lemma app'Swap[simp]:
  "app' (Swap, G, pc, maxs, rT, (ST,LT)) = (∃ T1 T2 ST'. ST = T1#T2#ST' )"
  ⟨proof⟩
```

```
lemma app'IAdd[simp]:
  "app' (IAdd, G, pc, maxs, rT, (ST,LT)) =
  (∃ ST'. ST = PrimT Integer#PrimT Integer#ST' )"
  ⟨proof⟩
```

```
lemma app'Ifcmpeq[simp]:
  "app' (Ifcmpeq b, G, pc, maxs, rT, (ST,LT)) =
  (∃ T1 T2 ST'. ST = T1#T2#ST' ∧ 0 ≤ b + int pc ∧
  ((∃ p. T1 = PrimT p ∧ T1 = T2) ∨
  (∃ r r'. T1 = RefT r ∧ T2 = RefT r')))"
  ⟨proof⟩
```

```
lemma app'Return[simp]:
  "app' (Return, G, pc, maxs, rT, (ST,LT)) =
  (∃ T ST'. ST = T#ST' ∧ G ⊢ T ≤ rT)"
  ⟨proof⟩
```

```
lemma app'Throw[simp]:
  "app' (Throw, G, pc, maxs, rT, (ST,LT)) =
  (∃ ST' r. ST = RefT r#ST' )"
  ⟨proof⟩
```

```
lemma app'Invoke[simp]:
  "app' (Invoke C mn fpTs, G, pc, maxs, rT, ST, LT) =
  (∃ apTs X ST' mD' rT' b'.
  ST = (rev apTs) @ X # ST' ∧
  length apTs = length fpTs ∧ is_class G C ∧
  (∀ (aT,fT) ∈ set(zip apTs fpTs). G ⊢ aT ≤ fT) ∧
  method (G,C) (mn,fpTs) = Some (mD', rT', b') ∧ G ⊢ X ≤ Class C)"
  (is "?app ST LT = ?P ST LT")
  ⟨proof⟩
```

```
lemma approx_loc_len [simp]:
  "approx_loc G hp loc LT ⇒ length loc = length LT"
  ⟨proof⟩
```

```
lemma approx_stk_len [simp]:
  "approx_stk G hp stk ST ⇒ length stk = length ST"
  ⟨proof⟩
```

lemma *isRefI* [intro, simp]: "G, hp $\vdash v :: \preceq \text{RefT } T \implies \text{isRef } v$ "
 ⟨proof⟩

lemma *isIntgI* [intro, simp]: "G, hp $\vdash v :: \preceq \text{PrimT Integer} \implies \text{isIntg } v$ "
 ⟨proof⟩

lemma *list_all2_approx*:
 " $\bigwedge s. \text{list_all2 } (\text{approx_val } G \text{ hp}) s (\text{map OK } S) =$
 $\text{list_all2 } (\text{conf } G \text{ hp}) s S$ "
 ⟨proof⟩

lemma *list_all2_conf_widen*:
 " $\text{wf_prog } mb \ G \implies$
 $\text{list_all2 } (\text{conf } G \text{ hp}) a \ b \implies$
 $\text{list_all2 } (\lambda x \ y. \ G \vdash x \preceq y) b \ c \implies$
 $\text{list_all2 } (\text{conf } G \text{ hp}) a \ c$ "
 ⟨proof⟩

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

theorem *no_type_error*:
 "assumes *welltyped*: " $\text{wt_jvm_prog } G \ \Phi$ " and *conforms*: " $G, \Phi \vdash_{\text{JVM}} s \ \checkmark$ "
 shows " $\text{exec_d } G \ (\text{Normal } s) \neq \text{TypeError}$ "
 ⟨proof⟩

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

theorem *welltyped_aggressive_imp_defensive*:
 " $\text{wt_jvm_prog } G \ \Phi \implies G, \Phi \vdash_{\text{JVM}} s \ \checkmark \implies G \vdash s \text{ -jvm} \rightarrow t$
 $\implies G \vdash (\text{Normal } s) \text{ -jvmd} \rightarrow (\text{Normal } t)$ "
 ⟨proof⟩

lemma *neq_TypeError_eq* [simp]: " $s \neq \text{TypeError} = (\exists s'. s = \text{Normal } s')$ "
 ⟨proof⟩

theorem *no_type_errors*:
 " $\text{wt_jvm_prog } G \ \Phi \implies G, \Phi \vdash_{\text{JVM}} s \ \checkmark$
 $\implies G \vdash (\text{Normal } s) \text{ -jvmd} \rightarrow t \implies t \neq \text{TypeError}$ "
 ⟨proof⟩

corollary *no_type_errors_initial*:
 fixes G (" Γ ") and Φ (" Φ ")
 assumes *wt*: " $\text{wt_jvm_prog } \Gamma \ \Phi$ "
 assumes *is_class*: " $\text{is_class } \Gamma \ C$ "
 and *method*: " $\text{method } (\Gamma, C) \ (m, []) = \text{Some } (C, b)$ "
 and *m*: " $m \neq \text{init}$ "
 defines *start*: " $s \equiv \text{start_state } \Gamma \ C \ m$ "

 assumes s : " $\Gamma \vdash (\text{Normal } s) \text{ -jvmd} \rightarrow t$ "
 shows " $t \neq \text{TypeError}$ "
 ⟨proof⟩

As corollary we get that the aggressive and the defensive machine are equivalent for welltyped

programs (if started in a conformant state or in the canonical start state)

```

corollary welltyped_commutates:
  fixes G ("Γ") and Phi ("Φ")
  assumes wt: "wt_jvm_prog Γ Φ" and *: "Γ, Φ ⊢ JVM s √"
  shows "Γ ⊢ (Normal s) -jvmd→ (Normal t) = Γ ⊢ s -jvm→ t"
  ⟨proof⟩

corollary welltyped_initial_commutates:
  fixes G ("Γ") and Phi ("Φ")
  assumes wt: "wt_jvm_prog Γ Φ"
  assumes is_class: "is_class Γ C"
    and method: "method (Γ, C) (m, []) = Some (C, b)"
    and m: "m ≠ init"
  defines start: "s ≡ start_state Γ C m"
  shows "Γ ⊢ (Normal s) -jvmd→ (Normal t) = Γ ⊢ s -jvm→ t"
  ⟨proof⟩

end

```

4.22 Kildall's Algorithm

theory Kildall **imports** SemilatAlg While_Combinator **begin**

consts

```
iter :: "'s binop ⇒ 's step_type ⇒
        's list ⇒ nat set ⇒ 's list × nat set"
propa :: "'s binop ⇒ (nat × 's) list ⇒ 's list ⇒ nat set ⇒ 's list * nat set"
```

primrec

```
"propa f [] ss w = (ss,w)"
"propa f (q'#qs) ss w = (let (q,t) = q';
                             u = t +_f ss!q;
                             w' = (if u = ss!q then w else insert q w)
                           in propa f qs (ss[q := u] w'))"
```

defs iter_def:

```
"iter f step ss w ==
  while (%(ss,w). w ≠ {})
    (%(ss,w). let p = SOME p. p ∈ w
              in propa f (step p (ss!p)) ss (w-{p}))
  (ss,w)"
```

constdefs

```
unstables :: "'s ord ⇒ 's step_type ⇒ 's list ⇒ nat set"
"unstables r step ss == {p. p < size ss ∧ ¬stable r step ss p}"
```

```
kildall :: "'s ord ⇒ 's binop ⇒ 's step_type ⇒ 's list ⇒ 's list"
"kildall r f step ss == fst(iter f step ss (unstables r step ss))"
```

consts merges :: "'s binop ⇒ (nat × 's) list ⇒ 's list ⇒ 's list"

primrec

```
"merges f [] ss = ss"
"merges f (p'#ps) ss = (let (p,s) = p' in merges f ps (ss[p := s +_f ss!p]))"
```

lemmas [simp] = Let_def semilat.le_iff_plus_unchanged [symmetric]

lemma (in semilat) nth_merges:

```
"∧ss. [p < length ss; ss ∈ list n A; ∀ (p,t) ∈ set ps. p < n ∧ t ∈ A] ⇒
  (merges f ps ss)!p = map snd [(p',t') ← ps. p' = p] ++_f ss!p"
(is "∧ss. [_; _; ?steptype ps] ⇒ ?P ss ps")
⟨proof⟩
```

lemma length_merges [rule_format, simp]:

```
"∀ss. size(merges f ps ss) = size ss"
⟨proof⟩
```


lemma (in semilat) merges_preserves_type_lemma:
 shows " $\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < n \wedge x \in A) \longrightarrow \text{merges } f \ ps \ xs \in \text{list } n \ A$ "
 <proof>

lemma (in semilat) merges_preserves_type [simp]:
 "[$xs \in \text{list } n \ A; \forall (p,x) \in \text{set } ps. p < n \wedge x \in A$]
 $\implies \text{merges } f \ ps \ xs \in \text{list } n \ A$ "
 <proof>

lemma (in semilat) merges_incr_lemma:
 " $\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A) \longrightarrow xs \leq[r] \text{merges } f \ ps \ xs$ "
 <proof>

lemma (in semilat) merges_incr:
 "[$xs \in \text{list } n \ A; \forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A$]
 $\implies xs \leq[r] \text{merges } f \ ps \ xs$ "
 <proof>

lemma (in semilat) merges_same_conv [rule_format]:
 " $(\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A) \longrightarrow (\text{merges } f \ ps \ xs = xs) = (\forall (p,x) \in \text{set } ps. x \leq_r xs!p))$ "
 <proof>

lemma (in semilat) list_update_le_listI [rule_format]:
 " $\text{set } xs \leq A \longrightarrow \text{set } ys \leq A \longrightarrow xs \leq[r] ys \longrightarrow p < \text{size } xs \longrightarrow x \leq_r ys!p \longrightarrow x \in A \longrightarrow xs[p := x +_f xs!p] \leq[r] ys$ "
 <proof>

lemma (in semilat) merges_pres_le_ub:
 assumes " $\text{set } ts \leq A$ " and " $\text{set } ss \leq A$ "
 and " $\forall (p,t) \in \text{set } ps. t \leq_r ts!p \wedge t \in A \wedge p < \text{size } ts$ " and " $ss \leq[r] ts$ "
 shows " $\text{merges } f \ ps \ ss \leq[r] ts$ "
 <proof>

lemma decomp_propa:
 " $\bigwedge ss \ w. (\forall (q,t) \in \text{set } qs. q < \text{size } ss) \implies \text{propa } f \ qs \ ss \ w = (\text{merges } f \ qs \ ss, \{q. \exists t. (q,t) \in \text{set } qs \wedge t +_f ss!q \neq ss!q\} \text{ Un } w)$ "
 <proof>

lemma (in semilat) stable_pres_lemma:
 shows "[$\text{pres_type step } n \ A; \text{bounded step } n; ss \in \text{list } n \ A; p \in w; \forall q \in w. q < n; \forall q. q < n \longrightarrow q \notin w \longrightarrow \text{stable } r \text{ step } ss \ q; q < n;$]

$\forall s'. (q, s') \in \text{set } (\text{step } p \text{ } (ss ! p)) \longrightarrow s' +_f ss ! q = ss ! q;$
 $q \notin w \vee q = p \parallel$
 $\implies \text{stable } r \text{ step } (\text{merges } f \text{ } (\text{step } p \text{ } (ss!p)) \text{ } ss) \text{ } q"$
 $\langle \text{proof} \rangle$

lemma (in semilat) merges_bounded_lemma:

$\parallel \text{mono } r \text{ step } n \text{ } A; \text{ bounded step } n;$
 $\forall (p', s') \in \text{set } (\text{step } p \text{ } (ss!p)). s' \in A; ss \in \text{list } n \text{ } A; ts \in \text{list } n \text{ } A; p < n;$
 $ss \leq[r] ts; \forall p. p < n \longrightarrow \text{stable } r \text{ step } ts \text{ } p \parallel$
 $\implies \text{merges } f \text{ } (\text{step } p \text{ } (ss!p)) \text{ } ss \leq[r] ts"$
 $\langle \text{proof} \rangle$

lemma termination_lemma: includes semilat

shows $\parallel ss \in \text{list } n \text{ } A; \forall (q, t) \in \text{set } qs. q < n \wedge t \in A; p \in w \parallel \implies$
 $ss <[r] \text{merges } f \text{ } qs \text{ } ss \vee$
 $\text{merges } f \text{ } qs \text{ } ss = ss \wedge \{q. \exists t. (q, t) \in \text{set } qs \wedge t +_f ss!q \neq ss!q\} \text{ Un } (w - \{p\}) < w"$
 $\langle \text{proof} \rangle$

lemma iter_properties[rule_format]: includes semilat

shows $\parallel \text{acc } r; \text{pres_type step } n \text{ } A; \text{mono } r \text{ step } n \text{ } A;$
 $\text{bounded step } n; \forall p \in w0. p < n; ss0 \in \text{list } n \text{ } A;$
 $\forall p < n. p \notin w0 \longrightarrow \text{stable } r \text{ step } ss0 \text{ } p \parallel \implies$
 $\text{iter } f \text{ step } ss0 \text{ } w0 = (ss', w')$
 \longrightarrow
 $ss' \in \text{list } n \text{ } A \wedge \text{stables } r \text{ step } ss' \wedge ss0 \leq[r] ss' \wedge$
 $(\forall ts \in \text{list } n \text{ } A. ss0 \leq[r] ts \wedge \text{stables } r \text{ step } ts \longrightarrow ss' \leq[r] ts)"$
 $\langle \text{proof} \rangle$

lemma kildall_properties: includes semilat

shows $\parallel \text{acc } r; \text{pres_type step } n \text{ } A; \text{mono } r \text{ step } n \text{ } A;$
 $\text{bounded step } n; ss0 \in \text{list } n \text{ } A \parallel \implies$
 $\text{kildall } r \text{ } f \text{ step } ss0 \in \text{list } n \text{ } A \wedge$
 $\text{stables } r \text{ step } (\text{kildall } r \text{ } f \text{ step } ss0) \wedge$
 $ss0 \leq[r] \text{kildall } r \text{ } f \text{ step } ss0 \wedge$
 $(\forall ts \in \text{list } n \text{ } A. ss0 \leq[r] ts \wedge \text{stables } r \text{ step } ts \longrightarrow$
 $\text{kildall } r \text{ } f \text{ step } ss0 \leq[r] ts)"$
 $\langle \text{proof} \rangle$

lemma is_bcv_kildall: includes semilat

shows $\parallel \text{acc } r; \text{top } r \text{ } T; \text{pres_type step } n \text{ } A; \text{bounded step } n; \text{mono } r \text{ step } n \text{ } A \parallel$
 $\implies \text{is_bcv } r \text{ } T \text{ step } n \text{ } A \text{ } (\text{kildall } r \text{ } f \text{ step})"$
 $\langle \text{proof} \rangle$

end

4.23 Kildall for the JVM

theory JVM imports Kildall Typing_Framework_JVM begin

constdefs

```

kiljvm :: "jvm_prog ⇒ nat ⇒ nat ⇒ ty ⇒ exception_table ⇒
          instr list ⇒ JVMType.state list ⇒ JVMType.state list"
"kiljvm G maxs maxr rT et bs ==
kildall (JVMType.le G maxs maxr) (JVMType.sup G maxs maxr) (exec G maxs rT et bs)"

wt_kil :: "jvm_prog ⇒ cname ⇒ ty list ⇒ ty ⇒ nat ⇒ nat ⇒
          exception_table ⇒ instr list ⇒ bool"
"wt_kil G C pTs rT mxs mxl et ins ==
check_bounded ins et ∧ 0 < size ins ∧
(let first = Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err));
 start = OK first#(replicate (size ins - 1) (OK None));
 result = kiljvm G mxs (1+size pTs+mxl) rT et ins start
 in ∀ n < size ins. result!n ≠ Err)"

wt_jvm_prog_kildall :: "jvm_prog ⇒ bool"
"wt_jvm_prog_kildall G ==
wf_prog (λG C (sig,rT,(maxs,maxl,b,et)). wt_kil G C (snd sig) rT maxs maxl et b) G"

```

theorem is_bcv_kiljvm:

```

"[[ wf_prog wf_mb G; bounded (exec G maxs rT et bs) (size bs) ]] ⇒
is_bcv (JVMType.le G maxs maxr) Err (exec G maxs rT et bs)
(size bs) (states G maxs maxr) (kiljvm G maxs maxr rT et bs)"
<proof>

```

lemma subset_replicate: "set (replicate n x) ⊆ {x}"

<proof>

lemma in_set_replicate:

"x ∈ set (replicate n y) ⇒ x = y"

<proof>

theorem wt_kil_correct:

assumes wf: "wf_prog wf_mb G"

assumes C: "is_class G C"

assumes pTs: "set pTs ⊆ types G"

assumes wtk: "wt_kil G C pTs rT maxs mxl et bs"

shows "∃ phi. wt_method G C pTs rT maxs mxl bs et phi"

<proof>

theorem wt_kil_complete:

assumes wf: "wf_prog wf_mb G"

assumes C: "is_class G C"

assumes pTs: "set pTs ⊆ types G"

```

    assumes wtm: "wt_method G C pTs rT maxs mxl bs et phi"

    shows "wt_kil G C pTs rT maxs mxl et bs"
  <proof>

```

```

theorem jvm_kildall_sound_complete:
  "wt_jvm_prog_kildall G = ( $\exists$ Phi. wt_jvm_prog G Phi)"
  <proof>

```

```

end

```

```

Implementation of finite sets by lists theory Executable_Set
imports List
begin

```

4.23.1 Definitional rewrites

```

lemma [code target: Set]:
  "A = B  $\longleftrightarrow$  A  $\subseteq$  B  $\wedge$  B  $\subseteq$  A"
  <proof>

```

```

declare subset_eq [code]

```

```

lemma [code]:
  "a  $\in$  A  $\longleftrightarrow$  ( $\exists$  x $\in$ A. x = a)"
  <proof>

```

```

definition
  filter_set :: "('a  $\Rightarrow$  bool)  $\Rightarrow$  'a set  $\Rightarrow$  'a set" where
    "filter_set P xs = {x $\in$ xs. P x}"

```

4.23.2 Operations on lists

Basic definitions

```

definition
  flip :: "('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b  $\Rightarrow$  'a  $\Rightarrow$  'c" where
    "flip f a b = f b a"

```

```

definition
  member :: "'a list  $\Rightarrow$  'a  $\Rightarrow$  bool" where
    "member xs x  $\longleftrightarrow$  x  $\in$  set xs"

```

```

definition
  insertl :: "'a  $\Rightarrow$  'a list  $\Rightarrow$  'a list" where
    "insertl x xs = (if member xs x then xs else x#xs)"

```

```

lemma [code target: List]: "member [] y  $\longleftrightarrow$  False"
and [code target: List]: "member (x#xs) y  $\longleftrightarrow$  y = x  $\vee$  member xs y"
  <proof>

```

```

fun
  drop_first :: "('a  $\Rightarrow$  bool)  $\Rightarrow$  'a list  $\Rightarrow$  'a list" where
    "drop_first f [] = []"

```

```

| "drop_first f (x#xs) = (if f x then xs else x # drop_first f xs)"
declare drop_first.simps [code del]
declare drop_first.simps [code target: List]

declare remove1.simps [code del]
lemma [code target: List]:
  "remove1 x xs = (if member xs x then drop_first ( $\lambda y. y = x$ ) xs else xs)"
<proof>

lemma member_nil [simp]:
  "member [] = ( $\lambda x. \text{False}$ )"
<proof>

lemma member_insertl [simp]:
  "x  $\in$  set (insertl x xs)"
<proof>

lemma insertl_member [simp]:
  fixes xs x
  assumes member: "member xs x"
  shows "insertl x xs = xs"
<proof>

lemma insertl_not_member [simp]:
  fixes xs x
  assumes member: " $\neg$  (member xs x)"
  shows "insertl x xs = x # xs"
<proof>

lemma foldr_remove1_empty [simp]:
  "foldr remove1 xs [] = []"
<proof>

```

Derived definitions

```

function unionl :: "'a list  $\Rightarrow$  'a list  $\Rightarrow$  'a list"
where
  "unionl [] ys = ys"
| "unionl xs ys = foldr insertl xs ys"
<proof>
termination <proof>

lemmas unionl_eq = unionl.simps(2)

function intersect :: "'a list  $\Rightarrow$  'a list  $\Rightarrow$  'a list"
where
  "intersect [] ys = []"
| "intersect xs [] = []"
| "intersect xs ys = filter (member xs) ys"
<proof>
termination <proof>

lemmas intersect_eq = intersect.simps(3)

```

```

function subtract :: "'a list  $\Rightarrow$  'a list  $\Rightarrow$  'a list"
where
  "subtract [] ys = ys"
| "subtract xs [] = []"
| "subtract xs ys = foldr remove1 xs ys"
<proof>
termination <proof>

```

```

lemmas subtract_eq = subtract.simps(3)

```

```

function map_distinct :: "('a  $\Rightarrow$  'b)  $\Rightarrow$  'a list  $\Rightarrow$  'b list"
where
  "map_distinct f [] = []"
| "map_distinct f xs = foldr (insertl o f) xs []"
<proof>
termination <proof>

```

```

lemmas map_distinct_eq = map_distinct.simps(2)

```

```

function unions :: "'a list list  $\Rightarrow$  'a list"
where
  "unions [] = []"
| "unions xs = foldr unionl xs []"
<proof>
termination <proof>

```

```

lemmas unions_eq = unions.simps(2)

```

```

consts intersects :: "'a list list  $\Rightarrow$  'a list"
primrec
  "intersects (x#xs) = foldr intersect xs x"

```

```

definition
  map_union :: "'a list  $\Rightarrow$  ('a  $\Rightarrow$  'b list)  $\Rightarrow$  'b list" where
    "map_union xs f = unions (map f xs)"

```

```

definition
  map_inter :: "'a list  $\Rightarrow$  ('a  $\Rightarrow$  'b list)  $\Rightarrow$  'b list" where
    "map_inter xs f = intersects (map f xs)"

```

4.23.3 Isomorphism proofs

```

lemma iso_member:
  "member xs x  $\longleftrightarrow$  x  $\in$  set xs"
  <proof>

```

```

lemma iso_insert:
  "set (insertl x xs) = insert x (set xs)"
  <proof>

```

```

lemma iso_remove1:
  assumes distinct: "distinct xs"
  shows "set (remove1 x xs) = set xs - {x}"
  <proof>

```

lemma iso_union:

"set (unionl xs ys) = set xs \cup set ys"
 <proof>

lemma iso_intersect:

"set (intersect xs ys) = set xs \cap set ys"
 <proof>

definition

subtract' :: "'a list \Rightarrow 'a list \Rightarrow 'a list" where
 "subtract' = flip subtract"

lemma iso_subtract:

fixes ys
 assumes distinct: "distinct ys"
 shows "set (subtract' ys xs) = set ys - set xs"
 and "distinct (subtract' ys xs)"
 <proof>

lemma iso_map_distinct:

"set (map_distinct f xs) = image f (set xs)"
 <proof>

lemma iso_unions:

"set (unions xss) = \bigcup set (map set xss)"
 <proof>

lemma iso_intersects:

"set (intersects (xs#xss)) = \bigcap set (map set (xs#xss))"
 <proof>

lemma iso_UNION:

"set (map_union xs f) = UNION (set xs) (set o f)"
 <proof>

lemma iso_INTER:

"set (map_inter (x#xs) f) = INTER (set (x#xs)) (set o f)"
 <proof>

definition

Blall :: "'a list \Rightarrow ('a \Rightarrow bool) \Rightarrow bool" where
 "Blall = flip list_all"

definition

Blex :: "'a list \Rightarrow ('a \Rightarrow bool) \Rightarrow bool" where
 "Blex = flip list_ex"

lemma iso_Ball:

"Blall xs f = Ball (set xs) f"
 <proof>

lemma iso_Bex:

"Blex xs f = Bex (set xs) f"
 <proof>

```
lemma iso_filter:
  "set (filter P xs) = filter_set P (set xs)"
  <proof>
```

4.23.4 code generator setup

<ML>

const serializations

```
consts_code
  "{}" ("{*[]*}")
  insert ("{*insertl*}")
  "op  $\cup$ " ("{*unionl*}")
  "op  $\cap$ " ("{*intersect*}")
  "op - :: 'a set  $\Rightarrow$  'a set  $\Rightarrow$  'a set" ("{* flip subtract *}")
  image ("{*map_distinct*}")
  Union ("{*unions*}")
  Inter ("{*intersects*}")
  UNION ("{*map_union*}")
  INTER ("{*map_inter*}")
  Ball ("{*Blall*}")
  Bex ("{*Blex*}")
  filter_set ("{*filter*}")
```

end

4.24 Example Welltypings

```
theory BVExample
imports "../JVM/JVMListExample" BVSpecTypeSafe JVM Executable_Set
begin
```

This theory shows type correctness of the example program in section 3.6 (p. 63) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

4.24.1 Setup

Since the types *cnam*, *vnam*, and *mname* are anonymous, we describe distinctness of names in the example by axioms:

```
axioms
  distinct_classes: "list_nam  $\neq$  test_nam"
  distinct_fields: "val_nam  $\neq$  next_nam"
```

Abbreviations for definitions we will have to use often in the proofs below:

```
lemmas name_defs    = list_name_def test_name_def val_name_def next_name_def
lemmas system_defs  = SystemClasses_def ObjectC_def NullPointerC_def
                      OutOfMemoryC_def ClassCastC_def
lemmas class_defs   = list_class_def test_class_def
```

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

```
lemma class_Object [simp]:
  "class E Object = Some (arbitrary, [], [])"
  <proof>
```

```
lemma class_NullPointer [simp]:
  "class E (Xcpt NullPointer) = Some (Object, [], [])"
  <proof>
```

```
lemma class_OutOfMemory [simp]:
  "class E (Xcpt OutOfMemory) = Some (Object, [], [])"
  <proof>
```

```
lemma class_ClassCast [simp]:
  "class E (Xcpt ClassCast) = Some (Object, [], [])"
  <proof>
```

```
lemma class_list [simp]:
  "class E list_name = Some list_class"
  <proof>
```

```
lemma class_test [simp]:
  "class E test_name = Some test_class"
  <proof>
```

```
lemma E_classes [simp]:
  "{C. is_class E C} = {list_name, test_name, Xcpt NullPointer,
```

Xcpt ClassCast, Xcpt OutOfMemory, Object}}"

⟨*proof*⟩

The subclass relation spelled out:

lemma *subcls1*:
 "subcls1 E = (λC D. (C, D) ∈ {(list_name, Object), (test_name, Object), (Xcpt NullPointerException, Object),
 (Xcpt ClassCast, Object), (Xcpt OutOfMemory, Object)}))"

⟨*proof*⟩

The subclass relation is acyclic; hence its converse is well founded:

lemma *notin_rtranc1*:
 "r** a b ⇒ a ≠ b ⇒ (λy. ¬ r a y) ⇒ False"

⟨*proof*⟩

lemma *acyclic_subcls1_E*: "acyclicP (subcls1 E)"

⟨*proof*⟩

lemma *wf_subcls1_E*: "wfP ((subcls1 E)⁻¹⁻¹)"

⟨*proof*⟩

Method and field lookup:

lemma *method_Object* [simp]:
 "method (E, Object) = empty"

⟨*proof*⟩

lemma *method_append* [simp]:
 "method (E, list_name) (append_name, [Class list_name]) =
 Some (list_name, PrimT Void, 3, 0, append_ins, [(1, 2, 8, Xcpt NullPointerException)])"

⟨*proof*⟩

lemma *method_makelist* [simp]:
 "method (E, test_name) (makelist_name, []) =
 Some (test_name, PrimT Void, 3, 2, make_list_ins, [])"

⟨*proof*⟩

lemma *field_val* [simp]:
 "field (E, list_name) val_name = Some (list_name, PrimT Integer)"

⟨*proof*⟩

lemma *field_next* [simp]:
 "field (E, list_name) next_name = Some (list_name, Class list_name)"

⟨*proof*⟩

lemma [simp]: "fields (E, Object) = []"

⟨*proof*⟩

lemma [simp]: "fields (E, Xcpt NullPointerException) = []"

⟨*proof*⟩

lemma [simp]: "fields (E, Xcpt ClassCast) = []"

⟨*proof*⟩

```
lemma [simp]: "fields (E, Xcpt OutOfMemory) = []"
  ⟨proof⟩
```

```
lemma [simp]: "fields (E, test_name) = []"
  ⟨proof⟩
```

```
lemmas [simp] = is_class_def
```

The next definition and three proof rules implement an algorithm to enumerate natural numbers. The command `apply (elim pc_end pc_next pc_0` transforms a goal of the form

$$pc < n \implies P \ pc$$

into a series of goals

$$P \ (0::'a)$$

$$P \ (Suc \ 0)$$

...

$$P \ n$$

```
constdefs
```

```
  intervall :: "nat  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  bool" ("_  $\in$  [_, _')")
  "x  $\in$  [a, b)  $\equiv$  a  $\leq$  x  $\wedge$  x < b"
```

```
lemma pc_0: "x < n  $\implies$  (x  $\in$  [0, n)  $\implies$  P x)  $\implies$  P x"
  ⟨proof⟩
```

```
lemma pc_next: "x  $\in$  [n0, n)  $\implies$  P n0  $\implies$  (x  $\in$  [Suc n0, n)  $\implies$  P x)  $\implies$  P x"
  ⟨proof⟩
```

```
lemma pc_end: "x  $\in$  [n,n)  $\implies$  P x"
  ⟨proof⟩
```

4.24.2 Program structure

The program is structurally wellformed:

```
lemma wf_struct:
  "wf_prog ( $\lambda G \ C \ mb. \ True$ ) E" (is "wf_prog ?mb E")
  ⟨proof⟩
```

4.24.3 Welltypings

We show welltypings of the methods `append_name` in class `list_name`, and `makelist_name` in class `test_name`:

```
lemmas eff_simps [simp] = eff_def norm_eff_def xcpt_eff_def
declare appInvoke [simp del]
```

```
constdefs
```

```
  phi_append :: method_type ("φa")
```

```

"φa ≡ map (λ(x,y). Some (x, map OK y)) [
  ( [], [Class list_name, Class list_name]),
  ( [Class list_name], [Class list_name, Class list_name]),
  ( [Class list_name], [Class list_name, Class list_name]),
  ( [Class list_name, Class list_name], [Class list_name, Class list_name]),
  ([NT, Class list_name, Class list_name], [Class list_name, Class list_name]),
  ( [Class list_name], [Class list_name, Class list_name]),
  ( [Class list_name, Class list_name], [Class list_name, Class list_name]),
  ( [PrimT Void], [Class list_name, Class list_name]),
  ( [Class Object], [Class list_name, Class list_name]),
  ( [], [Class list_name, Class list_name]),
  ( [Class list_name], [Class list_name, Class list_name]),
  ( [Class list_name, Class list_name], [Class list_name, Class list_name]),
  ( [], [Class list_name, Class list_name]),
  ( [PrimT Void], [Class list_name, Class list_name])]

```

```

lemma bounded_append [simp]:

```

```

  "check_bounded_append_ins [(Suc 0, 2, 8, Xcpt NullPointer)]"
  <proof>

```

```

lemma types_append [simp]: "check_types E 3 (Suc (Suc 0)) (map OK φa)"

```

```

  <proof>

```

```

lemma wt_append [simp]:

```

```

  "wt_method E list_name [Class list_name] (PrimT Void) 3 0 append_ins
    [(Suc 0, 2, 8, Xcpt NullPointer)] φa"
  <proof>

```

Some abbreviations for readability

syntax

```

Clist :: ty
Ctest :: ty

```

translations

```

"Clist" == "Class list_name"
"Ctest" == "Class test_name"

```

constdefs

```

phi_makelist :: method_type ("φm")
"φm ≡ map (λ(x,y). Some (x, y)) [
  ( [], [OK Ctest, Err , Err ]),
  ( [Clist], [OK Ctest, Err , Err ]),
  ( [Clist, Clist], [OK Ctest, Err , Err ]),
  ( [Clist], [OK Clist, Err , Err ]),
  ( [PrimT Integer, Clist], [OK Clist, Err , Err ]),
  ( [], [OK Clist, Err , Err ]),
  ( [Clist], [OK Clist, Err , Err ]),
  ( [Clist, Clist], [OK Clist, Err , Err ]),
  ( [Clist], [OK Clist, OK Clist, Err ]),
  ( [PrimT Integer, Clist], [OK Clist, OK Clist, Err ]),
  ( [], [OK Clist, OK Clist, Err ]),
  ( [Clist], [OK Clist, OK Clist, Err ]),
  ( [Clist, Clist], [OK Clist, OK Clist, Err ]),
  ( [Clist], [OK Clist, OK Clist, OK Clist]),

```

```

(
  (PrimT Integer, Clist], [OK Clist, OK Clist, OK Clist]),
(
  [], [OK Clist, OK Clist, OK Clist]),
(
  [Clist], [OK Clist, OK Clist, OK Clist]),
(
  [Clist, Clist], [OK Clist, OK Clist, OK Clist]),
(
  [PrimT Void], [OK Clist, OK Clist, OK Clist]),
(
  [], [OK Clist, OK Clist, OK Clist]),
(
  [Clist], [OK Clist, OK Clist, OK Clist]),
(
  [Clist, Clist], [OK Clist, OK Clist, OK Clist]),
(
  [PrimT Void], [OK Clist, OK Clist, OK Clist]))]

```

```

lemma bounded_makelist [simp]: "check_bounded make_list_ins []"
  <proof>

```

```

lemma types_makelist [simp]: "check_types E 3 (Suc (Suc (Suc 0))) (map OK  $\varphi_m$ )"
  <proof>

```

```

lemma wt_makelist [simp]:
  "wt_method E test_name [] (PrimT Void) 3 2 make_list_ins []  $\varphi_m$ "
  <proof>

```

The whole program is welltyped:

```

constdefs
  Phi :: prog_type ("Φ")
  "Φ C sg ≡ if C = test_name ∧ sg = (makelist_name, []) then  $\varphi_m$  else
    if C = list_name ∧ sg = (append_name, [Class list_name]) then  $\varphi_a$  else []"

```

```

lemma wf_prog:
  "wt_jvm_prog E Φ"
  <proof>

```

4.24.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

```

lemma "E, Φ ⊢ JVM start_state E test_name makelist_name √"
  <proof>

```

4.24.5 Example for code generation: inferring method types

```

constdefs
  test_kil :: "jvm_prog ⇒ cname ⇒ ty list ⇒ ty ⇒ nat ⇒ nat ⇒
    exception_table ⇒ instr list ⇒ JVMType.state list"
  "test_kil G C pTs rT mxs mxl et instr ==
    (let first = Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err));
      start = OK first#(replicate (size instr - 1) (OK None))
    in kiljvm G mxs (1+size pTs+mxl) rT et instr start)"

```

```

lemma [code]:
  "unstables r step ss = (UN p:{..

```

```

constdefs
  some_elem :: "'a set ⇒ 'a"
  "some_elem == (%S. SOME x. x : S)"

```

```

lemma [code]:
  "iter f step ss w =
    while (ss,w). w ≠ {}
      (ss,w). let p = some_elem w
              in propa f (step p (ss!p)) ss (w-{p}))
    (ss,w)"
  ⟨proof⟩

consts_code
  "some_elem" ("hd")

code_const some_elem
  (SML "hd")

lemma JVM_sup_unfold [code]:
  "JVMType.sup S m n = lift2 (Opt.sup
    (Product.sup (Listn.sup (JType.sup S))
      (λx y. OK (map2 (lift2 (JType.sup S)) x y))))"
  ⟨proof⟩

lemmas [code] =
  meta_eq_to_obj_eq [OF JType.sup_def [unfolded exec_lub_def]]
  meta_eq_to_obj_eq [OF JVM_le_unfold]

lemmas [code ind] = rtrancpl.rtrancpl_refl converse_rtrancpl_into_rtrancpl

code_module BV
contains
  test1 = "test_kil E list_name [Class list_name] (PrimT Void) 3 0
    [(Suc 0, 2, 8, Xcpt NullPointer)] append_ins"
  test2 = "test_kil E test_name [] (PrimT Void) 3 2 [] make_list_ins"

⟨ML⟩

end

```

```

theory AuxLemmas
imports "../J/JBasis"
begin

```

```

lemma app_nth_greater_len [rule_format (no_asm), simp]:
  "∀ ind. length pre ≤ ind → (pre @ a # post) ! (Suc ind) = (pre @ post) ! ind"

```

⟨proof⟩

lemma length_takeWhile: "v ∈ set xs ⇒ length (takeWhile (%z. z~v) xs) < length xs"
 ⟨proof⟩

lemma nth_length_takeWhile [simp]:
 "v ∈ set xs ⇒ xs ! (length (takeWhile (%z. z~v) xs)) = v"
 ⟨proof⟩

lemma map_list_update [simp]:
 "[x ∈ set xs; distinct xs] ⇒
 (map f xs) [length (takeWhile (λz. z ≠ x) xs) := v] =
 map (f(x:=v)) xs"
 ⟨proof⟩

lemma split_compose: "(split f) ∘ (λ (a,b). ((fa a), (fb b))) =
 (λ (a,b). (f (fa a) (fb b)))"
 ⟨proof⟩

lemma split_iter: "(λ (a,b,c). ((g1 a), (g2 b), (g3 c))) =
 (λ (a,p). ((g1 a), (λ (b, c). ((g2 b), (g3 c))) p)))"
 ⟨proof⟩

lemma singleton_in_set: "A = {a} ⇒ a ∈ A" ⟨proof⟩

lemma the_map_upd: "(the ∘ f(x↦v)) = (the ∘ f)(x:=v)"
 ⟨proof⟩

lemma map_of_in_set:
 "(map_of xs x = None) = (x ∉ set (map fst xs))"
 ⟨proof⟩

lemma map_map_upd [simp]:
 "y ∉ set xs ⇒ map (the ∘ f(y↦v)) xs = map (the ∘ f) xs"
 ⟨proof⟩

lemma map_map_upds [rule_format (no_asm), simp]:
 "∀ f vs. (∀ y∈set ys. y ∉ set xs) → map (the ∘ f(ys[↦]vs)) xs = map (the ∘ f) xs"

<proof>

lemma *map_upds_distinct* [rule_format (no_asm), simp]:
 "∀ f vs. length ys = length vs ⟶ distinct ys ⟶ map (the ∘ f(ys[↦]vs)) ys = vs"
<proof>

lemma *map_of_map_as_map_upd* [rule_format (no_asm)]: "distinct (map f zs) ⟶
 map_of (map (λ p. (f p, g p)) zs) = empty (map f zs [↦] map g zs)"
<proof>

lemma *map_upds_SomeD* [rule_format (no_asm)]:
 "∀ m ys. (m(xs[↦]ys)) k = Some y ⟶ k ∈ (set xs) ∨ (m k = Some y)"
<proof>

lemma *map_of_upds_SomeD*: "(map_of m (xs[↦]ys)) k = Some y
 ⟹ k ∈ (set (xs @ map fst m))"
<proof>

lemma *map_of_map_prop* [rule_format (no_asm)]:
 "(map_of (map f xs) k = Some v) ⟶
 (∀ x ∈ set xs. (P1 x)) ⟶
 (∀ x. (P1 x) ⟶ (P2 (f x))) ⟶
 (P2(k, v))"
<proof>

lemma *map_of_map2*: "∀ x ∈ set xs. (fst (f x)) = (fst x) ⟹
 map_of (map f xs) a = option_map (λ b. (snd (f (a, b)))) (map_of xs a)"
<proof>

lemma *option_map_of* [simp]: "(option_map f (map_of xs k) = None) = ((map_of xs k) = None)"
<proof>

end

theory *DefsComp*
imports "../JVM/JVMExec"
begin

constdefs
method_rT :: "cname × ty × 'c ⇒ ty"
"method_rT mtd == (fst (snd mtd))"

constdefs


```

gx :: "xstate ⇒ val option"  "gx ≡ fst"
gs :: "xstate ⇒ state"      "gs ≡ snd"
gh :: "xstate ⇒ aheap"       "gh ≡ fst∘snd"
gl :: "xstate ⇒ State.locals" "gl ≡ snd∘snd"

gmb :: "'a prog ⇒ cname ⇒ sig ⇒ 'a"
      "gmb G cn si ≡ snd(snd(the(method (G,cn) si)))"
gis :: "jvm_method ⇒ bytecode"
      "gis ≡ fst ∘ snd ∘ snd"

gjmb_pns  :: "java_mb ⇒ vname list"      "gjmb_pns ≡ fst"
gjmb_lvs  :: "java_mb ⇒ (vname×ty)list"  "gjmb_lvs ≡ fst∘snd"
gjmb_blk  :: "java_mb ⇒ stmt"            "gjmb_blk ≡ fst∘snd∘snd"
gjmb_res  :: "java_mb ⇒ expr"            "gjmb_res ≡ snd∘snd∘snd"
gjmb_plns :: "java_mb ⇒ vname list"
      "gjmb_plns ≡ λjmb. gjmb_pns jmb @ map fst (gjmb_lvs jmb)"

glvs :: "java_mb ⇒ State.locals ⇒ locvars"
      "glvs jmb loc ≡ map (the∘loc) (gjmb_plns jmb)"

lemmas gdefs = gx_def gh_def gl_def gmb_def gis_def glvs_def
lemmas gjmbdefs = gjmb_pns_def gjmb_lvs_def gjmb_blk_def gjmb_res_def gjmb_plns_def

lemmas galldefs = gdefs gjmbdefs

constdefs
  locvars_locals :: "java_mb prog ⇒ cname ⇒ sig ⇒ State.locals ⇒ locvars"
    "locvars_locals G C S lvs == the (lvs This) # glvs (gmb G C S) lvs"

  locals_locvars :: "java_mb prog ⇒ cname ⇒ sig ⇒ locvars ⇒ State.locals"
    "locals_locvars G C S lvs ==
    empty ((gjmb_plns (gmb G C S))[↦](tl lvs)) (This↦(hd lvs))"

  locvars_xstate :: "java_mb prog ⇒ cname ⇒ sig ⇒ xstate ⇒ locvars"
    "locvars_xstate G C S xs == locvars_locals G C S (gl xs)"

lemma locvars_xstate_par_dep:
  "lv1 = lv2 ⇒
  locvars_xstate G C S (xcpt1, hp1, lv1) = locvars_xstate G C S (xcpt2, hp2, lv2)"
  <proof>

lemma gx_conv [simp]: "gx (xcpt, s) = xcpt" <proof>

```

```
lemma gh_conv [simp]: "gh (xcpt, h, l) = h" <proof>
```

```
end
```

```
theory Index
imports AuxLemmas DefsComp
begin
```

```
constdefs
```

```
index :: "java_mb => vname => nat"
"index == λ (pn,lv,blk,res) v.
  if v = This
  then 0
  else Suc (length (takeWhile (λ z. z~=v) (pn @ map fst lv)))"
```

```
lemma index_length_pns: "
  [| i = index (pns,lvars,blk,res) vn;
    wf_java_mdecl G C ((mn,pTs),rT, (pns,lvars,blk,res));
    vn ∈ set pns |]
  ⇒ 0 < i ∧ i < Suc (length pns)"
<proof>
```

```
lemma index_length_lvars: "
  [| i = index (pns,lvars,blk,res) vn;
    wf_java_mdecl G C ((mn,pTs),rT, (pns,lvars,blk,res));
    vn ∈ set (map fst lvars) |]
  ⇒ (length pns) < i ∧ i < Suc((length pns) + (length lvars))"
<proof>
```

```
lemma select_at_index :
  "x ∈ set (gjmb_plns (gmb G C S)) ∨ x = This
  ⇒ (the (loc This) # glvs (gmb G C S) loc) ! (index (gmb G C S) x) =
    the (loc x)"
<proof>
```

```
lemma lift_if: "(f (if b then t else e)) = (if b then (f t) else (f e))"
<proof>
```

```
lemma update_at_index: "
  [| distinct (gjmb_plns (gmb G C S));
    x ∈ set (gjmb_plns (gmb G C S)); x ≠ This |] ⇒
  locvars_xstate G C S (Norm (h, l))[index (gmb G C S) x := val] =
    locvars_xstate G C S (Norm (h, l(x↦val)))"
```

⟨proof⟩

```
lemma index_of_var: "[[ xvar ∉ set pns; xvar ∉ set (map fst zs); xvar ≠ This ]]
  ⇒ index (pns, zs @ ((xvar, xval) # xys), blk, res) xvar = Suc (length pns + length
zs)"
⟨proof⟩
```

constdefs

```
disjoint_varnames :: "[vname list, (vname × ty) list] ⇒ bool"
```

```
"disjoint_varnames pns lvars ≡
distinct pns ∧ unique lvars ∧ This ∉ set pns ∧ This ∉ set (map fst lvars) ∧
(∀pn∈set pns. pn ∉ set (map fst lvars))"
```

```
lemma index_of_var2: "
  disjoint_varnames pns (lvars_pre @ (vn, ty) # lvars_post)
  ⇒ index (pns, lvars_pre @ (vn, ty) # lvars_post, blk, res) vn =
  Suc (length pns + length lvars_pre)"
⟨proof⟩
```

```
lemma wf_java_mdecl_disjoint_varnames:
  "wf_java_mdecl G C (S,rT,(pns,lvars,blk,res))
  ⇒ disjoint_varnames pns lvars"
⟨proof⟩
```

```
lemma wf_java_mdecl_length_pTs_pns:
  "wf_java_mdecl G C ((mn, pTs), rT, pns, lvars, blk, res)
  ⇒ length pTs = length pns"
⟨proof⟩
```

end

```
theory TranslCompTp
imports Index "../BV/JVMType"
begin
```

constdefs

```
comb :: "'a ⇒ 'b list × 'c, 'c ⇒ 'b list × 'd, 'a] ⇒ 'b list × 'd"
"comb == (λ f1 f2 x0. let (xs1, x1) = f1 x0;
```

```

                                (xs2, x2) = f2 x1
                                in (xs1 @ xs2, x2))"
comb_nil :: "'a ⇒ 'b list × 'a"
"comb_nil a == ([], a)"

syntax (xsymbols)
  "comb" :: "[ 'a ⇒ 'b list × 'c, 'c ⇒ 'b list × 'd, 'a ] ⇒ 'b list × 'd"
  (infixr "□" 55)

lemma comb_nil_left [simp]: "comb_nil □ f = f"
<proof>

lemma comb_nil_right [simp]: "f □ comb_nil = f"
<proof>

lemma comb_assoc [simp]: "(fa □ fb) □ fc = fa □ (fb □ fc)"
<proof>

lemma comb_inv: "(xs', x') = (f1 □ f2) x0 ⇒
  ∃ xs1 x1 xs2 x2. (xs1, x1) = (f1 x0) ∧ (xs2, x2) = f2 x1 ∧ xs' = xs1 @ xs2 ∧ x' = x2"
<proof>

syntax
  mt_of      :: "method_type × state_type ⇒ method_type"
  sttp_of    :: "method_type × state_type ⇒ state_type"

translations
  "mt_of"     => "fst"
  "sttp_of"   => "snd"

consts
  compTpExpr  :: "java_mb ⇒ java_mb prog ⇒ expr
                ⇒ state_type ⇒ method_type × state_type"

  compTpExprs :: "java_mb ⇒ java_mb prog ⇒ expr list
                ⇒ state_type ⇒ method_type × state_type"

  compTpStmt  :: "java_mb ⇒ java_mb prog ⇒ stmt
                ⇒ state_type ⇒ method_type × state_type"

constdefs
  nochangeST :: "state_type ⇒ method_type × state_type"
  "nochangeST sttp == ([Some sttp], sttp)"
  pushST :: "[ty list, state_type] ⇒ method_type × state_type"
  "pushST tps == (λ (ST, LT). ([Some (ST, LT)], (tps @ ST, LT)))"
  dupST :: "state_type ⇒ method_type × state_type"
  "dupST == (λ (ST, LT). ([Some (ST, LT)], (hd ST # ST, LT)))"
  dup_x1ST :: "state_type ⇒ method_type × state_type"
  "dup_x1ST == (λ (ST, LT). ([Some (ST, LT)],
    (hd ST # hd (tl ST) # hd ST # (tl (tl ST)), LT)))"
  popST :: "[nat, state_type] ⇒ method_type × state_type"

```

```

"popST n == (λ (ST, LT). ([Some (ST, LT)], (drop n ST, LT)))"
replST :: "[nat, ty, state_type] ⇒ method_type × state_type"
"replST n tp == (λ (ST, LT). ([Some (ST, LT)], (tp # (drop n ST), LT)))"

storeST :: "[nat, ty, state_type] ⇒ method_type × state_type"
"storeST i tp == (λ (ST, LT). ([Some (ST, LT)], (tl ST, LT [i := OK tp])))"

```

primrec

```

"compTpExpr jmb G (NewC c) = pushST [Class c]"

"compTpExpr jmb G (Cast c e) =
(compTpExpr jmb G e) □ (replST 1 (Class c))"

"compTpExpr jmb G (Lit val) = pushST [the (typeof (λv. None) val)]"

"compTpExpr jmb G (BinOp bo e1 e2) =
  (compTpExpr jmb G e1) □ (compTpExpr jmb G e2) □
  (case bo of
    Eq => popST 2 □ pushST [PrimT Boolean] □ popST 1 □ pushST [PrimT Boolean]
  | Add => replST 2 (PrimT Integer))"

"compTpExpr jmb G (LAcc vn) = (λ (ST, LT).
  pushST [ok_val (LT ! (index jmb vn))] (ST, LT))"

"compTpExpr jmb G (vn ::= e) =
  (compTpExpr jmb G e) □ dupST □ (popST 1)"

"compTpExpr jmb G ( {cn}e..fn ) =
  (compTpExpr jmb G e) □ replST 1 (snd (the (field (G,cn) fn)))"

"compTpExpr jmb G (FAss cn e1 fn e2 ) =
  (compTpExpr jmb G e1) □ (compTpExpr jmb G e2) □ dup_x1ST □ (popST 2)"

"compTpExpr jmb G ({C}a..mn({fpTs}ps)) =
  (compTpExpr jmb G a) □ (compTpExprs jmb G ps) □
  (replST ((length ps) + 1) (method_rT (the (method (G,C) (mn,fpTs)))))"

"compTpExprs jmb G [] = comb_nil"

"compTpExprs jmb G (e#es) = (compTpExpr jmb G e) □ (compTpExprs jmb G es)"

```

primrec

```

"compTpStmt jmb G Skip = comb_nil"

```

```

"compTpStmt jmb G (Expr e) = (compTpExpr jmb G e) □ popST 1"

"compTpStmt jmb G (c1;; c2) = (compTpStmt jmb G c1) □ (compTpStmt jmb G c2)"

"compTpStmt jmb G (If(e) c1 Else c2) =
  (pushST [PrimT Boolean]) □ (compTpExpr jmb G e) □ popST 2 □
  (compTpStmt jmb G c1) □ nochangeST □ (compTpStmt jmb G c2)"

"compTpStmt jmb G (While(e) c) =
  (pushST [PrimT Boolean]) □ (compTpExpr jmb G e) □ popST 2 □
  (compTpStmt jmb G c) □ nochangeST"

constdefs
  compTpInit :: "java_mb ⇒ (vname * ty)
    ⇒ state_type ⇒ method_type × state_type"
  "compTpInit jmb == (λ (vn,ty). (pushST [ty]) □ (storeST (index jmb vn) ty))"

consts
  compTpInitLvars :: "[java_mb, (vname × ty) list]
    ⇒ state_type ⇒ method_type × state_type"

primrec
  "compTpInitLvars jmb [] = comb_nil"
  "compTpInitLvars jmb (lv#lvars) = (compTpInit jmb lv) □ (compTpInitLvars jmb lvars)"

constdefs
  start_ST :: "opstack_type"
  "start_ST == []"

  start_LT :: "cname ⇒ ty list ⇒ nat ⇒ locvars_type"
  "start_LT C pTs n == (OK (Class C))#((map OK pTs))@(replicate n Err)"

  compTpMethod :: "[java_mb prog, cname, java_mb mdecl] ⇒ method_type"
  "compTpMethod G C == λ ((mn,pTs),rT, jmb).
    let (pns,lvars,blk,res) = jmb
    in (mt_of
      ((compTpInitLvars jmb lvars □
        compTpStmt jmb G blk □
        compTpExpr jmb G res □
        nochangeST)
      (start_ST, start_LT C pTs (length lvars))))"

  compTp :: "java_mb prog ⇒ prog_type"
  "compTp G C sig == let (D, rT, jmb) = (the (method (G, C) sig))
    in compTpMethod G C (sig, rT, jmb)"

constdefs
  ssize_sto :: "(state_type option) ⇒ nat"

```

```

"ssize_sto sto == case sto of None => 0 | (Some (ST, LT)) => length ST"

max_of_list :: "nat list => nat"
"max_of_list xs == foldr max xs 0"

max_ssize :: "method_type => nat"
"max_ssize mt == max_of_list (map ssize_sto mt)"

end

```

```

theory TranslComp imports TranslCompTp begin

```

```

consts
  compExpr  :: "java_mb => expr      => instr list"
  compExprs :: "java_mb => expr list => instr list"
  compStmt  :: "java_mb => stmt      => instr list"

```

```

primrec

```

```

"compExpr jmb (NewC c) = [New c]"

"compExpr jmb (Cast c e) = compExpr jmb e @ [Checkcast c]"

"compExpr jmb (Lit val) = [LitPush val]"

"compExpr jmb (BinOp bo e1 e2) = compExpr jmb e1 @ compExpr jmb e2 @
  (case bo of Eq => [Ifcmpeq 3, LitPush (Bool False), Goto 2, LitPush (Bool True)]
    | Add => [IAdd])"

"compExpr jmb (LAcc vn) = [Load (index jmb vn)]"

"compExpr jmb (vn ::= e) = compExpr jmb e @ [Dup , Store (index jmb vn)]"

```

```
"compExpr jmb ( {cn}e..fn ) = compExpr jmb e @ [Getfield fn cn]"
```

```
"compExpr jmb (FAss cn e1 fn e2 ) =
  compExpr jmb e1 @ compExpr jmb e2 @ [Dup_x1 , Putfield fn cn]"
```

```
"compExpr jmb (Call cn e1 mn X ps) =
  compExpr jmb e1 @ compExprs jmb ps @ [Invoke cn mn X]"
```

```
"compExprs jmb []      = []"
```

```
"compExprs jmb (e#es) = compExpr jmb e @ compExprs jmb es"
```

primrec

```
"compStmt jmb Skip = []"
```

```
"compStmt jmb (Expr e) = ((compExpr jmb e) @ [Pop])"
```

```
"compStmt jmb (c1;; c2) = ((compStmt jmb c1) @ (compStmt jmb c2))"
```

```
"compStmt jmb (If(e) c1 Else c2) =
  (let cnstf = LitPush (Bool False);
    cnd   = compExpr jmb e;
    thn   = compStmt jmb c1;
    els   = compStmt jmb c2;
    test  = Ifcmpeq (int(length thn +2));
    thnex = Goto (int(length els +1))
  in
    [cnstf] @ cnd @ [test] @ thn @ [thnex] @ els)"
```

```
"compStmt jmb (While(e) c) =
  (let cnstf = LitPush (Bool False);
    cnd   = compExpr jmb e;
    bdy   = compStmt jmb c;
    test  = Ifcmpeq (int(length bdy +2));
    loop  = Goto (-(int((length bdy) + (length cnd) +2)))
  in
    [cnstf] @ cnd @ [test] @ bdy @ [loop])"
```


constdefs

```

load_default_val :: "ty => instr"
"load_default_val ty == LitPush (default_val ty)"

compInit :: "java_mb => (vname * ty) => instr list"
"compInit jmb ==  $\lambda$  (vn,ty). [load_default_val ty, Store (index jmb vn)]"

compInitLvars :: "[java_mb, (vname  $\times$  ty) list]  $\Rightarrow$  bytecode"
"compInitLvars jmb lvars == concat (map (compInit jmb) lvars)"

compMethod :: "java_mb prog  $\Rightarrow$  cname  $\Rightarrow$  java_mb mdecl  $\Rightarrow$  jvm_method mdecl"
"compMethod G C jmdl == let (sig, rT, jmb) = jmdl;
                        (pns,lvars,blk,res) = jmb;
                        mt = (compTpMethod G C jmdl);
                        bc = compInitLvars jmb lvars @
                        compStmt jmb blk @ compExpr jmb res @
                        [Return]
                        in (sig, rT, max_ssize mt, length lvars, bc, [])"

compClass :: "java_mb prog => java_mb cdecl=> jvm_method cdecl"
"compClass G ==  $\lambda$  (C,cno,fdls,jmdls). (C,cno,fdls, map (compMethod G C) jmdls)"

comp :: "java_mb prog => jvm_prog"
"comp G == map (compClass G) G"

```

end

theory LemmasComp
imports TranslComp
begin

```

declare split_paired_All [simp del]
declare split_paired_Ex [simp del]

```

```

lemma split_pairs: " $(\lambda(a,b). (F a b)) (ab) = F (fst ab) (snd ab)$ "
<proof>

```

```

lemma c_hupd_conv:

```

"c_hupd h' (xo, (h,l)) = (xo, (if xo = None then h' else h),l)"
 <proof>

lemma gl_c_hupd [simp]: "(gl (c_hupd h xs)) = (gl xs)"
 <proof>

lemma c_hupd_xcpt_invariant [simp]: "gx (c_hupd h' (xo, st)) = xo"
 <proof>

lemma c_hupd_hp_invariant: "gh (c_hupd hp (None, st)) = hp"
 <proof>

lemma unique_map_fst [rule_format]: "($\forall x \in \text{set } xs. (\text{fst } x = \text{fst } (f \ x))$) \longrightarrow
 unique (map f xs) = unique xs"
 <proof>

lemma comp_unique: "unique (comp G) = unique G"
 <proof>

lemma comp_class_imp:
 "(class G C = Some(D, fs, ms)) \implies
 (class (comp G) C = Some(D, fs, map (compMethod G C) ms))"
 <proof>

lemma comp_class_None:
 "(class G C = None) = (class (comp G) C = None)"
 <proof>

lemma comp_is_class: "is_class (comp G) C = is_class G C"
 <proof>

lemma comp_is_type: "is_type (comp G) T = is_type G T"
 <proof>

lemma comp_classname: "is_class G C
 $\implies \text{fst } (\text{the } (\text{class } G \ C)) = \text{fst } (\text{the } (\text{class } (\text{comp } G) \ C))$ "
 <proof>

lemma comp_subcls1: "subcls1 (comp G) = subcls1 G"
 <proof>

lemma comp_widen: "widen (comp G) = widen G"
 <proof>

lemma comp_cast: "cast (comp G) = cast G"
 <proof>

lemma `comp_cast_ok`: "`cast_ok (comp G) = cast_ok G`"
 $\langle proof \rangle$

lemma `compClass_fst [simp]`: "`(fst (compClass G C)) = (fst C)`"
 $\langle proof \rangle$

lemma `compClass_fst_snd [simp]`: "`(fst (snd (compClass G C))) = (fst (snd C))`"
 $\langle proof \rangle$

lemma `compClass_fst_snd_snd [simp]`: "`(fst (snd (snd (compClass G C)))) = (fst (snd (snd C)))`"
 $\langle proof \rangle$

lemma `comp_wf_fdecl [simp]`: "`wf_fdecl (comp G) fd = wf_fdecl G fd`"
 $\langle proof \rangle$

lemma `compClass_forall [simp]`: "
 ($\forall x \in \text{set } (\text{snd } (\text{snd } (\text{snd } (\text{compClass } G \ C))))$). $P \text{ (fst } x \text{) (fst (snd } x \text{))}$) =
 ($\forall x \in \text{set } (\text{snd } (\text{snd } (\text{snd } C)))$). $P \text{ (fst } x \text{) (fst (snd } x \text{))}$)"
 $\langle proof \rangle$

lemma `comp_wf_mhead`: "`wf_mhead (comp G) S rT = wf_mhead G S rT`"
 $\langle proof \rangle$

lemma `comp_ws_cdecl`: "
`ws_cdecl (TranslComp.comp G) (compClass G C) = ws_cdecl G C`"
 $\langle proof \rangle$

lemma `comp_wf_syscls`: "`wf_syscls (comp G) = wf_syscls G`"
 $\langle proof \rangle$

lemma `comp_ws_prog`: "`ws_prog (comp G) = ws_prog G`"
 $\langle proof \rangle$

lemma `comp_class_rec`: "`wfP ((subcls1 G)^--1) \implies`
`class_rec (comp G) C t f =`
`class_rec G C t ($\lambda C' fs' ms' r'. f C' fs' (\text{map } (\text{compMethod } G \ C') \ ms') \ r')$)"`

lemma `comp_fields`: "`wfP ((subcls1 G)^--1) \implies`
`fields (comp G,C) = fields (G,C)`"
 $\langle proof \rangle$

lemma `comp_field`: "`wfP ((subcls1 G)^--1) \implies`
`field (comp G,C) = field (G,C)`"
 $\langle proof \rangle$

```

lemma class_rec_relation [rule_format (no_asm)]: "[ ws_prog G;
  ∀ fs ms. R (f1 Object fs ms t1) (f2 Object fs ms t2);
  ∀ C fs ms r1 r2. (R r1 r2) → (R (f1 C fs ms r1) (f2 C fs ms r2)) ]
  ⇒ ((class G C) ≠ None) →
  R (class_rec G C t1 f1) (class_rec G C t2 f2)"
⟨proof⟩

```

syntax

```
mtd_mb :: "cname × ty × 'c ⇒ 'c"
```

translations

```
"mtd_mb" => "Fun.comp snd snd"
```

```

lemma map_of_map_fst: "[ inj f;
  ∀ x ∈ set xs. fst (f x) = fst x; ∀ x ∈ set xs. fst (g x) = fst x ]
  ⇒ map_of (map g xs) k
  = option_map (λ e. (snd (g ((inv f) (k, e)))) (map_of (map f xs) k))"
⟨proof⟩

```

```

lemma comp_method [rule_format (no_asm)]: "[ ws_prog G; is_class G C ] ⇒
  ((method (comp G, C) S) =
  option_map (λ (D,rT,b). (D, rT, mtd_mb (compMethod G D (S, rT, b))))
  (method (G, C) S))"
⟨proof⟩

```

```

lemma comp_wf_mrT: "[ ws_prog G; is_class G D ] ⇒
  wf_mrT (TranslComp.comp G) (C, D, fs, map (compMethod G a) ms) =
  wf_mrT G (C, D, fs, ms)"
⟨proof⟩

```

```

lemma max_spec_preserves_length:
  "max_spec G C (mn, pTs) = {(md,rT),pTs'}"
  ⇒ length pTs = length pTs'"
⟨proof⟩

```

```

lemma ty_exprs_length [simp]: "(E ⊢ es[::]Ts → length es = length Ts)"
⟨proof⟩

```

```

lemma max_spec_preserves_method_rT [simp]:
  "max_spec G C (mn, pTs) = {(md,rT),pTs'}"
  ⇒ method_rT (the (method (G, C) (mn, pTs')))) = rT"
⟨proof⟩

```

```

declare compClass_fst [simp del]
declare compClass_fst_snd [simp del]
declare compClass_fst_snd_snd [simp del]

declare split_paired_All [simp add]
declare split_paired_Ex [simp add]

end

```

```

theory CorrComp
imports "../J/JTypeSafe" "LemmasComp"
begin

```

```

declare wf_prog_ws_prog [simp add]

```

```

lemma eval_evals_exec_xcpt:
  "(G ⊢ xs -ex>val-> xs' ⟶ gx xs' = None ⟶ gx xs = None) ∧
   (G ⊢ xs -exs[>]vals-> xs' ⟶ gx xs' = None ⟶ gx xs = None) ∧
   (G ⊢ xs -st-> xs' ⟶ gx xs' = None ⟶ gx xs = None)"
⟨proof⟩

```

```

lemma eval_xcpt: "G ⊢ xs -ex>val-> xs' ⟹ gx xs' = None ⟹ gx xs = None"
  (is "?H1 ⟹ ?H2 ⟹ ?T")
⟨proof⟩

```

```

lemma evals_xcpt: "G ⊢ xs -exs[>]vals-> xs' ⟹ gx xs' = None ⟹ gx xs = None"
  (is "?H1 ⟹ ?H2 ⟹ ?T")
⟨proof⟩

```

```

lemma exec_xcpt: "G ⊢ xs -st-> xs' ⟹ gx xs' = None ⟹ gx xs = None"
  (is "?H1 ⟹ ?H2 ⟹ ?T")
⟨proof⟩

```

```

theorem exec_all_trans: "[[exec_all G s0 s1]; (exec_all G s1 s2)] ⟹ (exec_all G s0 s2)"
⟨proof⟩

```

```

theorem exec_all_refl: "exec_all G s s"
⟨proof⟩

```

```

theorem exec_instr_in_exec_all:

```

```

"[[ exec_instr i G hp stk lvars C S pc frs = (None, hp', frs');
    gis (gmb G C S) ! pc = i ]] ==>
    G ⊢ (None, hp, (stk, lvars, C, S, pc) # frs) -jvm→ (None, hp', frs')"
<proof>

```

```

theorem exec_all_one_step: "
[[ gis (gmb G C S) = pre @ (i # post); pc0 = length pre;
  (exec_instr i G hp0 stk0 lvars0 C S pc0 frs) =
  (None, hp1, (stk1, lvars1, C, S, Suc pc0) # frs) ]]
==>
G ⊢ (None, hp0, (stk0, lvars0, C, S, pc0) # frs) -jvm→
  (None, hp1, (stk1, lvars1, C, S, Suc pc0) # frs)"
<proof>

```

constdefs

```

progression :: "jvm_prog ⇒ cname ⇒ sig ⇒
  aheap ⇒ opstack ⇒ locvars ⇒
  bytecode ⇒
  aheap ⇒ opstack ⇒ locvars ⇒
  bool"
("{_,_,_} ⊢ {_,_,_} >- _ → {_,_,_}" [61,61,61,61,61,61,90,61,61,61]60)
"{G,C,S} ⊢ {hp0, os0, lvars0} >- instrs → {hp1, os1, lvars1} ==
∀ pre post frs.
(gis (gmb G C S) = pre @ instrs @ post) →
G ⊢ (None, hp0, (os0, lvars0, C, S, length pre) # frs) -jvm→
  (None, hp1, (os1, lvars1, C, S, (length pre) + (length instrs)) # frs)"

```

lemma progression_call:

```

"[[ ∀ pc frs.
exec_instr instr G hp0 os0 lvars0 C S pc frs =
  (None, hp', (os', lvars', C', S', 0) # (fr pc) # frs) ∧
gis (gmb G C' S') = instrs' @ [Return] ∧
{G, C', S'} ⊢ {hp', os', lvars'} >- instrs' → {hp'', os'', lvars''} ∧
exec_instr Return G hp'' os'' lvars'' C' S' (length instrs')
  ((fr pc) # frs) =
  (None, hp2, (os2, lvars2, C, S, Suc pc) # frs) ]] ==>
{G, C, S} ⊢ {hp0, os0, lvars0} >- [instr] → {hp2, os2, lvars2}"
<proof>

```

lemma progression_transitive:

```

"[[ instrs_comb = instrs0 @ instrs1;
{G, C, S} ⊢ {hp0, os0, lvars0} >- instrs0 → {hp1, os1, lvars1};
{G, C, S} ⊢ {hp1, os1, lvars1} >- instrs1 → {hp2, os2, lvars2} ]]
==>
{G, C, S} ⊢ {hp0, os0, lvars0} >- instrs_comb → {hp2, os2, lvars2}"
<proof>

```

lemma progression_refl:

```
"{G, C, S} ⊢ {hp0, os0, lvars0} >- [] → {hp0, os0, lvars0}"
⟨proof⟩
```

```
lemma progression_one_step: "
  ∀ pc frs.
  (exec_instr i G hp0 os0 lvars0 C S pc frs) =
  (None, hp1, (os1, lvars1, C, S, Suc pc) # frs)
  ⇒ {G, C, S} ⊢ {hp0, os0, lvars0} >- [i] → {hp1, os1, lvars1}"
⟨proof⟩
```

constdefs

```
jump_fwd :: "jvm_prog ⇒ cname ⇒ sig ⇒
  aheap ⇒ locvars ⇒ opstack ⇒ opstack ⇒
  instr ⇒ bytecode ⇒ bool"
"jump_fwd G C S hp lvars os0 os1 instr instrs ==
  ∀ pre post frs.
  (gis (gmb G C S) = pre @ instr # instrs @ post) ⟶
  exec_all G (None, hp, (os0, lvars, C, S, length pre) # frs)
  (None, hp, (os1, lvars, C, S, (length pre) + (length instrs) + 1) # frs)"
```

```
lemma jump_fwd_one_step:
  "∀ pc frs.
  exec_instr instr G hp os0 lvars C S pc frs =
  (None, hp, (os1, lvars, C, S, pc + (length instrs) + 1) # frs)
  ⇒ jump_fwd G C S hp lvars os0 os1 instr instrs"
⟨proof⟩
```

lemma jump_fwd_progression_aux:

```
"[ instrs_comb = instr # instrs0 @ instrs1;
  jump_fwd G C S hp lvars os0 os1 instr instrs0;
  {G, C, S} ⊢ {hp, os1, lvars} >- instrs1 → {hp2, os2, lvars2} ]
  ⇒ {G, C, S} ⊢ {hp, os0, lvars} >- instrs_comb → {hp2, os2, lvars2}"
⟨proof⟩
```

lemma jump_fwd_progression:

```
"[ instrs_comb = instr # instrs0 @ instrs1;
  ∀ pc frs.
  exec_instr instr G hp os0 lvars C S pc frs =
  (None, hp, (os1, lvars, C, S, pc + (length instrs0) + 1) # frs);
  {G, C, S} ⊢ {hp, os1, lvars} >- instrs1 → {hp2, os2, lvars2} ]
  ⇒ {G, C, S} ⊢ {hp, os0, lvars} >- instrs_comb → {hp2, os2, lvars2}"
⟨proof⟩
```

constdefs

```
jump_bwd :: "jvm_prog ⇒ cname ⇒ sig ⇒
  aheap ⇒ locvars ⇒ opstack ⇒ opstack ⇒
  bytecode ⇒ instr ⇒ bool"
"jump_bwd G C S hp lvars os0 os1 instrs instr ==
```

```

 $\forall$  pre post frs.
(gis (gmb G C S) = pre @ instrs @ instr # post)  $\longrightarrow$ 
exec_all G (None, hp, (os0, lvars, C, S, (length pre) + (length instrs)) # frs)
  (None, hp, (os1, lvars, C, S, (length pre)) # frs)

```

lemma jump_bwd_one_step:

```

"  $\forall$  pc frs.
exec_instr instr G hp os0 lvars C S (pc + (length instrs)) frs =
  (None, hp, (os1, lvars, C, S, pc) # frs)
 $\implies$ 
jump_bwd G C S hp lvars os0 os1 instrs instr"
<proof>

```

lemma jump_bwd_progression:

```

"[ instrs_comb = instrs @ [instr];
{G, C, S}  $\vdash$  {hp0, os0, lvars0} >- instrs  $\rightarrow$  {hp1, os1, lvars1};
jump_bwd G C S hp1 lvars1 os1 os2 instrs instr;
{G, C, S}  $\vdash$  {hp1, os2, lvars1} >- instrs_comb  $\rightarrow$  {hp3, os3, lvars3} ]
 $\implies$  {G, C, S}  $\vdash$  {hp0, os0, lvars0} >- instrs_comb  $\rightarrow$  {hp3, os3, lvars3}"
<proof>

```

```

constdefs class_sig_defined :: "'c prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  bool"
"class_sig_defined G C S ==
is_class G C  $\wedge$  ( $\exists$  D rT mb. (method (G, C) S = Some (D, rT, mb)))

```

```

constdefs env_of_jmb :: "java_mb prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  java_mb env"
"env_of_jmb G C S ==
(let (mn, pTs) = S;
  (D, rT, (pns, lvars, blk, res)) = the(method (G, C) S) in
  (G, map_of lvars (pns[ $\mapsto$ ] pTs) (This  $\mapsto$  Class C)))

```

lemma env_of_jmb_fst [simp]: "fst (env_of_jmb G C S) = G"
 <proof>

lemma method_preserves [rule_format (no_asm)]:

```

"[ wf_prog wf_mb G; is_class G C;
 $\forall$  S rT mb.  $\forall$  cn  $\in$  fst ' set G. wf_mdecl wf_mb G cn (S, rT, mb)  $\longrightarrow$  (P cn S (rT, mb)) ]
 $\implies$   $\forall$  D.
  method (G, C) S = Some (D, rT, mb)  $\longrightarrow$  (P D S (rT, mb))"
<proof>

```


lemma *method_preserves_length*:

```
"[[ wf_java_prog G; is_class G C;
  method (G, C) (mn,pTs) = Some (D, rT, pns, lvars, blk, res)]]
  ==> length pns = length pTs"
<proof>
```

constdefs *wtpd_expr* :: "java_mb env => expr => bool"

```
"wtpd_expr E e == (∃ T. E ⊢ e :: T)"
wtpd_exprs :: "java_mb env => (expr list) => bool"
"wtpd_exprs E e == (∃ T. E ⊢ e [::] T)"
wtpd_stmt :: "java_mb env => stmt => bool"
"wtpd_stmt E c == (E ⊢ c √)"
```

lemma *wtpd_expr_newc*: "wtpd_expr E (NewC C) ==> is_class (prg E) C"

<proof>

lemma *wtpd_expr_cast*: "wtpd_expr E (Cast cn e) ==> (wtpd_expr E e)"

<proof>

lemma *wtpd_expr_lacc*: "[[wtpd_expr (env_of_jmb G C S) (LAcc vn);
 class_sig_defined G C S]]

```
==> vn ∈ set (gjmb_plns (gmb G C S)) ∨ vn = This"
```

<proof>

lemma *wtpd_expr_lass*: "wtpd_expr E (vn ::= e)"

```
==> (vn ≠ This) & (wtpd_expr E (LAcc vn)) & (wtpd_expr E e)"
```

<proof>

lemma *wtpd_expr_facc*: "wtpd_expr E ({fd}a..fn)"

```
==> (wtpd_expr E a)"
```

<proof>

lemma *wtpd_expr_fass*: "wtpd_expr E ({fd}a..fn:=v)"

```
==> (wtpd_expr E ({fd}a..fn)) & (wtpd_expr E v)"
```

<proof>

lemma *wtpd_expr_binop*: "wtpd_expr E (BinOp bop e1 e2)"

```
==> (wtpd_expr E e1) & (wtpd_expr E e2)"
```

<proof>

lemma *wtpd_exprs_cons*: "wtpd_exprs E (e # es)"

```
==> (wtpd_expr E e) & (wtpd_exprs E es)"
```

<proof>

lemma *wtpd_stmt_expr*: "wtpd_stmt E (Expr e) ==> (wtpd_expr E e)"

<proof>

lemma *wtpd_stmt_comp*: "wtpd_stmt E (s1;; s2) ==>

```
(wtpd_stmt E s1) & (wtpd_stmt E s2)"
```

<proof>

```

lemma wtpd_stmt_cond: "wtpd_stmt E (If(e) s1 Else s2) ==>
  (wtpd_expr E e) & (wtpd_stmt E s1) & (wtpd_stmt E s2)
  & (E⊢e::PrimT Boolean)"
<proof>

```

```

lemma wtpd_stmt_loop: "wtpd_stmt E (While(e) s) ==>
  (wtpd_expr E e) & (wtpd_stmt E s) & (E⊢e::PrimT Boolean)"
<proof>

```

```

lemma wtpd_expr_call: "wtpd_expr E ({C}a..mn({pTs'}ps))
  ==> (wtpd_expr E a) & (wtpd_exprs E ps)
  & (length ps = length pTs') & (E⊢a::Class C)
  & (∃ pTs md rT.
    E⊢ps[::]pTs & max_spec (prg E) C (mn, pTs) = {((md,rT),pTs')})"
<proof>

```

```

lemma wtpd_blk:
  "[[ method (G, D) (md, pTs) = Some (D, rT, (pns, lvars, blk, res));
    wf_prog wf_java_mdecl G; is_class G D ]]"
  ==> wtpd_stmt (env_of_jmb G D (md, pTs)) blk"
<proof>

```

```

lemma wtpd_res:
  "[[ method (G, D) (md, pTs) = Some (D, rT, (pns, lvars, blk, res));
    wf_prog wf_java_mdecl G; is_class G D ]]"
  ==> wtpd_expr (env_of_jmb G D (md, pTs)) res"
<proof>

```

```

lemma evals_preserves_length:
  "G⊢ xs -es[>]vs-> (None, s) ==> length es = length vs"
<proof>

```

```

lemma progression_Eq : "{G, C, S} ⊢
  {hp, (v2 # v1 # os), lvars}
  >- [Ifcmpeq 3, LitPush (Bool False), Goto 2, LitPush (Bool True)] →
  {hp, (Bool (v1 = v2) # os), lvars}"
<proof>

```

```
declare split_paired_All [simp del] split_paired_Ex [simp del]
⟨ML⟩
```

```
lemma distinct_method: "[ wf_java_prog G; is_class G C;
  method (G, C) S = Some (D, rT, pns, lvars, blk, res) ] ==>
  distinct (gjmb_plns (gmb G C S))"
⟨proof⟩
```

```
lemma distinct_method_if_class_sig_defined :
  "[ wf_java_prog G; class_sig_defined G C S ] ==>
  distinct (gjmb_plns (gmb G C S))"
⟨proof⟩
```

```
lemma method_yields_wf_java_mdecl: "[ wf_java_prog G; is_class G C;
  method (G, C) S = Some (D, rT, pns, lvars, blk, res) ] ==>
  wf_java_mdecl G D (S, rT, (pns, lvars, blk, res))"
⟨proof⟩
```

```
lemma progression_lvar_init_aux [rule_format (no_asm)]: "
  ∀ zs prfx lvals lvars0.
  lvars0 = (zs @ lvars) →
  (disjoint_varnames pns lvars0 →
  (length lvars = length lvals) →
  (Suc(length pns + length zs) = length prfx) →
  ({cG, D, S} ⊢
  {h, os, (prfx @ lvals)}
  >- (concat (map (compInit (pns, lvars0, blk, res)) lvars)) →
  {h, os, (prfx @ (map (λp. (default_val (snd p))) lvars))}))"
⟨proof⟩
```

```
lemma progression_lvar_init [rule_format (no_asm)]:
  "[ wf_java_prog G; is_class G C;
  method (G, C) S = Some (D, rT, pns, lvars, blk, res) ] ==>
  length pns = length pvs →
  (∀ lvals.
  length lvars = length lvals →
  {cG, D, S} ⊢
  {h, os, (a' # pvs @ lvals)}
  >- (compInitLvars (pns, lvars, blk, res) lvars) →
  {h, os, (locvars_xstate G C S (Norm (h, init_vars lvars(pns[↦]pvs)(This↦a')))))}"
⟨proof⟩
```

```
lemma state_ok_eval: "[xs::≤E; wf_java_prog (prg E); wtpd_expr E e;
  (prg E) ⊢ xs -e>v -> xs'] ==> xs'::≤E"
⟨proof⟩
```

lemma state_ok_evals: " $\llbracket xs :: \preceq E; wf_java_prog (prg E); wtpd_exprs E es; prg E \vdash xs \text{ -es}[\succ] vs \rightarrow xs' \rrbracket \implies xs' :: \preceq E$ "
 $\langle proof \rangle$

lemma state_ok_exec: " $\llbracket xs :: \preceq E; wf_java_prog (prg E); wtpd_stmt E st; prg E \vdash xs \text{ -st} \rightarrow xs' \rrbracket \implies xs' :: \preceq E$ "
 $\langle proof \rangle$

lemma state_ok_init:
 $\llbracket wf_java_prog G; (x, h, l) :: \preceq (env_of_jmb G C S); is_class G dynT; method (G, dynT) (mn, pTs) = Some (md, rT, pns, lvars, blk, res); list_all2 (conf G h) pvs pTs; G, h \vdash a' :: \preceq Class md \rrbracket$
 \implies
 $(np a' x, h, init_vars lvars (pns[\mapsto] pvs) (This \mapsto a')) :: \preceq (env_of_jmb G md (mn, pTs))$
 $\langle proof \rangle$

lemma ty_exprs_list_all2 [rule_format (no_asm)]:
 $(\forall Ts. (E \vdash es [::] Ts) = list_all2 (\lambda e T. E \vdash e :: T) es Ts)$
 $\langle proof \rangle$

lemma conf_bool: " $G, h \vdash v :: \preceq PrimT Boolean \implies \exists b. v = Bool b$ "
 $\langle proof \rangle$

lemma class_expr_is_class: " $\llbracket E \vdash e :: Class C; ws_prog (prg E) \rrbracket \implies is_class (prg E) C$ "
 $\langle proof \rangle$

lemma max_spec_widen: " $max_spec G C (mn, pTs) = \{((md, rT), pTs')\} \implies list_all2 (\lambda T T'. G \vdash T \preceq T') pTs pTs'$ "
 $\langle proof \rangle$

lemma eval_conf: " $\llbracket G \vdash s \text{ -e}[\succ] v \rightarrow s'; wf_java_prog G; s :: \preceq E; E \vdash e :: T; gx s' = None; prg E = G \rrbracket \implies G, gh s' \vdash v :: \preceq T$ "
 $\langle proof \rangle$

lemma evals_preserves_conf:
 $\llbracket G \vdash s \text{ -es}[\succ] vs \rightarrow s'; G, gh s \vdash t :: \preceq T; E \vdash es [::] Ts; wf_java_prog G; s :: \preceq E; prg E = G \rrbracket \implies G, gh s' \vdash t :: \preceq T$
 $\langle proof \rangle$

lemma eval_of_class: " $\llbracket G \vdash s \text{ -e}[\succ] a' \rightarrow s'; E \vdash e :: Class C; wf_java_prog G; s :: \preceq E; gx s' = None; a' \neq Null; G = prg E \rrbracket \implies (\exists lc. a' = Addr lc)$ "
 $\langle proof \rangle$

lemma dynT_subcls:

```
"[ a' ≠ Null; G, h ⊢ a' :: ≤ Class C; dynT = fst (the (h (the_Addr a'))));
  is_class G dynT; ws_prog G ] ⇒ G ⊢ dynT ≤C C"
⟨proof⟩
```

lemma method_defined: "[

```
  m = the (method (G, dynT) (mn, pTs));
  dynT = fst (the (h a)); is_class G dynT; wf_java_prog G;
  a' ≠ Null; G, h ⊢ a' :: ≤ Class C; a = the_Addr a';
  ∃ pTsa md rT. max_spec G C (mn, pTsa) = {(md, rT), pTs} ]
⇒ (method (G, dynT) (mn, pTs)) = Some m"
⟨proof⟩
```

theorem compiler_correctness:

```
"wf_java_prog G ⇒
  (G ⊢ xs -ex>val-> xs' →
   gx xs = None → gx xs' = None →
   (∀ os CL S.
    (class_sig_defined G CL S) →
    (wtpd_expr (env_of_jmb G CL S) ex) →
    (xs :: ≤ (env_of_jmb G CL S)) →
    ( {TranslComp.comp G, CL, S} ⊢
      {gh xs, os, (locvars_xstate G CL S xs)}
      >- (compExpr (gmb G CL S) ex) →
      {gh xs', val#os, locvars_xstate G CL S xs'}))) ∧
```

```
(G ⊢ xs -exs[>]vals-> xs' →
  gx xs = None → gx xs' = None →
  (∀ os CL S.
   (class_sig_defined G CL S) →
   (wtpd_exprs (env_of_jmb G CL S) exs) →
   (xs :: ≤ (env_of_jmb G CL S)) →
   ( {TranslComp.comp G, CL, S} ⊢
     {gh xs, os, (locvars_xstate G CL S xs)}
     >- (compExprs (gmb G CL S) exs) →
     {gh xs', (rev vals)@os, (locvars_xstate G CL S xs')}))) ∧
```

```
(G ⊢ xs -st-> xs' →
  gx xs = None → gx xs' = None →
  (∀ os CL S.
   (class_sig_defined G CL S) →
   (wtpd_stmt (env_of_jmb G CL S) st) →
   (xs :: ≤ (env_of_jmb G CL S)) →
   ( {TranslComp.comp G, CL, S} ⊢
```

```

    {gh xs, os, (locvars_xstate G CL S xs)}
    >- (compStmt (gmb G CL S) st) →
    {gh xs', os, (locvars_xstate G CL S xs')}}))"
  <proof>

```

```

theorem compiler_correctness_eval: "
  [| G ⊢ (None, hp, loc) -ex > val-> (None, hp', loc');
    wf_java_prog G;
    class_sig_defined G C S;
    wtpd_expr (env_of_jmb G C S) ex;
    (None, hp, loc) :: ⋚ (env_of_jmb G C S) |] ⇒
  {(TranslComp.comp G), C, S} ⊢
    {hp, os, (locvars_locals G C S loc)}
    >- (compExpr (gmb G C S) ex) →
    {hp', val#os, (locvars_locals G C S loc')}}"
  <proof>

```

```

theorem compiler_correctness_exec: "
  [| G ⊢ Norm (hp, loc) -st-> Norm (hp', loc');
    wf_java_prog G;
    class_sig_defined G C S;
    wtpd_stmt (env_of_jmb G C S) st;
    (None, hp, loc) :: ⋚ (env_of_jmb G C S) |] ⇒
  {(TranslComp.comp G), C, S} ⊢
    {hp, os, (locvars_locals G C S loc)}
    >- (compStmt (gmb G C S) st) →
    {hp', os, (locvars_locals G C S loc')}}"
  <proof>

```

```

declare split_paired_All [simp] split_paired_Ex [simp]
  <ML>

```

```

declare wf_prog_ws_prog [simp del]

```

```

end

```

```

theory TypeInf
imports "../J/WellType"
begin

```

lemma *NewC_invers*: "E \vdash NewC C::T
 $\implies T = \text{Class } C \wedge \text{is_class } (\text{prg } E) C$ "
 <proof>

lemma *Cast_invers*: "E \vdash Cast D e::T
 $\implies \exists C. T = \text{Class } D \wedge E \vdash e::C \wedge \text{is_class } (\text{prg } E) D \wedge \text{prg } E \vdash C \preceq? \text{Class } D$ "
 <proof>

lemma *Lit_invers*: "E \vdash Lit x::T
 $\implies \text{typeof } (\lambda v. \text{None}) x = \text{Some } T$ "
 <proof>

lemma *LAcc_invers*: "E \vdash LAcc v::T
 $\implies \text{localT } E v = \text{Some } T \wedge \text{is_type } (\text{prg } E) T$ "
 <proof>

lemma *BinOp_invers*: "E \vdash BinOp bop e1 e2::T'
 $\implies \exists T. E \vdash e1::T \wedge E \vdash e2::T \wedge$
 (if bop = Eq then T' = PrimT Boolean
 else T' = T \wedge T = PrimT Integer)"
 <proof>

lemma *LAss_invers*: "E \vdash v::e::T'
 $\implies \exists T. v \sim= \text{This} \wedge E \vdash \text{LAcc } v::T \wedge E \vdash e::T' \wedge \text{prg } E \vdash T' \preceq T$ "
 <proof>

lemma *FAcc_invers*: "E \vdash {fd}a..fn::fT
 $\implies \exists C. E \vdash a::\text{Class } C \wedge \text{field } (\text{prg } E, C) \text{ fn} = \text{Some } (fd, fT)$ "
 <proof>

lemma *FAss_invers*: "E \vdash {fd}a..fn:=v::T'
 $\implies \exists T. E \vdash \text{{fd}a..fn}::T \wedge E \vdash v::T' \wedge \text{prg } E \vdash T' \preceq T$ "
 <proof>

lemma *Call_invers*: "E \vdash {C}a..mn({pTs'}ps)::rT
 $\implies \exists pTs \text{ md.}$
 $E \vdash a::\text{Class } C \wedge E \vdash ps[::]pTs \wedge \text{max_spec } (\text{prg } E) C (\text{mn}, pTs) = \{((\text{md}, rT), pTs')\}$ "
 <proof>

lemma *Nil_invers*: "E \vdash [] [::] Ts $\implies Ts = []$ "
 <proof>

lemma *Cons_invers*: "E \vdash e#es[::]Ts \implies
 $\exists T Ts'. Ts = T\#Ts' \wedge E \vdash e::T \wedge E \vdash es[::]Ts'$ "
 <proof>

lemma *Expr_invers*: "E \vdash Expr e $\surd \implies \exists T. E \vdash e::T$ "
 <proof>

lemma *Comp_invers*: "E \vdash s1;; s2 $\surd \implies E \vdash s1\sqrt{} \wedge E \vdash s2\sqrt{}$ "
 <proof>

```

lemma Cond_invers: "E⊢If(e) s1 Else s2√
  ⇒ E⊢e::PrimT Boolean ∧ E⊢s1√ ∧ E⊢s2√"
<proof>

```

```

lemma Loop_invers: "E⊢While(e) s√
  ⇒ E⊢e::PrimT Boolean ∧ E⊢s√"
<proof>

```

```

declare split_paired_All [simp del]
declare split_paired_Ex [simp del]

```

```

lemma uniqueness_of_types: "
  (∀ (E::'a prog × (vname ⇒ ty option)) T1 T2.
    E⊢e :: T1 → E⊢e :: T2 → T1 = T2) ∧
  (∀ (E::'a prog × (vname ⇒ ty option)) Ts1 Ts2.
    E⊢es [::] Ts1 → E⊢es [::] Ts2 → Ts1 = Ts2)"
<proof>

```

```

lemma uniqueness_of_types_expr [rule_format (no_asm)]: "
  (∀ E T1 T2. E⊢e :: T1 → E⊢e :: T2 → T1 = T2)"
<proof>

```

```

lemma uniqueness_of_types_exprs [rule_format (no_asm)]: "
  (∀ E Ts1 Ts2. E⊢es [::] Ts1 → E⊢es [::] Ts2 → Ts1 = Ts2)"
<proof>

```

```

constdefs
  inferred_tp  :: "[java_mb env, expr] ⇒ ty"
  "inferred_tp E e == (SOME T. E⊢e :: T)"
  inferred_tps :: "[java_mb env, expr list] ⇒ ty list"
  "inferred_tps E es == (SOME Ts. E⊢es [::] Ts)"

```

```

lemma inferred_tp_wt: "E⊢e :: T ⇒ (inferred_tp E e) = T"
<proof>

```

```

lemma inferred_tps_wt: "E⊢es [::] Ts ⇒ (inferred_tps E es) = Ts"
<proof>

```

```

end

```


theory Altern imports BVSpec begin

constdefs

```

check_type :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  JVMType.state  $\Rightarrow$  bool"
"check_type G mxs mxr s  $\equiv$  s  $\in$  states G mxs mxr"

wt_instr_altern :: "[instr,jvm_prog,ty,method_type,nat,nat,p_count,
                    exception_table,p_count]  $\Rightarrow$  bool"
"wt_instr_altern i G rT phi mxs mxr max_pc et pc  $\equiv$ 
app i G mxs rT pc et (phi!pc)  $\wedge$ 
check_type G mxs mxr (OK (phi!pc))  $\wedge$ 
( $\forall$  (pc',s')  $\in$  set (eff i G pc et (phi!pc)). pc' < max_pc  $\wedge$  G  $\vdash$  s' <= phi!pc)"

wt_method_altern :: "[jvm_prog,cname,ty list,ty,nat,nat,instr list,
                    exception_table,method_type]  $\Rightarrow$  bool"
"wt_method_altern G C pTs rT mxs mxl ins et phi  $\equiv$ 
let max_pc = length ins in
0 < max_pc  $\wedge$ 
length phi = length ins  $\wedge$ 
check_bounded ins et  $\wedge$ 
wt_start G C pTs mxl phi  $\wedge$ 
( $\forall$  pc. pc < max_pc  $\longrightarrow$  wt_instr_altern (ins!pc) G rT phi mxs (1+length pTs+mxl) max_pc
et pc)"

```

lemma wt_method_wt_method_altern :

```

"wt_method G C pTs rT mxs mxl ins et phi  $\longrightarrow$  wt_method_altern G C pTs rT mxs mxl ins
et phi"
<proof>

```

lemma check_type_check_types [rule_format]:

```

"( $\forall$  pc. pc < length phi  $\longrightarrow$  check_type G mxs mxr (OK (phi ! pc)))
 $\longrightarrow$  check_types G mxs mxr (map OK phi)"
<proof>

```

lemma wt_method_altern_wt_method [rule_format]:

```

"wt_method_altern G C pTs rT mxs mxl ins et phi  $\longrightarrow$  wt_method G C pTs rT mxs mxl ins
et phi"
<proof>

```

end

theory CorrCompTp

```
imports LemmasComp TypeInf "../BV/JVM" "../BV/Altern"
begin
```

```
declare split_paired_All [simp del]
declare split_paired_Ex [simp del]
```

```
constdefs
```

```
  initied_LT :: "[cname, ty list, (vname × ty) list] ⇒ locvars_type"
  "initied_LT C pTs lvars == (OK (Class C))#((map OK pTs))@(map (Fun.comp OK snd) lvars)"
  is_initied_LT :: "[cname, ty list, (vname × ty) list, locvars_type] ⇒ bool"
  "is_initied_LT C pTs lvars LT == (LT = (initied_LT C pTs lvars))"

  local_env :: "[java_mb prog, cname, sig, vname list, (vname × ty) list] ⇒ java_mb env"
  "local_env G C S pns lvars ==
    let (mn, pTs) = S in (G, map_of lvars (pns[↦]pTs) (This↦Class C))"
```

```
lemma local_env_fst [simp]: "fst (local_env G C S pns lvars) = G"
⟨proof⟩
```

```
lemma wt_class_expr_is_class: "⌊ ws_prog G; E ⊢ expr :: Class cname;
  E = local_env G C (mn, pTs) pns lvars ⌋
  ⇒ is_class G cname "
⟨proof⟩
```

4.24.6 index

```
lemma local_env_snd: "
  snd (local_env G C (mn, pTs) pns lvars) = map_of lvars (pns[↦]pTs) (This↦Class C)"
⟨proof⟩
```

```
lemma index_in_bounds: " length pns = length pTs ⇒
  snd (local_env G C (mn, pTs) pns lvars) vname = Some T
  ⇒ index (pns, lvars, blk, res) vname < length (initied_LT C pTs lvars)"
⟨proof⟩
```

```
lemma map_upds_append [rule_format (no_asm)]:
  "∀ x1s m. (length k1s = length x1s
  → m(k1s[↦]x1s)(k2s[↦]x2s) = m ((k1s@k2s)[↦] (x1s@x2s)))"
⟨proof⟩
```

```
lemma map_of_append [rule_format]:
  "∀ ys. (map_of ((rev xs) @ ys) = (map_of ys) ((map fst xs) [↦] (map snd xs)))"
⟨proof⟩
```

```
lemma map_of_as_map_upds: "map_of (rev xs) = empty ((map fst xs) [↦] (map snd xs))"
⟨proof⟩
```

lemma map_of_rev: "unique xs \implies map_of (rev xs) = map_of xs"
 <proof>

lemma map_upds_rev [rule_format]: " \forall xs. (distinct ks \longrightarrow length ks = length xs
 \longrightarrow m (rev ks \mapsto] rev xs) = m (ks \mapsto] xs))"
 <proof>

lemma map_upds_takeWhile [rule_format]:
 " \forall ks. (empty(rev ks \mapsto]rev xs)) k = Some x \longrightarrow length ks = length xs \longrightarrow
 xs ! length (takeWhile (λ z. z \neq k) ks) = x"
 <proof>

lemma local_env_initiated_LT: "[snd (local_env G C (mn, pTs) pns lvars) vname = Some T;
 length pns = length pTs; distinct pns; unique lvars]
 \implies (initiated_LT C pTs lvars ! index (pns, lvars, blk, res) vname) = OK T"
 <proof>

lemma initiated_LT_at_index_no_err: " i < length (initiated_LT C pTs lvars)
 \implies initiated_LT C pTs lvars ! i \neq Err"
 <proof>

lemma sup_loc_update_index: "
 [G \vdash T \preceq T'; is_type G T'; length pns = length pTs; distinct pns; unique lvars;
 snd (local_env G C (mn, pTs) pns lvars) vname = Some T']
 \implies
 comp G \vdash
 initiated_LT C pTs lvars [index (pns, lvars, blk, res) vname := OK T] \leq 1
 initiated_LT C pTs lvars"
 <proof>

4.24.7 Preservation of ST and LT by compTpExpr / compTpStmt

lemma sttp_of_comb_nil [simp]: "sttp_of (comb_nil sttp) = sttp"
 <proof>

lemma mt_of_comb_nil [simp]: "mt_of (comb_nil sttp) = []"
 <proof>

lemma sttp_of_comb [simp]: "sttp_of ((f1 \square f2) sttp) = sttp_of (f2 (sttp_of (f1 sttp)))"
 <proof>

lemma mt_of_comb: "(mt_of ((f1 \square f2) sttp)) =
 (mt_of (f1 sttp)) @ (mt_of (f2 (sttp_of (f1 sttp))))"
 <proof>

lemma mt_of_comb_length [simp]: "[n1 = length (mt_of (f1 sttp)); n1 \leq n]
 \implies (mt_of ((f1 \square f2) sttp) ! n) = (mt_of (f2 (sttp_of (f1 sttp))) ! (n - n1))"
 <proof>

```

lemma compTpExpr_Exprs_LT_ST: "
  [| jmb = (pns, lvars, blk, res);
    wf_prog wf_java_mdecl G;
    wf_java_mdecl G C ((mn, pTs), rT, jmb);
    E = local_env G C (mn, pTs) pns lvars |]
  ==>
    (∀ ST LT T.
      E ⊢ ex :: T →
      is_initiated_LT C pTs lvars LT →
      sttp_of (compTpExpr jmb G ex (ST, LT)) = (T # ST, LT))
  ∧
    (∀ ST LT Ts.
      E ⊢ exs [::] Ts →
      is_initiated_LT C pTs lvars LT →
      sttp_of (compTpExprs jmb G exs (ST, LT)) = ((rev Ts) @ ST, LT))"

```

⟨proof⟩

```

lemmas compTpExpr_LT_ST [rule_format (no_asm)] =
  compTpExpr_Exprs_LT_ST [THEN conjunct1]

```

```

lemmas compTpExprs_LT_ST [rule_format (no_asm)] =
  compTpExpr_Exprs_LT_ST [THEN conjunct2]

```

```

lemma compTpStmt_LT_ST [rule_format (no_asm)]: "
  [| jmb = (pns, lvars, blk, res);
    wf_prog wf_java_mdecl G;
    wf_java_mdecl G C ((mn, pTs), rT, jmb);
    E = (local_env G C (mn, pTs) pns lvars) |]
  ==> (∀ ST LT.
    E ⊢ s√ →
    (is_initiated_LT C pTs lvars LT)
    → sttp_of (compTpStmt jmb G s (ST, LT)) = (ST, LT))"

```

⟨proof⟩

```

lemma compTpInit_LT_ST: "
  sttp_of (compTpInit jmb (vn, ty) (ST, LT)) = (ST, LT[(index jmb vn) := OK ty])"
⟨proof⟩

```

```

lemma compTpInitLvars_LT_ST_aux [rule_format (no_asm)]:
  "∀ pre lvars_pre lvars0.
    jmb = (pns, lvars0, blk, res) ∧
    lvars0 = (lvars_pre @ lvars) ∧
    (length pns) + (length lvars_pre) + 1 = length pre ∧
    disjoint_varnames pns (lvars_pre @ lvars)
  →"

```

```
sttp_of (compTpInitLvars jmb lvars (ST, pre @ replicate (length lvars) Err))
  = (ST, pre @ map (Fun.comp OK snd) lvars)"
⟨proof⟩
```

```
lemma compTpInitLvars_LT_ST:
  "[ jmb = (pns, lvars, blk, res); wf_java_mdecl G C ((mn, pTs), rT, jmb) ]
  ⇒ (sttp_of (compTpInitLvars jmb lvars (ST, start_LT C pTs (length lvars))))
  = (ST, inited_LT C pTs lvars)"
⟨proof⟩
```

```
lemma max_of_list_elem: "x ∈ set xs ⇒ x ≤ (max_of_list xs)"
⟨proof⟩
```

```
lemma max_of_list_sublist: "set xs ⊆ set ys
  ⇒ (max_of_list xs) ≤ (max_of_list ys)"
⟨proof⟩
```

```
lemma max_of_list_append [simp]:
  "max_of_list (xs @ ys) = max (max_of_list xs) (max_of_list ys)"
⟨proof⟩
```

```
lemma app_mono_mxs: "[ app i G mxs rT pc et s; mxs ≤ mxs' ]
  ⇒ app i G mxs' rT pc et s"
⟨proof⟩
```

```
lemma err_mono [simp]: "A ⊆ B ⇒ err A ⊆ err B"
⟨proof⟩
```

```
lemma opt_mono [simp]: "A ⊆ B ⇒ opt A ⊆ opt B"
⟨proof⟩
```

```
lemma states_mono: "[ mxs ≤ mxs' ]
  ⇒ states G mxs mxr ⊆ states G mxs' mxr"
⟨proof⟩
```

```
lemma check_type_mono: "[ check_type G mxs mxr s; mxs ≤ mxs' ]
  ⇒ check_type G mxs' mxr s"
⟨proof⟩
```

```
lemma wt_instr_prefix: "
  [ wt_instr_altern (bc ! pc) cG rT mt mxs mxr max_pc et pc;
    bc' = bc @ bc_post; mt' = mt @ mt_post;
    mxs ≤ mxs'; max_pc ≤ max_pc';
```

```

    pc < length bc; pc < length mt;
    max_pc = (length mt) ]
  => wt_instr_altern (bc' ! pc) cG rT mt' mxs' mxr max_pc' et pc"
<proof>

```

```

lemma pc_succs_shift: "pc' ∈ set (succs i (pc'' + n))
  => ((pc' - n) ∈ set (succs i pc''))"
<proof>

```

```

lemma pc_succs_le: "[ pc' ∈ set (succs i (pc'' + n));
  ∀ b. ((i = (Goto b) ∨ i = (Ifcmpeq b)) → 0 ≤ (int pc'' + b)) ]
  => n ≤ pc'"
<proof>

```

```

constdefs
  offset_xcentry :: "[nat, exception_entry] ⇒ exception_entry"
  "offset_xcentry ==
    λ n (start_pc, end_pc, handler_pc, catch_type).
      (start_pc + n, end_pc + n, handler_pc + n, catch_type)"

  offset_xctable :: "[nat, exception_table] ⇒ exception_table"
  "offset_xctable n == (map (offset_xcentry n))"

```

```

lemma match_xcentry_offset [simp]: "
  match_exception_entry G cn (pc + n) (offset_xcentry n ee) =
  match_exception_entry G cn pc ee"
<proof>

```

```

lemma match_xctable_offset: "
  (match_exception_table G cn (pc + n) (offset_xctable n et)) =
  (option_map (λ pc'. pc' + n) (match_exception_table G cn pc et))"
<proof>

```

```

lemma match_offset [simp]: "
  match G cn (pc + n) (offset_xctable n et) = match G cn pc et"
<proof>

```

```

lemma match_any_offset [simp]: "
  match_any G (pc + n) (offset_xctable n et) = match_any G pc et"
<proof>

```

```

lemma app_mono_pc: "[ app i G mxs rT pc et s; pc' = pc + n ]
  => app i G mxs rT pc' (offset_xctable n et) s"
<proof>

```

syntax

empty_et :: exception_table

translations

"empty_et" => "[]"

lemma xcpt_names_Nil [simp]: "(xcpt_names (i, G, pc, [])) = []"
 <proof>

lemma xcpt_eff_Nil [simp]: "(xcpt_eff i G pc s []) = []"
 <proof>

lemma app_jumps_lem: "[[app i cG mxs rT pc empty_et s; s=(Some st)]]
 ==> $\forall b. ((i = (\text{Goto } b) \vee i = (\text{Ifcmpeq } b)) \longrightarrow 0 \leq (\text{int } pc + b))$ "
 <proof>

lemma wt_instr_offset: "
 [[$\forall pc''. pc'' < \text{length } mt.$
 wt_instr_altern ((bc@bc_post) ! pc'') cG rT (mt@mt_post) mxs mxr max_pc empty_et pc'';

 bc' = bc_pre @ bc @ bc_post; mt' = mt_pre @ mt @ mt_post;
 length bc_pre = length mt_pre; length bc = length mt;
 length mt_pre \leq pc; pc < length (mt_pre @ mt);
 mxs \leq mxs'; max_pc + length mt_pre \leq max_pc']]
 ==> wt_instr_altern (bc' ! pc) cG rT mt' mxs' mxr max_pc' empty_et pc"
 <proof>

constdefs

start_sttp_resp_cons :: "[state_type \Rightarrow method_type \times state_type] \Rightarrow bool"
 "start_sttp_resp_cons f ==
 (\forall sttp. let (mt', sttp') = (f sttp) in (\exists mt'_rest. mt' = Some sttp # mt'_rest))"

start_sttp_resp :: "[state_type \Rightarrow method_type \times state_type] \Rightarrow bool"
 "start_sttp_resp f == (f = comb_nil) \vee (start_sttp_resp_cons f)"

lemma start_sttp_resp_comb_nil [simp]: "start_sttp_resp comb_nil"
 <proof>

lemma start_sttp_resp_cons_comb_cons [simp]: "start_sttp_resp_cons f

$\Rightarrow \text{start_sttp_resp_cons } (f \sqcap f')$
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_comb_cons_r}$: " $\llbracket \text{start_sttp_resp } f; \text{start_sttp_resp_cons } f' \rrbracket$
 $\Rightarrow \text{start_sttp_resp_cons } (f \sqcap f')$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_comb}$ [simp]: " $\text{start_sttp_resp_cons } f$
 $\Rightarrow \text{start_sttp_resp } (f \sqcap f')$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_comb}$: " $\llbracket \text{start_sttp_resp } f; \text{start_sttp_resp } f' \rrbracket$
 $\Rightarrow \text{start_sttp_resp } (f \sqcap f')$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_nochangeST}$ [simp]: " $\text{start_sttp_resp_cons nochangeST}$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_pushST}$ [simp]: " $\text{start_sttp_resp_cons (pushST } Ts)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_dupST}$ [simp]: " $\text{start_sttp_resp_cons dupST}$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_dup_x1ST}$ [simp]: " $\text{start_sttp_resp_cons dup_x1ST}$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_popST}$ [simp]: " $\text{start_sttp_resp_cons (popST } n)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_replST}$ [simp]: " $\text{start_sttp_resp_cons (replST } n \text{ } tp)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_storeST}$ [simp]: " $\text{start_sttp_resp_cons (storeST } i \text{ } tp)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_compTpExpr}$ [simp]: " $\text{start_sttp_resp_cons (compTpExpr jmb } G \text{ } ex)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_cons_compTpInit}$ [simp]: " $\text{start_sttp_resp_cons (compTpInit jmb } lv)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_nochangeST}$ [simp]: " $\text{start_sttp_resp nochangeST}$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_pushST}$ [simp]: " $\text{start_sttp_resp (pushST } Ts)$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_dupST}$ [simp]: " $\text{start_sttp_resp dupST}$ "
 $\langle \text{proof} \rangle$

lemma $\text{start_sttp_resp_dup_x1ST}$ [simp]: " $\text{start_sttp_resp dup_x1ST}$ "

<proof>

lemma start_sttp_resp_popST [simp]: "start_sttp_resp (popST n)"

<proof>

lemma start_sttp_resp_replST [simp]: "start_sttp_resp (replST n tp)"

<proof>

lemma start_sttp_resp_storeST [simp]: "start_sttp_resp (storeST i tp)"

<proof>

lemma start_sttp_resp_compTpExpr [simp]: "start_sttp_resp (compTpExpr jmb G ex)"

<proof>

lemma start_sttp_resp_compTpExprs [simp]: "start_sttp_resp (compTpExprs jmb G exs)"

<proof>

lemma start_sttp_resp_compTpStmt [simp]: "start_sttp_resp (compTpStmt jmb G s)"

<proof>

lemma start_sttp_resp_compTpInitLvars [simp]: "start_sttp_resp (compTpInitLvars jmb lvars)"

<proof>

4.24.8 length of compExpr/ compTpExprs

lemma length_comb [simp]: "length (mt_of ((f1 \square f2) sttp)) =
length (mt_of (f1 sttp)) + length (mt_of (f2 (sttp_of (f1 sttp))))"

<proof>

lemma length_comb_nil [simp]: "length (mt_of (comb_nil sttp)) = 0"

<proof>

lemma length_nochangeST [simp]: "length (mt_of (nochangeST sttp)) = 1"

<proof>

lemma length_pushST [simp]: "length (mt_of (pushST Ts sttp)) = 1"

<proof>

lemma length_dupST [simp]: "length (mt_of (dupST sttp)) = 1"

<proof>

lemma length_dup_x1ST [simp]: "length (mt_of (dup_x1ST sttp)) = 1"

<proof>

lemma length_popST [simp]: "length (mt_of (popST n sttp)) = 1"

<proof>

lemma length_replST [simp]: "length (mt_of (replST n tp sttp)) = 1"

<proof>

lemma length_storeST [simp]: "length (mt_of (storeST i tp sttp)) = 1"

<proof>

```

lemma length_compTpExpr_Exprs [rule_format]: "
  (∀ sttp. (length (mt_of (compTpExpr jmb G ex sttp)) = length (compExpr jmb ex)))
  ∧ (∀ sttp. (length (mt_of (compTpExprs jmb G exs sttp)) = length (compExprs jmb exs)))"
⟨proof⟩

lemma length_compTpExpr: "length (mt_of (compTpExpr jmb G ex sttp)) = length (compExpr
jmb ex)"
⟨proof⟩

lemma length_compTpExprs: "length (mt_of (compTpExprs jmb G exs sttp)) = length (compExprs
jmb exs)"
⟨proof⟩

lemma length_compTpStmt [rule_format]: "
  (∀ sttp. (length (mt_of (compTpStmt jmb G s sttp)) = length (compStmt jmb s)))"
⟨proof⟩

lemma length_compTpInit: "length (mt_of (compTpInit jmb lv sttp)) = length (compInit
jmb lv)"
⟨proof⟩

lemma length_compTpInitLvars [rule_format]:
  "∀ sttp. length (mt_of (compTpInitLvars jmb lvars sttp)) = length (compInitLvars jmb
lvars)"
⟨proof⟩

```

4.24.9 Correspondence bytecode - method types

syntax

```

ST_of :: "state_type ⇒ opstack_type"
LT_of :: "state_type ⇒ locvars_type"

```

translations

```

"ST_of" => "fst"
"LT_of" => "snd"

```

lemma states_lower:

```

"⌊ OK (Some (ST, LT)) ∈ states cG mxs mxr; length ST ≤ mxs ⌋
⇒ OK (Some (ST, LT)) ∈ states cG (length ST) mxr"

```

⟨proof⟩

lemma check_type_lower:

```

"⌊ check_type cG mxs mxr (OK (Some (ST, LT))); length ST ≤ mxs ⌋
⇒ check_type cG (length ST) mxr (OK (Some (ST, LT)))"

```

⟨proof⟩

constdefs

```

bc_mt_corresp :: "
  [bytecode, state_type ⇒ method_type × state_type, state_type, jvm_prog, ty, nat, p_count]
  ⇒ bool"

```

```

"bc_mt_corresp bc f sttp0 cG rT mxr idx ==
let (mt, sttp) = f sttp0 in
(length bc = length mt ∧
 ((check_type cG (length (ST_of sttp0)) mxr (OK (Some sttp0))) →
  (∀ mxs.
    mxs = max_ssize (mt@[Some sttp]) →
    (∀ pc. pc < idx →
      wt_instr_altern (bc ! pc) cG rT (mt@[Some sttp]) mxs mxr (length mt + 1) empty_et
    pc)
  ∧
  check_type cG mxs mxr (OK ((mt@[Some sttp]) ! idx))))))"

```

```

lemma bc_mt_corresp_comb: "
  [[ bc' = (bc1@bc2); l' = (length bc');
    bc_mt_corresp bc1 f1 sttp0 cG rT mxr (length bc1);
    bc_mt_corresp bc2 f2 (sttp_of (f1 sttp0)) cG rT mxr (length bc2);
    start_sttp_resp f2]]
  ⇒ bc_mt_corresp bc' (f1 □ f2) sttp0 cG rT mxr l'"
⟨proof⟩

```

```

lemma bc_mt_corresp_zero [simp]: "[[ length (mt_of (f sttp)) = length bc; start_sttp_resp
f]]
  ⇒ bc_mt_corresp bc f sttp cG rT mxr 0"
⟨proof⟩

```

constdefs

```

mt_sttp_flatten :: "method_type × state_type ⇒ method_type"
"mt_sttp_flatten mt_sttp == (mt_of mt_sttp) @ [Some (sttp_of mt_sttp)]"

```

```

lemma mt_sttp_flatten_length [simp]: "n = (length (mt_of (f sttp)))
  ⇒ (mt_sttp_flatten (f sttp)) ! n = Some (sttp_of (f sttp))"
⟨proof⟩

```

```

lemma mt_sttp_flatten_comb: "(mt_sttp_flatten ((f1 □ f2) sttp)) =
  (mt_of (f1 sttp)) @ (mt_sttp_flatten (f2 (sttp_of (f1 sttp))))"
⟨proof⟩

```

```

lemma mt_sttp_flatten_comb_length [simp]: "[[ n1 = length (mt_of (f1 sttp)); n1 ≤ n ]]
  ⇒ (mt_sttp_flatten ((f1 □ f2) sttp) ! n) = (mt_sttp_flatten (f2 (sttp_of (f1 sttp)))
! (n - n1))"
⟨proof⟩

```

```

lemma mt_sttp_flatten_comb_zero [simp]: "start_sttp_resp f
  ⇒ (mt_sttp_flatten (f sttp)) ! 0 = Some sttp"
⟨proof⟩

```

lemma `int_outside_right`: " $0 \leq (m::\text{int}) \implies m + (\text{int } n) = \text{int } ((\text{nat } m) + n)$ "
 $\langle \text{proof} \rangle$

lemma `int_outside_left`: " $0 \leq (m::\text{int}) \implies (\text{int } n) + m = \text{int } (n + (\text{nat } m))$ "
 $\langle \text{proof} \rangle$

lemma `less_Suc [simp]` : " $n \leq k \implies (k < \text{Suc } n) = (k = n)$ "
 $\langle \text{proof} \rangle$

lemmas `check_type_simps` = `check_type_def states_def JVMType.sl_def`
`Product.esl_def stk_esl_def reg_sl_def upto_esl_def Listn.sl_def Err.sl_def`
`JType.esl_def Err.esl_def Err.le_def Listn.le_def Product.le_def Product.sup_def Err.sup_def`
`Opt.esl_def Listn.sup_def`

lemma `check_type_push`: " $\llbracket \text{is_class } cG \text{ cname}; \text{check_type } cG (\text{length } ST) \text{ mxr } (OK (\text{Some } (ST, LT))) \rrbracket$
 $\implies \text{check_type } cG (\text{Suc } (\text{length } ST)) \text{ mxr } (OK (\text{Some } (\text{Class } \text{cname} \# ST, LT)))$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_New`: " $\llbracket \text{is_class } cG \text{ cname} \rrbracket$
 $\implies \text{bc_mt_corresp } [\text{New } \text{cname}] (\text{pushST } [\text{Class } \text{cname}]) (ST, LT) \text{ cG } rT \text{ mxr } (\text{Suc } 0)$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_Pop`: " $\text{bc_mt_corresp } [\text{Pop}] (\text{popST } (\text{Suc } 0)) (T \# ST, LT) \text{ cG } rT \text{ mxr } (\text{Suc } 0)$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_Checkcast`: " $\llbracket \text{is_class } cG \text{ cname}; \text{sttp} = (ST, LT);$
 $(\exists rT \text{ STo. } ST = \text{RefT } rT \# \text{STo}) \rrbracket$
 $\implies \text{bc_mt_corresp } [\text{Checkcast } \text{cname}] (\text{replST } (\text{Suc } 0) (\text{Class } \text{cname})) \text{ sttp } cG \text{ rT } \text{mxr } (\text{Suc } 0)$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_LitPush`: " $\llbracket \text{typeof } (\lambda v. \text{None}) \text{ val} = \text{Some } T \rrbracket$
 $\implies \text{bc_mt_corresp } [\text{LitPush } \text{val}] (\text{pushST } [T]) \text{ sttp } cG \text{ rT } \text{mxr } (\text{Suc } 0)$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_LitPush_CT`: " $\llbracket \text{typeof } (\lambda v. \text{None}) \text{ val} = \text{Some } T \wedge cG \vdash T \preceq T';$
 $\text{is_type } cG \text{ T'} \rrbracket$
 $\implies \text{bc_mt_corresp } [\text{LitPush } \text{val}] (\text{pushST } [T']) \text{ sttp } cG \text{ rT } \text{mxr } (\text{Suc } 0)$ "
 $\langle \text{proof} \rangle$

lemma `bc_mt_corresp_Load`: " $\llbracket i < \text{length } LT; LT ! i \neq \text{Err}; \text{mxr} = \text{length } LT \rrbracket$
 $\implies \text{bc_mt_corresp } [\text{Load } i]$

$$\langle \text{proof} \rangle \quad (\lambda(ST, LT). \text{pushST } [\text{ok_val } (LT \ ! \ i)] \ (ST, LT)) \ (ST, LT) \ cG \ rT \ mxr \ (\text{Suc } 0)"$$

lemma *bc_mt_corresp_Store_init*: "[i < length LT]
 \implies *bc_mt_corresp* [Store i] (storeST i T) (T # ST, LT) cG rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Store*: "[i < length LT; cG \vdash LT[i := OK T] \leq LT]
 \implies *bc_mt_corresp* [Store i] (popST (Suc 0)) (T # ST, LT) cG rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Dup*: "
bc_mt_corresp [Dup] dupST (T # ST, LT) cG rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Dup_x1*: "
bc_mt_corresp [Dup_x1] dup_x1ST (T1 # T2 # ST, LT) cG rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_IAdd*: "
bc_mt_corresp [IAdd] (replST 2 (PrimT Integer))
 (PrimT Integer # PrimT Integer # ST, LT) cG rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Getfield*: "[wf_prog wf_mb G;
 field (G, C) vname = Some (cname, T); is_class G C]
 \implies *bc_mt_corresp* [Getfield vname cname]
 (replST (Suc 0) (snd (the (field (G, cname) vname))))
 (Class C # ST, LT) (comp G) rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Putfield*: "[wf_prog wf_mb G;
 field (G, C) vname = Some (cname, Ta); G \vdash T \preceq Ta; is_class G C]
 \implies *bc_mt_corresp* [Putfield vname cname] (popST 2) (T # Class C # T # ST, LT)
 (comp G) rT mxr (Suc 0)"
 $\langle \text{proof} \rangle$

lemma *Call_app*: "[wf_prog wf_mb G; is_class G cname;
 STs = rev pTsa @ Class cname # ST;
 max_spec G cname (mname, pTsa) = {((md, T), pTs')}]
 \implies app (Invoke cname mname pTs') (comp G) (length (T # ST)) rT 0 empty_et (Some (STs,
 LTs))"
 $\langle \text{proof} \rangle$

lemma *bc_mt_corresp_Invoke*: "[wf_prog wf_mb G;

```

max_spec G cname (mname, pTsa) = {((md, T), fpTs)};
is_class G cname ]
⇒ bc_mt_corresp [Invoke cname mname fpTs] (replST (Suc (length pTsa)) T)
    (rev pTsa @ Class cname # ST, LT) (comp G) rT mxr (Suc 0)"
⟨proof⟩

```

```

lemma wt_instr_Ifcmpeq: "⌊ Suc pc < max_pc;
  0 ≤ (int pc + i); nat (int pc + i) < max_pc;
  (mt_sttp_flatten f ! pc = Some (ts#ts'#ST,LT)) ∧
  ((∃ p. ts = PrimT p ∧ ts' = PrimT p) ∨ (∃ r r'. ts = RefT r ∧ ts' = RefT r'));
  mt_sttp_flatten f ! Suc pc = Some (ST,LT);
  mt_sttp_flatten f ! nat (int pc + i) = Some (ST,LT);
  check_type (TranslComp.comp G) mxs mxr (OK (Some (ts # ts' # ST, LT))) ⌋
  ⇒ wt_instr_altern (Ifcmpeq i) (comp G) rT (mt_sttp_flatten f) mxs mxr max_pc empty_et
pc"
⟨proof⟩

```

```

lemma wt_instr_Goto: "⌊ 0 ≤ (int pc + i); nat (int pc + i) < max_pc;
  mt_sttp_flatten f ! nat (int pc + i) = (mt_sttp_flatten f ! pc);
  check_type (TranslComp.comp G) mxs mxr (OK (mt_sttp_flatten f ! pc)) ⌋
  ⇒ wt_instr_altern (Goto i) (comp G) rT (mt_sttp_flatten f) mxs mxr max_pc empty_et
pc"
⟨proof⟩

```

```

lemma bc_mt_corresp_comb_inside: "
  ⌊
    bc_mt_corresp bc' f' sttp0 cG rT mxr l1;
    bc' = (bc1@bc2@bc3); f' = (f1 □ f2 □ f3);
    l1 = (length bc1); l12 = (length (bc1@bc2));
    bc_mt_corresp bc2 f2 (sttp_of (f1 sttp0)) cG rT mxr (length bc2);
    length bc1 = length (mt_of (f1 sttp0));
    start_sttp_resp f2; start_sttp_resp f3
  ⌋
  ⇒ bc_mt_corresp bc' f' sttp0 cG rT mxr l12"
⟨proof⟩

```

constdefs

```

contracting :: "(state_type ⇒ method_type × state_type) ⇒ bool"
"contracting f == (∀ ST LT.
  let (ST', LT') = sttp_of (f (ST, LT))
  in (length ST' ≤ length ST ∧ set ST' ⊆ set ST ∧
    length LT' = length LT ∧ set LT' ⊆ set LT))"

```

```
lemma set_drop_Suc [rule_format]: "∀ xs. set (drop (Suc n) xs) ⊆ set (drop n xs)"
⟨proof⟩
```

```
lemma set_drop_le [rule_format,simp]: "∀ n xs. n ≤ m ⟶ set (drop m xs) ⊆ set (drop n xs)"
⟨proof⟩
```

```
lemma set_drop [simp] : "set (drop m xs) ⊆ set xs"
⟨proof⟩
```

```
lemma contracting_popST [simp]: "contracting (popST n)"
⟨proof⟩
```

```
lemma contracting_nochangeST [simp]: "contracting nochangeST"
⟨proof⟩
```

```
lemma check_type_contracting: "[[ check_type cG mxs mxr (OK (Some sttp)); contracting f]]
  ⟹ check_type cG mxs mxr (OK (Some (sttp_of (f sttp))))"
⟨proof⟩
```

```
lemma bc_mt_corresp_comb_wt_instr: "
  [[ bc_mt_corresp bc' f' sttp0 cG rT mxr l1;
    bc' = (bc1@[inst]@bc3); f' = (f1 □ f2 □ f3);
    l1 = (length bc1);
    length bc1 = length (mt_of (f1 sttp0));
    length (mt_of (f2 (sttp_of (f1 sttp0)))) = 1;
    start_sttp_resp_cons f1; start_sttp_resp_cons f2; start_sttp_resp f3;

    check_type cG (max_ssize (mt_sttp_flatten (f' sttp0))) mxr
      (OK ((mt_sttp_flatten (f' sttp0)) ! (length bc1)))
  ⟶
    wt_instr_altern inst cG rT
      (mt_sttp_flatten (f' sttp0))
      (max_ssize (mt_sttp_flatten (f' sttp0)))
      mxr
      (Suc (length bc'))
      empty_et
      (length bc1);
    contracting f2
  ]]
  ⟹ bc_mt_corresp bc' f' sttp0 cG rT mxr (length (bc1@[inst]))"
⟨proof⟩
```

```
lemma compTpExpr_LT_ST_rewr [simp]: "[[
  wf_java_prog G;
```

```

wf_java_mdecl G C ((mn, pTs), rT, (pns, lvars, blk, res));
local_env G C (mn, pTs) pns lvars ⊢ ex :: T;
is_initiated_LT C pTs lvars LT
⇒ sttp_of (compTpExpr (pns, lvars, blk, res) G ex (ST, LT)) = (T # ST, LT)"
⟨proof⟩

```

```

lemma wt_method_compTpExpr_Exprs_corresp: "
  [| jmb = (pns, lvars, blk, res);
   wf_prog wf_java_mdecl G;
   wf_java_mdecl G C ((mn, pTs), rT, jmb);
   E = (local_env G C (mn, pTs) pns lvars) |]
⇒
  (∀ ST LT T bc' f'.
    E ⊢ ex :: T →
    (is_initiated_LT C pTs lvars LT) →
    bc' = (compExpr jmb ex) →
    f' = (compTpExpr jmb G ex)
    → bc_mt_corresp bc' f' (ST, LT) (comp G) rT (length LT) (length bc'))
  ∧
  (∀ ST LT Ts.
    E ⊢ exs [::] Ts →
    (is_initiated_LT C pTs lvars LT)
    → bc_mt_corresp (compExprs jmb exs) (compTpExprs jmb G exs) (ST, LT) (comp G) rT (length
    LT) (length (compExprs jmb exs)))"
⟨proof⟩

```

```

lemmas wt_method_compTpExpr_corresp [rule_format (no_asm)] =
  wt_method_compTpExpr_Exprs_corresp [THEN conjunct1]

```

```

lemma wt_method_compTpStmt_corresp [rule_format (no_asm)]: "
  [| jmb = (pns, lvars, blk, res);
   wf_prog wf_java_mdecl G;
   wf_java_mdecl G C ((mn, pTs), rT, jmb);
   E = (local_env G C (mn, pTs) pns lvars) |]
⇒
  (∀ ST LT T bc' f'.
    E ⊢ s√ →
    (is_initiated_LT C pTs lvars LT) →
    bc' = (compStmt jmb s) →
    f' = (compTpStmt jmb G s)
    → bc_mt_corresp bc' f' (ST, LT) (comp G) rT (length LT) (length bc'))"
⟨proof⟩

```



```

lemma wt_method_compTpInit_corresp: "[[ jmb = (pns,lvars,blk,res);
  wf_java_mdecl G C ((mn, pTs), rT, jmb); mxr = length LT;
  length LT = (length pns) + (length lvars) + 1; vn ∈ set (map fst lvars);
  bc = (compInit jmb (vn,ty)); f = (compTpInit jmb (vn,ty));
  is_type G ty ]]]
  ⇒ bc_mt_corresp bc f (ST, LT) (comp G) rT mxr (length bc)"
<proof>

```

```

lemma wt_method_compTpInitLvars_corresp_aux [rule_format (no_asm)]: "
  ∀ lvars_pre lvars0 ST LT.
  jmb = (pns,lvars0,blk,res) ∧
  lvars0 = (lvars_pre @ lvars) ∧
  length LT = (length pns) + (length lvars0) + 1 ∧
  wf_java_mdecl G C ((mn, pTs), rT, jmb)
  → bc_mt_corresp (compInitLvars jmb lvars) (compTpInitLvars jmb lvars) (ST, LT) (comp
G) rT
  (length LT) (length (compInitLvars jmb lvars))"
<proof>

```

```

lemma wt_method_compTpInitLvars_corresp: "[[ jmb = (pns,lvars,blk,res);
  wf_java_mdecl G C ((mn, pTs), rT, jmb);
  length LT = (length pns) + (length lvars) + 1; mxr = (length LT);
  bc = (compInitLvars jmb lvars); f = (compTpInitLvars jmb lvars) ]]]
  ⇒ bc_mt_corresp bc f (ST, LT) (comp G) rT mxr (length bc)"
<proof>

```

```

lemma wt_method_comp_wo_return: "[[ wf_prog wf_java_mdecl G;
  wf_java_mdecl G C ((mn, pTs), rT, jmb);
  bc = compInitLvars jmb lvars @ compStmt jmb blk @ compExpr jmb res;
  jmb = (pns,lvars,blk,res);
  f = (compTpInitLvars jmb lvars □ compTpStmt jmb G blk □ compTpExpr jmb G res);
  sttp = (start_ST, start_LT C pTs (length lvars));
  li = (length (inited_LT C pTs lvars))
  ]]]
  ⇒ bc_mt_corresp bc f sttp (comp G) rT li (length bc)"
<proof>

```

```

lemma check_type_start: "[ wf_mhead cG (mn, pTs) rT; is_class cG C ]
  ⇒ check_type cG (length start_ST) (Suc (length pTs + mxl))
    (OK (Some (start_ST, start_LT C pTs mxl)))"
<proof>

```

```

lemma wt_method_comp_aux: "[ bc' = bc @ [Return]; f' = (f □ nochangeST);
  bc_mt_corresp bc f sttp0 cG rT (1+length pTs+mxl) (length bc);
  start_sttp_resp_cons f';
  sttp0 = (start_ST, start_LT C pTs mxl);
  mxs = max_ssize (mt_of (f' sttp0));
  wf_mhead cG (mn, pTs) rT; is_class cG C;
  sttp_of (f sttp0) = (T # ST, LT);

  check_type cG mxs (1+length pTs+mxl) (OK (Some (T # ST, LT))) →
  wt_instr_altern Return cG rT (mt_of (f' sttp0)) mxs (1+length pTs+mxl)
    (Suc (length bc)) empty_et (length bc)

  ]
  ⇒ wt_method_altern cG C pTs rT mxs mxl bc' empty_et (mt_of (f' sttp0))"
<proof>

```

```

lemma wt_instr_Return: "[fst f ! pc = Some (T # ST, LT); (G ⊢ T ≤ rT); pc < max_pc;
  check_type (TranslComp.comp G) mxs mxr (OK (Some (T # ST, LT)))

  ]
  ⇒ wt_instr_altern Return (comp G) rT (mt_of f) mxs mxr max_pc empty_et pc"
<proof>

```

```

theorem wt_method_comp: "
  [ wf_java_prog G; (C, D, fds, mths) ∈ set G; jmdcl ∈ set mths;
  jmdcl = ((mn,pTs), rT, jmb);
  mt = (compTpMethod G C jmdcl);
  (mxs, mxl, bc, et) = mtd_mb (compMethod G C jmdcl) ]
  ⇒ wt_method (comp G) C pTs rT mxs mxl bc et mt"

```

<proof>

```

lemma comp_set_ms: "(C, D, fs, cms) ∈ set (comp G)
  ⇒ ∃ ms. (C, D, fs, ms) ∈ set G ∧ cms = map (compMethod G C) ms"
<proof>

```

4.24.10 Main Theorem

```

theorem wt_prog_comp: "wf_java_prog G ⇒ wt_jvm_prog (comp G) (compTp G)"
<proof>

```

```
declare split_paired_All [simp add]  
declare split_paired_Ex [simp add]  
  
end
```


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